The topological structures in strongly coupled QGP with chiral fermions on the lattice

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In collaboration with:
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Outline

1. The $U_A(1)$ puzzle in QCD: a way to resolve it
2. Our results
3. Topological structures and $U_A(1)$
4. Implications for experiments
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The $U_A(1)$ puzzle

- **Origin:**
  Anomalous $U_A(1)$ not an exact symmetry of QCD yet may affect the order of phase transition for $N_f = 2$ [Pisarski & Wilczek, 83].

- In model QFT with same symmetries as QCD, it is not possible to quantify the $U_A(1)$ effects in observables.

- Need lattice studies with fermions having exact chiral/flavour symmetry + reproduce exactly anomaly on the lattice.
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Why is it important?

- $m_{u,d} \ll \Lambda_{QCD}$, chiral symmetry drives phase transition at $\mu_B \to 0$
- The singular part of free energy should show critical scaling $\to$ hints of criticality from lattice studies [BI-BNL collaboration, 09].
Why is it important?

- Criticality at $\mu = 0$ changes on whether $U_A(1)$ is effectively restored
  
  [Pelissetto & Vicari, 13, Nakayama & Ohtsuki, 14].
  
  - $O(4)$ critical exponents for $U_A(1)$ broken
  - $U(2) \times U(2)$ if $U_A(1)$ effectively restored

- Effects should be visible in higher order fluctuations measured in the experiments [Karsch & Redlich, 11]
Why is it important?

- Could affect the EoS relevant for anomalous hydrodynamics with chiral imbalance?
  - Softening of $\eta'$ mass near freezeout? [Grahl & Rischke, 14,15]
  - Consequences for the critical end-point at finite $\mu_B$?
  - Lattice QCD can answer such questions from first principles + confirmation from Heavy-Ion experiments
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The major issues with the lattice studies so far

- Finite volume effects → ensure presence of topological objects in a box.
- Most studies done with lattice fermions with only a remnant of continuum chiral symmetry + explicitly broken $U_A(1)$ [S. Chandrasekharan, 96, H. Ohno et. al 12, V. Dick et. al., 15].
- Studies done with chiral fermions are in a fixed topological sector + small volume [JLQCD collaboration, 13].
- Lattice cut-off effects need careful consideration [G. Cossu et. al, 14].
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A no-go theorem on the lattice disfavours ultra-local chiral fermion operator [Nielsen & Ninomiya, 82].

Overlap fermions [Narayanan & Neuberger, 94, Neuberger, 98] sacrifice ultra-locality

\[ D_{ov} = M(1 + \gamma_5 \text{sgn}(\gamma_5 D_W(-M))) , \text{sgn}(A) = A/\sqrt{A.A}. \]

But have an exact chiral symmetry under non-local chiral transformations + index theorem at finite “a” [Ginsparg & Wilson, 82, Luscher, 98].

\[ \{\gamma_5, D\} = aD\gamma_5D \]
One can also start from 5D world+ put a defect to localize chiral fermions on the 4D brane.

Domain wall fermions \cite{Kaplan 92, Shamir 95} in the limit $N_5 \to \infty$

$$D_{DW} = M(1 - \gamma_5 \text{sgn}(\ln |T|)),$$

$$T = (1 + a_5 \gamma_5 D_{WP}^+)^{-1}(1 - a_5 \gamma_5 D_{WP}^-).$$
Observables sensitive to $U_A(1)$ breaking...

- Not an exact symmetry $\rightarrow$ no order-parameter
- Look at the difference of the integrated 2 point correlators [Shuryak, 94]

$$\chi_\pi - \chi_\delta = \int d^4x \ [\langle i\pi^+(x)i\pi^-(0) \rangle - \langle \delta^+(x)\delta^-(0) \rangle]$$

- Equivalently study $\rho(\lambda, m_f)$ of the Dirac operator [Cohen, 95, Hatsuda & Lee, 95]

$$\chi_\pi - \chi_\delta \xrightarrow{V \to \infty} \int_0^\infty d\lambda \, \frac{4m_f^2 \rho(\lambda, m_f)}{(\lambda^2 + m_f^2)^2}, \quad \langle \bar{\psi}\psi \rangle \xrightarrow{V \to \infty} \int_0^\infty d\lambda \, \frac{2m_f \rho(\lambda, m_f)}{(\lambda^2 + m_f^2)}$$

- Chiral symmetry restored: $\lim_{m_f \to 0} \lim_{V \to \infty} \rho(0, m_f) \to 0 \Rightarrow U_A(1)$ restored.

- Chiral symmetry restored $+ U_A(1)$ broken if

$$\lim_{\lambda \to 0} \rho(\lambda, m_f) \to \delta(\lambda)m_f^\alpha, \quad 1 < \alpha < 2.$$
Spectral density of Dirac operator at finite $T$

- Very little known. Only recently there are very interesting results
  [Aoki, Fukaya & Taniguchi, 12].

- Assuming $\rho(\lambda, m)$ to be analytic in $m^2, \lambda$, look at chiral Ward identities of $n$-point function of scalar & pseudo-scalar currents.

- $\rho(\lambda, m \to 0) \sim \lambda^3 \Rightarrow U_A(1)$ breaking effects invisible in these sectors for upto 6-point functions.

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Look for non-analyticities $+$ analytic rise in the infrared QCD Dirac spectrum
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Numerical details

- Möbius domain wall fermions on 5D hypercube with $N = 32$ sites along each spatial 4-dim, $N_5 = 16$ and $N_\tau = 8$ sites along temporal dim.

- Volumes, $V = N^3 a^3$, Temperature, $T = \frac{1}{N_\tau a}$, $a$ is the lattice spacing.

- Box size: $m_\pi V^{1/3} > 4$

- 2 light+1 heavy flavour

- Input $m_s$ physical $\approx 100$ MeV and $m_s/m_l = 27, 12$
  $\Rightarrow m_\pi = 135, 200$ MeV. [Columbia-BNL-LLNL, 13, 14].

- The sign function and chiral symmetry maintained as precise as $10^{-10}$. 
QCD Dirac spectrum at finite $T$

- General features: Near zero mode peak + bulk.
- No gap observed up to $1.2 \ T_C$ for physical quark mass [V. Dick et. al. in prep].
We fit to the ansatz: \( \rho(\lambda) = \frac{Ae}{\lambda^2 + A} + B\lambda^\gamma \).

- Bulk rises linearly as \( \lambda \) near \( T_c \).
- No gap even when quark mass reduced!
The rise of the bulk is $\gamma \sim 2 \rightarrow$ Still not consistent with $\lambda^3$.

Infrared modes becomes rarer with a small peak.
A closer look at the near-zero modes

- The near-zero modes sensitive to the sea quark mass $m_\pi$ heavier but the peak survives!
- Falls by more than a third at $1.2T_c$. 

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Comparing eigenspectra for different lattice fermions

- The bulk spectra of staggered quarks (HISQ) consistent with DW spectrum with heavier quark mass at $1.2 T_c$.
- More near-zero states in HISQ than domain wall..broken anomalous $U_A(1)$?
- Bulk spectrum insensitive to lattice discretization.

![Graph showing comparison of eigenspectra for different lattice fermions. The graph illustrates the relationship between $m_s \rho / T^4$ and $\lambda / m_s$ for different quark mass conditions.](image)
Fate of $U_A(1)$ near $T_c$

- Contribution to $U_A(1)$ breaking in 2-point correlation functions mainly come from small eigenvalues.
- First 50 eigenvalues produce most of the breaking obtained from inversion of DW Dirac operator [Columbia-BNL-LLNL, 13,14].

![Graph showing correlation between $T$ and $(\chi_\pi - \chi_\delta)/T^2$.](slide)
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What are the constituents of the hot QCD medium?

- At $T = 0$, anomaly effects related to instantons [t’Hooft, 76].

- Near chiral crossover transition $T_c$, a medium consisting of interacting instantons can explain chiral symmetry breaking $\Rightarrow$ Instanton Liquid Model [Shuryak, 82].

- At $T \gg T_c$, medium is like a dilute gas of instantons [Gross, Pisarski & Yaffe, 81].

- What is the medium made up of for $T_c \leq T \leq 2T_c$?
Near-zero modes due to a dilute instanton gas?
Small residual interactions at $1.2T_c$.
The dilute gas picture sets in QCD already at $1.5T_c$ [V. Dick et. al., 15].
Topological susceptibility above $T_c$

- Have strong sensitivity to sea-quark mass.
- Naive fit produces a $T^{-2.5}$ fall unlike DIGM which predicts $T^{-7}$ for QCD.
- At high $T$, topological objects with -ve E,M charges get excited [Shuryak & Sulejmanpasic 12].
- At finite $T$, $n_{\text{instanton}} \sim T^{-7}$ and $n_{\text{dyons}} \sim T^{-2.3}$.
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![Graph showing susceptibility vs temperature with two fits: $T^{-2}$ and $T^{-2.8}$]
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![Graph showing the relationship between $\chi_t^{1/4}$ and $T$. The graph includes a fit for $T^{-2.8}$ with $\chi^2$/dof = 0.2 and another fit for $T^{-2}$ with $\chi^2$/dof = 1.4.]
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Topological susceptibility above $T_c$

- A better observable: 
  \[
  \frac{\langle Q^4 \rangle - 3 \langle Q^2 \rangle^2}{\langle Q^2 \rangle}
  \]

- At $T = 0$ QCD consistent with large $N_c$ expansion of $\chi_t$ [M. Ce et. al, 15].
- Departure from large $N_c$ expectations but a slow rise towards DIG $\gtrsim T_c \rightarrow$ effects of residual interactions or fractional topological charges? [See R. Larsen’s talk].

![Graph showing $(\langle Q^4 \rangle - 3 \langle Q^2 \rangle^2)/\langle Q^2 \rangle$ vs. $T$ (MeV)]

- $m_\pi = 135$ MeV
- Dilute gas
- Large $N_c$
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Applications for anomalous hydrodynamics

- Hints of charge separation due to anomaly observed from hydrodynamic simulations. [Y. Hirono et al, 13, 14, X. Liao & Y. Yin, 15].

The Equation of State at freezeout is a crucial input.
Eos at finite $\mu_5$ used $\Rightarrow j_5$ is not conserved current.
Instead quantify effects of local CP odd fluctuations by $\chi_t$ known from LQCD

$$F(\theta) = F(0) + \frac{\theta^2}{2} \frac{\partial^2 F}{\partial \theta^2} \bigg|_{\theta \to 0} + ..$$
On large volume lattice we found that $U_A(1)$ broken for $T \leq 1.2T_c$.

- Infrared eigenvalues contribute dominantly to its breaking.
- Consists of near-zero+tail of the bulk modes. The latter quite robust insensitive to lattice cut-off effects.
- Near-zero modes require a careful study.
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QCD medium at $T \gtrsim T_c$: Summary

- The medium made of weakly interacting topological structures.
- $T$-dependence of $\chi_t$ suggest small residual interactions between them.
- Hints of structures with magnetic charges? → preliminary, needs careful study for further conclusions.
- Consequences for phenomenology of strongly coupled QGP?

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