The topological structures in strongly coupled QGP with chiral fermions on the lattice

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In collaboration with:

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Outline

1. The $U_A(1)$ puzzle in QCD: a way to resolve it
2. Our results
3. Topological structures and $U_A(1)$
4. Implications for experiments

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The $U_A(1)$ puzzle

- Origin:
  Anomalous $U_A(1)$ not an exact symmetry of QCD yet may affect the order of phase transition for $N_f = 2$ [Pisarski & Wilczek, 83].

- In model QFT with same symmetries as QCD, it is not possible to quantify the $U_A(1)$ effects in observables.

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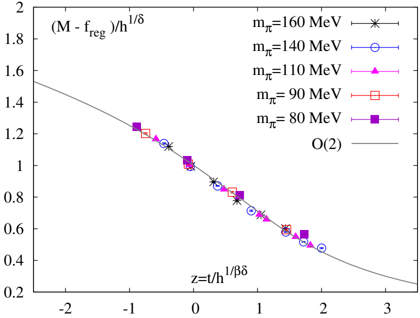
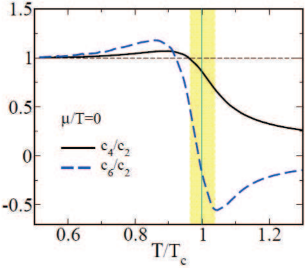
Why is it important?

- $m_{u,d} << \Lambda_{QCD}$, chiral symmetry drives phase transition at $\mu_B \to 0$
- The singular part of free energy should show critical scaling $\to$ hints of criticality from lattice studies [BI-BNL collaboration, 09].



Why is it important?

- Criticality at $\mu = 0$ changes on whether $U_A(1)$ is effectively restored [Pelissetto & Vicari, 13, Nakayama & Ohtsuki, 14].
    - $O(4)$ critical exponents for $U_A(1)$ broken
    - $U(2) \times U(2)$ if $U_A(1)$ effectively restored
- Effects should be visible in higher order fluctuations measured in the experiments [Karsch & Redlich, 11]



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- Could affect the EoS relevant for anomalous hydrodynamics with chiral imbalance?
- Softening of η' mass near freezeout? [Grahl & Rischke, 14,15]
- Consequences for the critical end-point at finite μ_B?
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The major issues with the lattice studies so far

- Finite volume effects $\rightarrow$ ensure presence of topological objects in a box.
- Most studies done with lattice fermions with only a remnant of continuum chiral symmetry + explicitly broken $U_A(1)$ [S. Chandrasekharan, 96, H. Ohno et. al 12, V. Dick et. al., 15].
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- Lattice cut-off effects need careful consideration [G. Cossu et. al, 14]

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Chiral fermions on the lattice

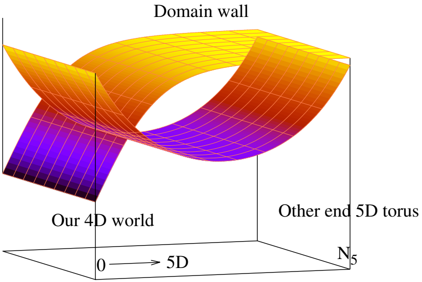
- A no-go theorem on the lattice disfavours ultra-local chiral fermion operator [Nielsen & Ninomiya, 82]
- Overlap fermions [Narayanan & Neuberger, 94, Neuberger, 98] sacrifice ultra-locality

$$D_{ov} = M(1 + \gamma_5 \, \mathbf{sgn}(\gamma_5 D_W(-M))) \ , \ \mathbf{sgn}(A) = A/\sqrt{A.A}.$$

- But have an exact chiral symmetry under non-local chiral transformations + index theorem at finite "a" [Ginsparg & Wilson, 82, Luscher, 98].

$$\{\gamma_5, D\} = aD\gamma_5 D$$

Chiral fermions on the lattice



- One can also start from 5D world+ put a defect to localize chiral fermions on the 4D brane.
- Domain wall fermions [Kaplan 92, Shamir 95] in the limit $N_5 \to \infty$
  $D_{DW} = M(1 - \gamma_5 \, \mathbf{sgn}(\ln|T|))$, $T = (1 + a_5\gamma_5 D_W P_+)^{-1}(1 - a_5\gamma_5 D_W P_-)$.

Observables sensitive to $U_A(1)$ breaking..

- Not an exact symmetry→ no order-parameter
- Look at the difference of the integrated 2 point correlators [Shuryak, 94]

$$\chi_\pi - \chi_\delta = \int d^4x \; \left[\langle i\pi^+(x) i\pi^-(0)\rangle - \langle \delta^+(x)\delta^-(0)\rangle\right]$$

- Equivalently study $\rho(\lambda, m_f)$ of the Dirac operator [Cohen, 95, Hatsuda & Lee, 95]

$$\chi_\pi - \chi_\delta \stackrel{V\to\infty}{\to} \int_0^\infty d\lambda \frac{4m_f^2 \;\rho(\lambda, m_f)}{(\lambda^2 + m_f^2)^2} \;,\; \langle\bar{\psi}\psi\rangle \stackrel{V\to\infty}{\to} \int_0^\infty d\lambda \frac{2m_f \;\rho(\lambda, m_f)}{(\lambda^2 + m_f^2)}$$

- Chiral symmetry restored: $\lim_{m_f\to 0} \lim_{V\to\infty} \rho(0, m_f) \to 0 \Rightarrow U_A(1)$ restored.
- Chiral symmetry restored $+U_A(1)$ broken if

$$\lim_{\lambda\to 0} \rho(\lambda, m_f) \to \delta(\lambda) m_f^\alpha \;, 1 < \alpha < 2.$$

Spectral density of Dirac operator at finite T

- Very little known. Only recently there are very interesting results [Aoki, Fukaya & Taniguchi, 12].
- Assuming $\rho(\lambda, m)$ to be analytic in m^2, λ, look at chiral Ward identities of n-point function of scalar & pseudo-scalar currents.
- $\rho(\lambda, m \to 0) \sim \lambda^3 \Rightarrow U_A(1)$ breaking effects invisible in these sectors for upto 6-point functions.
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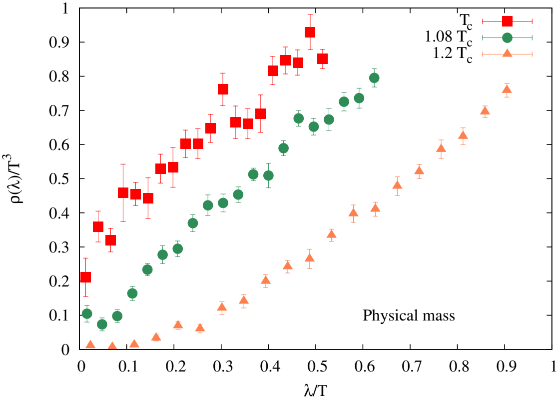
Numerical details

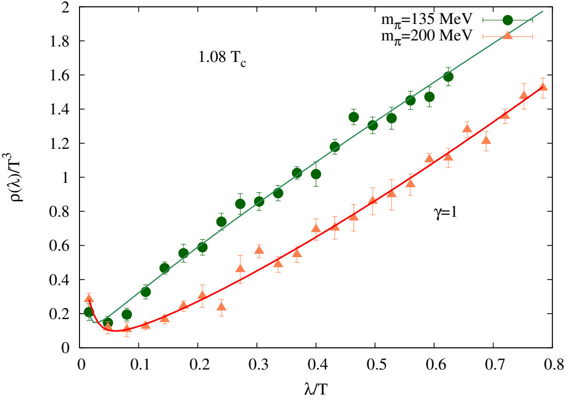
- Möbius domain wall fermions on 5D hypercube with $N = 32$ sites along each spatial 4-dim, $N_5 = 16$ and $N_\tau = 8$ sites along temporal dim.
- Volumes, $V = N^3 a^3$, Temperature, $T = \frac{1}{N_\tau a}$, a is the lattice spacing.
- Box size: $m_\pi \ V^{1/3} > 4$
- 2 light+1 heavy flavour
- Input m_s physical ≈ 100 MeV and $m_s/m_l = 27, 12$ $\Rightarrow m_\pi = 135, 200$ MeV. [Columbia-BNL-LLNL, 13,14].
- The sign function and chiral symmetry maintained as precise as 10^{-10}.

- General features: Near zero mode peak +bulk.
- No gap observed upto 1.2 T_c for physical quark mass [V. Dick et. al. in prep].



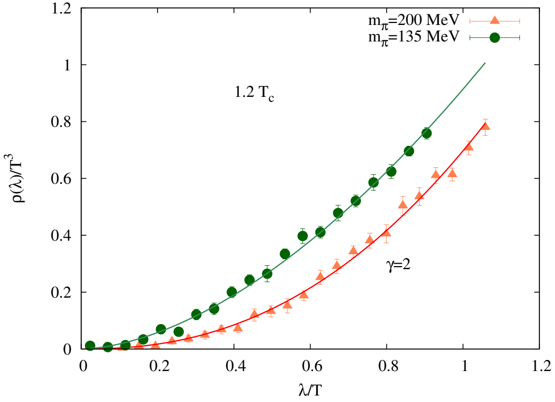
General Characteristics

- We fit to the ansatz: $\rho(\lambda) = \frac{A\epsilon}{\lambda^2+A} + B\lambda^\gamma$.
- Bulk rises linearly as λ near T_c.
- No gap even when quark mass reduced!



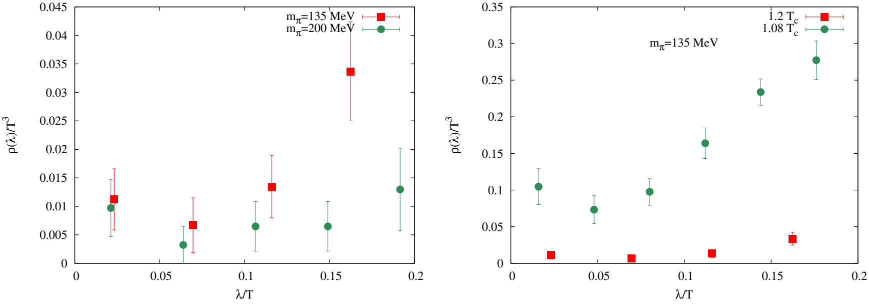
General Characteristics

- The rise of the bulk is $\gamma \sim 2$ $\rightarrow$ Still not consistent with λ^3.
- Infrared modes becomes rarer with a small peak.



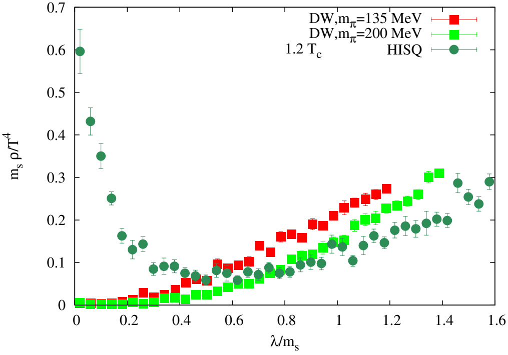
A closer look at the near-zero modes

- The near-zero modes sensitive to the sea quark mass $\rightarrow$ sparse when m_π heavier but the peak survives!
- Falls by more than a third at $1.2\,T_c$.



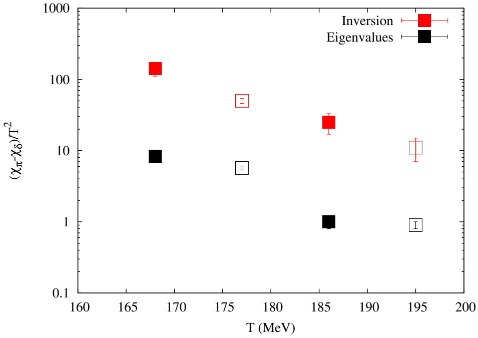
Comparing eigenspectra for different lattice fermions

- The bulk spectra of staggered quarks(HISQ) consistent with DW spectrum with heavier quark mass at $1.2\,T_c$.
- More near-zero states in HISQ than domain wall..broken anomalous $U_A(1)$?
- Bulk spectrum insensitive to lattice discretization.



Fate of $U_A(1)$ near T_c

- Contribution to $U_A(1)$ breaking in 2-point correlation functions mainly come from small eigenvalues.
- First 50 eigenvalues produce most of the breaking obtained from inversion of DW Dirac operator [Columbia-BNL-LLNL, 13,14].



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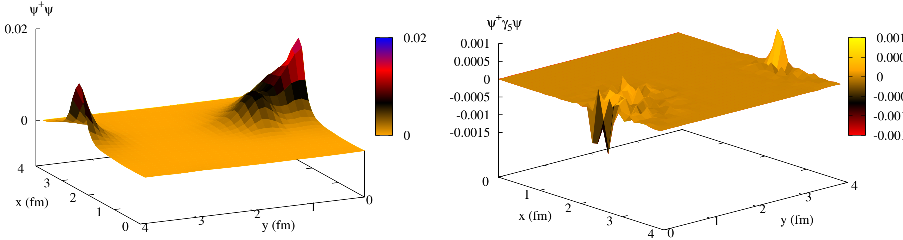
What are the constituents of the hot QCD medium?

- At $T = 0$, anomaly effects related to instantons [t'Hooft, 76].
- Near chiral crossover transition T_c, a medium consisting of interacting instantons can explain chiral symmetry breaking $\Rightarrow$ Instanton Liquid Model [Shuryak, 82].
- At $T >> T_c$, medium is like a dilute gas of instantons [Gross, Pisarski & Yaffe, 81].
- What is the medium made up of for $T_c \leq T \leq 2T_c$?

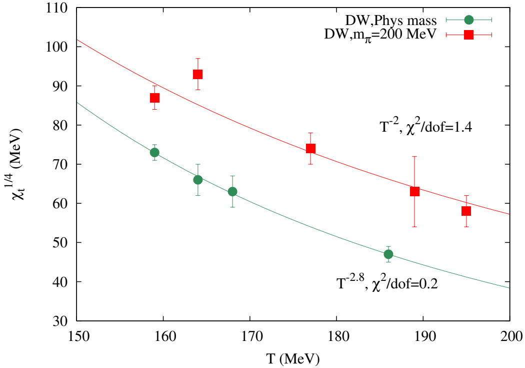
A closer look at near-zero modes



- Near-zero modes due to a dilute instanton gas?
- Small residual interactions at $1.2\,T_c$.
- The dilute gas picture sets in QCD already at $1.5\,T_c$ [V. Dick et. al., 15].

Topological susceptibility above T_c

- Have strong sensitivity to sea-quark mass.
- Naive fit produces a $T^{-2.5}$ fall unlike DIGM which predicts T^{-7} for QCD.
- At high T, topological objects with -ve E,M charges get excited [Shuryak & Sulejmanpasic 12].
- At finite T, $n_{\text{instanton}} \sim T^{-7}$ and $n_{\text{dyons}} \sim T^{-2.3}$.



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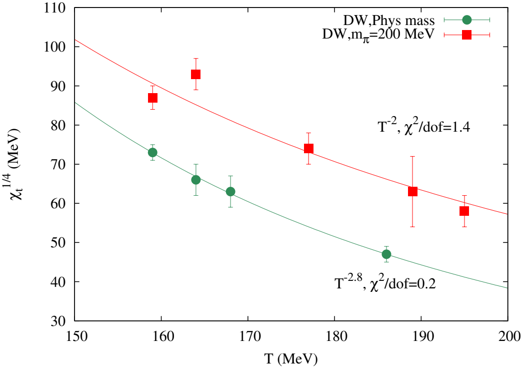

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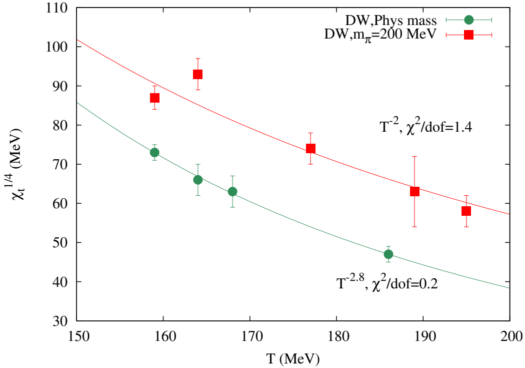

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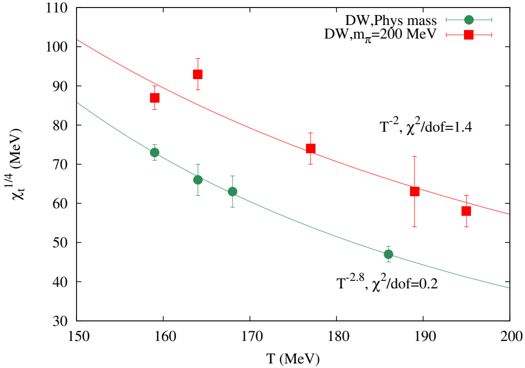


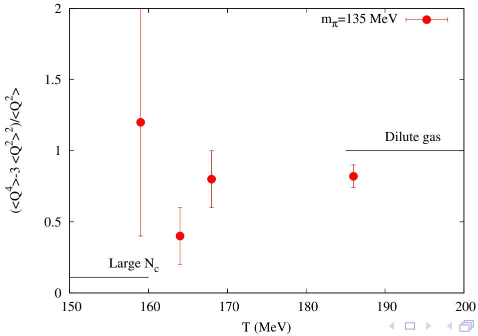
Topological susceptibility above T_c

- A better observable:

$$\frac{< Q^4 > -3 < Q^2 >^2}{< Q^2 >}$$

- At $T = 0$ QCD consistent with large N_c expansion of χ_t [M. Ce et. al, 15].
- Departure from large N_c expectations but a slow rise towards DIG $\gtrsim T_c \rightarrow$ effects of residual interactions or fractional topological charges?

[See R. Larsen's talk].



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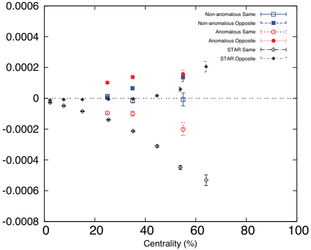
Applications for anomalous hydrodynamics

- Hints of charge separation due to anomaly observed from hydrodynamic simulations. [Y. Hirono et. al, 13, 14, X. Liao & Y. Yin, 15].

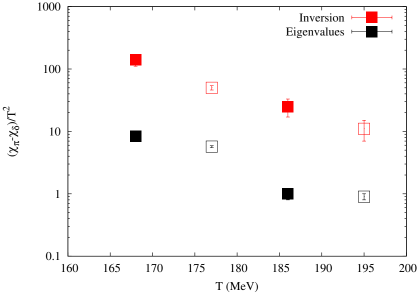


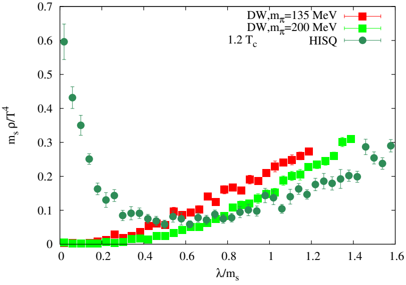
- The Equation of State at freezeout is a crucial input.
- Eos at finite μ_5 used $\Rightarrow j_5$ is not conserved current.
- Instead quantify effects of local CP odd fluctuations by χ_t known from LQCD

$$F(\theta) = F(0) + \frac{\theta^2}{2}\frac{\partial^2 F}{\partial \theta^2}|_{\theta \to 0} + ..$$

Summary

- On large volume lattice we found that $U_A(1)$ broken for $T \leq 1.2\,T_c$.
- Infrared eigenvalues contribute dominantly to its breaking.
- Consists of near-zero+tail of the bulk modes. The latter quite robust insensitive to lattice cut-off effects.
- Near-zero modes require a careful study.





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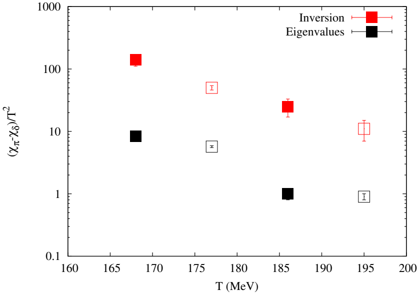

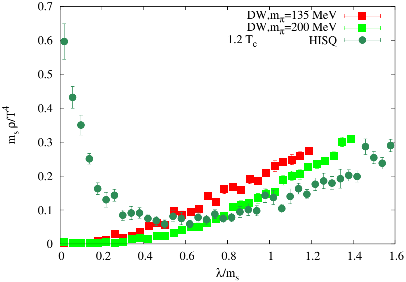

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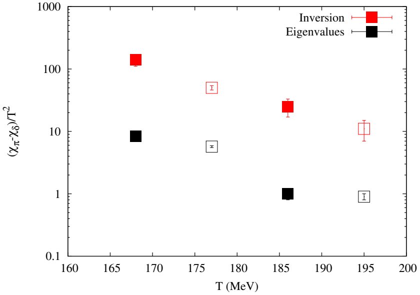

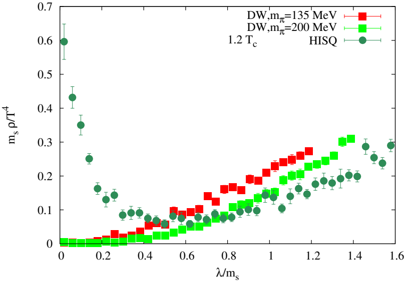

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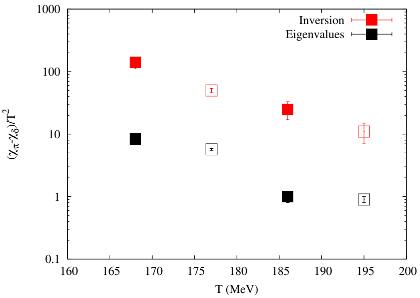
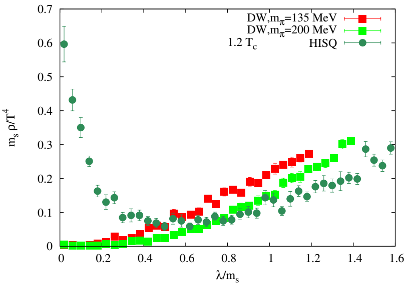
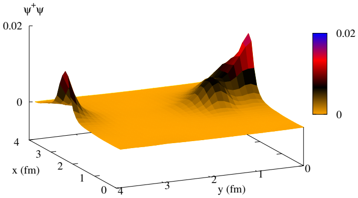


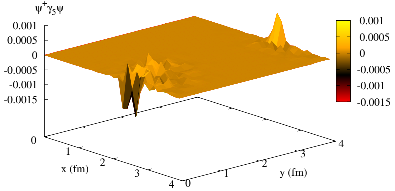


QCD medium at $T \gtrsim T_c$: Summary

- The medium made of weakly interacting topological structures.
- T-dependence of χ_t suggest small residual interactions between them.
- Hints of structures with magnetic charges? $\rightarrow$ preliminary, needs careful study for further conclusions.
- Consequences for phenomenology of strongly coupled QGP?

  [J. Xu, J. Liao & M. Gyulassy, 15].





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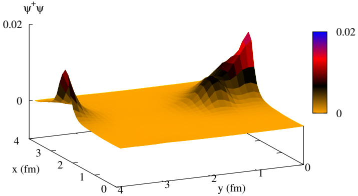

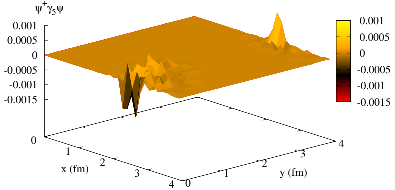

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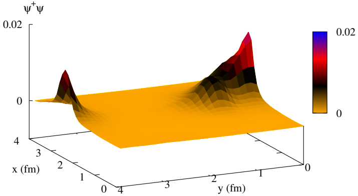

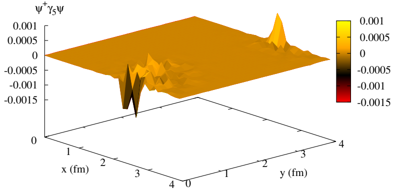

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