Dynamic critical behavior for the relativistic $O(N)$ model in the framework of the real-time functional RG and applications to QCD

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INTRODUCTION

- Understanding the dynamics at the critical point is important to set the QCD critical point.
- Dynamic universality classes are much more complicated than static ones.
- Real-time functional RG is a new possibility to treat dynamic fluctuations in a systematic manner.

Closed-time path (CTP) formalism:

$$Z = \mathrm{tr}\,[\hat\rho_D] = \mathrm{tr}\left[\mathrm{e}^{-\mathrm{i}\hat H(t_f-t_i)}\hat\rho_D \mathrm{e}^{\mathrm{i}\hat H(t_f-t_i)}\right] = \int \mathcal{D}\varphi_+\mathcal{D}\varphi_- \langle\varphi_+;t_i|\hat\rho_D|\varphi_-;t_i\rangle \exp\mathrm{i}\left(S[\varphi_+]-S[\varphi_-]\right).$$

Microscopic action is given by the relativistic $O(N)$ model:

$$S[\varphi]=\int \mathrm{d}t\,\mathrm{d}^d\boldsymbol{x}\left(\frac{1}{2}(\partial_\mu\varphi_a)^2-\frac{m^2}{2}\varphi_a^2-\frac{\lambda}{4}(\varphi_a^2)^2\right)$$

Goals:

- Develop the renormalization group for CTP effective actions: Real-time FRG.
- Compute the dynamic critical phenomenon using the real-time FRG.

FLUCTUATION-DISSIPATION THEOREM & SCALING OPERATORS

Introduce classical and quantum fields

$$\varphi=(\varphi_++\varphi_-)/2,\ \tilde\varphi=\varphi_+-\varphi_-.$$

Fluctuation-Dissipation Theorem: Retarded and statistical propagators are closely related.

$$\langle\varphi(p)\varphi(-p)\rangle=\coth\frac{\beta p^0}{2}\mathrm{Im}\left(\mathrm{i}\langle\varphi(p)\tilde\varphi(-p)\rangle\right).$$

We require that the coarse-graining procedure respect FDT at any RG scales [1].

Take the derivative expansion of the inverse retarded propagator.

$$\Gamma^{(2\mathrm{pt})}(p)=\begin{pmatrix}0 & Z^{\parallel}\omega^2-Z^{\perp}\boldsymbol{p}^2+\mathrm{i}\Omega\omega\\ Z^{\parallel}\omega^2-Z^{\perp}\boldsymbol{p}^2-\mathrm{i}\Omega\omega & \mathrm{i}\Omega\omega\coth(\beta\omega/2)\end{pmatrix}.$$

The same Ω must appear thanks to FDT!

Local interaction approximation:
The interaction part of the CTP effective action is assumed to be local,

$$\mathcal{U}=m^2\sigma_2+\lambda_{1,2}\left(\sigma_1-v^2/2\right)\sigma_2+\lambda_{2,3}\sigma_2\sigma_3,$$

with the $O(N)$ invariants,

$$\sigma_1=\frac{1}{2}\phi^a\phi_a,\quad \sigma_2=\phi^a\tilde\phi_a,\quad \sigma_3=\frac{1}{2}\tilde\phi^a\tilde\phi_a,$$

Another term, such as

$$\mathrm{i}\lambda_{1,3}\sigma_1\sigma_3,$$

may be included as a local interaction from the viewpoint of symmetry, but FDT states that

$$\lambda_{1,3}=0.$$

In the UV region (or $T=0$), furthermore, FDT gives another constraint:

$$\lambda_{1,2}=4\lambda_{2,3}=\lambda/3.$$

FUNCTIONAL RENORMALIZATION GROUP

FRG modifies the CTP action as $S[\varphi,\tilde\varphi]\to S[\varphi,\tilde\varphi]+\Delta_k S[\varphi,\tilde\varphi]$, where

$$\Delta_k S[\varphi,\tilde\varphi]=-\int_{x,y}\tilde\varphi^a(x)R_{k,ab}(x,y)\varphi^b(y).$$

The function R_k is a momentum-dependent mass term, i.e.

$$R_{k,ab}(x^0,y^0;\boldsymbol{p})=R_k(\boldsymbol{p})\delta(x^0-y^0)\delta_{ab},$$

The initial density matrix must also be modified to respect FDT.

Renormalization Group equation:
The 1PI effective action Γ_k follows the one-loop exact RG equation [2]:

$$\frac{\partial}{\partial s}\Gamma_k=i\int_p \mathrm{Tr}\left\{\frac{\partial}{\partial s}R_k(\boldsymbol{p})\,\mathrm{Re}G_k^{\mathrm{R}}(\omega,\boldsymbol{p})\right\},$$

with $s=\ln k/\Lambda$. It connects quantum and classical effective actions:

$$\Gamma_k\to\Gamma\quad\text{as}\quad s\to-\infty,\qquad \Gamma_k\to S\quad\text{as}\quad s\to 0.$$

REAL-TIME RG FLOW

Scaling properties: At low energies and momenta, we assume the scaling property

$$\omega\sim|\boldsymbol{p}|^z.$$

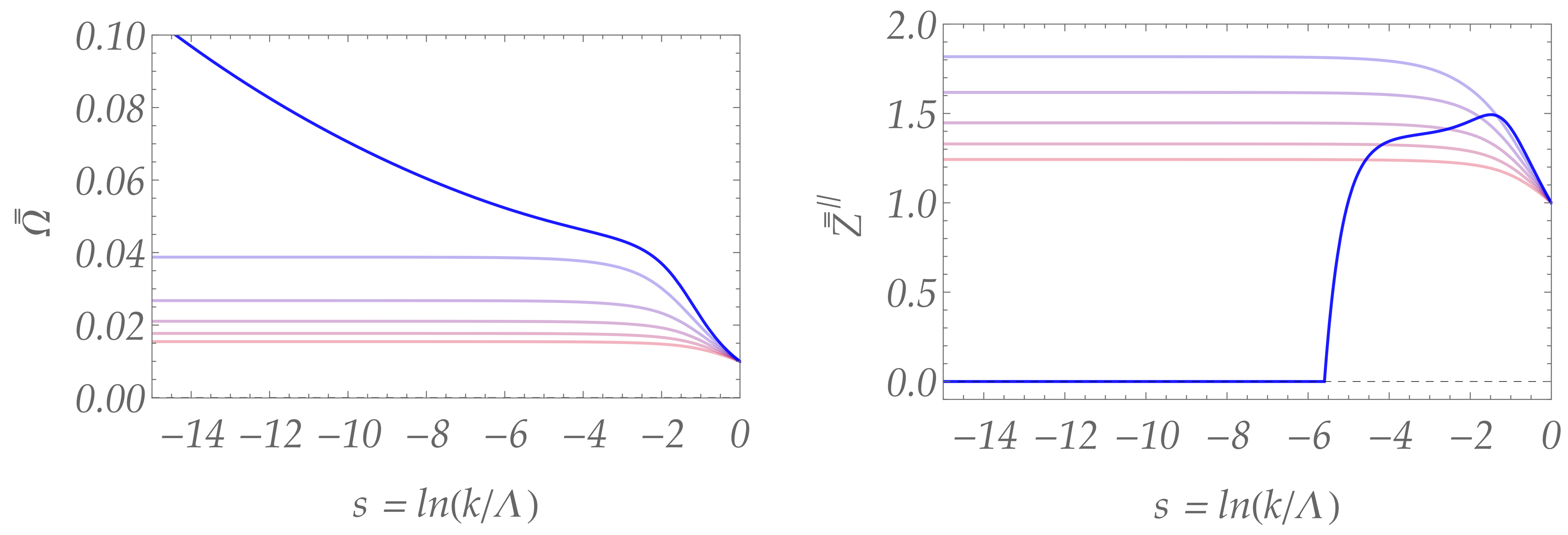
Scale-invariance of the lowest order derivatives tells the scaling dimensions of classical/quantum fields:

$$[\varphi]=\frac{1}{2}(d-2+\eta^\perp),\ [\tilde\varphi]=[\varphi]+z.$$

The scaling dim. of quantum fields is larger than that of classical fields by z.

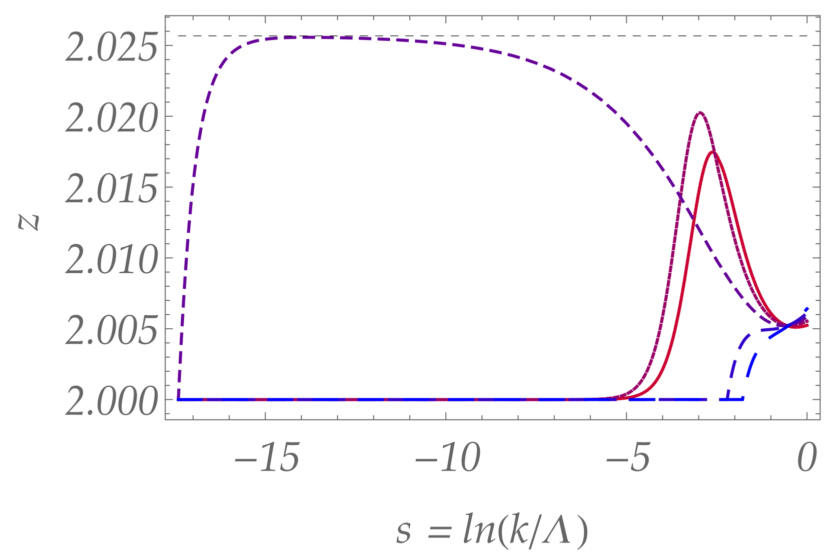
$$z=1 \quad \text{UV fixed point}$$
$$z=2+c\eta^\perp \quad \text{IR fixed point}$$

RG flow at the critical temperature $T_{\Lambda,\mathrm{cr}}$: ($N=1$ at $T\neq 0$.)



As the RG scale is lowered, $Z^{\parallel}$ goes inside the negative region.
This behaviors is out of the scope of our truncation, and we put $Z^{\parallel}=0$ at low-energy scales.

Dynamic scaling exponent:



We find the plateau of the dynamic scaling exponent at [1]

$$\mathbf{z\simeq 2.025}$$

Expansion around the upper critical dimension:
Set $d=4-\varepsilon$. One can check the regulator dependence of static and dynamic scaling exponents:

	Regulator function	$\eta^\perp/\left(\frac{N+2}{(N+8)^2}\varepsilon^2\right)$	$(z-2)/\eta^\perp$
Exponential cutoff	$r_{\mathrm{exp}}=(e^y-1)^{-1}$	1/2	0.73
Litim cutoff	$r_{\mathrm{opt}}=(1/y-1)\,\theta(1-y)$	1/2	1/2
Sharp cutoff	$r_{\mathrm{sharp}}=1/\theta(y-1)-1$	∞	−1
Effective theory		1/2	0.73

PROSPECTS TOWARD APPLICATIONS TO QCD

Dispersion relation at high-temperature phase of the order-parameter field φ:

$$\omega(\boldsymbol{p})\simeq\mathrm{i}\frac{Z^\perp}{\Omega T}(m_R^2+\boldsymbol{p}^2).$$

Microscopic QFT and the macroscopic dynamics (**model A**) is connected within our *ansatz*.

What's the next step?

The time-development is governed by Hamiltonian. ⇒ There are many conserved quantities:

- Energy-momentum tensor: $T^{0\mu}$
- Noether charge (Baryon number): j^0

Mode couplings between φ and j^0, $T^{0,\mu}$ affects the dynamic critical behavior significantly [3].

1. We must develop a new technique of FRG in order to treat the hydrodynamic modes governed by composite operators $T^{0,\mu}$, j^0. Can we describe model H in our approach?
2. If the experimental length scale $\xi\ll 1/m_\pi$, pions behave as massless and affects critical behaviors. What happens if $\xi\sim 1/m_\pi$?

Developing microscopic approaches, we will be able to answer these questions.

References

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