Parametric instability of classical Yang-Mills fields

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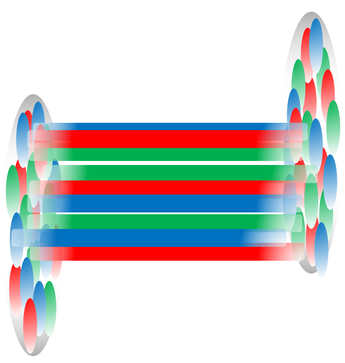
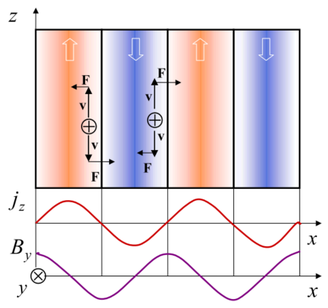
Abstract Plasma instabilities play an important role in thermalization. Recently, classical gluon fields in a nonexpanding geometry are found to show parametric instability under a longitudinally polarized background. The growth rates of low momentum modes are large enough compared with other instabilities proposed so far. Surprisingly, parametric instability survives even in an expanding geometry. We introduce the conformal coordinates which enable us to map an expanding problem into a nonexpanding problem. We find that fluctuations with finite longitudinal momentum can grow exponentially.

1. Instabilities in Heavy Ion Collisions

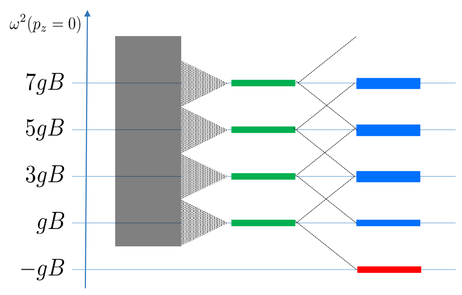


glasama

in the earliest stage of HIC,
the system is under **strong fields**
(color electromagnetic flux tubes)



Nielsen-Olesen instability



nonAbelian Weibel instability

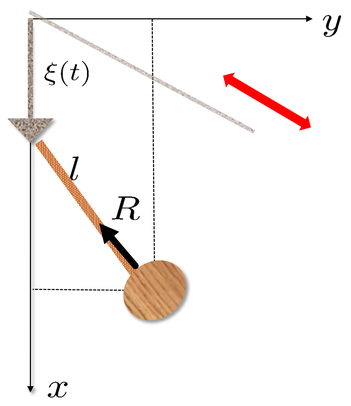
Yet another instability ?

2. Parametric Instabilities in Physics

Parametric instabilities play important roles in many fields of physics

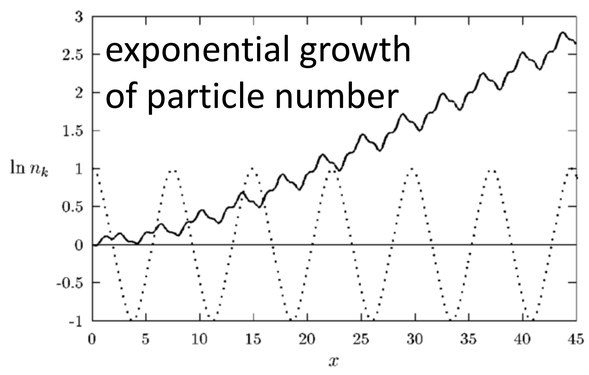
pendulum with time-dependent length

$$\ddot{y} + \omega_0^2 y = \lambda \sin(2\omega_0 t) y$$



preheating in cosmology

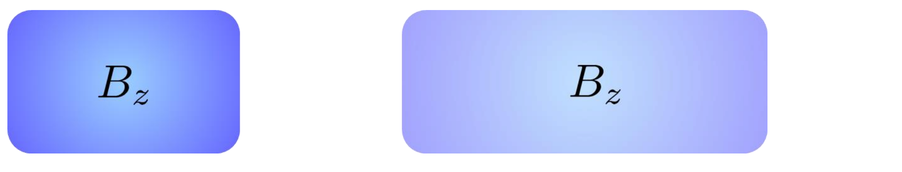
$$\ddot{\chi}_k + \left(k^2 + \frac{g^2}{\lambda}\,\mathrm{cn}^2(t)\right)\chi_k = 0$$



Greene, Kofman, Linde, Starobinsky (1997)

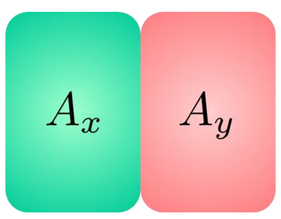
classical Yang-Mills theory

let us consider a simplified situation



- SU(2) Yang-Mills
- temporal gauge
- **non-expanding** or **expanding** geometry
- **homogenous** magnetic field

Berges, Scheffler, Schlichting, Sexty (2013)



background field configuration

$$A_i^a = \sqrt{B(t)}\left(\delta^{a2}\delta_{ix} + \delta^{a1}\delta_{iy}\right)$$

EOMs of fluctuation

$$\frac{d^2\vec{a}}{dt^2} + \Omega^2[B(t)]\vec{a} = 0$$

background color magnetic field is periodic function of time

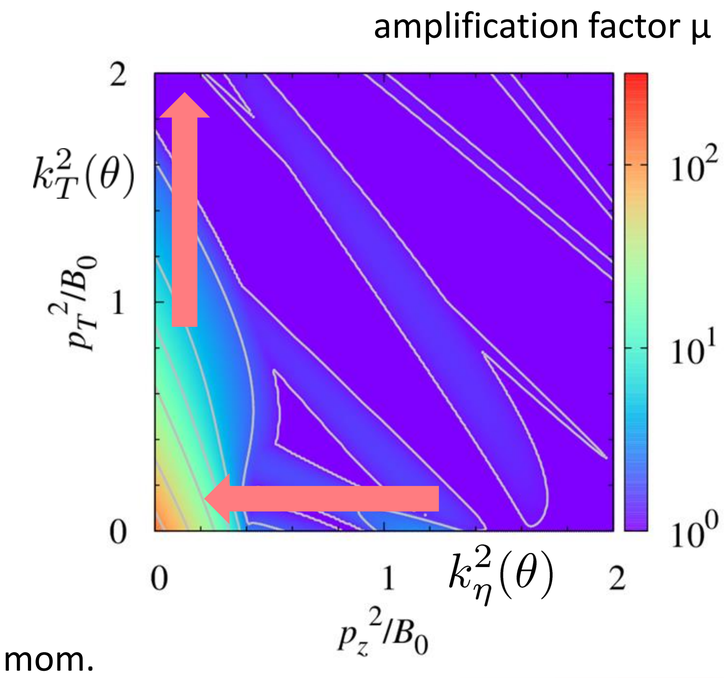
$$B(t) = B_0 \mathrm{cn}^2(\sqrt{B_0}t\,; 1/\sqrt{2})$$

3. Instability Band

S.T, Iida, Kunihiro, Ohnishi PRD 91 (2015)

the solution is unstable if
$|\mu| > 1$

(nonexpanding case)
broad instability band around zero momentum region



(expanding case)
transverse and longitudinal mom.
effectively depend on time

$$k_T^2 = \frac{1}{4\theta^2} + \frac{2}{3}\theta p_T^2 \qquad k_\eta^2 = \frac{1+9p_\eta^2}{4\theta^2}$$

How about expanding case ?
zero mode is still unstable
finite mom. modes can also be unstable

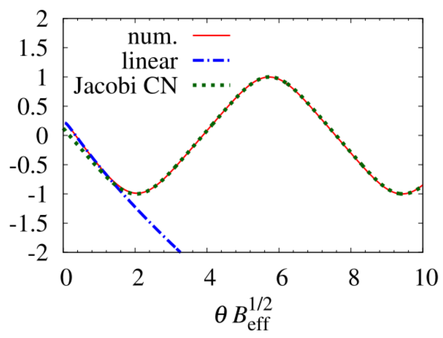
4. Instabilities in an Expanding System

$$\partial_\tau \to \tau^{-1/3}\partial_\theta$$
$$A_i^a \to \tau^{-1/3} A_i^a$$
$$A_\eta^a \to \tau^{-1/3} A_\eta^a$$

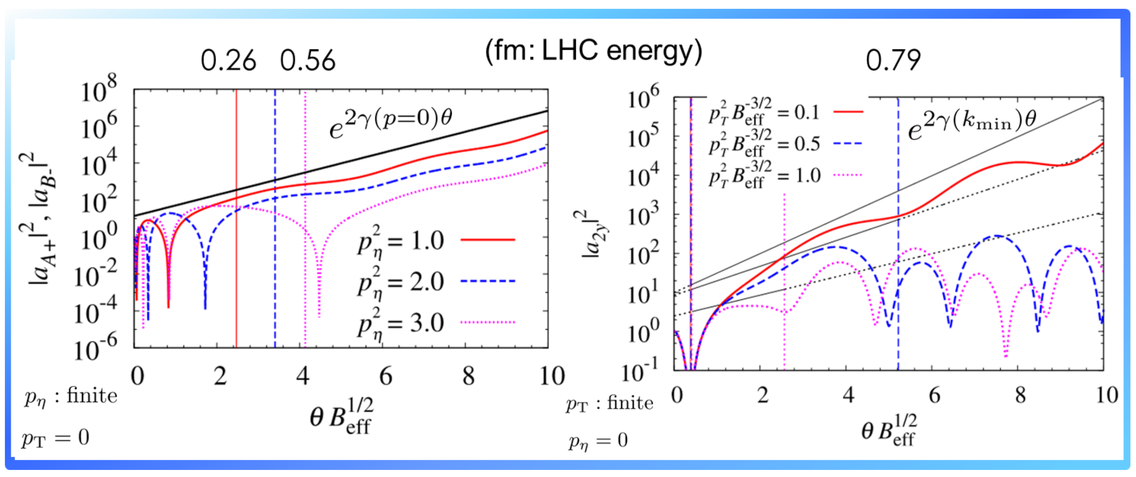
EOM of background field

$$\partial_\theta^2 \tilde{A} + \frac{1}{4\theta^2}\tilde{A} + \tilde{A}^3 = 0$$

almost periodic in conformal time



parametric instability survives in an expanding geometry



amplification factor / 0.7 fm (@LHC)

- zero mode (nonexpanding) ~120
- zero mode (expanding) ~30

SUMMARY

- We investigate instabilities of classical Yang-Mills fields under time dependent color magnetic field in nonexpanding and expanding geometries
- In a nonexpanding geometry, zero momentum mode is most unstable due to parametric instability
- The parametric instability survives even in an expanding geometry