Entanglement Entropies of Several Bosonic Field Theories

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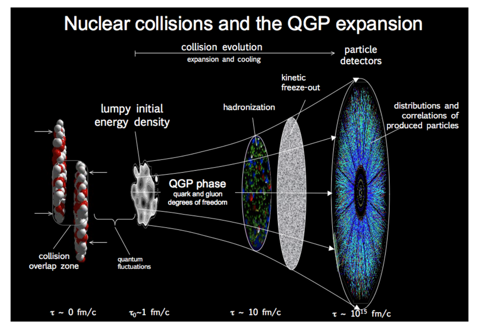
Introduction



In high-energy heavy ion collision process, the ratio of particle species implies that there exist thermal distributions of intermediate particles. The microscopic mechanism of the evolution from initial colliding nuclear to early thermalization is still in the state of study. The problems about how thermal entropy is produced in collision and whether time reversal symmetry is broken consequentially demand more theoretical explanation.

Entanglement entropy is a candidate to produce thermal entropy and preserve time reversal symmetry during deconfinement phase transition. The study of entanglement entropy, thus, attracts lots of interesting.

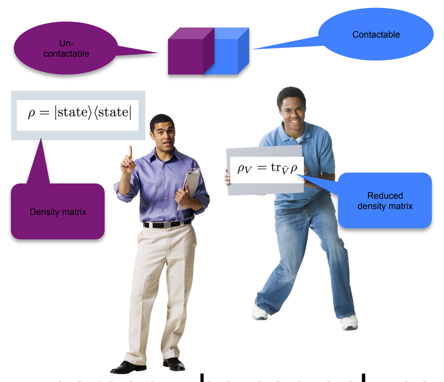
In condense matter physics, entanglement entropy has shown its power to describe quantum phase transition and corresponding critical phenomenon, such as scaling behavior. Our work is focused on its application in high-energy physics. The calculation of entanglement entropy for general field theory, then, becomes more and more important.

Entanglement Entropy and Replica Method

Quantum state can be entangled as

$$|S=0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

which cannot be decomposed into direct product of two individual spin states. Entanglement entropy is measured to characterize level of entanglement between sub-systems.

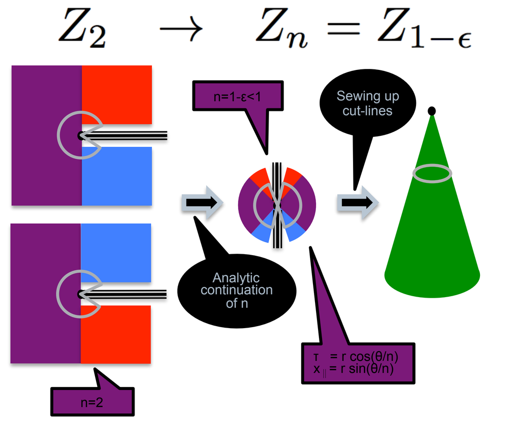


The region of quantum field theory can be divided into two parts: contactable part and un-contactable part. Person who can communicate with both two parts uses density matrix to describe physical information of the state while person who can only communicate with one part uses reduced density matrix. Entanglement entropy is defined as Von-Neumann entropy of reduced density matrix.

$$S_{\text{ent.}}(V|\bar{V}) = -\text{tr}(\rho_V \log \rho_V)$$

Calculation of Von-Neumann entropy is replaced by derivative of free energy on the cone with respect to deficit angle in replica trick. Cone is the analytic continuation of n-sheeted manifold.

$$Z_2 \quad \rightarrow \quad Z_n = Z_{1-\epsilon}$$



$$S_{\text{ent.}} \quad = \quad -\partial_{n\rightarrow 1}(\log Z_n - n \log Z_1)$$

O(N) σ - Model

System is in symmetric phase when mass squared is positive.

$$\mathcal{L}_{\mathrm{E}} = \sum_{i=1}^{N}\left[\frac{1}{2}(\partial\phi^i)^2 + \frac{1}{2}\mu^2(\phi^i)^2\right] + \frac{\lambda}{4}\left[\sum_{i=1}^{N}(\phi^i)^2\right]^2$$

with 4-dimensional counter-terms at one loop level. $\delta\mathcal{L}^{(4\mathrm{d})} = \sum_{i=1}^{N}\left[\frac{1}{2}\delta\mu^2(\phi^i)^2\right]$

Negative mass squared induces symmetry breaking.

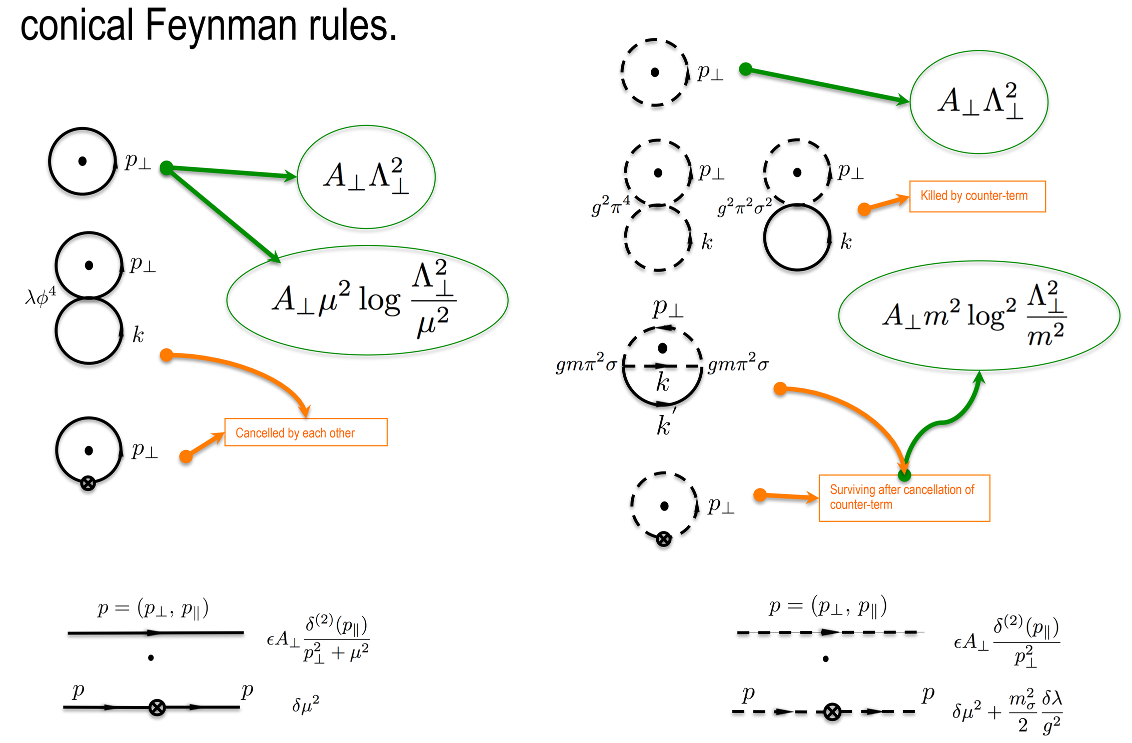
$$\mathcal{L}_{\mathrm{E}} = \sum_{i=1}^{N-1}\frac{1}{2}(\partial\pi^i)^2 + \frac{1}{2}(\partial\sigma)^2 + \frac{1}{2}m_\sigma^2\sigma^2 + \frac{g}{\sqrt{2}}m_\sigma\left(\sum_{i=1}^{N-1}(\pi^i)^2\sigma + \sigma^3\right) + \frac{g^2}{4}\left(\left[\sum_{i=1}^{N-1}(\pi^i)^2\right]^2 + \sigma^4 + 2\sum_{i=1}^{N-1}(\pi^i)^2\sigma^2\right)$$

with 4-dimensional counter-terms one loop level.

$$\delta\mathcal{L}^{(4\mathrm{d})} = \left(\delta\mu^2 + \frac{m_\sigma^2}{2}\frac{\delta\lambda}{g^2}\right)\sum_{i=1}^{N-1}\frac{1}{2}(\pi^i)^2 + \left(\delta\mu^2 + \frac{3m_\sigma^2}{2}\frac{\delta\lambda}{g^2}\right)\frac{1}{2}m_\sigma^2\sigma^2 + \frac{m_\sigma}{\sqrt{2}g}\left(\delta\mu^2 + \frac{m_\sigma^2}{2}\frac{\delta\lambda}{g^2}\right)\sigma$$

Bubble Diagrams on Cone

Free energy on cone is contributed from bubble diagrams with conical Feynman rules.

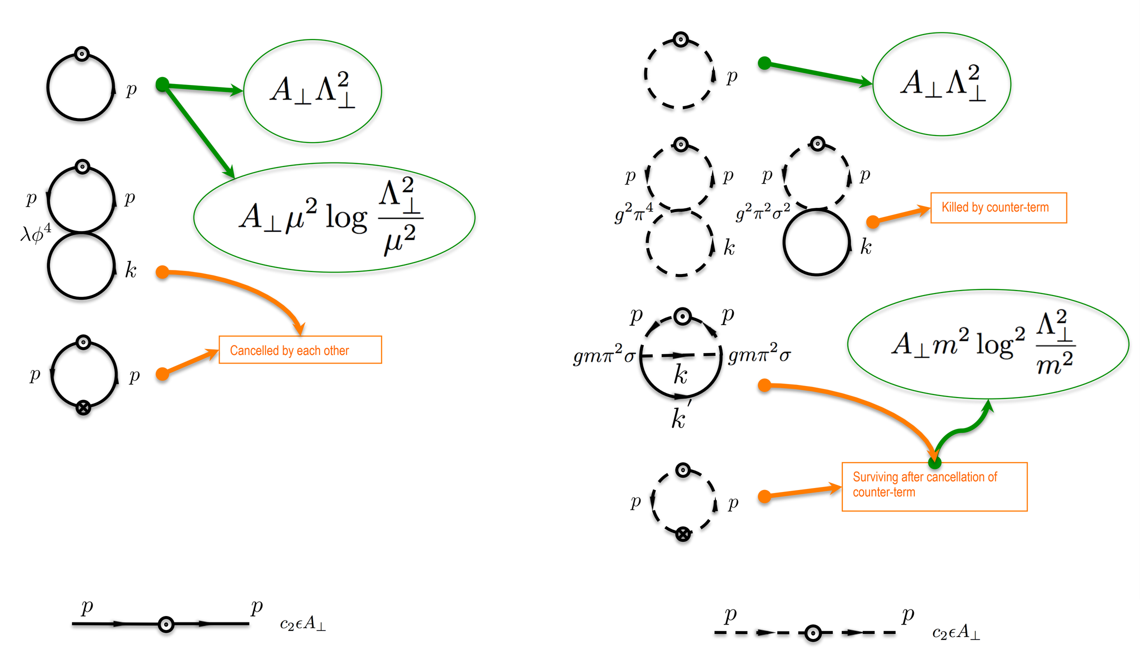


Renormalization of Entanglement Entropy

Different sub-leading divergences in symmetric phase and symmetry breaking phase can be renormalized by counter-terms on interface with the same valued renormalization parameters.

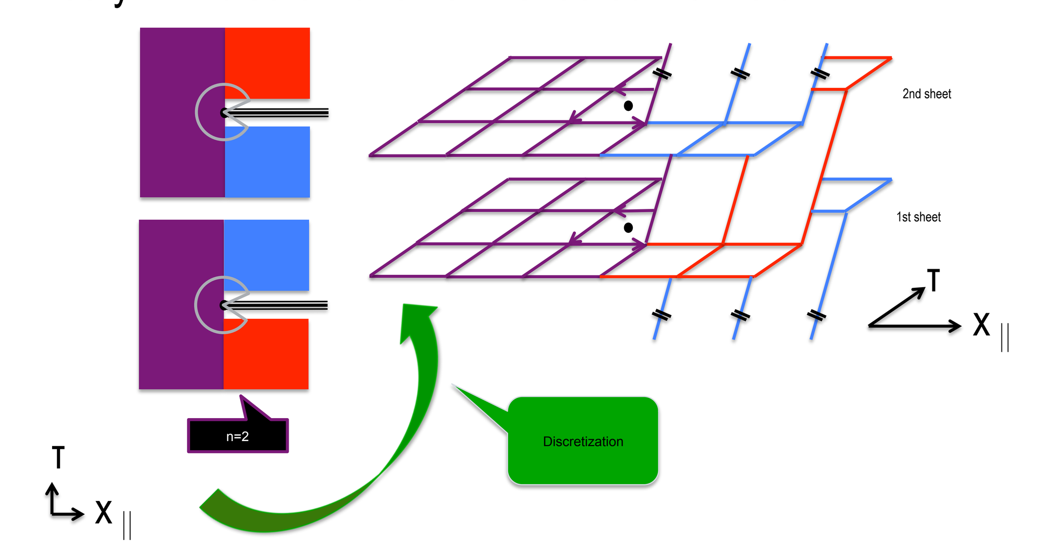
Symmetric phase: $\delta\mathcal{L}^{(2\mathrm{d})} = \epsilon c_0\Lambda^2\delta^{(2)}(x_\parallel) + \epsilon c_2\sum_{i=1}^{N}(\phi^i)^2\delta^{(2)}(x_\parallel)$

Symmetry breaking phase: $\delta\mathcal{L}^{(2\mathrm{d})} = \epsilon c_0\Lambda^2\delta^{(2)}(x_\parallel) + \epsilon c_2\sum_{i=1}^{N-1}(\pi^i)^2\delta^{(2)}(x_\parallel) + \epsilon c_2\sigma^2\delta^{(2)}(x_\parallel)$



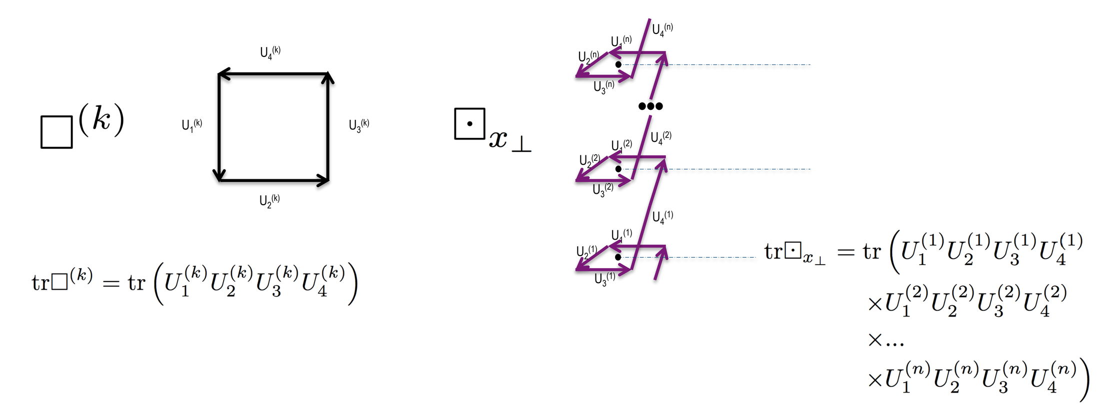
SU(N) Gauge Theory

SU(N) gauge theory can be solved by strong coupling expansion on lattice. We calculate entanglement entropy for SU(N) gauge theory with discretization of n-sheeted manifold.



Replica method is used for lattice gauge theory by introducing compact action on n-sheeted manifold.

$$S_n[\Box] = -\frac{\beta}{N}\sum_{k=1}^{n}\sum_{\Box}\text{Re tr}\Box^{(k)} - \frac{\beta}{N}\frac{1}{n}\sum_{x_\perp}\text{Re tr}\boxdot_{x_\perp}$$

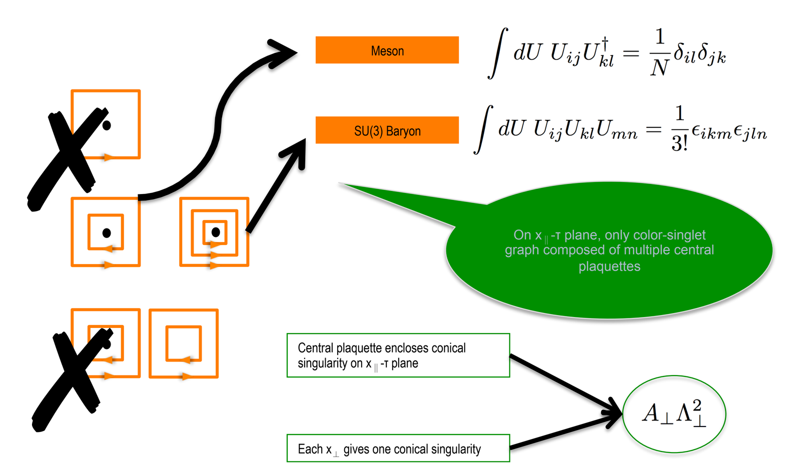


$$\text{tr}\Box^{(k)} = \text{tr}\left(U_1^{(k)}U_2^{(k)}U_3^{(k)}U_4^{(k)}\right)$$

$$\text{tr}\boxdot_{x_\perp} = \text{tr}\left(U_1^{(1)}U_2^{(1)}U_3^{(1)}U_4^{(1)} \times U_1^{(2)}U_2^{(2)}U_3^{(2)}U_4^{(2)} \times \ldots \times U_1^{(n)}U_2^{(n)}U_3^{(n)}U_4^{(n)}\right)$$

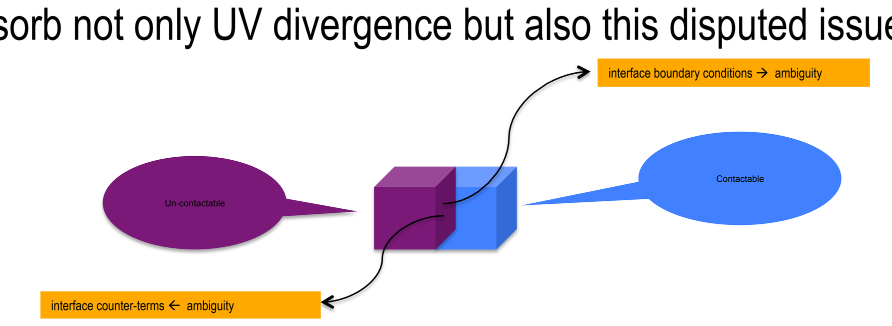
Leading Result in Strong Coupling Expansion

Entanglement entropy in strong coupling expansion series is given by diagrams with a pair of central plaquettes conjugating to each other like meson and triple central plaquettes like baryon.



Ambiguity and Interface Counter-terms

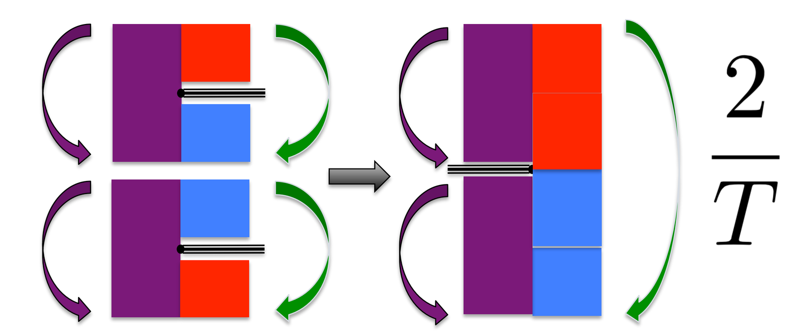
The compact action on n-sheeted manifold has ambiguity term. Different boundary conditions are distinguished by this edge mode term on interface. It is related to the problem that how to decompose Hilbert space of gauge theory into contactable part and un-contactable part. We find interface counter-terms can absorb not only UV divergence but also this disputed issue.



Quantities Independent on Counter-terms

Calculation of entanglement entropy depends on interface counter-terms. The physical quantities related to entanglement but irrelevant with counter-terms includes:

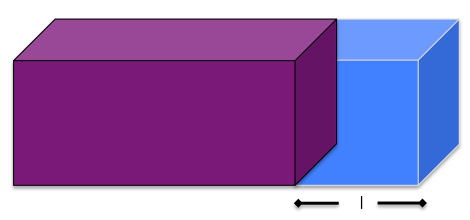
- **Finite temperature dependence of entanglement entropy**

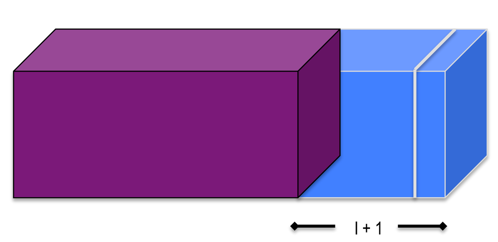


$S_{\text{ent.}}(T)-S_{\text{ent.}}(T=0)$

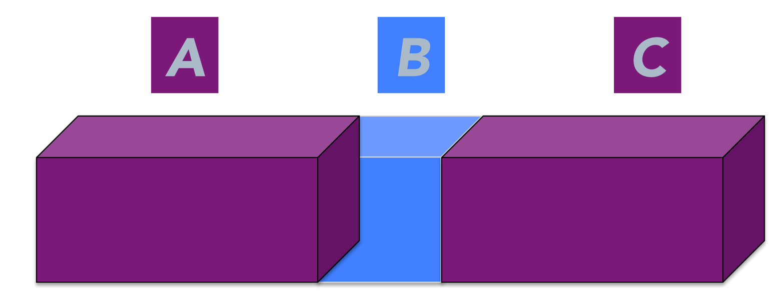
- **Susceptibility to sub-system's size of entanglement entropy**

$S_{\text{ent.}}(l+1)-S_{\text{ent.}}(l)$





- **Mutual information**



$I = S_{\text{ent.}}(A|BC)+S_{\text{ent.}}(AB|C)-S_{\text{ent.}}(B)$

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