

# Initial state of the HIC: thermalization and isotropization

Aleksi Kurkela

QM2015, Kobe

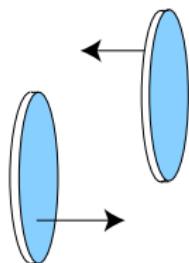


---

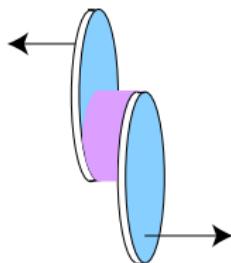
Universitetet  
i Stavanger

# Where are we at?

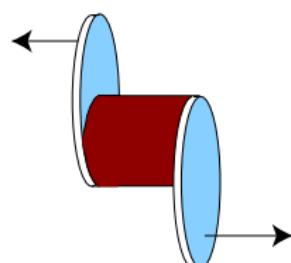
Lorentz contracted nuclei



Pre-thermal plasma



Locally thermalised plasma



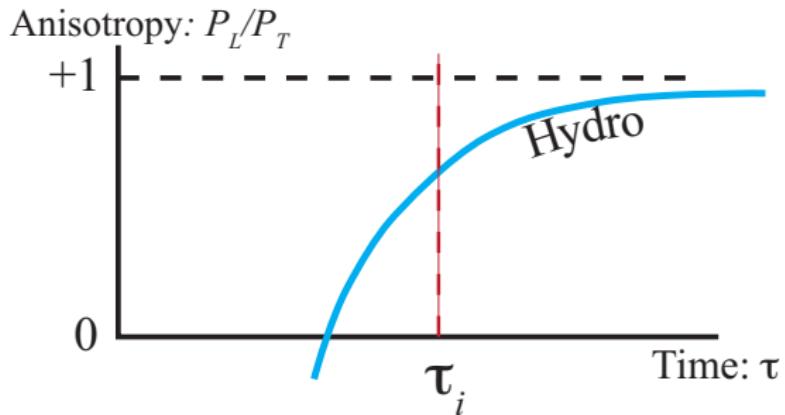
- Soft physics of HIC described by relativistic hydrodynamics

$$\partial_\mu T^{\mu\nu} = 0$$

- Gradient expansion around local thermal equilibrium

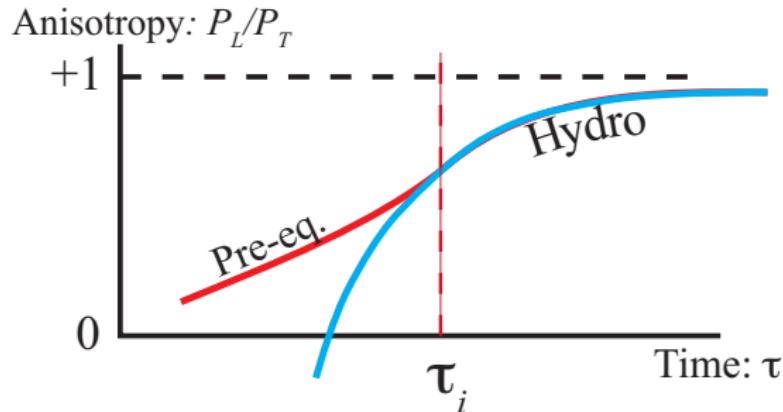
$$T^{\mu\nu} = T_{\text{Thermal equilibrium}}^{\mu\nu} - \eta(\epsilon)\sigma^{\mu\nu} - \zeta(\epsilon)\{g^{\mu\nu} + u^\mu u^\nu\}(\nabla \cdot u) + \dots$$

## Where are we at?



- Strong anisotropy  $P_L/P_T \ll 1$ , sign of large corrections
- At early times *pre-equilibrium* evolution
- Hydro simulations start at *initialization time*  $\tau_i$

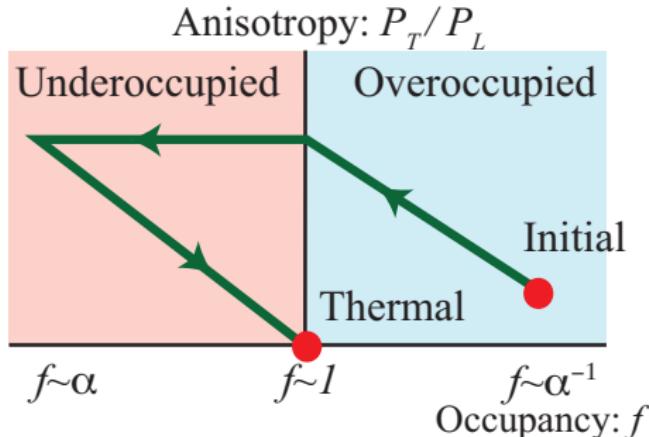
## Where are we at?



- If prethermal evolution converges smoothly to hydro, independence of unphysical  $\tau_i$
- Explicit example: Strong coupling  $\mathcal{N} = 4$  SYM  
Chesler, Yaffe PRL 106 (2011) 021601; van der Schee et al. PRL 111 (2013) 22, 222302,  
[arXiv:1507.08195](https://arxiv.org/abs/1507.08195)

This has proven to be challenging in QCD, even at weak coupling

# Bottom-up thermalization at weak coupling



- Color Glass Condensate: Initial condition overoccupied

McLerran, Venugopalan PRD49 (1994) 2233-2241 , PRD49 (1994) 3352-3355 ; Gelis et. al Int.J.Mod.Phys. E16 (2007) 2595-2637 , Ann.Rev.Nucl.Part.Sci. 60 (2010) 463-489

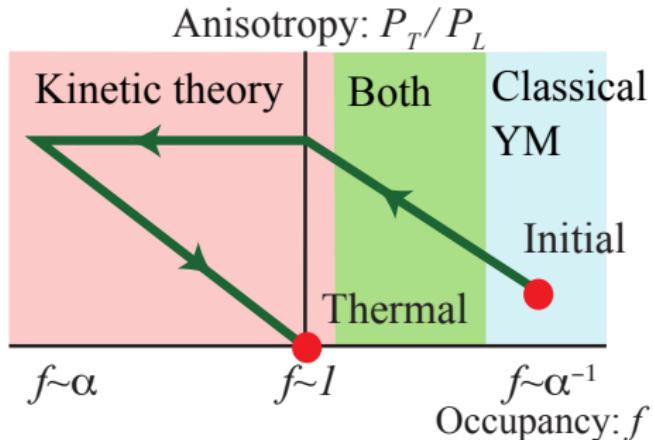
$$f(Q_s) \sim 1/\alpha_s, \quad Q_s \sim 2\text{GeV}$$

- Expansion makes system underoccupied before thermalizing

Baier et al Phys.Lett. B502 (2001) 51-58; AK, Moore JHEP 1111 (2011) 120

$$f(Q_s) \ll 1$$

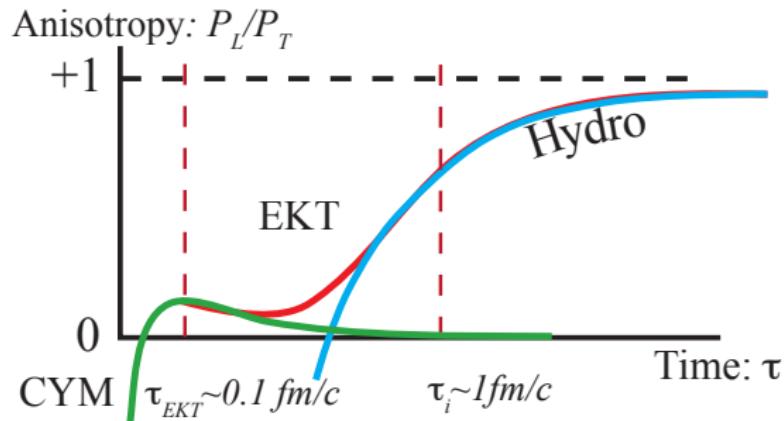
# Bottom-up thermalization at weak coupling



- Degrees of freedom:
  - $f \gg 1$ : Classical Yang-Mills theory (CYM)
  - $f \ll 1/\alpha_s$ : (Semi-)classical particles, Eff. Kinetic Theory (EKT)
- Transmutation of fields to particles: Field-particle duality  
Son, Mueller PLB582 (2004) 279-287; Jeon PRC72 (2005) 014907; Mathieu et al EPJ. C74 (2014) 2873 ; AK, Moore PRD89 (2014) 7, 074036

$$1 \ll f \ll 1/\alpha_s$$

## Strategy at weak coupling



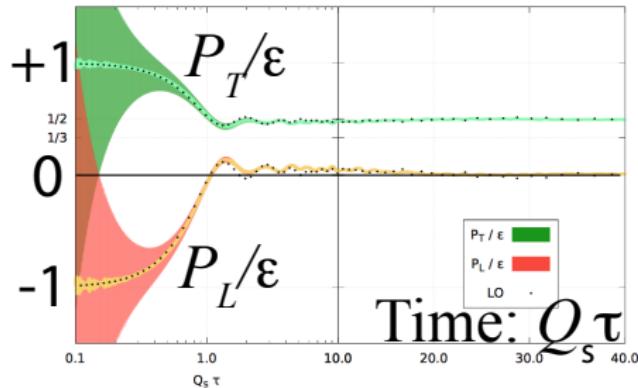
Strategy: Switch from CYM to EKT at  $\tau_{EKT}$ ,

$$1 \ll f \ll 1/\alpha_s$$

From EKT to hydro at  $\tau_i$ ,

$$P_L/P_T \sim 1$$

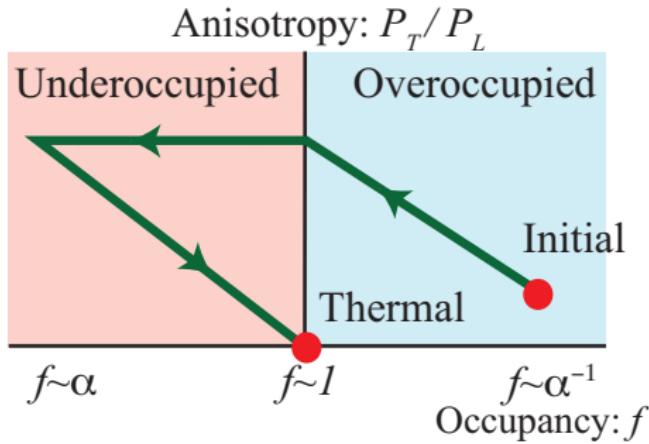
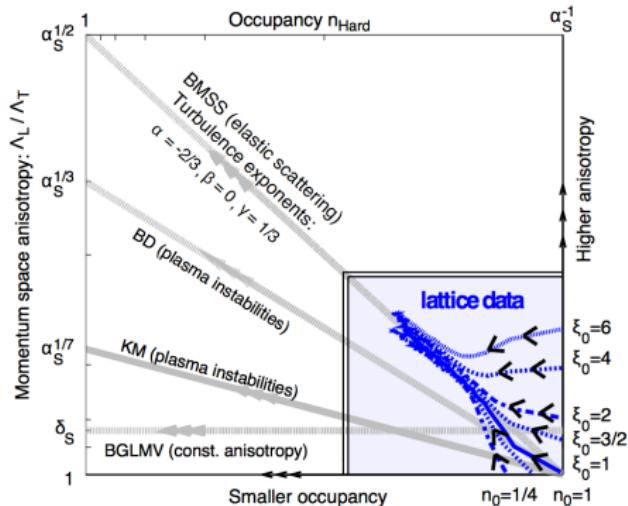
## Early times $0 < Q_s \tau \lesssim 1$ : classical evolution



Epelbaum & Gelis, PRL. 111 (2013) 23230

- Melting of the coherent boost invariant CGC fields
  - The initial condition from CGC: MV-model, JIMWLK
  - After  $\tau \sim 1/Q_s$ , fields decohere,  $P_L > 0$
- Analytic control at early times [Kapusta's talk](#), Chen et al. arXiv:1507.03524

# Later times $Q_s \tau > 1$ : classical evolution



Berges et al. Phys.Rev. D89 (2014) 7, 074011

- Numerical demonstration of classical/overoccupied part of the diagram
- Classical theory never thermalises or isotropizes
- Before  $f \sim 1$ , must switch to kinetic theory

# Effective kinetic theory of Arnold, Moore, Yaffe

JHEP 0301 (2003) 030

$$\frac{df}{dt} = -C_{2 \leftrightarrow 2}[f] - C_{1 \leftrightarrow 2}[f]$$

- Soft and collinear divergences lead to nontrivial matrix elements

soft: screening, Hard-loop; collinear: LPM, ladder resum

$$= \text{Re} \left( \left[ \begin{array}{c|c} \text{gluon} & \text{curly gluon} \\ \text{gluon} & \text{gluon} \end{array} \right] \right)^* \left( \left[ \begin{array}{c|c} \text{curly gluon} & \text{gluon} \\ \text{gluon} & \text{gluon} \end{array} \right] \right)$$

- No free parameters; LO accurate in the  $\alpha_s \rightarrow 0, \alpha_s f \rightarrow 0$  limit.
- Used for LO transport coefficients in QCD, jet energy loss  
Arnold et al. JHEP 0305 (2003) 051; Moore, York PRD79 (2009) 054011; Ghiglieri, Teaney 1502.03730; AK, Wiedemann PLB740 (2015) 172-178; Iancu, Wu 1506.07871, Talk by Wu

# Effective kinetic theory of Arnold, Moore, Yaffe

JHEP 0301 (2003) 030

$$\frac{df}{dt} = -C_{2 \leftrightarrow 2}[f] - C_{1 \leftrightarrow 2}[f]$$

- Soft and collinear divergences lead to nontrivial matrix elements

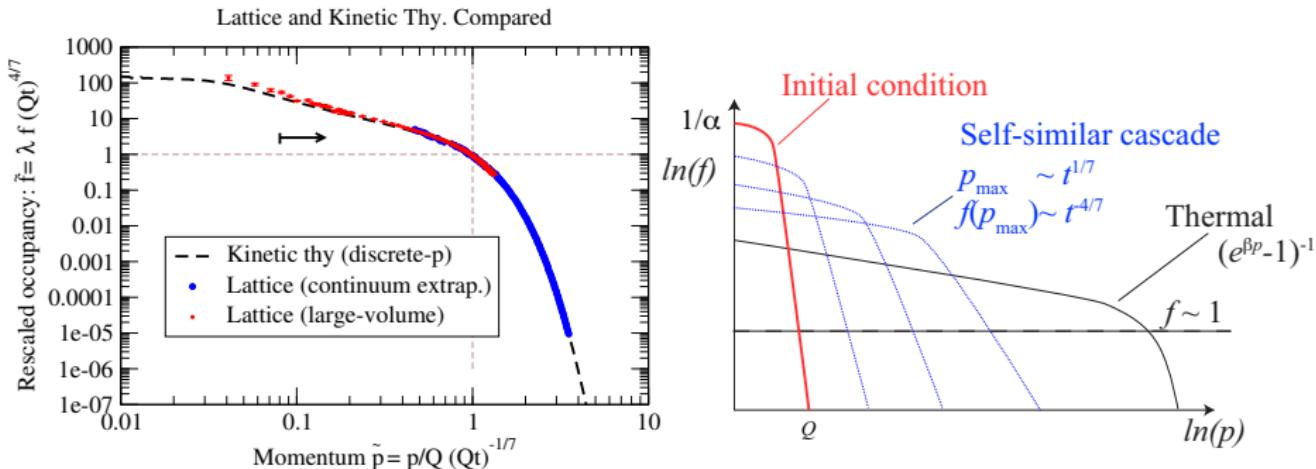
soft: screening, Hard-loop; collinear: LPM, ladder resum

$$= \text{Re} \left( \begin{array}{c} \text{ladder diagram} \\ \times \end{array} \right)^* \left( \begin{array}{c} \text{ladder diagram} \\ \times \end{array} \right)$$

- No free parameters; LO accurate in the  $\alpha_s \rightarrow 0, \alpha_s f \rightarrow 0$  limit.
- Used for LO transport coefficients in QCD, jet energy loss  
Arnold et al. JHEP 0305 (2003) 051; Moore, York PRD79 (2009) 054011; Ghiglieri, Teaney 1502.03730; AK, Wiedemann PLB740 (2015) 172-178; Iancu, Wu 1506.07871, Talk by Wu
- Now also available in NLO  $\mathcal{O}(\sqrt{\alpha_s})$

Talk by Ghiglieri, 1509.07773

# Comparison between CYM and EKT: Isotropic

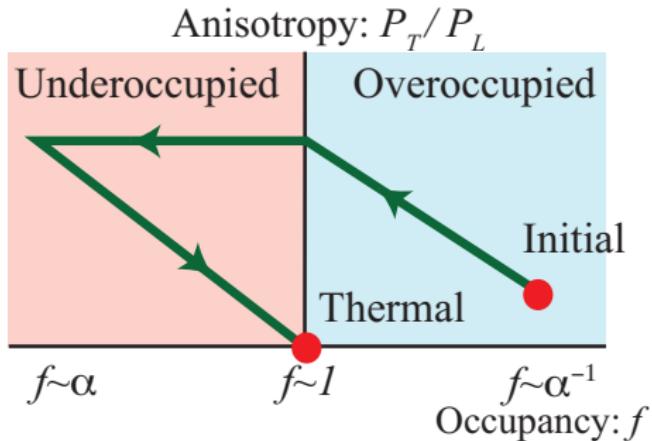
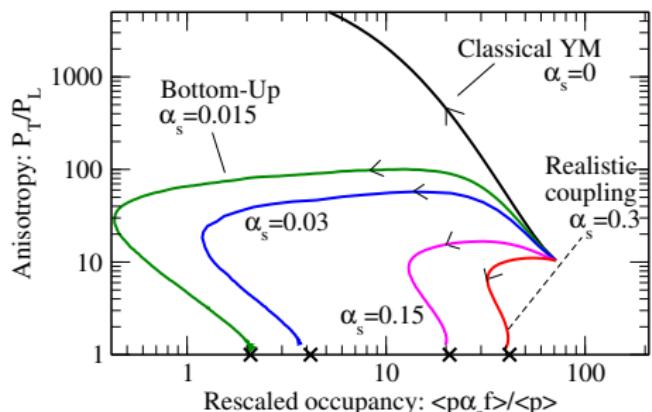


- Quantitative agreement between CYM and EKT for

$$1 \ll f \ll 1/\alpha_s$$

# Route to equilibrium in EKT

AK, Zhu, PRL in press

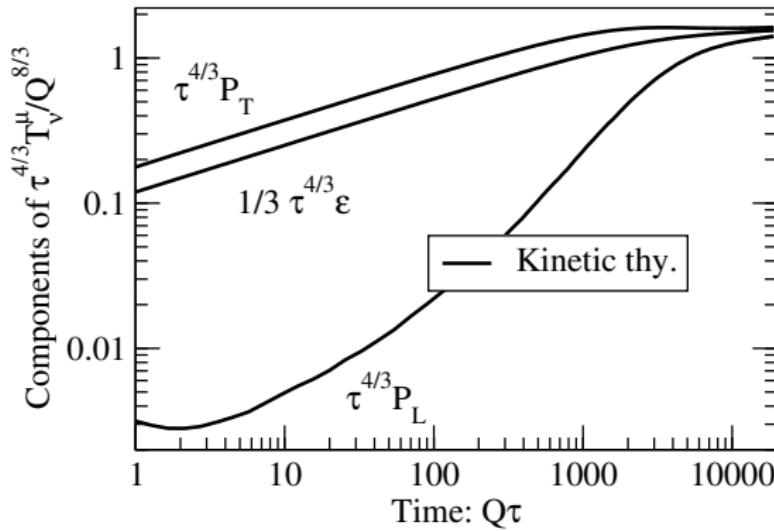


- Initial condition ( $f \sim 1/\alpha_s$ ) from classical field theory calculation  
Lappi PLB703 (2011) 325-330
- In the classical limit ( $\alpha_s \rightarrow 0, \alpha_s f$  fixed), no thermalization
- At small values of couplings, clear Bottom-Up behaviour
- Features become less defined as  $\alpha_s$  grows

# Smooth approach to hydrodynamics

AK, Zhu, PRL in press

$$\alpha_s = 0.03$$

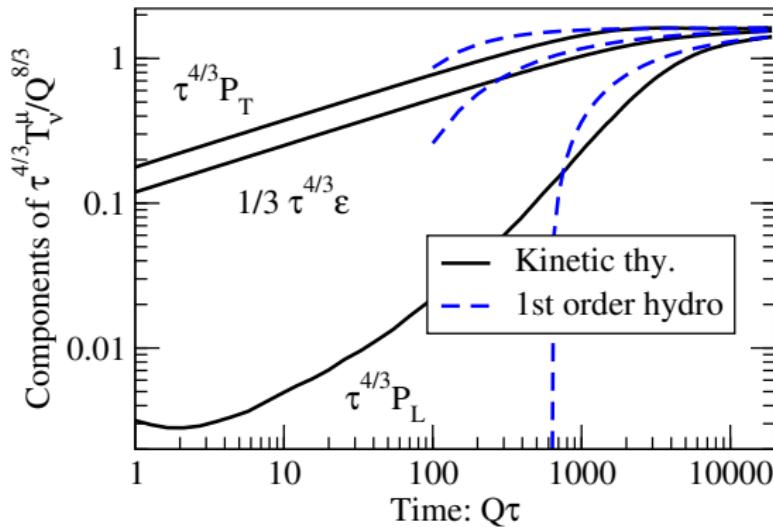


- Kinetic theory converges to hydro smoothly and automatically

# Smooth approach to hydrodynamics

AK, Zhu, PRL in press

$$\alpha_s = 0.03$$

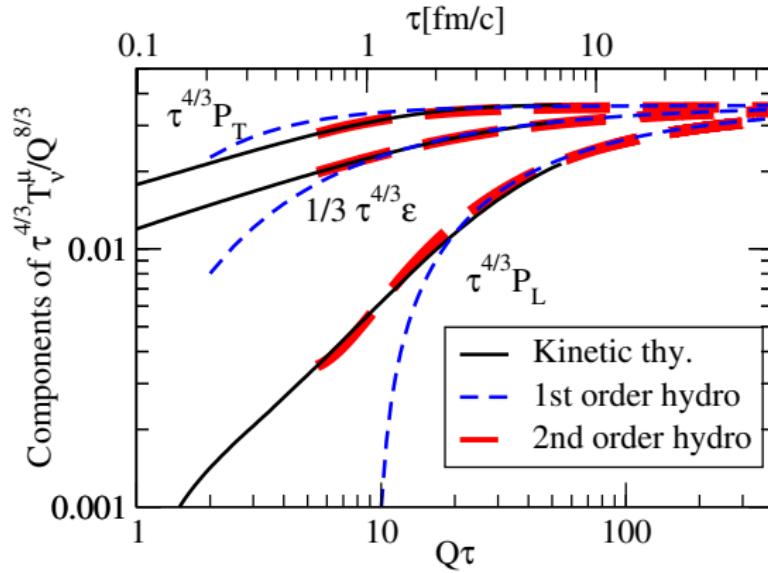


- Kinetic theory converges to hydro smoothly and automatically
- Hydro prediction fixed by perturbative  $\eta/s$

# Smooth approach to hydrodynamics

AK, Zhu, PRL in press

$$\alpha_s = 0.3$$



- For realistic couplings, hydrodynamics reached around  $\lesssim 1\text{fm}/c$ .
- Hydro seems to give a good description even when  $P_L/P_T \sim 1/5$

# Caveats

- Fermions
- Transverse dynamics, preflow
- Plasma instabilities, anisotropic screening
  - Numerically small effect
- Improved initial CYM simulations for initial condition of EKT

AK, Moore, JHEP 1112 (2011) 044 , JHEP 1111 (2011) 120

Berges et al. Phys.Rev. D89 (2014) 7, 074011

and

- Potentially large NLO corrections
  - Caron-Huot, Moore PRL 100 (2008) 052301, Ghiglieri et al. JHEP 1305 (2013) 010, JHEP 1412 (2014) 029, 1502.03730, 1509.07773
    - But  $T(\tau_i) \sim 3T_c$  in perturbative region

Qualitative  $\Rightarrow$  Quantitative

# Other recent directions

Other theories:

- Several studies in scalar theory
  - Classical [Berges et al. 1508.03073](#)
  - Kinetic theory [Epelbaum et al. JHEP 1509 \(2015\) 117](#)
- Exact solutions to kinetic theory in relaxation time approximation [Denicol et al. PRL 113 \(2014\) 20, 202301](#)
- Properties of kinetic theory with generic elastic and inelastic terms [talk by Blaizot](#)

Strong coupling  $\mathcal{N} = 4$  SYM

[Talk by Chesler](#)

Approaching from the hydro side:

- What limits the convergence of hydrodynamics [Heller, Spalinski PRL 115, 072501 \(2015\)](#)
- Modifying hydro to work further from equilibrium, aHydro [Talk by Strickland](#)

## Where are we going?

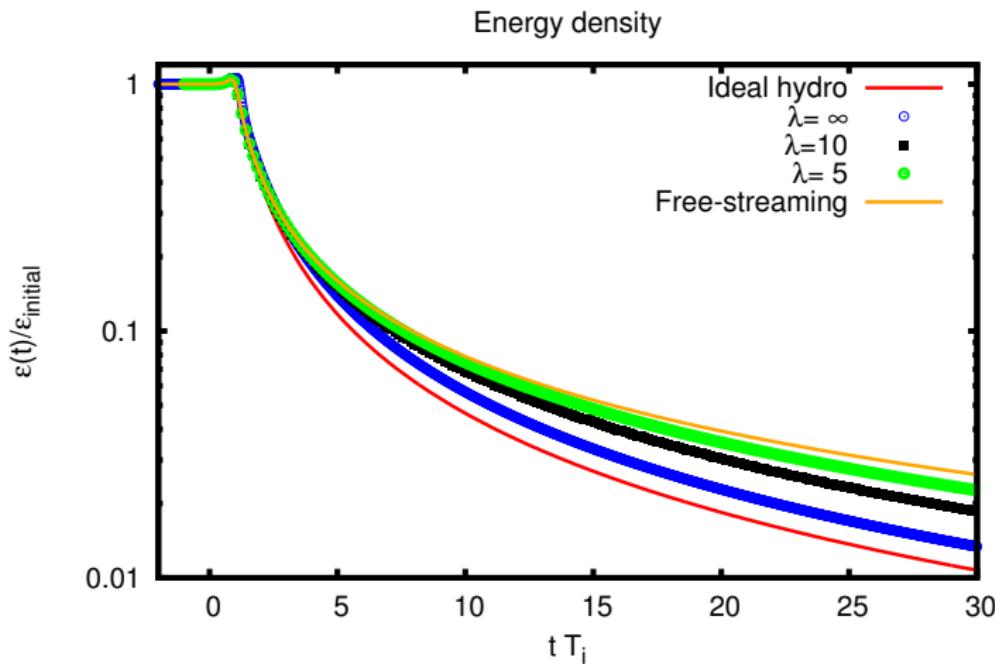
- Combination of classical Yang-Mills simulations and effective kinetic theory allows to follow the time evolution from highly occupied initial condition to thermal equilibrium.
- Weak coupling thermalization extrapolated to realistic couplings shows agreement with hydro around

$$\tau_i \sim 1\text{fm}/c$$

- Unified description of soft and hard physics: hydro, jets, etc.

# Weakly or strongly coupled thermalization?

Apples to apples comparison of weak and strong coupling



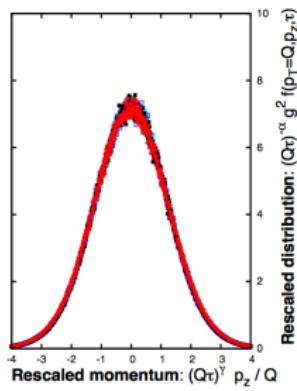
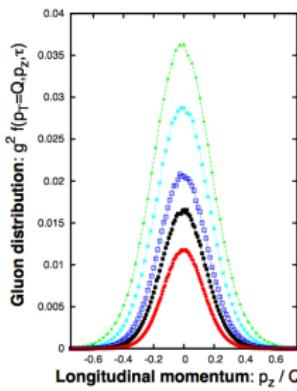
Backup sildes

# Comparison between CYM and EKT: Expanding

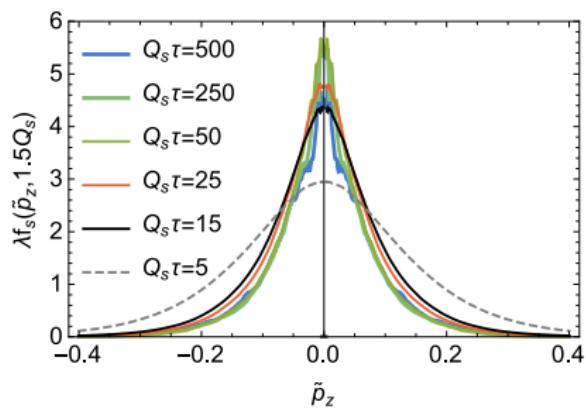
In non-pert classical regime  $1 \ll f \ll 1/\alpha_s$

$$f(p_z, p_\perp, \tau) = (Q_s \tau)^{-2/3} f_S((Q_s \tau)^{1/3} p_z, p_\perp),$$

CYM  $\alpha_s f \ll 1$  limit but  $f \gg 1$



EKT  $f \gg 1$  limit but  $f \ll 1/\alpha_s$



# Radiative break-up

AK, Lu, PRL 113 (2014) 18, 182301

