### Introduction

One of the most important results in heavy ion collisions is the anisotropic distribution of particle production in transverse direction. This collective phenomenon is known as anisotropic flow, and successfully parameterized by a Fourier decomposition [1]

\[
\frac{dN}{dp_{t}} = \frac{1}{2\pi} \sum_{m,n} v_m \cos(n\psi - m\phi)
\]

where the coefficient \( v_m \) is the magnitude of anisotropy, and the \( \psi \) is the symmetry plane. The second order coefficient \( v_2 \), named elliptic flow, is well described as the result of spatial response of almond–shaped initial nuclear overlap region.

Recently, the correlation between two different flow harmonics were reported and named “Symmetric 2–harmonic 4–particle Cumulants (SC)”. The correlation between \( v_2 \), \( v_2 \) and between \( v_2 \), \( v_2 \) are measured by multi particle correlations [7].

\[
\langle \langle \langle \cos(m\psi + n\phi - m\psi - n\phi) \rangle \rangle \rangle - \langle \langle \langle \cos(m\psi - n\phi) \rangle \rangle \rangle \langle \langle \cos(n\psi - m\phi) \rangle \rangle - \langle \langle \langle \cos(m\psi - n\phi) \rangle \rangle \rangle \langle \langle \langle \cos(n\psi - m\phi) \rangle \rangle \rangle
\]

The above equation is non–zero only if there are non–zero flow fluctuations, and the fluctuation of harmonics \( v_m \) and \( v_n \) is correlated.

(Yes see “Measurements of Correlations between Anisotropic Flow Harmonics in Pb–Pb Collisions in ALICE” by You Zhou on Wed Contribution ID : 476)

### Correlation between flow harmonics

As shown in Fig. 2 the SC results suggest correlations between \( v_2 \) and \( v_2 \), and anti–correlations between \( v_2 \) and \( v_2 \).

Hydrodynamic models describe the \( v_n \) reasonably well, but can’t describe SC (4,2) and SC (3,2) simultaneously by using a single \( \eta/s \) parameterization.

The SC measurement is a new observable which quantifies the relationship between event–by–event fluctuations of two different flow harmonics, and it can provide additional constrains on the \( \eta/s \) than single \( v_n \) measurements.

The correlation between different flow harmonics may provide new insight into both the early stage dynamics and transport properties of the QGP.

The SC results can also be cross checked with moments [8] by using SP (scalar product) method instead of multi particle cumulants method.

### Measure fluctuations with moments

The statistical properties of flow are contained in its moments, which is defined as the average values of products of complex flow coefficient \( v_n \).

\[
M_n = \left\langle \prod_n v_n^* v_n \right\rangle
\]

The scaled moments, which is defined as

\[
\eta^{\text{sd}} = \frac{\langle v_n^* \rangle}{\langle v_n \rangle}
\]

can be used to study flow fluctuations as a function of centrality. This values can be also obtained from the multi–particle cumulant method:

\[
\eta^{\text{mc}} = \frac{\langle v_n^* \rangle}{\langle v_n \rangle^2} + 2 \cdot \frac{\langle v_n \rangle^2}{\langle v_n^* \rangle^2}
\]

Not only the correlation between the magnitude of two different flow harmonics, but also the direction of flow correlation (event plane correlation) and non–linear coefficients of flow (induced term from lower harmonics) can be studied [9, 10].

### Flow fluctuations

But this smooth almond shape model would yield in \( v_2 = 0 \) and is not enough to explain higher flow harmonics. As actual collision shape is not perfectly almond–shape, there is another contribution coming from event–by–event fluctuations [5, 6] of the complex shape of overlap region.

Because these fluctuations can generate any type of anisotropy, they can give rise to any harmonics of \( v_n \).

### Reference