Deconfinement and chiral crossover with Dirac-mode expansion in QCD

Takahiro M. Doi[1], Krytzytof Redlich[2], Chihiro Sasaki[2,3] and Hideo Suganuma[1]

1: Kyoto University, 2: Wroclaw University, 3: FIAS

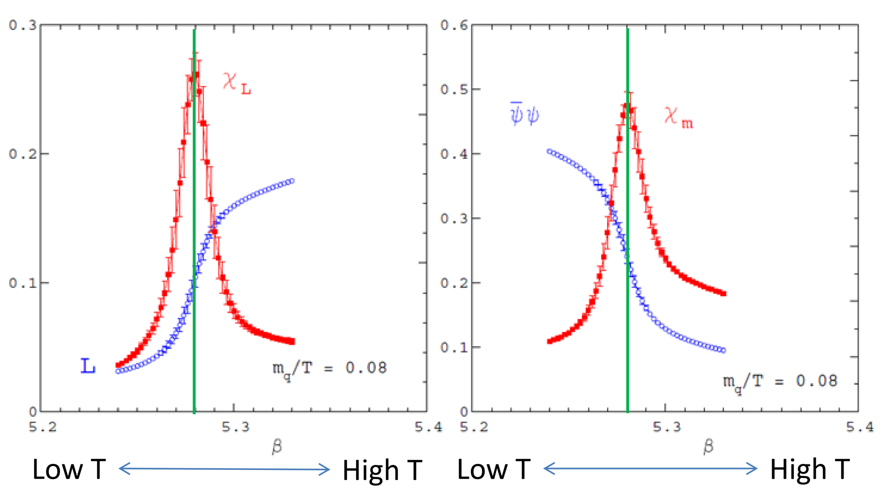
Based on arXiv: 1505.05752

Abstract

Based on the analytical formulae in the lattice QCD, we investigate the contribution from each Dirac mode to the Polyakov loop fluctuation, which is very good probe for the deconfinement transition even in the presence of dynamical quarks. While the low-lying Dirac modes are important modes for chiral symmetry breaking, these modes are not important to quantify the Polyakov loop fluctuations. Our results suggest that confinement and chiral symmetry breaking are not necessarily one-to-one corresponding in QCD.

Introduction

Deconfinement and chiral-restoration transition temperatures are very close



F. Karsch, Lect. Notes Phys. 583, 209 (2002)

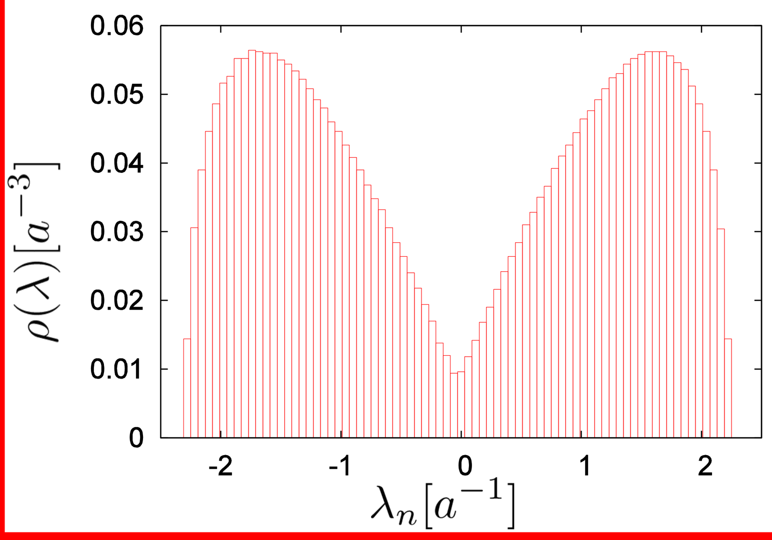
$\langle L \rangle, \chi_L$: Polyakov loop and its susceptibility
$\langle \bar{\psi}\psi \rangle, \chi_m$: chiral condensate and its susceptibility

Transition temperatures are almost same.

Q. Are two phenomena strongly correlated?

A. Nontrivial !!

Dirac mode: eigenmode of Dirac operator $\hat{\not{D}}$



Dirac operator: $\hat{\not{D}}$ eigenvalue equation: $\hat{\not{D}}|n\rangle = i\lambda_n|n\rangle$
Dirac eigenmode: $|n\rangle$ Dirac eigenvalue: $i\lambda_n$
eigenvalue density: $\rho(\lambda) = \frac{1}{V}\sum_n \delta(\lambda - \lambda_n)$

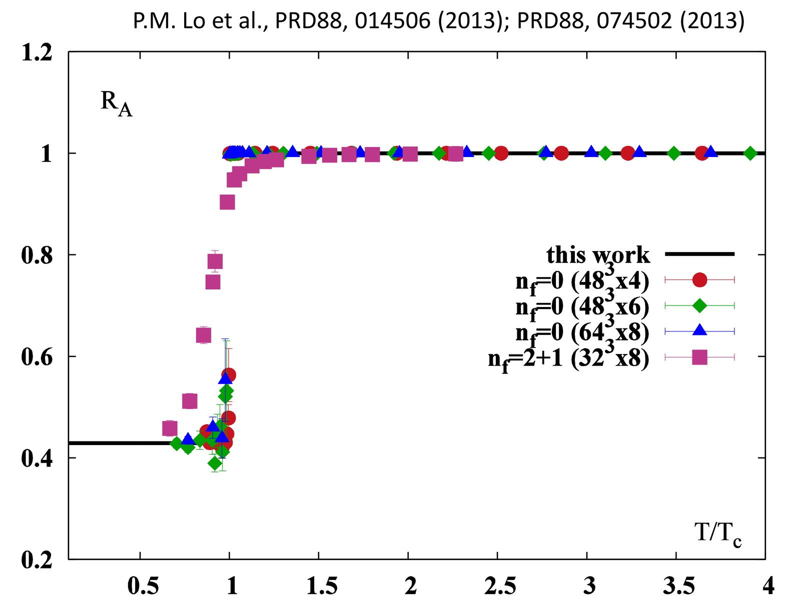
Low-lying Dirac mode: essence of chiral symmetry breaking

c.f.) Banks-Casher relation:

$$\langle \bar{q}q \rangle = -\lim_{m\to 0}\lim_{V\to\infty} \pi\langle \rho(0) \rangle$$

Polyakov loop fluctuations: good probe for quark deconfinement

P.M. Lo et al., PRD88, 014506 (2013); PRD88, 074502 (2013)



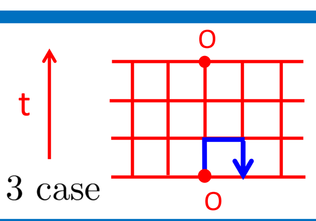
- Z3 rotated Polyakov loop(PL): $\tilde{L} = L\mathrm{e}^{2\pi k i/3}$
- longitudinal, transverse PL: $L_L \equiv \mathrm{Re}(\tilde{L})$, $L_T \equiv \mathrm{Im}(\tilde{L})$
- Polyakov loop susceptibilities (Y=L or T):
  $T^3\chi_A = \frac{N_\sigma^3}{N_\tau^3}[\langle |L|^2 \rangle - \langle |L| \rangle^2]$ $\quad T^3\chi_Y = \frac{N_\sigma^3}{N_\tau^3}[\langle (L_Y)^2 \rangle - \langle L_Y \rangle^2]$
- Ratios of Polyakov loop susceptibilities: $R_A \equiv \frac{\chi_A}{\chi_L}$, $R_T \equiv \frac{\chi_T}{\chi_L}$
- temperature, spatial and temporal lattice size: $T,\ N_\sigma,\ N_\tau$

Polyakov loop fluctuation R_A: Good probe for deconfinement transition

Analytical relation between Polyakov loop and Dirac mode

Setup

In this study, we use
- standard square lattice
- with ordinary periodic boundary condition for gluons,
- with the odd temporal length N_t (temporally odd-number lattice)



Derivation

Consider the functional trace on the temporally odd-number lattice:

$$I \equiv \mathrm{Tr}_{c,\gamma}(\hat{U}_4\, \hat{\not{D}}^{N_4-1}) \quad (N_4 : \mathrm{odd}) \quad (\mathrm{Tr}_{c,\gamma} \equiv \Sigma_s \mathrm{tr}_c \mathrm{tr}_\gamma)$$

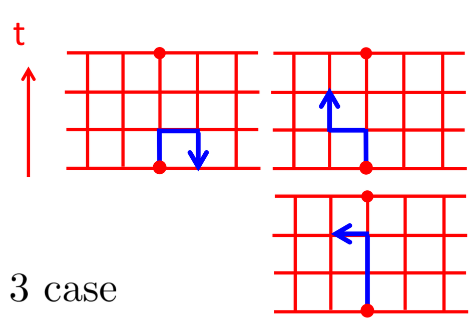
$|s\rangle$: site

Dirac operator : $\hat{\not{D}} = \frac{1}{2}\sum_\mu \gamma_\mu(\hat{U}_\mu - \hat{U}_{-\mu})$ Link variable operator : $\langle s|\hat{U}_\mu|s'\rangle = U_\mu(s)\delta_{s+\hat{\mu},s'}$

$I \equiv \mathrm{Tr}_{c,\gamma}(\hat{U}_4\, \hat{\not{D}}^{N_4-1})$ includes many trajectories on the square lattice.
The length of the trajectories is N_4 , i.e., odd.

Any closed loop needs even-number link-variables on square lattice.

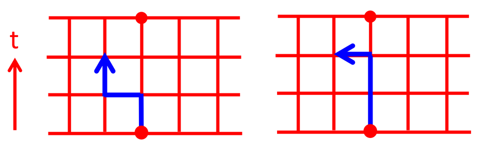
In this functional trace $I \equiv \mathrm{Tr}_{c,\gamma}(\hat{U}_4\, \hat{\not{D}}^{N_4-1})$,
it is impossible to form a closed loop on the square lattice,
because the length of the trajectories, N_4, is odd.



$N_4 = 3$ case
length of trajectories: $N_t = 3$
odd

Almost all trajectories are gauge-variant & give no contribution.

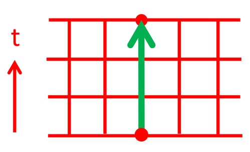
$N_4 = 3$ case



gauge variant
(no contribution)

Only the exception is the Polyakov loop.

$N_4 = 3$ case



gauge invariant !!

I is proportional to the Polyakov loop.

$$I = \mathrm{Tr}_{c,\gamma}(\hat{U}_4\, \hat{\not{D}}^{N_4-1}) = \frac{12V}{(2a)^{N_4-1}} L_P \quad \cdots ①$$

On the other hand, $I = \mathrm{Tr}_{c,\gamma}(\hat{U}_4\, \hat{\not{D}}^{N_4-1}) = \sum_n \langle n|\hat{U}_4\, \hat{\not{D}}^{N_4-1}|n\rangle = i^{N_4-1}\sum_n \lambda_n^{N_4-1}\langle n|\hat{U}_4|n\rangle$ ···②

from ①、②

$$L_P = \frac{(2i)^{N_4-1}}{12V}\sum_n \lambda_n^{N_4-1}\langle n|\hat{U}_4|n\rangle$$

relation between Polyakov loop and Dirac modes

TMD, H. Suganuma, T. Iritani, Phys. Rev. D 90, 094505 (2014)

$$L_P = \frac{(2ai)^{N_4-1}}{12V}\sum_n \lambda_n^{N_4-1}\langle n|\hat{U}_4|n\rangle \quad (N_4 \text{ is odd})$$

- Low-lying Dirac-modes are important for CSB (Banks-Casher relation) $(\lambda_n \sim 0)$
- Low-lying Dirac-modes have little contribution to Polyakov loop

Low-lying Dirac modes are not important for the Polyakov loop.

In fact, from similar analysis, we can derivate the similar relation between Wilson loop and Dirac mode.
Therefore, low-lying Dirac-modes have little contribution to the string tension σ, or the confining force.

Stronger statement: main results

relation between Polyakov loop fluctuation and Dirac modes

TMD, K. Redlich, C. Sasaki and H. Suganuma, arXiv: 1505.05752 [hep-lat]

$$R_A = \frac{\left\langle \left|\sum_n \lambda_n^{N_\tau-1}\langle n|\hat{U}_4|n\rangle\right|^2 \right\rangle - \left\langle \left|\sum_n \lambda_n^{N_\tau-1}\langle n|\hat{U}_4|n\rangle\right| \right\rangle^2}{\left\langle \left(\sum_n \lambda_n^{N_\tau-1}\mathrm{Re}\left(\mathrm{e}^{2\pi k i/3}\langle n|\hat{U}_4|n\rangle\right)\right)^2 \right\rangle - \left\langle \sum_n \lambda_n^{N_\tau-1}\mathrm{Re}\left(\mathrm{e}^{2\pi k i/3}\langle n|\hat{U}_4|n\rangle\right) \right\rangle^2}$$

- The ratio R_A is a good "order parameter" for deconfinement transition.
- Since the damping factor $\lambda_n^{N_\tau-1}$ is very small with small $|\lambda_n| \simeq 0$, low-lying Dirac modes (with small $|\lambda_n| \simeq 0$) are not important for R_A, which are important modes for chiral symmetry breaking.

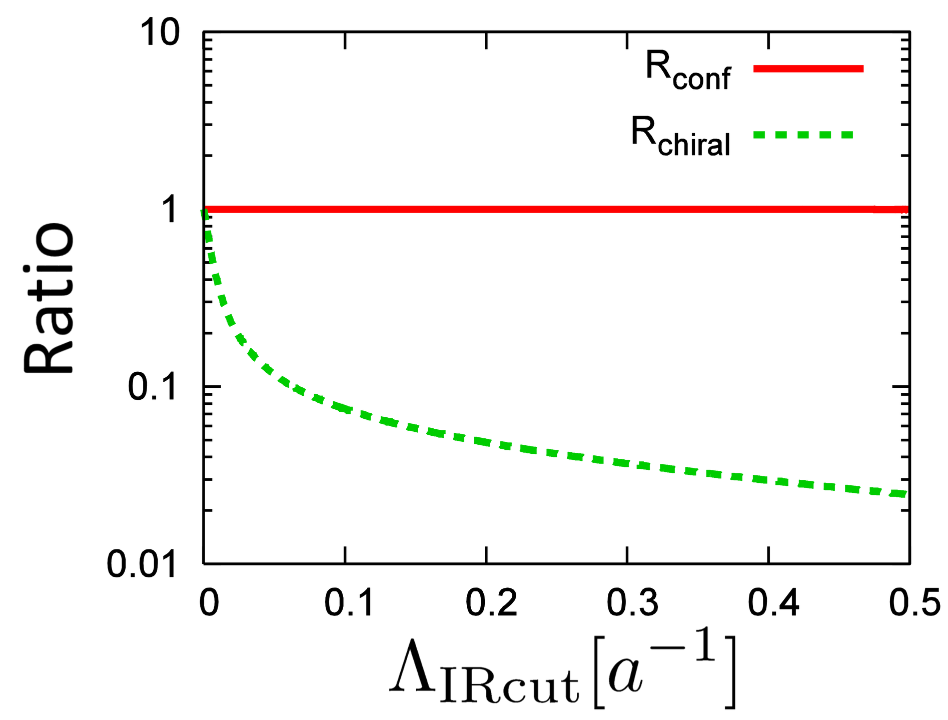
Low-lying Dirac modes are not important for Polyakov loop fluctuation.

This results suggest that...

No direct, one-to-one correspondence between confinement and chiral symmetry breaking in QCD.

Numerical analysis

Contribution to $\langle \bar{q}q \rangle$ and R_A from low-lying Dirac modes



lattice setup
- quenched SU(3) lattice QCD
- standard plaquette action
- gauge coupling: $\beta = \frac{2N_c}{g^2} = 5.6$
- lattice size: $N_\sigma^3 \times N_\tau = 10^3 \times 5$
  ⇔lattice spacing : $a \simeq 0.25$ fm
- periodic boundary condition for link-variables and Dirac operator

- R_{chiral} is reduced by removing low-lying Dirac modes.
- R_{conf} is almost unchanged.

It is also numerically confirmed that low-lying Dirac modes are important for chiral symmetry breaking and not important for quark confinement.

definition for Λ-dependent (IR-cut) quantities
- susceptibilities:
  $(\chi)_\Lambda = \frac{1}{T^3}\frac{N_\sigma^3}{N_\tau^3}[\langle Y_\Lambda^2 \rangle - \langle Y_\Lambda \rangle^2]$, $Y \equiv |L|,\ L_L,\ L_T$
- chiral condensate: $\langle \bar{\psi}\psi \rangle_\Lambda = -\frac{1}{V}\sum_{|\lambda_n|\geq\Lambda}\frac{2m}{\lambda_n^2 + m^2}$
- RA: $(R_A)_\Lambda = \frac{(\chi_A)_\Lambda}{(\chi_L)_\Lambda}$
- Quantities indicating the effect of removal of low-lying Dirac modes:
  $R_{\mathrm{conf}} = \frac{(R_A)_\Lambda}{R_A}$, $R_{\mathrm{chiral}} = \frac{\langle \bar{\psi}\psi \rangle_\Lambda}{\langle \bar{\psi}\psi \rangle}$

Summary

We derive the relation between the Polyakov loop fluctuations and the Dirac mode on the temporally odd-number lattice. From this relation, we conclude that low-lying Dirac modes are not essential modes for the Polyakov loop fluctuation, which are sensitive probe for the deconfinement transition.
Our results suggest ***no direct one-to-one correspondence between confinement and chiral symmetry breaking in QCD***.