# Characterizing flow fluctuations with moments

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## 1 Introduction

We present a complete set of multiparticle correlation observables for ultrarelativistic heavy-ion collisions. These include moments of the distribution of the anisotropic flow in a single harmonic, and also mixed moments, which contain the information on correlations between event planes of different harmonics. We explain how these moments can be measured using just two symmetric subevents separated by a rapidity gap. We illustrate the method with realistic simulations in the AMPT model.

## 2 Moments and the two-subevents method

Consider the single-particle distribution in the azimuthal angle $\varphi$:

$$P(\varphi) = \frac{1}{2\pi}\sum_{n=-\infty}^{+\infty} V_n e^{-in\varphi}, \quad (1)$$

where $V_n$ is the (complex) anisotropic flow coefficient in the $n$th harmonic. Note that $V_n^* = V_{-n}$ and $V_0 = 1$. One usually defines $v_n \equiv |V_n|$. Statistical properties of $V_n$ are contained in its moments:

$$\mathcal{M} \equiv \left\langle \prod_n (V_n)^{k_n}(V_n^*)^{l_n} \right\rangle, \qquad k_n \text{ and } l_n : \text{integers}, \quad (2)$$

where angular brackets denote an average over events.

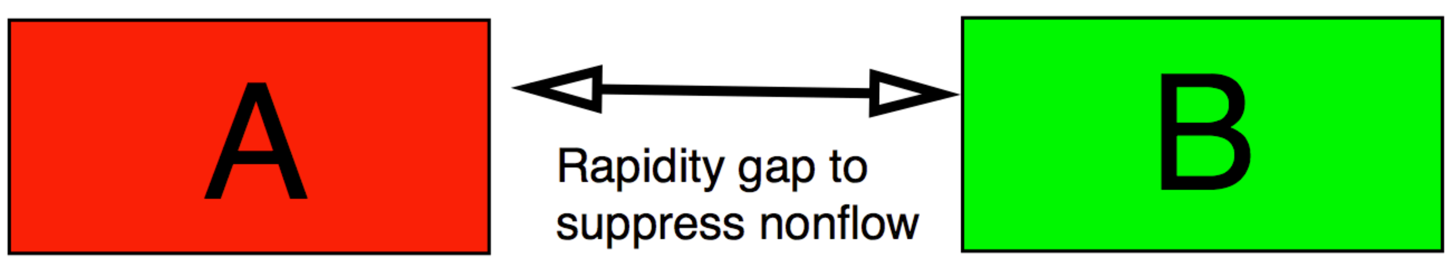
Note that moments involve in general *event-plane angles* through the *phase* of $V_n$. Azimuthal symmetry implies that the only nontrivial moments satisfy $\sum_n nk_n = \sum_n nl_n$.

Method to measure moments: The flow vector in a reference detector is

$$Q_n \equiv \frac{1}{N}\sum_{j=1}^{N} e^{in\varphi_j}, \quad (3)$$

where the sum runs over $N$ particles seen in the reference detector in a given event, and $\varphi_j$ are their azimuthal angles.

Use two reference detectors A and B symmetric around midrapidity:




$$\mathcal{M} = \left\langle \prod_n (Q_{nA})^{k_n}(Q_{nB}^*)^{l_n} \right\rangle + A \leftrightarrow B. \quad (4)$$

$\mathcal{M}$ provides a complete set of multiparticle correlation observables.

## 3 Applications of the method

- *Direct* measurement of even moments of a single flow harmonic $\langle v_n^{2k}\rangle$. Contain *all* the information on event-by-event $v_n$ fluctuations.

  [An alternative *indirect* method is to extract them from cumulants, e.g., $v_n\{4\}^4 = 2\langle v_n^2\rangle^2 - \langle v_n^4\rangle$.]

- *Event-plane correlations* can be written simply in terms of moments [1].

- *Standard candles* = correlations between the magnitudes of $v_n$ and $v_m$ [2]. Unlike event-plane correlations, these observables do not involve event-plane angles:

$$\langle v_n^2 v_m^2\rangle / \langle v_n^2\rangle\langle v_m^2\rangle, \quad (5)$$

$< 1$ means negative correlation. For instance, between $v_2$ and $v_3$ (Fig. 1).

$> 1$ means positive correlation. For instance, between $v_2$ and $v_4$, because of the $v_4$ induced by $v_2$ [3].

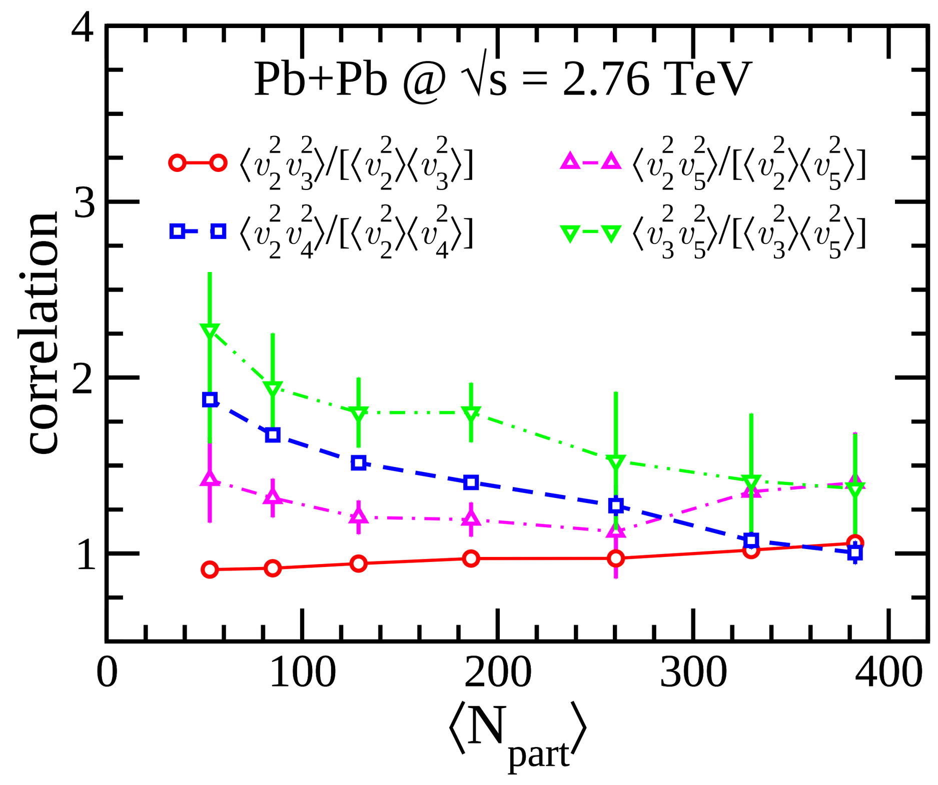



Figure 1: Correlations between $v_n^2$ and $v_m^2$ in the AMPT model.

## 4 New observables to test the nonlinear response

In hydrodynamics, one typically models $V_4$ (and similarly $V_5$) as the sum of two *independent* terms: (1) from fluctuations and (2) from *nonlinearity* induced by $V_2$ [3]:

$$\begin{aligned} V_4 &= V_{4l} + \chi_4 V_2^2, \\ V_5 &= V_{5l} + \chi_5 V_2 V_3, \end{aligned} \quad (6)$$

where $\chi_4$, $\chi_5$ are constant in a centrality class.

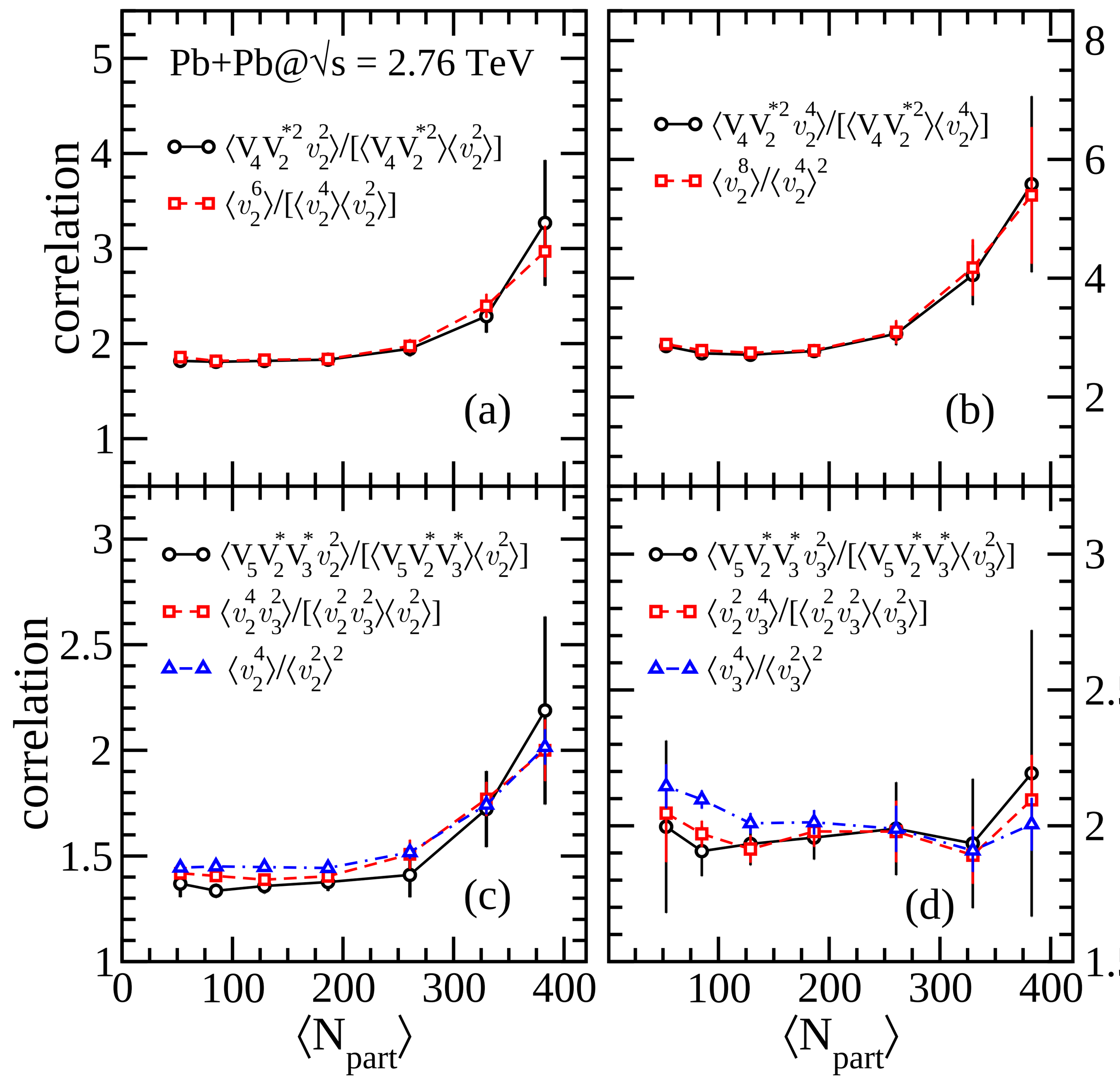



Figure 2: Correlations calculated in AMPT, and tests of Eqs. (7). Circles connected by solid lines correspond to the left-hand sides, and squares connected by dashed lines correspond to the right-hand sides of these equations.

This implies the following relations between moments:

$$\frac{\langle V_4(V_2^*)^2 v_2^2\rangle}{\langle V_4(V_2^*)^2\rangle\langle v_2^2\rangle} = \frac{\langle v_2^6\rangle}{\langle v_2^4\rangle\langle v_2^2\rangle}, \quad \frac{\langle V_4(V_2^*)^2 v_2^4\rangle}{\langle V_4(V_2^*)^2\rangle\langle v_2^4\rangle} = \frac{\langle v_2^8\rangle}{\langle v_2^4\rangle^2},$$

$$\frac{\langle V_5V_2^*V_3^* v_2^2\rangle}{\langle V_5V_2^*V_3^*\rangle\langle v_2^2\rangle} = \frac{\langle v_2^4 v_3^2\rangle}{\langle v_2^2v_3^2\rangle\langle v_2^2\rangle}, \quad \frac{\langle V_5V_2^*V_3^* v_3^2\rangle}{\langle V_5V_2^*V_3^*\rangle\langle v_3^2\rangle} = \frac{\langle v_2^2 v_3^4\rangle}{\langle v_2^2v_3^2\rangle\langle v_3^2\rangle}. \quad (7)$$

Note that the left-hand sides involve event-plane correlations while the right-hand sides do not, so that these relations are highly non-trivial.

All these relations are supported to a good approximation by the AMPT simulations (Fig. 2) [4]. It is important to test if experimental data confirm these predictions.

## References

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