The q-Statistics and QCD Thermodynamics at LHC

Raghunath Sahoo[1], Trambak Bhattacharyya[1], Jean Cleymans[2], Arvind Khuntia[1], Prakhar Garg[1], Pragati Sahoo[1], Pooja Pareek[1]

Email : Raghunath.Sahoo@cern.ch

[1]Discipline of Physics, School of Basic Science, Indian Institute of Technology Indore, M.P. 452017, India

[2]UCT-CERN Research Centre and Department of Physics, University of Cape Town, Rondebosch 7701, South Africa

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Introduction

- Tsallis distribution gives excellent fits to the transverse momentum distributions observed in the Relativistic Heavy Ion Collider (RHIC) and in the Large Hadron Collider (LHC) experiments in a broad range of transverse momentum with three parameters q, Tsallis temperature T and volume V. The parameter q is referred to as the Tsallis parameter.
- The q value shows the degree of deviation of different thermodynamic variables those obtained from a thermalized Boltzmann distribution.
- Using those variables, we can study the behavior of speed of sound in physical hadron gas, when there is a deviation in q values from 1, using Tsallis non-extensive statistics.

Tsallis Non-extensive Stastistics

- The Tsallis-Boltzmann distribution function is given by,

$$f = \left[1 + (q-1)\frac{E-\mu}{T}\right]^{-\frac{1}{q-1}} \xrightarrow{(q \rightarrow 1)} e^{-\frac{E-\mu}{T}}$$

where T , μ and E are the Tsallis temperature, chemical potential and energy respectively.

- As in most of the cases q is close to 1 , we can do the Taylor expansion of the Tsallis Boltzmann distribution, which is given by,

$$\left[1+(q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}} \simeq \mathrm{e}^{-\frac{E-\mu}{T}}\left\{1+(q-1)\frac{1}{2}\frac{E-\mu}{T}\left(-2+\frac{E-\mu}{T}\right)\right.$$
$$\left. + \frac{(q-1)^2}{2!}\frac{1}{12}\left[\frac{E-\mu}{T}\right]^2\left[24-20\frac{E-\mu}{T}+3\left(\frac{E-\mu}{T}\right)^2\right] + \mathcal{O}\{(q-1)^3\} + \ldots\right\}$$

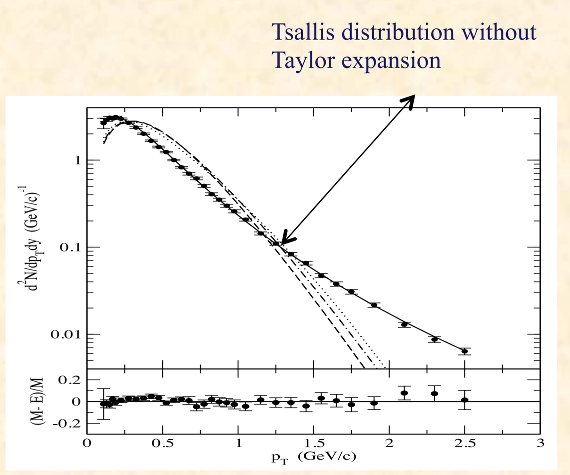
here, the first term is the Boltzmann term and rest are higher order terms in (q-1).

The invariant yield using the Taylor series expansion is given by,

$$E\frac{dN}{d^3p} \simeq \mathcal{C}Ee^{-\Phi} + \mathcal{C}E\frac{x}{1!}\frac{\Phi}{2}(-2+\Phi)e^{-\Phi} + \mathcal{C}E\frac{x^2}{2!}\frac{\Phi^2}{12}\left(24-20\Phi+3\Phi^2\right)e^{-\Phi}$$

$$\Rightarrow \frac{dN}{p_T dp_T dy d\phi} \simeq \mathcal{C}Ee^{-\Phi} + \mathcal{C}E\frac{x}{1!}\frac{\Phi}{2}(-2+\Phi)e^{-\Phi} + \mathcal{C}E\frac{x^2}{2!}\frac{\Phi^2}{12}\left(24-20\Phi+3\Phi^2\right)e^{-\Phi}$$

where q-1=x, (E-μ)/T=ϕ .



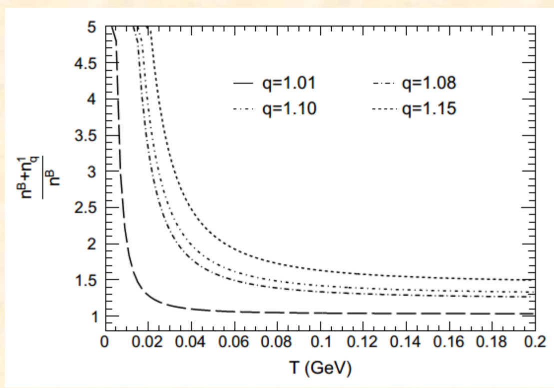
Fits to the normalized differential charged particle yields as measured by the ALICE collaboration in p+p collisions at $\sqrt{s_{NN}}$ = 0.9 TeV .

- We can fit the above distribution function for p+p collisions and see the deviation from the thermalized Boltzmann distribution.

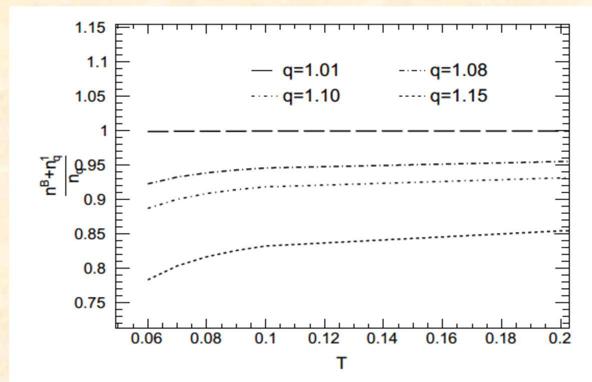
Deviation of thermodynamic quantities from Boltzmann Statistics

- As we increase the q-values from 1, the contribution to number density, energy density and pressure from the first order expansion increases.
- This shows the deviation of the system from a thermalized Boltzmann distribution.

Number density

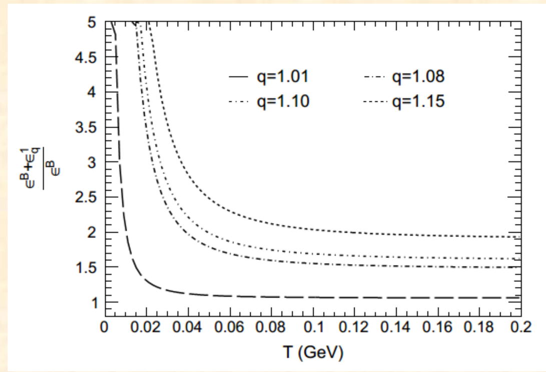


The particle density calculated to first order in (q-1) normalized to particle density in Boltzmann gas.

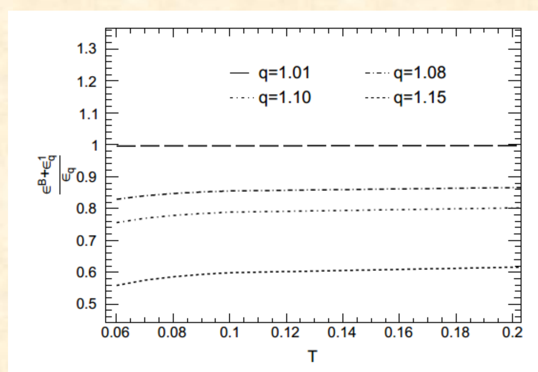


The particle density calculated to first order in (q-1) normalized to particle density in Tsallis gas.

Energy density

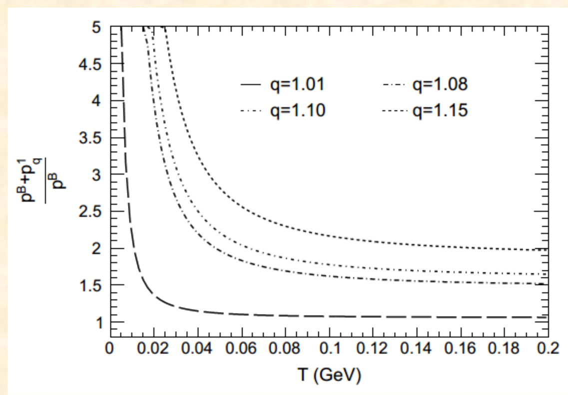


The energy density calculated to first order in (q-1) normalized to energy density in Boltzmann gas.

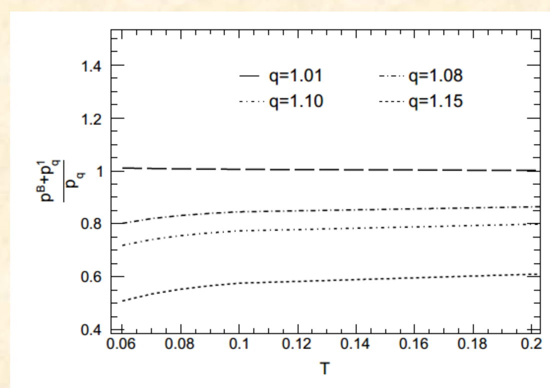


The energy density calculated to first order in (q-1) normalized to energy density in Tsallis gas.

Pressure



The pressure calculated to first order in (q-1) normalized to pressure in Boltzman gas.



The pressure calculated to first order in (q-1) normalized to pressure in Tsallis gas.

Inclusion of Radial Flow

- To include flow in Tsallis Boltzmann distribution function with Taylor expansion, we have taken the cylindrical symmetry in which it has a explicit dependence on flow parameter v. Here, we have replaced f(E) by $f(p^{\mu}u_{\mu})$, where $(p^{\mu}u_{\mu})$ is a Lorentz invariant quantity and

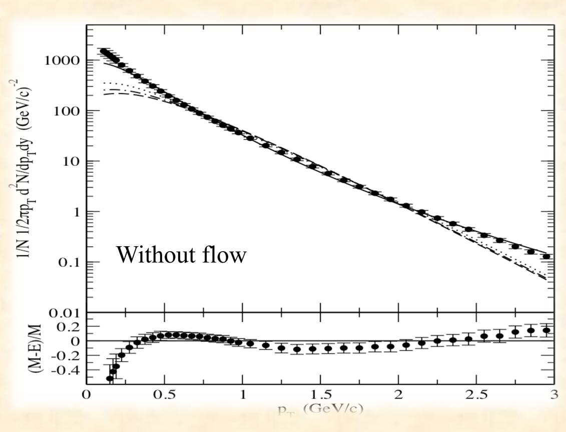
$$p^{\mu} = (m_T cosh y, p_T cos\phi, p_T sin\phi, m_T sinh y)$$
$$u_{\mu} = (\gamma cosh\xi, \gamma v cos\alpha, \gamma v sin\alpha, \gamma sinh\xi) .$$

The expression for transverse momentum distribution with radial flow up to first order in (q-1) is given by,

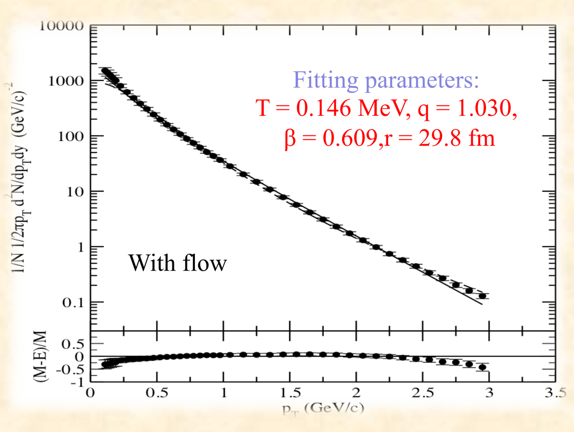
$$\frac{1}{p_T}\frac{dN}{dp_T dy} = \frac{gV}{(2\pi)^2}\left\{2T[rI_0(s)K_1(r) - sI_1(s)K_0(r)] - (q-1)Tr^2I_0(s)[K_0(r)+K_2(r)]\right.$$
$$+4(q-1)\,TrsI_1(s)K_1(r) - (q-1)Ts^2K_0(r)[I_0(s)+I_2(s)] + \frac{(q-1)}{4}Tr^3I_0(s)[K_3(r)+3K_1(r)]$$
$$-\frac{3(q-1)}{2}Tr^2s[K_2(r)+K_0(r)]I_1(s) + \frac{3(q-1)}{2}Ts^2r[I_0(s)+I_2(s)]K_1(r)$$
$$\left.-\frac{(q-1)}{4}Ts^3[I_3(s)+3I_1(s)]K_0(r)\right\}$$

where $r = \gamma m_T/T$, $s = \gamma v p_T/T$

$I_n(x)$ and $K_n(x)$ are the modified Bessel's function of first and second kind respectively.



The charged pion particle spectra fit for Pb+Pb at $\sqrt{s_{NN}}$= 2.76 TeV



The charged pion particle spectra fit for Pb+Pb at $\sqrt{s_{NN}}$= 2.76 TeV

Speed of Sound in a Physical Hadron Resonance Gas

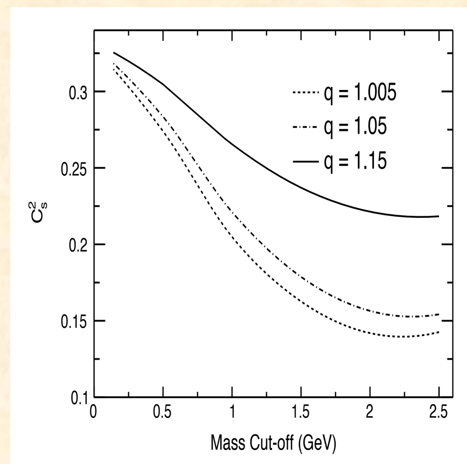
- A physical resonance gas consists of hadrons, which obey Fermi-Dirac statistics and Bose-Einstein statistics.
- Tsallis form of the Fermi-Dirac and Bose-Einstein distributions are,

$$f_T(E) \equiv \frac{1}{\exp_q\left(\frac{E-\mu}{T}\right) \mp 1}$$

Here, μ = 0, $x \cong E/T$
(+) sign -> Bose-Einstein
(-) sign -> Fermi-Dirac

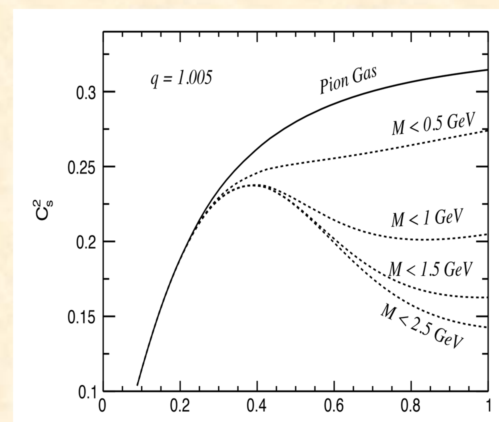
where, $\exp_q(x) \equiv \begin{cases} [1+(q-1)x]^{1/(q-1)} & \text{if} \quad x > 0 \\ [1+(1-q)x]^{1/(1-q)} & \text{if} \quad x \leq 0 \end{cases}$

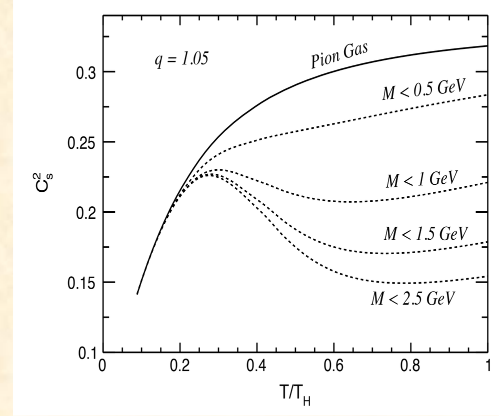
- We plot the speed of sound as a function of T/T_H for different values of q.



c_s^2 as a function of mass cut off

- As we add heavier and heavier resonances to the system the speed of sound decreases.





c_s^2 as a function of T/T_H

- The speed of sound for physical resonance gas becomes independent of mass cut off above 2 GeV.

Summary and Conclusions

- Using Taylor approximation we study the degree of deviation of transverse momentum spectra from a thermalized Boltzmann distribution.
- Inclusion of radial flow in Tsallis distribution gives an analytical way in describing the transverse momentum spectra up to 3 GeV in Pb+Pb collisions at $\sqrt{s_{NN}}$= 2.76 TeV .
- For physical hadron resonance gas, a small deviation in q values from 1, using Tsallis non-extensive statistics gives a significant change in speed of sound.
- The speed of sound in physical resonance gas does not depend on the mass cut-off taken in the system.
- Above 2 GeV, the speed of sound is independent of mass cut off for a physical resonance gas.

References

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