The q-Statistics and QCD Thermodynamics at LHC

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Summary and Conclusions

Using Taylor approximation we study the degree of deviation of transverse momentum spectra from a thermalized Boltzmann distribution.

Inclusion of radial flow in Tsallis distribution gives an analytical way in describing the transverse momentum spectra up to 3 GeV in Pb+Pb collisions at \( \sqrt{s_{NN}} = 2.76 \) TeV.

For physical hadron resonance gas, a small deviation in \( q \) values from 1, using Tsallis non-extensive statistics gives a significant change in speed of sound.

The speed of sound in physical resonance gas does not depend on the mass cut-off taken in the system.

Above 2 GeV, the speed of sound is independent of mass cut-off for a physical resonance gas.

References


Introduction

- Tsallis distribution gives excellent fits to the transverse momentum distributions observed in the Relativistic Heavy Ion Collider (RHIC) and in the Large Hadron Collider (LHC) experiments in a broad range of transverse momentum with three parameters \( q \), Tsallis temperature \( T \) and volume \( V \). The parameter \( q \) is referred to as the Tsallis parameter.
- The \( q \) values shows the degree of deviation of different thermodynamic variables those obtained from a thermalized Boltzmann distribution.
- Using those variables, we can study the behavior of speed of sound in physical hadron gas, when there is a deviation in \( q \) values from 1, using Tsallis non-extensive statistics.

Tsallis Non-extensive Statistics

- The Tsallis-Boltzmann distribution function is given by:
  \[ f = \left[ 1 + (q-1) \exp^{-\frac{E-\mu}{T}} \right]^{-\frac{1}{q-1}} \exp^{-\frac{E-\mu}{T}} \]
  where \( T \), \( \mu \) and \( E \) are the Tsallis temperature, chemical potential and energy respectively.
- As in most of the cases, \( q \) is close to 1, we can do the Taylor expansion of the Tsallis Boltzmann distribution, which is given by:
  \[ \left[ 1 + (q-1) \exp^{-\frac{E-\mu}{T}} \right]^{-\frac{1}{q-1}} \exp^{-\frac{E-\mu}{T}} = 1 - \frac{(q-1)(E-\mu)}{T} + \frac{q-1}{2} \frac{(E-\mu)^2}{T^2} + \cdots \]
  here, the first term is the Boltzmann term and rest are higher order terms in \( q-1 \).
- The invariant yield using the Taylor series expansion is given by:
  \[ \frac{dN}{dy} = \left[ 1 + (q-1) \exp^{-\frac{E-\mu}{T}} \right]^{-\frac{1}{q-1}} \exp^{-\frac{E-\mu}{T}} \]
  where \( q-1 \approx (E-\mu)/T \approx \phi \).
- We can fit the above distribution function for \( p+p \) collisions and see the deviation from the thermalized Boltzmann distribution.

Deviation of thermodynamic quantities from Boltzmann Statistics

- As we increase the \( q \)-values from 1, the contribution to number density, energy density and pressure from the first order expansion increases.

For more details see : arXiv:1507.08434

Speed of Sound in a Physical Hadron Resonance Gas

- A physical resonance gas consists of hadrons, which obey Fermi-Dirac statistics and Bose-Einstein statistics.
- Tsallis form of the Fermi-Dirac and Bose-Einstein distributions are,
  \[ f_{f}(x) = \exp^{\frac{x-\mu}{T}} \quad \text{and} \quad f_{s}(x) = \frac{1}{1 + \exp^{\frac{x-\mu}{T}}} \]
  where,
  \[ \exp^{\frac{x-\mu}{T}} \quad \text{---> Bose-Einstein} \]
  \[ \frac{1}{1 + \exp^{\frac{x-\mu}{T}}} \quad \text{---> Fermi-Dirac} \]
- We plot the speed of sound as a function of \( T/T \mu \) for different values of \( q \).
- As we add heavier and heavier resonances to the system, the speed of sound decreases.
- The speed of sound for physical resonance gas becomes independent of mass cut off above 2 GeV.

Inclusion of Radial Flow

- To include flow in Tsallis Boltzmann distribution function with Taylor expansion, we have taken the cylindrical symmetry in which it has a explicit dependence on flow parameter \( v \). Here, we have replaced \( f(E) \) by \( f(p_{T}\mu_{s}) \), where \( (p_{T}\mu_{s}) \) is a Lorentz invariant quantity and
  \[ p_{T} = (m_{\pi}u_{\pi}, m_{\rho}u_{\rho}, p_{T}, u_{\mu}) \]
  \[ u_{\mu} = (\gamma_{\pi}u_{\pi}, 1, 0, 0) \]
  The expression for transverse momentum distribution with radial flow up to first order in \( (q-1) \) is given by:
  \[ \frac{dN}{dy} = \left[ 1 + (q-1) \exp^{-\frac{(1+v_{s})p_{T}\mu_{s}}{T}} \right]^{-\frac{1}{q-1}} \exp^{-\frac{(1+v_{s})p_{T}\mu_{s}}{T}} \]
  where \( r = m_{\pi}T, s = \gamma_{\pi}p_{T}/T \)
  \[ I_{0}(s) \text{ and } K_{0}(s) \text{ are the modified Bessel’s function of first and second kind respectively.} \]

References