Early Time Dynamics of Gluon Fields in High Energy Nuclear Collisions

Joe Kapusta (University of Minnesota)

with

Guangyao Chen (Iowa State University)

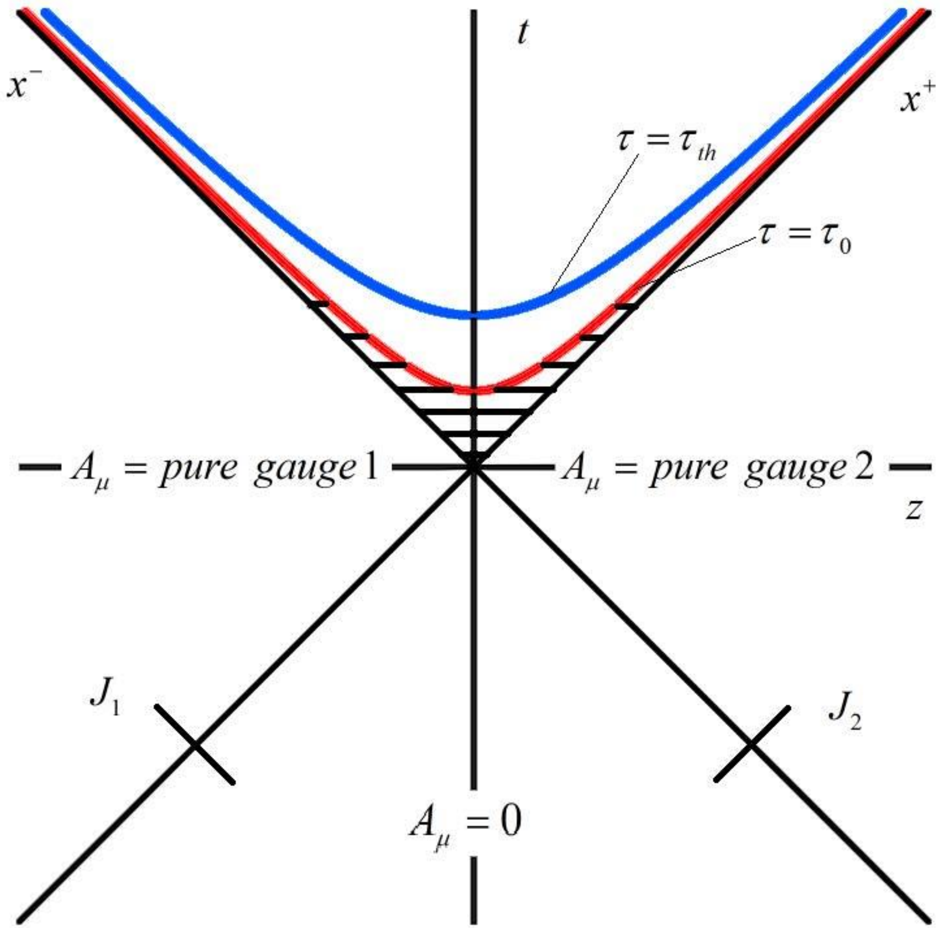
Rainer Fries (Texas A&M University)

Yang Li (University of Minnesota – Duluth)

G. Chen, R. J. Fries, J. I. Kapusta, and Y. Li, arXiv:1507.03524
G. Chen and R. J. Fries, Phys. Lett. B **723**, 417 (2013)
R. J. Fries, J. I. Kapusta, and Y. Li, arXiv:nucl-th/0604054

Solve the classical Yang-Mills field equations for very early times

- McLerran-Venugopalan model
- Boost invariant
- Color charge density can vary in the transverse direction
- Interesting early time behavior with nontrivial flow fields
- Provide initial conditions for viscous hydrodynamics



Initial conditions along the forward light cone are known.

$$A^i_{\perp(0)}(x_\perp) = A^i_1(x_\perp) + A^i_2(x_\perp)$$

$$A_{(0)}(x_\perp) = -\frac{ig}{2}\left[A^i_1(x_\perp), A^i_2(x_\perp)\right]$$

Classical Yang-Mills equations in the forward light cone with dependence on proper time and transverse spatial coordinates

$$\frac{1}{\tau}\frac{\partial}{\partial\tau}\frac{1}{\tau}\frac{\partial}{\partial\tau}\tau^2 A - [D^i,[D^i,A]] = 0\,,$$

$$ig\tau\left[A,\frac{\partial}{\partial\tau}A\right] - \frac{1}{\tau}\left[D^i,\frac{\partial}{\partial\tau}A_\perp^i\right] = 0\,,$$

$$\frac{1}{\tau}\frac{\partial}{\partial\tau}\tau\frac{\partial}{\partial\tau}A_\perp^i - ig\tau^2\left[A,[D^i,A]\right] - [D^j,F^{ji}] = 0$$

$$F^{+-} = -\frac{1}{\tau}\frac{\partial}{\partial\tau}\tau^2 A,$$

$$F^{i\pm} = -x^\pm\left(\frac{1}{\tau}\frac{\partial}{\partial\tau}A_\perp^i \mp [D^i,A]\right),$$

$$F^{ij} = \partial^i A_\perp^j - \partial^j A_\perp^i - ig[A_\perp^i,A_\perp^j].$$

Expand in a power series in proper time

$$A(\tau,\vec{x}_\perp) = \sum_{n=0}^{\infty} \tau^n A_{(n)}(\vec{x}_\perp),$$

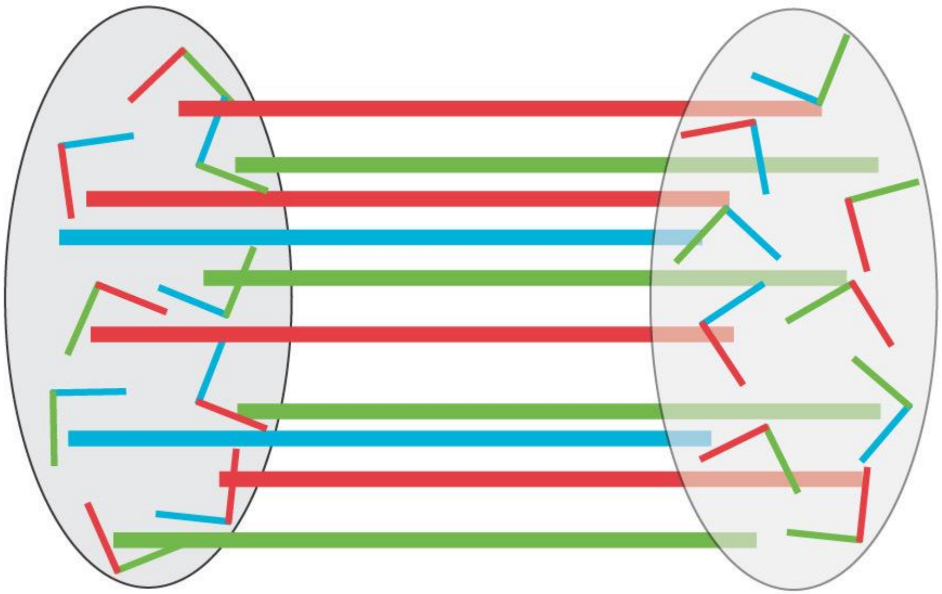
$$A^i_\perp(\tau,\vec{x}_\perp) = \sum_{n=0}^{\infty} \tau^n A^i_{\perp(n)}(\vec{x}_\perp)$$

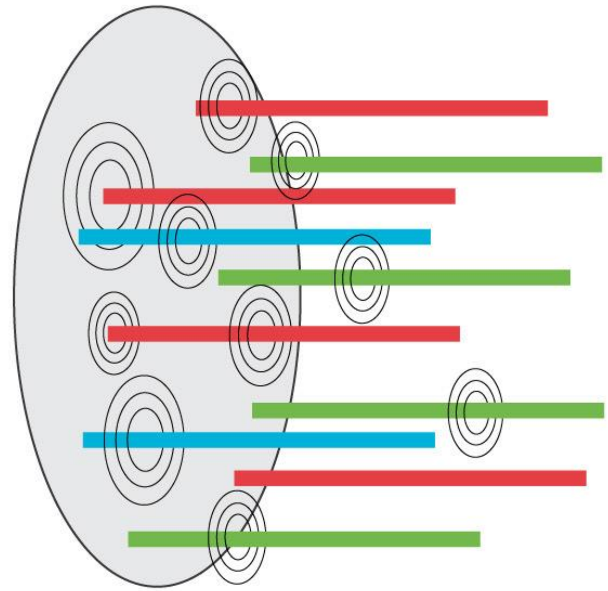
$$A_{(n)} = \frac{1}{n(n+2)} \sum_{k+l+m=n-2} \left[D^i_{(k)}, \left[D^i_{(l)}, A_{(m)}\right]\right]$$

$$A^i_{\perp(n)} = \frac{1}{n^2}\left(\sum_{k+l=n-2} \left[D^j_{(k)}, F^{ji}_{(l)}\right] + ig \sum_{k+l+m=n-4} \left[A_{(k)}, \left[D^i_{(l)}, A_{(m)}\right]\right]\right)$$

Several scales will enter : the color charge per unit area μ with $Q_s^2 \sim g^4\mu$, a transverse IR regulator (gluon mass) m, and a transverse UV cutoff Q^2.

Note: Series converges to the known solution in the Abelian limit.





The color charges and transverse fields initially present generate longitudinal color electric and magnetic fields after impact.

Transverse fields between the receding nuclei appear shortly thereafter. Fields from Faraday’s and Ampere’s Laws are shown.

(Abelian version)

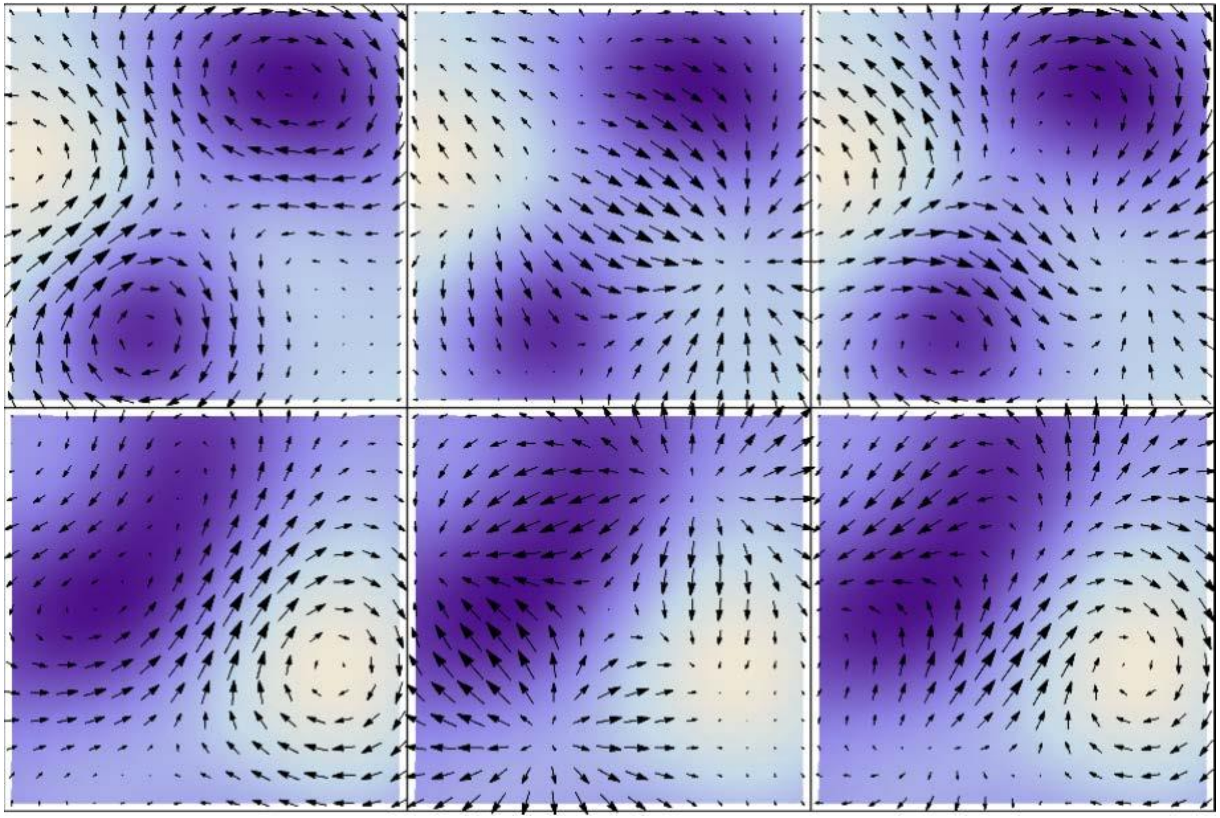
One event with random seeding of initial color charges.
(Abelian version)

Electric field (arrows)

Longitudinal magnetic field (shading)

Magnetic field (arrows)

Longitudinal electric field (shading)



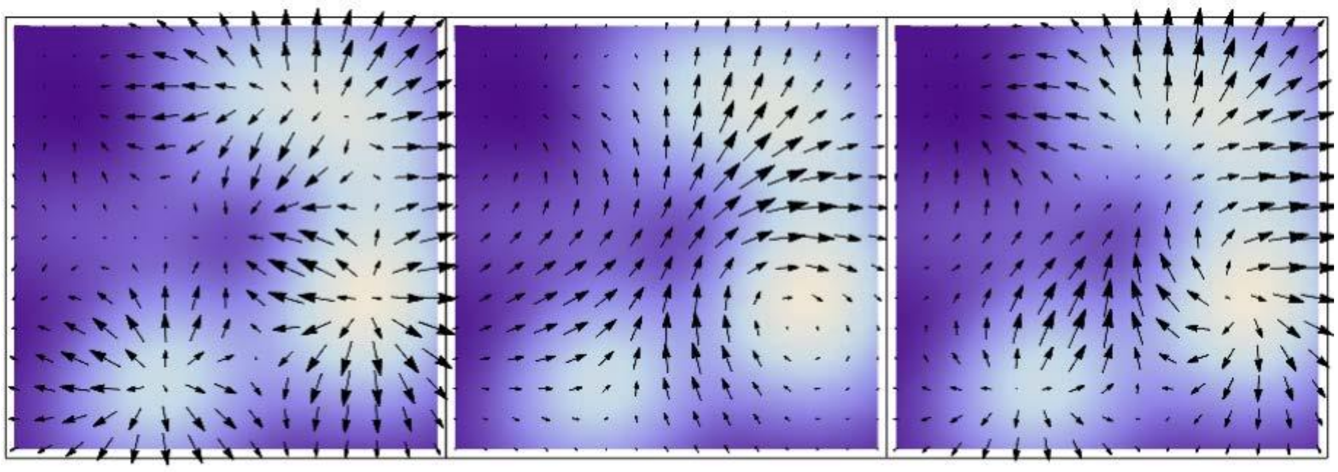
rapidity even

rapidity odd

sum (eta=1)

One event with random seeding of initial color charges.

Transverse energy flow (arrows) and energy density (shading). (Abelian version)



rapidity even

rapidity odd

sum (eta = 1)

Averaging Over the Initial Color Charges

Solve field equations for arbitrary color charges of both nuclei, then average over color charges to obtain event averages of any observable.

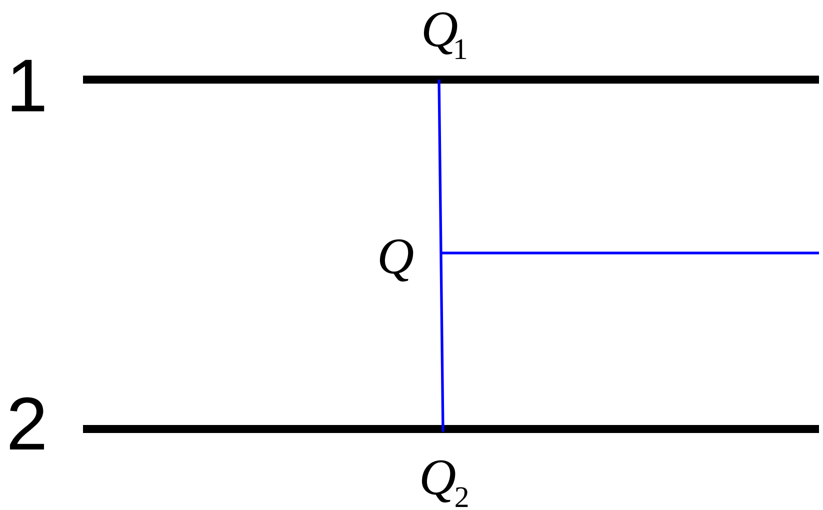
$$\langle O \rangle_{\rho_1,\rho_2} = \int d[\rho_1] d[\rho_2] O(\rho_1,\rho_2) w(\rho_1) w(\rho_2)$$

↑ Gaussian of width μ

Energy-momentum tensor solved up to and including 4th order in time and to 1st order in transverse gradients of the color charges per unit area.

Initial Energy Density

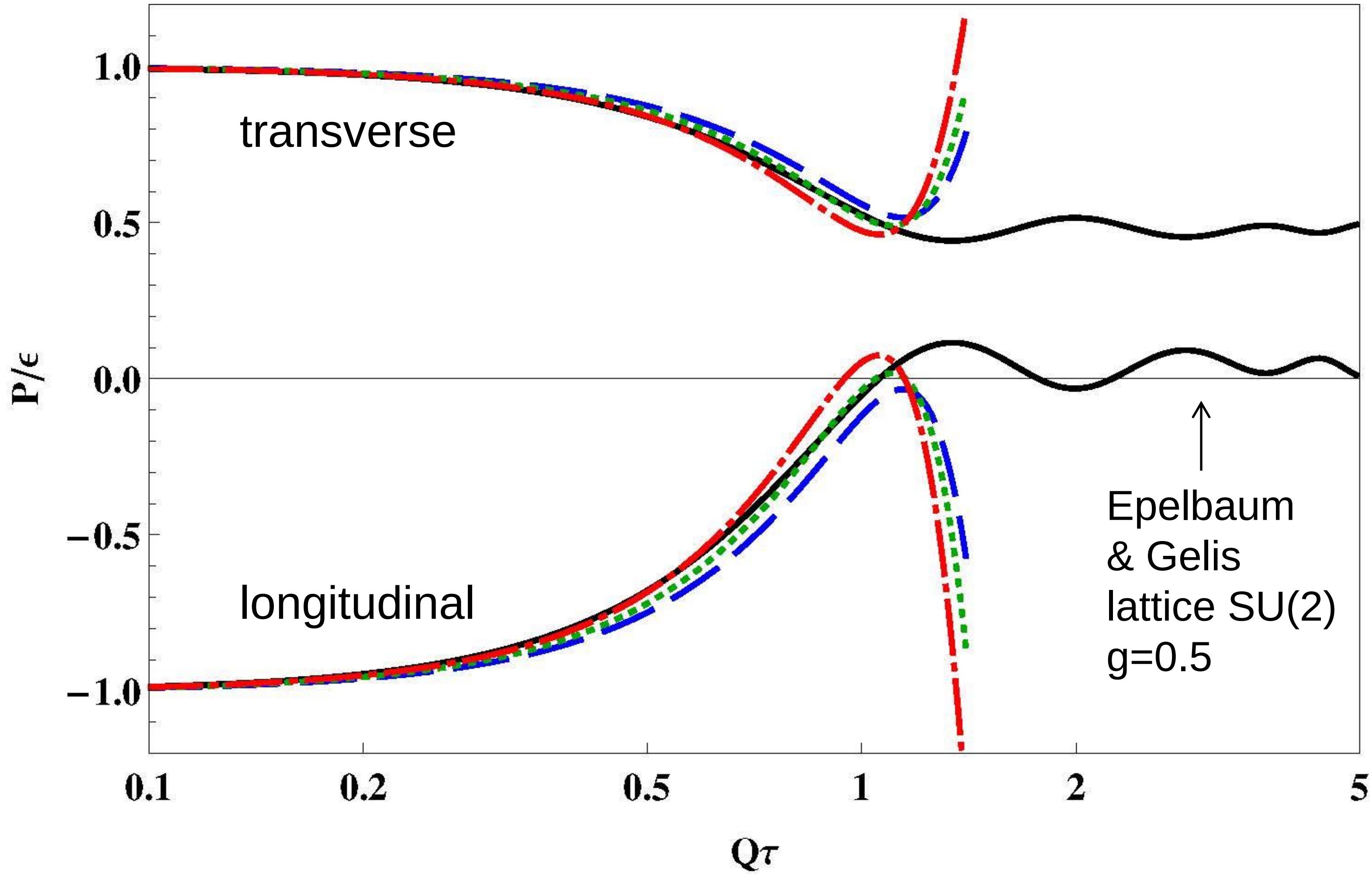
$$\varepsilon_0(\vec{x}_\perp) = \frac{2\pi N_c \alpha_s^3}{N_c^2 - 1} \mu_1(\vec{x}_\perp)\mu_2(\vec{x}_\perp) \ln\left(\frac{Q_1^2}{\hat{m}^2}\right) \ln\left(\frac{Q_2^2}{\hat{m}^2}\right)$$



$$\alpha_s(M^2) = \frac{1}{\beta_2 \ln(M^2/\Lambda_{\rm QCD}^2)}$$

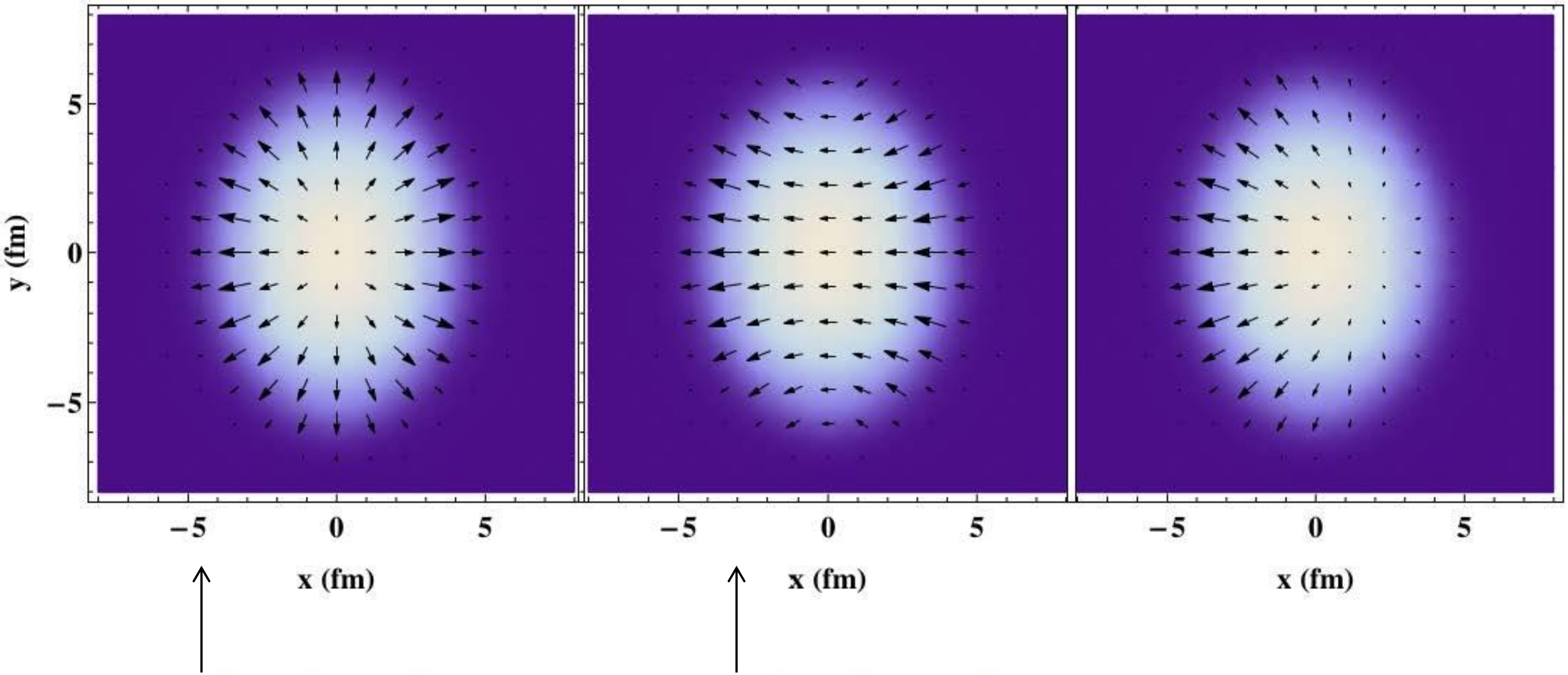
$$\varepsilon_0(\vec{x}_\perp) \approx \frac{2\pi N_c \alpha_s(Q^2)}{\beta_2^2 (N_c^2 - 1)} \mu_1(\vec{x}_\perp)\mu_2(\vec{x}_\perp)$$

Slab collisions up to and including 4th order in time.



$$\ln(Q^2/\hat{m}^2) = 0.8, 0.9, 1.0$$

Pb-Pb collisions at b=6 fm



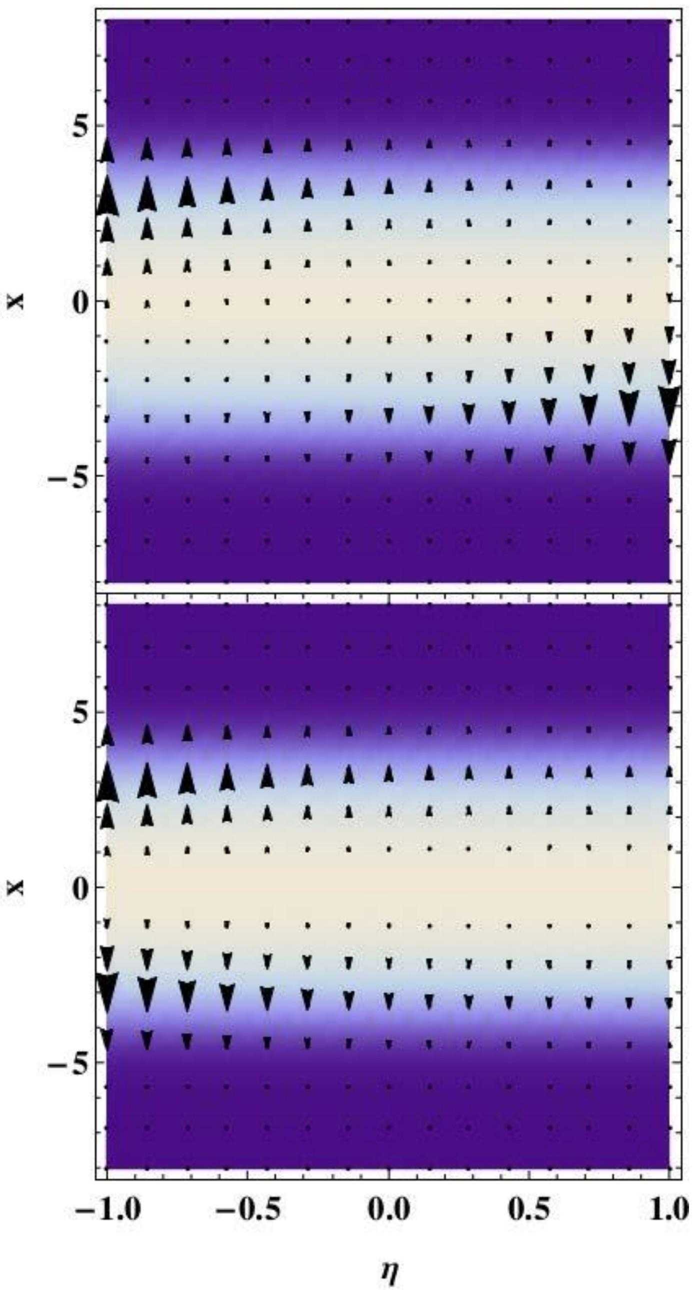
$$T^{0i}_{\rm even} = \frac{\tau}{2}\,\alpha^i\left(1-\frac{1}{2a}(Q\tau)^2\right)\cosh\eta \qquad T^{0i}_{\rm odd} = \frac{\tau}{2}\,\beta^i\left(1-\frac{9}{16a}(Q\tau)^2\right)\sinh\eta$$

sum at $\eta = 1$

Pb-Pb collision
with b=6 fm

Transverse energy flow T^{0i}

Pb-Ca collision
with b=0
Pb on right



The Future

Calculate to all orders in proper time
– done by Ming Li to leading order in Q^2

Implement into 2^{nd} order viscous hydrodynamics
– in progress by several groups
– how does thermalization occur?

Finite energy solutions especially for the RHIC beam energy scan
– in progress by Rainer Fries, Sener Ozonder, and Ming Li

Thank you!

Supported by:

Office of Science, U.S. Department of Energy

U.S. National Science Foundation