Testing hydrodynamics with an exact solution of the Boltzmann equation subject to Gubser flow

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Abstract

Relativistic hydrodynamics plays an important role in the quantitative description of the space-time evolution of the strongly coupled QGP created in Ultrarelativistic Heavy-Ion Collisions. Thus, it is necessary to have under control the physical assumptions made in the hydrodynamical modeling. In this work we present a new exact solution to the relativistic Boltzmann equation within the relaxation time approximation (RTA). This solution describes a system undergoing boost-invariant longitudinal and azimuthally symmetric radial expansion for arbitrary shear viscosity to entropy density ratio. The resulting solution is invariant under the SO(3)$_q$ ⊗SO(1,1)⊗Z$_2$ group symmetry (Gubser symmetry). We test the efficiency of various hydrodynamic approximation methods by comparing the evolution of the moments of the exact solution (such as energy density and shear viscous tensor) with the corresponding solutions of the macroscopic hydrodynamic equations. In addition, we briefly discuss the phase-space evolution of this new exact solution and the physical constraints on its applicability.

The Gubser flow

- Gubser flow is invariant under the following symmetry group

$$SO(3)_q \otimes SO(1,1) \otimes Z_2$$

Special Conformal Transformation + Rotations — Longitudinal boost invariance — Reflections along the beam line

- Symmetries become manifest by considering the conformal map between Minkowski and the 3 dim. de Sitter space times a line

$$ds^2 = -d\tau^2 + \tau^2 d\varsigma^2 + dr^2 + r^2 d\phi^2 \qquad \tau = \sqrt{t^2 - z^2} \qquad \varsigma = \tanh^{-1}(z/t)$$

↓ Weyl rescaling

$$d\hat{s}^2 = \frac{ds^2}{\tau^2}$$

↓ Coordinate transformation

$$d\hat{s}^2 = \underbrace{-d\rho^2 + \cosh^2\rho\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)}_{dS_3} + \underbrace{d\varsigma^2}_{R}$$

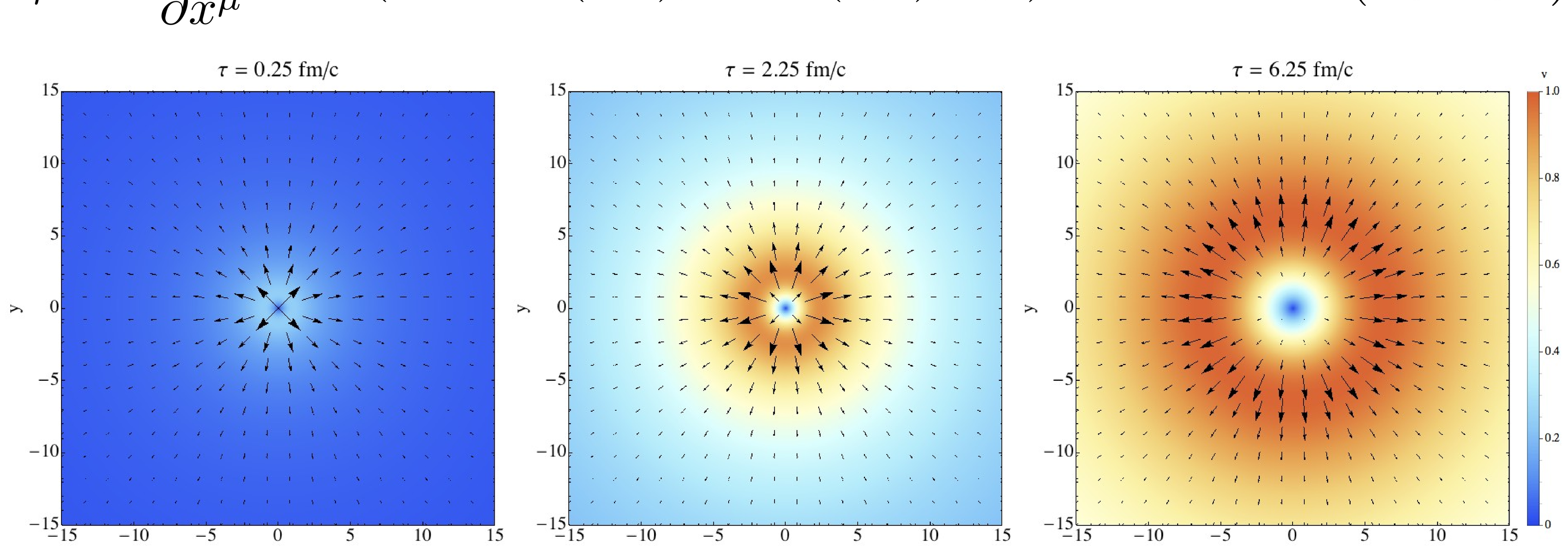
$$\rho = -\sinh^{-1}\left(\frac{1 - q^2\tau^2 + q^2 r^2}{2q\tau}\right) \qquad \theta = \tanh^{-1}\left(\frac{2qr}{1 + q^2\tau^2 - q^2 r^2}\right)$$

- In de Sitter space, the fluid velocity is a static flow

$$\hat{u}^\mu = (1, 0, 0, 0)$$

- The flow velocity in Minkowski space is

$$u_\mu = \tau \frac{\partial \hat{x}^\nu}{\partial x^\mu} \hat{u}_\nu = (-\cosh\kappa(\tau, r), \sinh\kappa(\tau, r), 0, 0) \qquad \kappa(\tau, r) = \tanh^{-1}\left(\frac{2q^2 r\tau}{1 + q^2\tau^2 + q^2 r^2}\right)$$



Gubser solution to the Boltzmann equation

- We solve the Boltzmann equation in the de Sitter space where the Gubser symmetry is manifest.
- In general, the symmetries of a physical system impose strong constraints on the number of independent variables of the distribution function and on the particular combination on which the dependent variables appear. The symmetries of the Gubser flow imply

$$f(\rho, \theta, \phi, \varsigma, \hat{p}_\theta, \hat{p}_\phi, \hat{p}_\varsigma) \longrightarrow f(\rho, \hat{p}_\Omega^2, \hat{p}_\varsigma) \qquad \hat{p}_\Omega^2 = \hat{p}_\theta^2 + \frac{\hat{p}_\phi^2}{\sin^2\theta}$$

- The RTA Boltzmann equation in the de Sitter space simplifies to

$$\frac{\partial}{\partial\rho} f\left(\rho, \hat{p}_\Omega^2, \hat{p}_\varsigma\right) = -\frac{\hat{T}(\rho)}{c}\left(f\left(\rho, \hat{p}_\Omega^2, \hat{p}_\varsigma\right) - f_{eq}\left(\hat{p}^\rho / \hat{T}(\rho)\right)\right) \qquad \hat{p}^\rho = \sqrt{\hat{p}_\varsigma^2 + \frac{\hat{p}_\Omega^2}{\cosh^2\rho}} \qquad c = 5\frac{\eta}{\mathcal{S}}$$

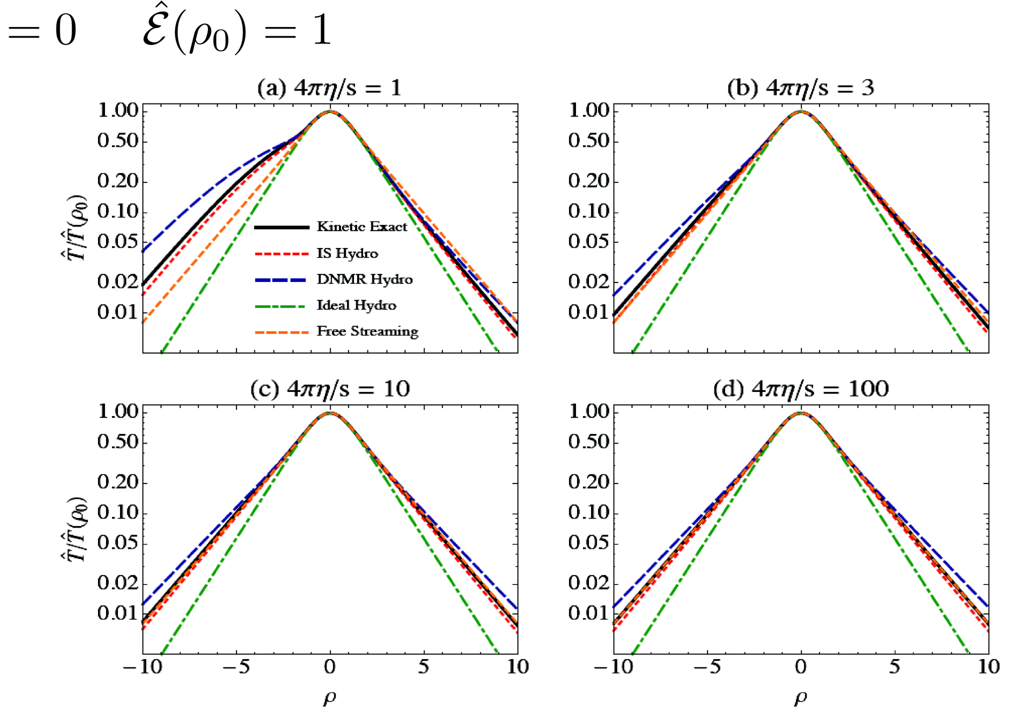
- The exact solution of this equation is

$$f(\rho, \hat{p}_\Omega^2, \hat{p}_\varsigma) = D(\rho, \rho_0) f_0(\rho_0, \hat{p}_\Omega^2, \hat{p}_\varsigma) + \frac{1}{c}\int_{\rho_0}^{\rho} d\rho'\, D(\rho, \rho')\hat{T}(\rho') f_{eq}(\rho', \hat{p}_\Omega^2, \hat{p}_\varsigma)$$

$$f_0 = f_{eq} = e^{\hat{u}\cdot\hat{p}/\hat{T}} \qquad D(\rho_2, \rho_1) = \exp\left(-\int_{\rho_1}^{\rho_2} d\rho' \frac{\hat{T}(\rho')}{c}\right)$$

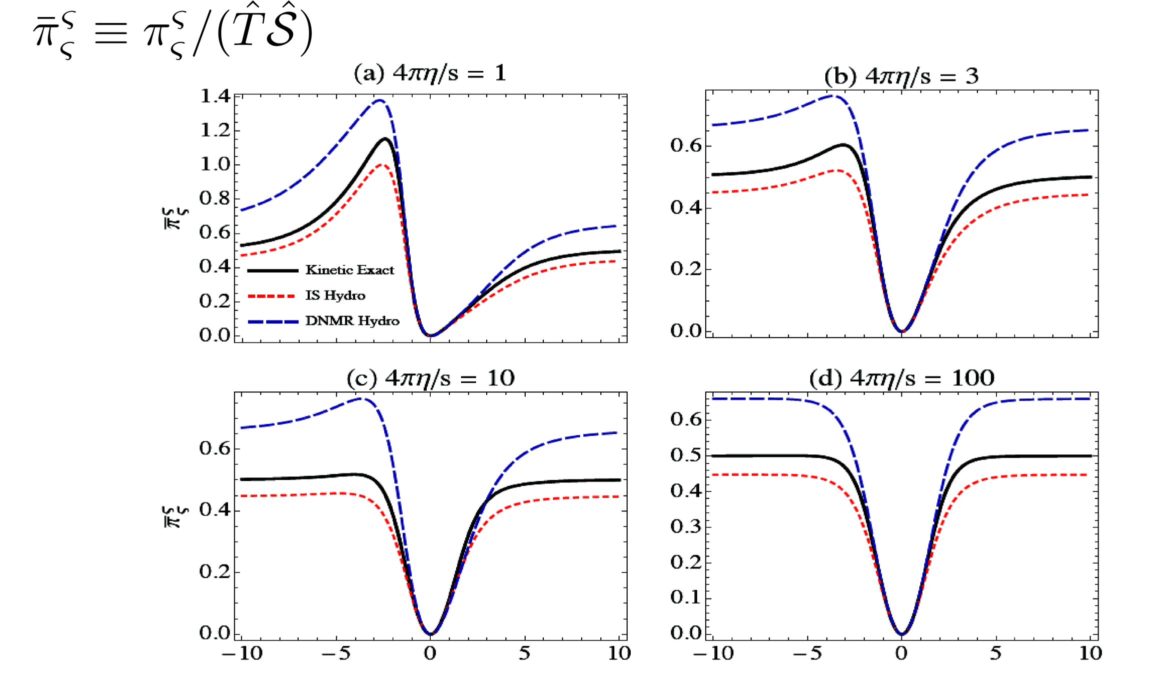
Testing hydrodynamics in the de Sitter space

$\rho_0 = 0 \quad \hat{\mathcal{E}}(\rho_0) = 1$



- DNMR theory gives the best description near ρ=0.
- IS theory gives a better approximation at large values of ρ .
- The exact solution describes the free streaming limit which correspond to large values of η/S .

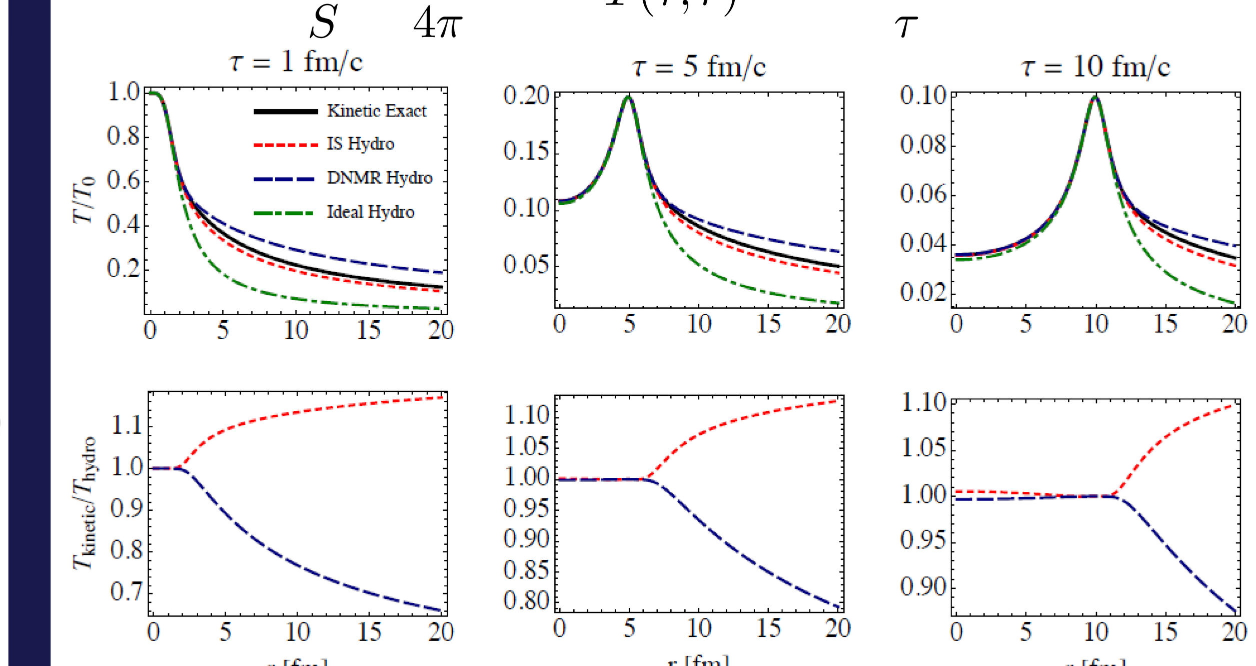
$\bar{\pi}_\varsigma^\varsigma \equiv \pi_\varsigma^\varsigma / (\hat{T}\hat{\mathcal{S}})$



- Hydrodynamical approaches work better for small values of η/S.
- There is a approximate agreement at a qualitative ($\mathcal{O}$(30%)) level at large values of η/S between the exact solution and 2[nd] order hydrodynamical approaches.

Testing hydrodynamics in Minkowski space

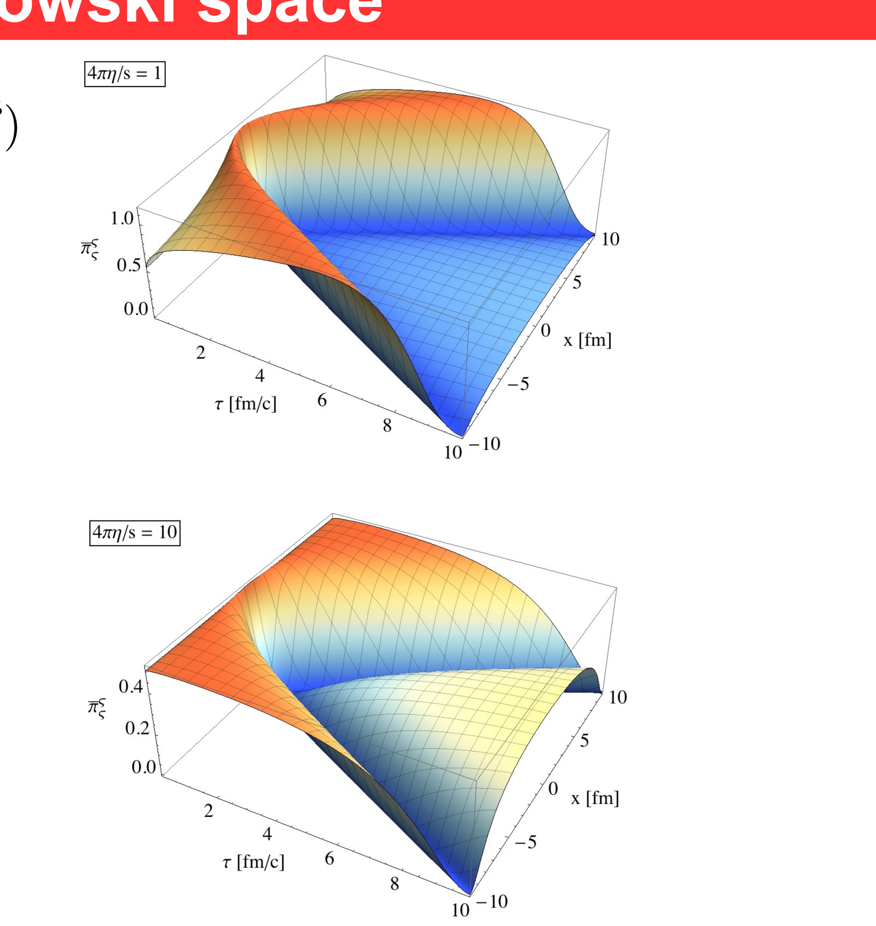
$$\frac{\eta}{S} = \frac{1}{4\pi} \qquad T(\tau, r) = \frac{\hat{T}(\rho(\tau, r))}{\tau}$$



The maximum deviations between the temperature obtained from exact solution and the 2[nd] order hydrodynamical approaches is

- for IS theory the order of 10-15% .
- for DNMR theory the order of 15-30% .

$\bar{\pi}_\varsigma^\varsigma \equiv \pi_\varsigma^\varsigma / (\hat{T}\hat{\mathcal{S}})$



The assumed value of η/S has a strong effect on the spacetime evolution of the shear stress.

Conclusions

- We obtain a new solution to the RTA Boltzmann equation for a system that undergoes under the Gubser flow for an arbitrary shear viscosity over entropy ratio. The solution is invariant under the Gubser group symmetry.
- With this exact solution we are able to test the validity and accuracy of different hydrodynamical approaches in situations that resemble heavy-ion collisions where the fireball expands simultaneously along the longitudinal and azimuthally symmetric transverse direction.

References

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