Strong-Coupling Effects in a Plasma of Confining Gluons

Radoslaw Ryblewski

in collaboration with:
W. Florkowski (INP PAN, Cracow), N. Su (Goethe-U. Frankfurt), K. Tywoniuk (ICC, Barcelona U.)

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RHIC and LHC HIC data suggest that QGP is a strongly-interacting dissipative fluid

⇒ relativistic dissipative fluid dynamics (equation of state?, transport coefficients?)

equation of state (at high $T$)

- lattice QCD
  (M. Cheng et al., PRD 77, 014511 (2008); S. Borsanyi et al., JHEP 11, 077 (2010); S. Borsanyi et al., JHEP 07, 056 (2012))

- re-summed perturbation theory

transport coefficients
(W. Israel, J. M. Stewart, Ann. Phys. (NY) 118, 341 (1979); A. Muronga, PRC 69, 034903 (2004); A. El et al., PRC 81, 041901(R) (2010); G. S. Denicol et al., PRL 105, 162501 (2010); A. Jaiswal, PRC 87, 051901(R) (2013), . . .)

\[
\dot{\Pi} + \frac{\Pi}{\tau_{\Pi}} = -\frac{\zeta}{\tau_{\Pi}} \theta - \delta_{\Pi\Pi} \Pi \theta + \lambda \Pi \Pi \pi^{\mu\nu} \sigma_{\mu\nu}
\]

\[
\dot{\pi}^{\mu\nu} + \frac{\pi^{\mu\nu}}{\tau_{\pi}} = 2 \frac{\eta}{\tau_{\pi}} \sigma^{\mu\nu} + 2\pi^{\mu}_{\gamma} \pi^{\nu}\gamma - \delta_{\Pi\Pi} \Pi^{\mu\nu} \theta - \tau_{\Pi\Pi} \pi^{\mu}_{\gamma} \sigma^{\nu}\gamma + \lambda \Pi \Pi \sigma^{\mu\nu}
\]

viscous hydrodynamics kinetic theory wise with RTA (G. S. Denicol et al., PRL 105, 162501 (2010); A. Jaiswal, PRC 87, 051901(R) (2013))

$\zeta \rightarrow$ bulk viscosity  \hspace{1cm} $\eta \rightarrow$ shear viscosity

by force of the Kubo formulas, they are sensitive to the long-distance dynamics of the underlying microscopic theory
Motivation

- transport coefficients (cont’d)
  - pQCD (P.B. Arnold et al., JHEP 11, 001 (2000); P.B. Arnold et al., JHEP 05, 051 (2003); G. D. Moore, O. Saremi, JHEP 09, 015 (2008); P.B. Arnold et al., PRD 74, 085021 (2006);)
  - low energy theorems (D. Kharzeev, K. Tuchin, JHEP 09, 093 (2008); F. Karsch et al., PLB 663, 217 (2008);)
  - lQCD (H. B. Meyer, PRL 100, 162001 (2008); H. B. Meyer, PRD 76, 101701 (2007);)
  - N = 4 supersymmetric YM plasma with broken conformal symmetry (G. Policastro et al., PRL 87, 081601 (2001); P. Benincasa et al., NPB 733, 160 (2006); A. Buchel, PRD 72, 106002 (2005); S. I. Finazzo et al., JHEP 02, 051 (2015))
  - HRG+HS (G.P. Kadam, H. Mishra, NPA 934 (2014) 133-147)
  - parton-hadron-string dynamics transport approach (V. Ozvenchuk et al., PRC 87, no. 6, 064903 (2013))
  - ...

\[ \zeta \text{ and } \eta \text{ are comparable around } T_C, \text{ large uncertainties} \]
Gribov’s confining gluons

- Gribov quantization of YM theory - fixing the infrared (IR) residual gauge transformations remaining after Faddeev-Popov procedure

\[ a \text{ new scale } \gamma_G \text{ that leads to an IR-improved dispersion relation for gluons (Coulomb gauge)} \]

(\text{V. Gribov, NPB 139, 1 (1978); D. Zwanziger, NPB 323, 513 (1989);})

\[ E(k) = k \rightarrow E(k) = \sqrt{k^2 + \frac{\gamma_G^4}{k^2}} \]

- attracted a lot of attention recently
  (G. Burgio et al., PRL 102, 032002 (2009);
  N. Su, K. Tywoniuk, PRL. 114, 161601 (2015);
  D. E. Kharzeev, E. M. Levin, PRL 114, 242001 (2015);
  ...)

- reduction of the physical phase space due to the large energy cost of the excitation of soft gluons

\[ \downarrow \]

essential feature of the confinement
(\text{V. Gribov, NPB 139, 1 (1978); R.P. Feynman, NPB 188, 479 (1981); D. Zwanziger, NPB 485, 185 (1997);})

- what are the residual long-range correlations present at finite temperature?
Covariant setup

**local rest frame**

\[ E(k) = \sqrt{k^2 + \frac{\gamma_G^4}{k^2}} \]

**comoving frame**

\[ E(k \cdot u) = \sqrt{(k \cdot u)^2 + \frac{\gamma_G^4}{(k \cdot u)^2}} \]

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**thermodynamics of the system of confining gluons**

(D. Zwanziger, PRL 94, 182301 (2005);


\[ \varepsilon = g_0 \int \frac{d^3 k}{(2\pi)^3} E(k) f(k) \quad \rightarrow \quad \varepsilon = \int dK E(k \cdot u) f(x, k) \]

\[ P = \frac{g_0}{3} \int \frac{d^3 k}{(2\pi)^3} |k| \frac{\partial E(k)}{\partial |k|} f(k) \quad \rightarrow \quad P = \frac{1}{3} \int dK \frac{(k \cdot u)^2}{E(k \cdot u)} \left( 1 - \frac{\gamma_G^4}{(k \cdot u)^4} \right) f(x, k) \]

\[ \int dK (\ldots) = g_0 \int \frac{d^3 k}{(2\pi)^3 k^0} (k \cdot u) (\ldots) \]

\[ g_0 = 2(N_c^2 - 1) \quad (SU(N_c)) \]
in vacuum $\gamma_G^2 = \mu^2 \exp \left( \frac{5}{6} - \frac{64 \pi^2}{3 N_c g^2} \right)$
(D. Zwanziger, PRL 94, 182301 (2005);

in high-T limit $\gamma_G \rightarrow \frac{d}{d+1} \frac{N_c}{4 \sqrt{2\pi}} g^2 T$
(D. Zwanziger, PRD 76, 125014 (2007);

$T \approx (2 - 4) T_c \quad \Rightarrow \quad \gamma_G \approx \text{const.}$

$\gamma_G(T)$ derived from the gap equation with running coupling from lQCD
(K. Fukushima, N. Su, PRD 88, 076008 (2013);
J. O. Andersen et al., PRL 104, 122003 (2010));
consider the case of a transversely homogeneous boost-invariant system

assume the Bjorken flow of matter in longitudinal direction (boost-invariance)

\[ u^\mu = \left( \frac{t}{\tau}, 0, 0, \frac{z}{\tau} \right) \]

introduce convenient boost-invariant variables

\[ v = k^0 t - k_z z \]
\[ w = k_z t - k^0 z \]

EOM follow from the conservation of \( T^{\mu\nu} \)

\[ T^{\mu\nu} = \int dK k^\mu k^\nu f(x, k) \]
\[ f = f(\tau, w, k_\perp) \]

within the assumed symmetries the \( T^{\mu\nu} \) has the spherically anisotropic form

\[ T^{\mu\nu} = (\varepsilon + P_\perp) u^\mu u^\nu - P_\perp g^{\mu\nu} + (P_\parallel - P_\perp) z_\mu z_\nu \]
\[ z^\mu = \left( \frac{z}{\tau}, 0, 0, \frac{t}{\tau} \right) \]

\[ \partial_\mu T^{\mu\nu} = 0 \quad \Rightarrow \quad \frac{d\varepsilon}{d\tau} + \frac{\varepsilon + P_\parallel}{\tau} = 0 \]
Non-equilibrium fluid dynamics of GZ plasma

(A. Muronga, PRC 69, 034903 (2004);
R. Baier et al., PRC 73, 064903 (2006);
C. Sasaki, K. Redlich, PRC 79, 055207)

\[
\frac{d\varepsilon}{d\tau} + \frac{1}{\tau} (\varepsilon + P_{GZ} + \Pi - \pi) = 0 \iff \frac{d\varepsilon}{d\tau} + \frac{\varepsilon + P||}{\tau} = 0
\]

**dissipative fluxes**

\[
\pi = \frac{4}{3} \frac{\eta}{\tau}
\]

\[
\Pi = -\frac{\zeta}{\tau}
\]

\[
\pi = \frac{2}{3} (P|| - P_\perp)
\]

\[
\Pi = P - P_{GZ} = \frac{1}{3} (P|| + 2P_\perp) - P_{GZ}
\]

(Navier-Stokes dissipative fluid dynamics)

**what is the form of \(\zeta\) and \(\eta\) for GZ plasma?** close to equilibrium one can use the linear response approximation \(f \approx f_{GZ} + \delta f + \cdots\)

\[
\zeta(T, \gamma_G) = \frac{g_0 \gamma_G^5}{3\pi^2} \frac{\tau_{rel}}{T} \int_0^\infty dy \left[ c_s^2 - \frac{1}{3} \frac{y^4 - 1}{y^4 + 1} \right] f_{GZ}(1 + f_{GZ})
\]

\[
\eta(T, \gamma_G) = \frac{1}{10} \frac{g_0 \gamma_G^5}{3\pi^2} \frac{\tau_{rel}}{T} \int_0^\infty dy \frac{(y^4 - 1)^2}{y^4 + 1} f_{GZ}(1 + f_{GZ})
\]
Kinetic equation in the relaxation-time approximation (cross-check)

\[
\frac{d\epsilon}{d\tau} + \frac{\epsilon + P_\parallel}{\tau} = \int dK E(\tau, w, k_\perp) \frac{\partial f(\tau, w, k_\perp)}{\partial \tau}
\]

(0+1)D kinetic equation in RTA

P. L. Bhatnagar et al., Phys. Rev. 94, 511 (1954);
G. Baym, PLB 138, 18 (1984);
G. Baym, NPA 418, 525C (1984);

\[
\frac{\partial f(\tau, w, k_\perp)}{\partial \tau} = \frac{f_{GZ}(\tau, w, k_\perp) - f(\tau, w, k_\perp)}{\tau_{\text{rel}}(\tau)}
\]

satisfied as long as Landau matching condition is satisfied \(\epsilon_{GZ} = \epsilon\)

formal solution

W. Florkowski et al., NPA 916, 249 (2013);
W. Florkowski et al., PRC 88, 024903 (2013);
W. Florkowski et al., PRC 89, 054908 (2014);

\[
f(\tau, w, k_\perp) = f_0(w, k_\perp)D(\tau, \tau_0)
\]

\[
+ \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau_{\text{rel}}(\tau')} D(\tau, \tau') f_{GZ}(\tau', w, k_\perp)
\]

\[
\epsilon = \int dK E(\tau, w, k_\perp) f(\tau, w, k_\perp)
\]

\[
P_\parallel = \int dK \frac{w^2}{\tau^2 E(\tau, w, k_\perp)} \left[ 1 - \frac{\gamma_A^G}{(w^2/\tau^2 + k_{\perp}^2)^2} \right] f
\]

\[
P_{\perp} = \int dK \frac{k_{\perp}^2}{2 E(\tau, w, k_\perp)} \left[ 1 - \frac{\gamma_A^G}{(w^2/\tau^2 + k_{\perp}^2)^2} \right] f
\]

\[
D(\tau_2, \tau_1) = \exp \left[ -\int_{\tau_1}^{\tau_2} \frac{d\tau}{\tau_{\text{rel}}(\tau)} \right]
\]
Results: Bulk and shear viscosity

perfect agreement with the exact solutions of the RTA Boltzmann equation
Results: $\zeta/\eta$ dependence

$$\frac{\zeta}{\eta} = 15 \left( \frac{1}{3} - c_s^2 \right)^2$$

- photon gas coupled to hot matter
  (S. Weinberg, Astrophys. J. 168, 175 (1971);

- scalar theory
  (A. Hosoya et al., Ann. Phys. (N.Y.) 154, 229 (1984);
  R. Horsley, W. Schoenmaker, NPB 280, 716 (1987);

- weakly-coupled QCD (large-$T$ limit)
  (P. B. Arnold et al., JHEP 0011 (2000) 001;
  P. B. Arnold et al., JHEP 0305 (2003) 051;
  P. B. Arnold et al., PRD 74 (2006) 085021;)

$$\frac{\zeta}{\eta} = \kappa \left( \frac{1}{3} - c_s^2 \right)$$

- strongly-coupled nearly-conformal gauge theory plasma using gauge theory–gravity duality (large-$T$ limit)
  ($\kappa = 2$, $\kappa = 4.558 - 4.935$)
  (P. Benincasa et al., NPB 733, 160 (2006);
  A. Buchel, PRD 72, 106002 (2005);)

- Gribov’s plasma of confining gluons

approximate linear scaling in phenomenologically important regime, in line with strongly-coupled approaches
recently discovered massless mode of the QGP, in line with the holographic quasinormal mode

(N. Su and K. Tywoniuk, PRL 114 (2015), no. 16 161601;
Results: bulk and shear viscosity – large-T and small-T limits

- \( T \gg \gamma_G \)
  \[ \frac{\zeta}{s} = \frac{5}{8 \sqrt{2}\pi^3} \frac{\gamma_G^3 \tau_{\text{rel}}}{T^2} + O\left(\frac{1}{T^4}\right) \]
  \[ \frac{\eta}{s} = \frac{\tau_{\text{rel}}}{5} + O\left(\frac{1}{T}\right) \]

- \( \zeta/\eta \) for GZ plasma
  \[ \frac{\zeta}{\eta} = \kappa \left(\frac{1}{3} - c_s^2\right) \]
  \[ \kappa = \frac{5}{2} \quad (T \gg \gamma_G) \]
  \[ \frac{\zeta}{\eta} = \frac{5}{3} \quad (T \ll \gamma_G) \]

- \( \zeta/\eta \) for massive Bose-Einstein gas
  \( \zeta/\eta = \kappa \left(\frac{1}{3} - c_s^2\right)^{3/2} \]
  \[ \kappa = \frac{5 \sqrt{15}}{2} \approx 5.81 \quad (T \gg m) \]
  \[ \frac{\zeta}{\eta} = \frac{2}{3} \quad (T \ll m) \]
Summary

- **dynamic and non-equilibrium description** of a plasma consisting of confining gluons (obtained from the Gribov quantization of SU(3) YM theory) introduced for the first time

- the expressions for the shear and bulk viscosities of the Gribov-Zwanziger plasma were derived and checked against exact solutions of the kinetic equation

- several studied features suggest that the GZ plasma is a strongly coupled system:
  - $\zeta/\eta$ T-scaling which is in line with the strong-coupling methods results was found
  - large $\zeta/s$

- Gribov-Zwanziger quantization can be used to address in- and out-of-equilibrium physics in a unified way, which can be useful for future phenomenological applications to ultrarelativistic heavy-ion collisions

Outlook

- include the running $\gamma_G(T)$

- use our formula in the hydrodynamic symulations
Thank you for your attention!
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