Compact Stars with a Dyson-Schwinger Quark Model

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I. INTRODUCTION

The possible appearence of quark matter (QM) in the interior of massive neutron stars (NS) and the hypothetical stable strange quark matter and strange stars are important questions both in studying QCD in extreme conditions and compact stars. The value of the maximum mass of a NS is probably one of the physical quantities that are most sensitive to the EOS of QM. Unfortunately, the QM EOS is poorly known at zero temperature and at the high baryonic density appropriate for NS. One has, therefore, to rely on models of QM, which are usually too far from real QCD. Thus, we try to develop quark models in the framework of the Dyson-Schwinger equations(DSE)[1–3], which is well based on QCD. We study the DSE quark models with a density dependent gluon propagator and different quark-gluon vertices, i.e. the Rainbow approximation, the Ball-Chiu (BC) vertex[4] and the 1BC vertex[5, 6]. To study possible hadron-quark phase transition and hybrid star structure, we combine a definite baryonic EOS, developed within the Brueckner-Hartree-Fock (BHF) many-body approach of nuclear matter. Due to uncertainty of parameters in our model, we also investigate the hypothetical strange quark matter and strange stars.

II. EQUATION OF STATE OF DENSE MATTER

A. EOS of hadronic matter within Brueckner theory

The Brueckner-Bethe-Goldstone theory is based on a linked cluster expansion of the energy per nucleon of nuclear matter (see Ref. [7], chapter 1 and references therein). The basic input quantities in the Bethe-Goldstone equation are the nucleon-nucleon (NN) two-body potentials V. In this work, we chose the EoS with the Bonn B potential, supplemented by a compatible microscopic three body forces [8]. Due to the hyperonic EOS in this theory is too soft, we neglect hyperons in this work.

B. Quark Matter

We work in the formalism of DSE in Euclidean space. The DSE of the quark propagator reads

$$S(p;\mu)^{-1} = Z_2\,[ip + i\gamma_4(p_4+i\mu) + m_q] + Z_1 \int^\Lambda \frac{d^4q}{(2\pi)^4}\, g^2(\mu) D_{\rho\sigma}(p-q;\mu) \frac{\lambda^a}{2}\gamma_\rho S(q;\mu) \Gamma^a_\sigma(q,p;\mu) \quad (1)$$

in which models of gluon propagator and quark-gluon vertex are needed.

For the gluon propagator, we employ the Landau gauge form with an infrared dominant interaction modified by the chemical potential

$$Z_1 g^2 D_{\rho\sigma}(k) = \pi d\, \frac{k^2}{\omega^6} exp(-\frac{k^2+\alpha\mu_q^2}{\omega^2})\,(\delta_{\rho\sigma} - \frac{k_\rho k_\sigma}{k^2})\,, \quad (2)$$

As to the quark gluon vertex, we investigate the 'Rainbow approximation', i.e. $\Gamma_\nu(q,k) = \gamma_\nu$. the 1BC vertex and the Ball-Chiu (BC)vertex, which satisfied the Ward Identity of QED and is well successful in studying QCD. The form of BC vertex at finite chemical potential is developed in [4],

$$\begin{aligned} i\Gamma_\sigma(k,\ell;\mu) &= i\Sigma_A(k,\ell;\mu)\gamma_\sigma^\perp + i\Sigma_C(k,\ell;\mu)\gamma_\sigma^\parallel \\ &\quad +(\tilde{k}+\tilde{\ell})_\sigma \left[\frac{i}{2}\gamma^\perp\cdot(\tilde{k}+\tilde{\ell})\Delta_A(\tilde{k},\tilde{\ell};\mu) + \frac{i}{2}\gamma^\parallel\cdot(\tilde{k}+\tilde{\ell})\Delta_C(\tilde{k},\tilde{\ell};\mu) + \Delta_B(\tilde{k},\tilde{\ell};\mu)\right] \quad (3) \\ \Sigma_F(k,\ell;\mu) &= \frac{1}{2}\left[F(\vec{k}^2,k_4;\mu) + F(\vec{\ell}^2,\ell_4;\mu)\right] \\ \Delta_F(\tilde{k},\tilde{\ell};\mu) &= \frac{F(\vec{k}^2,k_4;\mu) - F(\vec{\ell}^2,\ell_4;\mu)}{\tilde{k}^2-\tilde{\ell}^2} \end{aligned}$$

where $\tilde{k} = k + u$, $\gamma^\perp = \gamma - \hat{u}\gamma\cdot\hat{u}$, $\gamma^\parallel = \hat{u}\gamma\cdot\hat{u}$, with $\hat{u}^2 = 1$. The 1BC vertex is only the first row of BC vertex Eq.(3).

The EOS of cold QM is given following Refs. [4, 9]. We express the quark number density and other thermodynamical quantities as

$$\begin{aligned} n_q(\mu) &= 6\int \frac{d^3p}{(2\pi)^3}\, f_q(|p|;\mu)\,, & (4) \\ f_q(|p|;\mu) &= \frac{1}{4\pi}\int_{-\infty}^{\infty} dp_4\, \mathrm{tr_D}\left[-\gamma_4 S_q(p;\mu)\right]\,, & (5) \\ P_q(\mu_q) &= P_q(\mu_{q,0}) + \int_{\mu_{q,0}}^{\mu_q} \mathrm{d}\mu\, n_q(\mu)\,. & (6) \\ P_Q(\mu_u,\mu_d,\mu_s) &= \sum_{q=u,d,s} \tilde{P}_q(\mu_q) - B_{DS} & (7) \\ \tilde{P}_q(\mu_q) &\equiv \int_{\mu_{q,0}}^{\mu_q} \mathrm{d}\mu\, n_q(\mu)\,, & (8) \\ B_{DS} &\equiv -\sum_{q=u,d,s} P_q(\mu_{q,0})\,. & (9) \end{aligned}$$

III. NUMERICAL RESULTS

A. Parameter Setting

In our model, we set $\omega = 0.5GeV$, $d = 1GeV^2$(with Rainbow approximation), $d = 0.5GeV^2$(with BC vertex), $m_{u,d} = 0$, $m_s = 0.115GeV$, which is fitted by hadron properties in vacuum. α and B_{DS} are phenomenological parameter. In the following section for studying the hadron-quark phase transition, we investigate effects of different α and vertices, but keep $B_{DS} = 90MeVfm^{-3}$ a constant. We constrain the parameter α with different vertex ansätze by requiring the same phase transition point under the Maxwell construction (see Fig.1). In Sec. III(E), to study the possibility of absolute stable strange quark matter, we extend the parameter space of α and B_{DS}.

B. Hadron-Quark Phase Transition in Beta Stable Matter

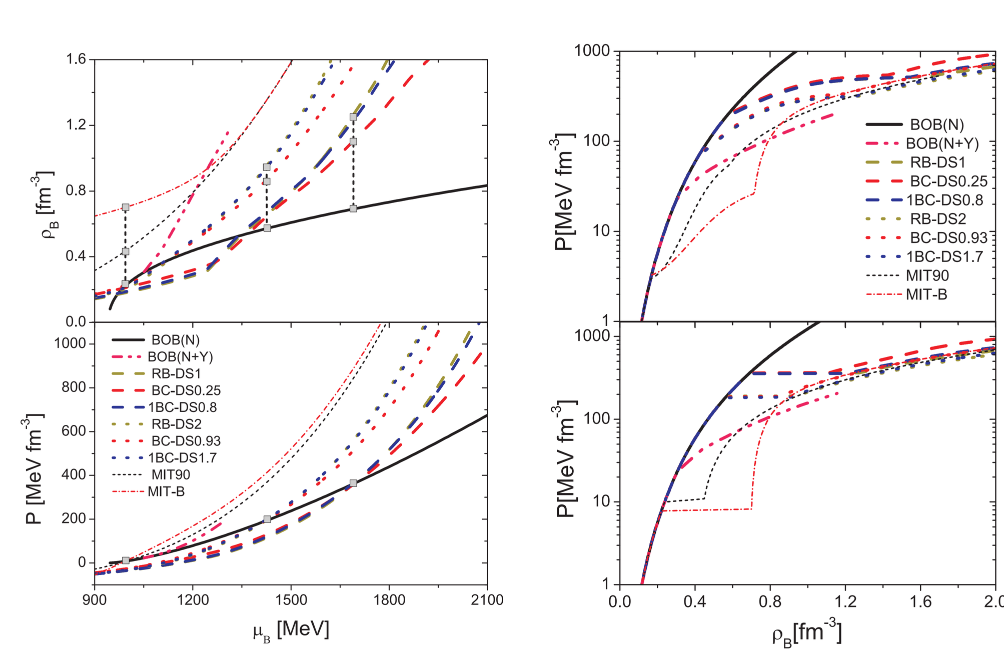

FIG. 1: The EOS of beta stable matter with hadron quark phase transition.

In Fig.1, we show EOS of beta stable matter with hadron quark phase transition. On the left panel, baryon number density and pressure depending on baryon chemical potential are shown. The crossing points (left lower panel) and vertical lines (left upper panel) represent the phase transition under Maxwell construction. On the right panel, we show the pressure depending on baryon number densities, with Glendenning construction (right upper panel) and Maxwell construction (right lower panel) respectively. For comparison, we also show the results of a hyperon EOS and quark mater EOS within MIT bag models [5].

C. Hybrid Stars Structure

A static NS structure can be obtained by solving the standard TOV equation, combing our EOS of the beta stable matter. Since the EOSs under Maxwell construction give unstable star structure, we only show our results in Fig.2 with the EOS under Glendenning construction (the right upper panel of Fig.1)

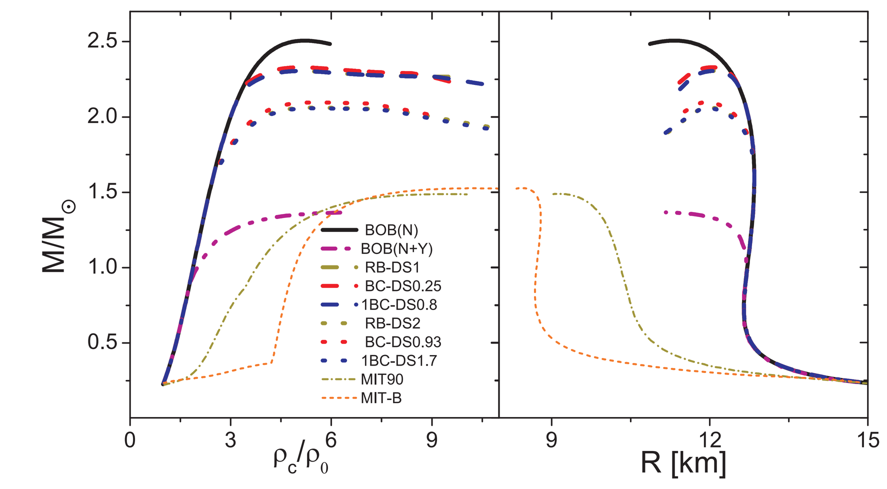

FIG. 2: Gravitational NS mass vs. the central baryon density (left panel) and the radius (right panel).

D. Rotation Effects on NS Structure

We investigate the effects of rotation on NS with the numerical code by N. Stergioulas, (available online http://www.gravity.phys.uwm.edu/rns/). In Fig.3 we show the R-M relation of rotating NS with our EOS RB1(left panel) and RB2(right panel). The rotating frequencies Ω are in units of $10^4 rad/s$ and Ω_k represents the Kepler frequency.

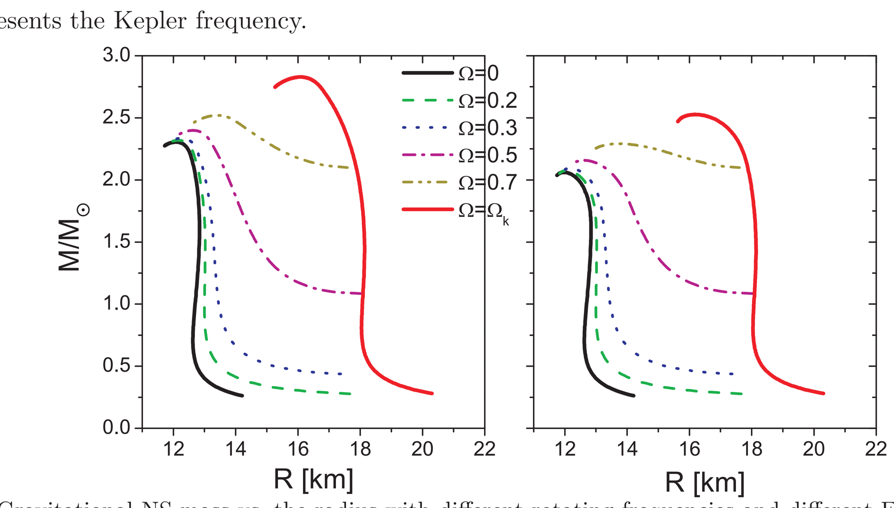

FIG. 3: Gravitational NS mass vs. the radius with different rotating frequencies and different EOS (RB1 for the left and RB2 for the right).

E. Strange Quark Matter and Strange Stars

Under the strange matter hypothesis, the ground state with $P = 0$ is not the hadron matter but the strange quark matter with a large density. The stability of the strange quark matter require that the baryon chemical potential $\mu_B < 930.4MeV$. Meanwhile, the stability of nuclei and neutrons against symmetric two-flavor quark matter require that $\mu_B > 939.6MeV$ for the two-flavor quark matter at $P = 0$ [10]. With BC vertex, only the results with $\alpha = 2$ and $B_{DS} = 50Mevfm^{-3}$ approximately fulfill the above requirements, see Table.I. In Table.II we show the boundary of B_{DS} under the above constrains with different vertices and α. We can see that only when $\alpha = 2$ (for BC vertex) and $\alpha = 5$(for rainbow approximation), $B_{min} < B_{max}$ and stable strange quark matter is possible.

α	$B_{DS}[MeVfm^{-3}]$	$\mu_B(N_f=3)[GeV]$	$n_B[fm^{-3}]$	u[%]	s[%]	$\mu_B(N_f=2)[GeV]$
0.25	50	0.928	0.182	0.339	0	0.906
	70	1.926	0.227	0.339	0	1.002
	90	1.107	0.268	0.34	0	1.080
1	5	0.929	0.188	0.338	0	0.906
	70	1.021	0.26	0.336	0.082	1.002
	90	1.088	0.344	0.334	0.174	1.077
2	5	0.919	0.232	0.334	0.159	0.939
	7	0.992	0.316	0.334	0.226	1.032
	9	1.05	0.388	0.333	0.255	1.11

TABLE I: Properties of quark matter at $P = 0$.

	BC			RB			
α	0.25	1	2	0.5	4	5	20
$B_{min}[Mevfm^{-3}]$	56.5	56.1	50.2	48.6	53.5	55.1	63.1
$B_{max}[Mevfm^{-3}]$	50.5	50.2	52.8	43.3	51.8	56.6	79

TABLE II: Constraints on B_{DS} for strange quark matter hypothesis.

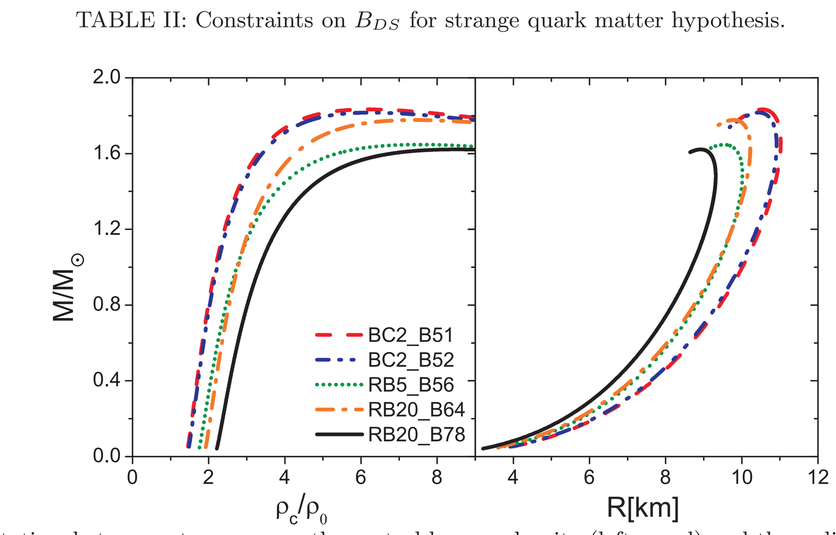

FIG. 4: Gravitational strange star mass vs. the central baryon density (left panel) and the radius (right panel).

Combing the EOS of stable strange quark matter, we solve the standard TOV equations to obtain the structure of static bare strange stars. The results are shown in Fig.4. The maximum mass of strange quark stars are about 1.6-1.8 solar mass, a little lower than 2-solar mass.

IV. SUMMARY AND REMARKS

We develop a quark model based on the Dyson-Schwinge equations of quark propagator at finite chemical potentials with various quark-gluon vertex and a chemical potential dependent effective interaction. We obtain the EoS of cold dense quark matter under beta-equilibrium and discuss the hadron-quark phase transition, in combination with a hadronic EOS given by BBG theory. We find that the phase transition and the equation of state of the quark or mixed phase and consequently the resulting hybrid star mass and radius depend mainly on a global reduction of the effective interaction due to effects of both the quark-gluon vertex and gluon propagator, but are not sensitive to details of the vertex ansatz.

We also investigate the rotation effects on NS. With observed pulsar frequencies up to now (lower than 3000rad/s), the rotating effects are still smaller on heavy NS.

To investigate the strange quark matter hypothesis, we extend our parameter space. We found possible range of parameter for stable strange quark matter, while the corresponding maximum strange quark stars are about 1.6-1.8 solar mass, a little lower than 2-solar mass. Though the possibility of a 2-solar mass strange star cannot be excluded, considering other effects such as rotation, the maximum mass of strange stars can hardly be larger than 2-solar mass.

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