Introduction

The original proposal of Matsui and Satz [1] was based on the idea of color screening

\[ V(r) = -\frac{N_a e^{-mr}}{r}, \]

(1)

if the heavy quarks are separated by a distance bigger than the Debye radius they cannot see each other.

Latter on it was found that quarkonium potential in perturbation theory had an imaginary part [2] of the size of the real part. This represents the process of a color singlet decaying into a color octet.

The opposite process, namely two heavy quarks from different origin recombining into a bound state, is also possible.

- It is desirable to go from a static to a dynamic picture of quarkonium.
- Quantum evolution can be studied [3, 4] but in practice it is very challenging to solve the equations, specially when we have many heavy quarks.
- At large times the evolution can be studied using a Langevin equation. This has been studied in QED [5]. Our aim here is to derive the QCD case.

Langevin dynamics in QCD

After a rigorous derivation using open quantum systems techniques we get for heavy quarks

\[ M\ddot{r}_f = -\frac{C_F N_c}{6} \nabla^2 \left[ \frac{D(0)}{b t} + \Theta_a(t) + \sum_{b\neq a} \Theta_{ab}(r_a - r_b, t) \right] \]

(5)

and for heavy antiquarks

\[ M\ddot{r}_f = -\frac{C_F N_c}{6} \nabla^2 \left[ \frac{D(0)}{b t} + \Theta_a(t) + \sum_{b\neq a} \Theta_{ab}(r_a - r_b, t) \right] \]

(6)

where

\[ D(r) = \nabla \cdot V(r) \]

(7)

and \( \Theta_a \), \( \Theta_{ab} \) and \( \Theta_{aa} \) are random fields. \( \psi_a \) is analogous to \( \xi \) in QED

\[ \langle \Theta_{aa}(r_a - r_b, t) \rangle = 0 \]

(8)

\[ \langle \Theta_{ab}(r_a - r_b, t) \Theta_{bp}(r_b - r_p, t') \rangle = \frac{C_F N_c}{6} \nabla_i \nabla_j \left( R V(r_i - r_j) \right) \delta_{ac} \delta_{bd} (t - t') \]

(9)

and \( \Theta_{aa} \) and \( \Theta_{ab} \) behave in a similar way.

Interpretation

- When two heavy quarks exchange a gluon they will get out of the color coherent state and they will arrive to another one with a decay width \( \alpha \sim (D(0) - D(r_a - r_b)) \). In order to influence the large time evolution they have to exchange another gluon (to go back to the incoherent state) before the intermediate state decays.

Differences between QED and QCD

\[ e^- + \gamma \rightarrow e^- \]

The electron changes momentum, but the electric charge is not modified. a, b is color in fundamental representation. A in the adjoint. The quark changes color.

- QED. Medium introduces uncertainty in position. Force between quarks depends on the position of the particles.
- QCD. Medium introduces uncertainty in position and in color. Force depends on the position and on color state.

Color decoherence, expected at large times

- The color state of each heavy quark is independent of the color state of the others. Maximize entropy.
- A heavy quark has equal probability to be in any color state.

This implies that whenever I have a heavy quark and a heavy antiquark pair

\[ P_{\text{singlet}} = \frac{1}{N_c^2}, \quad P_{\text{octet}} = \frac{N_c^2 - 1}{N_c^2} \]

(3)

- Singlet. The force is attractive.
- Octet. The force is repulsive and \( \frac{1}{N_c} \) weaker than that of the singlet.

\[ F_{\text{singlet}}(t) P_{\text{singlet}} + F_{\text{octet}}(t) P_{\text{octet}} = 0 \]

(4)

The mean value is 0 but we have fluctuations around this value → The force between heavy quarks is a stochastic force.

Bibliography


Numerical results for one heavy quark pair

![Figure: Relative distance for the simulation of 10 trajectories for a heavy quark and a heavy antiquark which were initially in a Dirac delta state. A cut-off is introduced to regulate the potential at short distances. \( k \) is defined as \( \sqrt{C_F N_c/6} \).

Numerical results for 50 heavy quark pairs

![Figure: Example of an event with 50 pairs created in a Dirac delta state and distributed randomly over a 5fm × 5fm plane.]}