Classical and quantum evolution of quarkonium in a plasma

Jean-Paul Blaizot and Miguel Ángel Escobedo (presenter)

Institute de Physique Théorique, CNRS-CEA

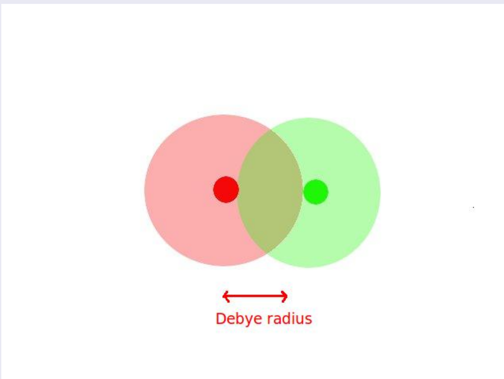
29th September, 2015

Introduction

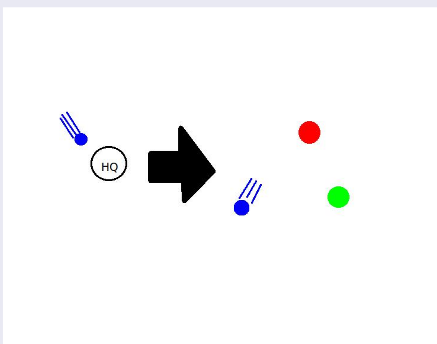
The original proposal of Matsui and Satz [1] was based on the idea of color screening

$$V(r) = -\alpha_s \frac{e^{-m_D r}}{r}, \qquad (1)$$

if the heavy quarks are separated by a distance bigger than the Debye radius they can not see each other.



Latter on it was found that quarkonium potential in perturbation theory had an imaginary part [2] of the size of the real part. This represents the process of a color singlet decaying into a color octet.



The opposite process, namely two heavy quarks from different origin recombining into a bound state, is also possible.

- It is desirable to go from a static to a dynamic picture of quarkonium. Compute time evolution of heavy quarks.
- Quantum evolution can be studied [3, 4] but in practice it is very challenging to solve the equations, specially when we have many heavy quarks.
- At large times the evolution can be studied using a Langevin equation. This has been studied in QED [5]. Our aim here is to derive the QCD case.

Langevin dynamics in QED

$$M\ddot{\mathbf{R}} = -M\gamma(\mathbf{R})\dot{\mathbf{R}} + \mathbf{F}(\mathbf{R}) + \xi(\mathbf{R},t) \qquad (2)$$

- Drag. The velocity of the heavy particle tends to be reduced as a result of collisions with particles in the medium.
- Force between heavy particles. A classical force that depends on the electric charge and the position of the heavy particles.
- Stochastic force. Random force due to the particles in the medium. Related to drag by fluctuation-dissipation theorem.

Differences between QED and QCD

$$e^- + \gamma \to e^-$$

The electron changes momentum, but the electric charge is not modified.

$$q_a + g_A \to q_b$$

a, b is color in fundamental representation. A in the adjoint. The quark changes color.

- QED. Medium introduces uncertainty in position. Force between quarks depends on the position of the particles.
- QCD. Medium introduces uncertainty in position and in color. Force depends on the position and on color state.

Color decoherence, expected at large times

- The color state of each heavy quark is independent of the color state of the others. Maximize entropy.
- A heavy quark has equal probability to be in any color state.

This implies that whenever I have a heavy quark and a heavy antiquark pair

$$P_{singlet} = \frac{1}{N_c^2} \quad P_{octet} = \frac{N_c^2 - 1}{N_c^2} \qquad (3)$$

- Singlet. The force is attractive.
- Octet. The force is repulsive and $\frac{1}{N_c^2-1}$ weaker than that of the singlet.

$$F_{singlet}(\mathbf{r})P_{singlet} + F_{octet}(\mathbf{r})P_{octet} = 0 \qquad (4)$$

The mean value is 0 but we have fluctuations around this value → The force between heavy quarks is a stochastic force.

Langevin dynamics in QCD

After a rigorous derivation using open quantum systems techniques we get for heavy quarks

$$M\ddot{\mathbf{r}}_a = -\frac{C_F\nabla^2 D(\mathbf{0})}{6T}\dot{\mathbf{r}}_a + \boldsymbol{\Psi}_a(t) + \sum_{b\neq a}^{Np} \boldsymbol{\Theta}^{pp}_{ab}(\mathbf{r_a}-\mathbf{r_b},t) + \sum_{\hat{b}}^{Na} \boldsymbol{\Theta}^{pa}_{a\hat{b}}(\mathbf{r_a}-\hat{\mathbf{r}}_{\hat{b}},t), \qquad (5)$$

and for heavy antiquarks

$$M\ddot{\hat{\mathbf{r}}}_{\hat{a}} = -\frac{C_F\nabla^2 D(\mathbf{0})}{6T}\dot{\hat{\mathbf{r}}}_{\hat{a}} + \boldsymbol{\Psi}_{\hat{a}}(t) + \sum_{\hat{b}\neq\hat{a}}^{Na} \boldsymbol{\Theta}^{aa}_{\hat{a}\hat{b}}(\hat{\mathbf{r}}_{\hat{a}}-\hat{\mathbf{r}}_{\hat{b}},t) + \sum_{j}^{Np} \boldsymbol{\Theta}^{pa}_{\hat{a}b}(\hat{\mathbf{r}}_{\hat{a}}-\mathbf{r}_b,t) \qquad (6)$$

where

$$D(\mathbf{r}) = \Im V(\mathbf{r}) \qquad (7)$$

and Ψ_a, Θ^{pp},Θ^{pa} and Θ^{aa} are random fields. Ψ_a is analogous to ξ in QED

$$\langle \Theta^{pp}_{i,ab}(\mathbf{r}_a - \mathbf{r}_b, t)\rangle = 0 \qquad (8)$$

$$\langle \Theta^{pp}_{i,ab}(\mathbf{r}_a - \mathbf{r}_b, t)\Theta^{pp}_{j,cd}(\mathbf{r}_c - \mathbf{r}_d, t')\rangle = -\frac{C_F \nabla_\mathbf{i}(\Re V(\mathbf{r}_a - \mathbf{r}_b))\nabla_\mathbf{j}(\Re V(\mathbf{r}_c - \mathbf{r}_d))}{N_c^2(D(\mathbf{0}) - D(\mathbf{r}_a - \mathbf{r}_b))}\delta_{ac}\delta_{bd}\delta(t-t') \qquad (9)$$

and Θ^{pa} and Θ^{aa} behave in a similar way.

Interpretation

- When two heavy quarks exchange a gluon they will get out of the color incoherent state and they will arrive to another one with a decay width $\propto -(D(\mathbf{0}) - D(\mathbf{r}_a - \mathbf{r}_b))$. In order to influence the large time evolution they have to exchange another gluon (to go back to the incoherent state) before the intermediate state decays.

Numerical results for one heavy quark pair

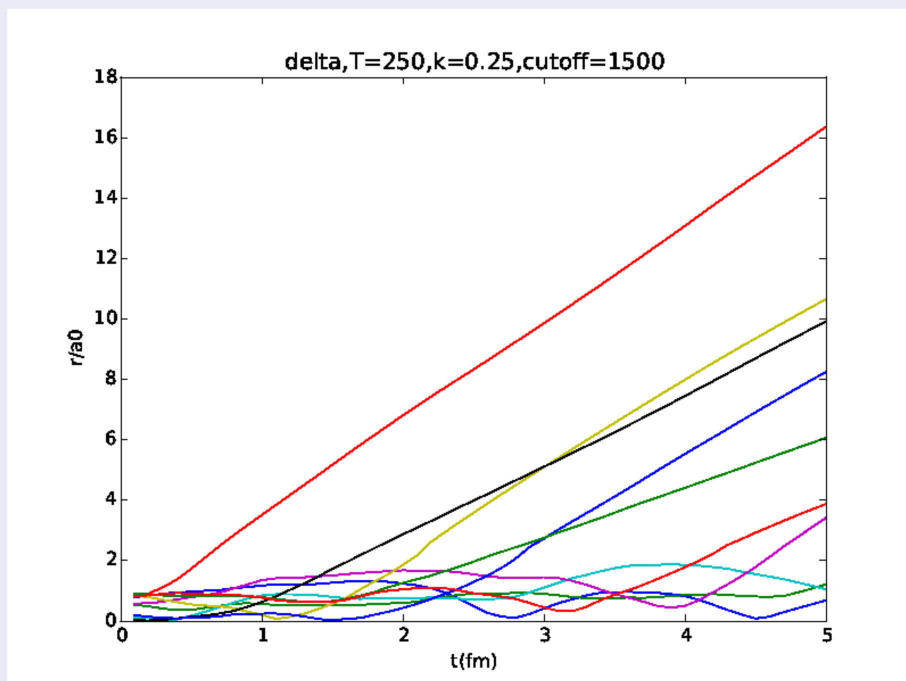


Figure: Relative distance for the simulation of 10 trajectories for a heavy quark and a heavy antiquark which were initially in a dirac delta state. A cut-off is introduced to regulate the potential at short distances. k is defined as $\frac{C_F\nabla^2 D(\mathbf{0})}{3T^3}$.

Numerical results for 50 heavy quark pairs

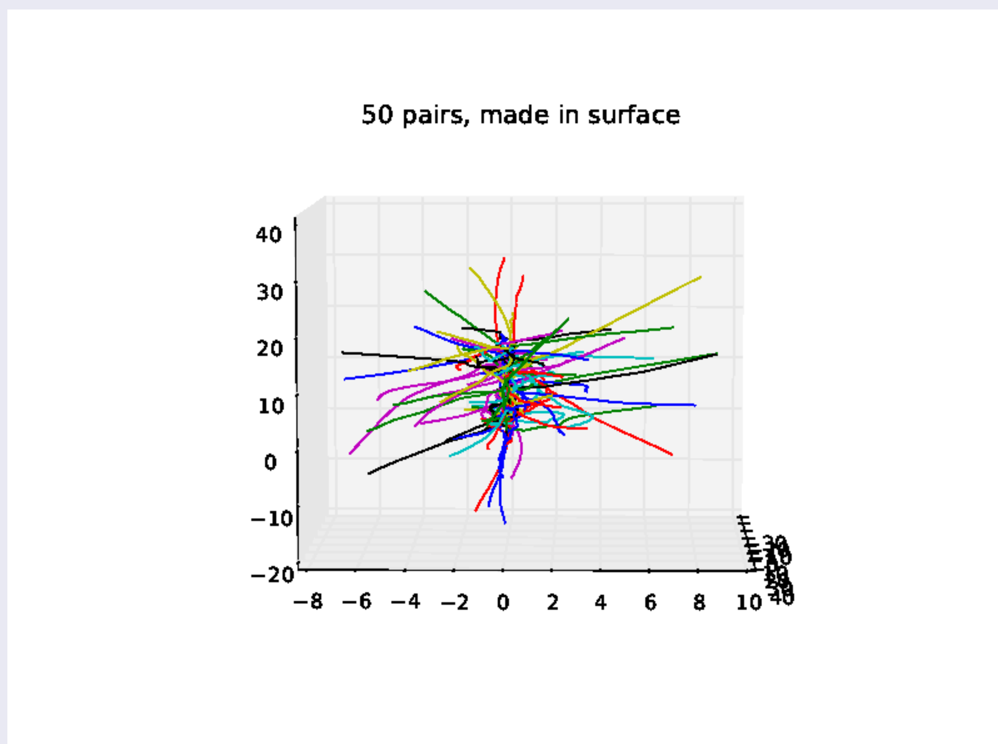


Figure: Example of an event with 50 pairs created in a Dirac delta state and distributed randomly over a $5fm \times 5fm$ plane.

Bibliography

[1] T. Matsui and H. Satz, Phys. Lett. B **178** (1986) 416.

[2] M. Laine, O. Philipsen, P. Romatschke and M. Tassler, JHEP **0703** (2007) 054 [hep-ph/0611300].

[3] Y. Akamatsu, Phys. Rev. D **91** (2015) 5, 056002 [arXiv:1403.5783 [hep-ph]].

[4] N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, In preparation.

[5] J. P. Blaizot, D. De Boni, P. Faccioli and G. Garberoglio, arXiv:1503.03857 [nucl-th].