Ivan Vitev

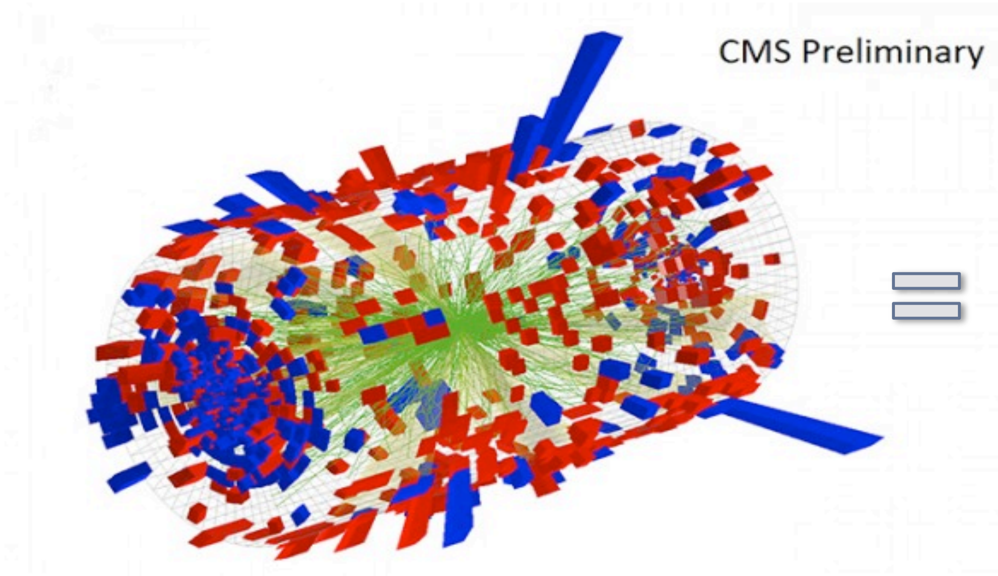
Soft-Collinear Effective Theory for hadronic and nuclear collisions: The evolution of jet quenching form RHIC to the highest LHC energies

Quark Matter 2015, September - October 2015
Kobe Fashion Mart, Kobe, Japan

Outline of the talk

- An effective theory for jet propagation in matter $SCET_G$. A tool to improve upon the the energy loss approach. Medium induced splitting kernels
- Connecting the energy loss and the QCD evolution approaches. Illustration at RHIC and predictions for the LHC
- Evolution and resummation for jet substructure observables and jet shapes. NLL results in p+p collisions
- Quenching of reconstructed jets, modification of the differential jet shapes beyond the energy loss approach

Logs, Legs, and Loops



S. Chatrchyan et al. PLB (2013)

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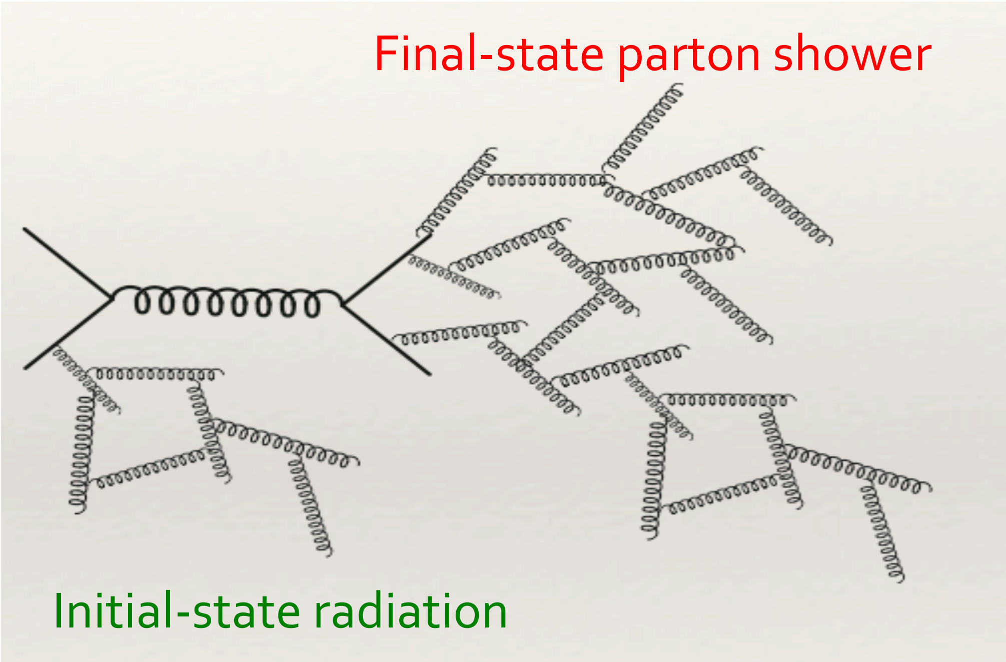


- Traditional energy loss approach, phenomenologically successful but cannot be systematically improved, higher orders and resummation
- In HEP significant effort has been devoted to understanding the parton shower. We demonstrate how this parton shower technology can be applied to heavy ion reactions, NLO, NLL, etc
- The same techniques should be applied to hard probes: particles, jets, and heavy flavor

The big picture

- Jet physics presents a multiscale problem, EFT treatment

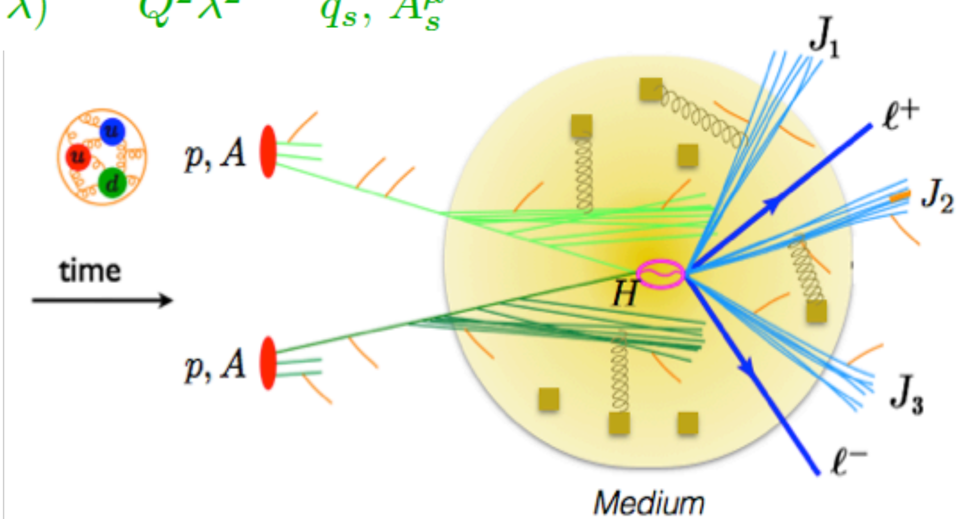
C. Bauer et al. (2001)

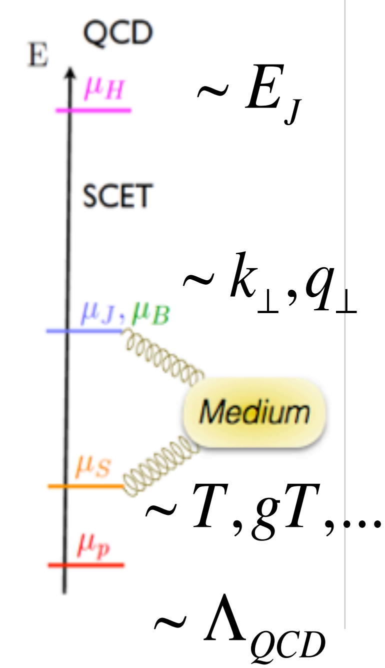
D. Pirol et al. (2004)

SCET (Soft Colliner Effective Theory)

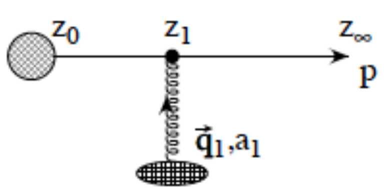
modes	$p^\mu = (+,-,\perp)$	p^2	fields
collinear	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$	$\xi_n,\ A_n^\mu$
soft	$Q(\lambda, \lambda, \lambda)$	$Q^2\lambda^2$	$q_s,\ A_s^\mu$

$$\sigma = \mathrm{Tr}(HS) \otimes \prod_{i=1}^{n_B} B_i \otimes \prod_{j=1}^{N} J_j$$





- Factorization, with modified J, B, S



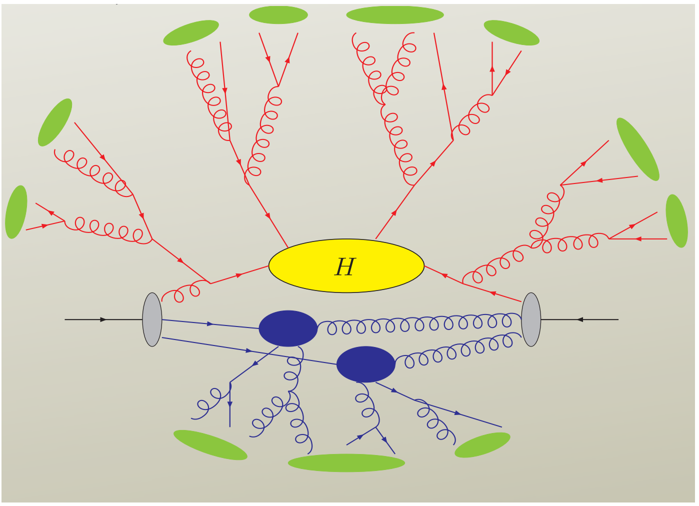
$q = (\lambda^2, \lambda^2, \lambda)Q$

Glauber gluons to mediate physical interactions with the QCD medium

A. Idilbi et al. (2008)

Ovanesyan et al. (2011)

In-medium parton splittings, properties, and DGLAP evolution



G. Altarelli et al. (1977)

- Implemented in DGLAP evolution equations

$$\frac{dN(tot.)}{dxd^2k_\perp} = \frac{dN(vac.)}{dxd^2k_\perp} + \frac{dN(med.)}{dxd^2k_\perp}$$

G. Ovanesyan et al. (2012)

$$\left(\frac{dN}{dx\,d^2\boldsymbol{k}_\perp}\right)_{q\to qg} = \frac{\alpha_s}{2\pi^2} C_F \frac{1+(1-x)^2}{x} \int \frac{d\Delta z}{\lambda_g(z)} \int d^2\mathbf{q}_\perp \frac{1}{\sigma_{el}} \frac{d\sigma_{el}^{\rm medium}}{d^2\mathbf{q}_\perp} \left[-\left(\frac{A_\perp}{A_\perp^2}\right)^2 + \frac{B_\perp}{B_\perp^2}\cdot\left(\frac{B_\perp}{B_\perp^2} - \frac{C_\perp}{C_\perp^2}\right) \right.$$

$$\times\left(1-\cos[(\Omega_1-\Omega_2)\Delta z]\right) + \frac{C_\perp}{C_\perp^2}\cdot\left(2\frac{C_\perp}{C_\perp^2} - \frac{A_\perp}{A_\perp^2} - \frac{B_\perp}{B_\perp^2}\right)\left(1-\cos[(\Omega_1-\Omega_3)\Delta z]\right)$$

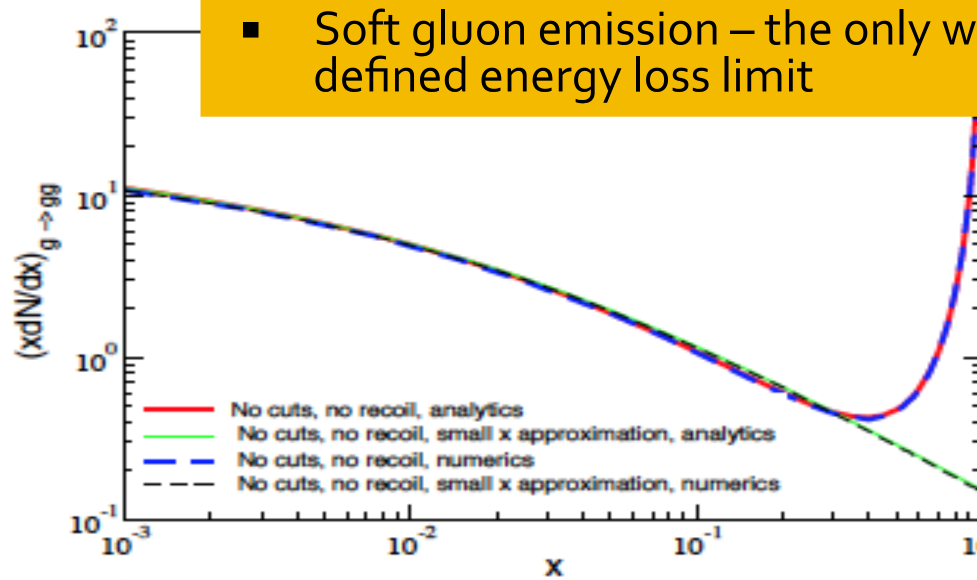
$$+\frac{B_\perp}{B_\perp^2}\cdot\frac{C_\perp}{C_\perp^2}\left(1-\cos[(\Omega_2-\Omega_3)\Delta z]\right) + \frac{A_\perp}{A_\perp^2}\cdot\left(\frac{A_\perp}{A_\perp^2} - \frac{D_\perp}{D_\perp^2}\right)\cos[\Omega_4\Delta z]$$

$$\left. +\frac{A_\perp}{A_\perp^2}\cdot\frac{D_\perp}{D_\perp^2}\cos[\Omega_5\Delta z] + \frac{1}{N_c^2}\frac{B_\perp}{B_\perp^2}\cdot\left(\frac{A_\perp}{A_\perp^2} - \frac{B_\perp}{B_\perp^2}\right)\left(1-\cos[(\Omega_1-\Omega_2)\Delta z]\right)\right].$$

$N.B.\ x \to 1-x$ $\qquad A,..D,\Omega_1..\Omega_5 - functions(x,k_\perp,q_\perp)$

As in vacuum, a total of 4 splitting functions

- Soft gluon emission – the only well defined energy loss limit



QCD evolution in the soft gluon energy loss limit

$$P_{q\to qg} = \frac{2C_F}{x_+} + \left(\frac{2C_F}{x} g[x,Q,L,\mu]\right)_+ ,$$

$$P_{g\to gg} = \frac{2C_A}{x_+} + \left(\frac{2C_A}{x} g[x,Q,L,\mu]\right)_+ ,$$

$$P_{g\to q\bar{q}} = 0,$$

$$P_{q\to gq} = 0,$$

- If a connection is to be found between the energy loss and the evolution approach, it is in the soft gluon limit
- We solve the DGLAP evolution equations analytically

$$D_{h/c}^{\rm med.}(z,Q) = {\rm e}^{-2C_R \frac{\alpha_s}{\pi}\left[\ln\frac{Q}{Q_0}\right]\{[n(z)-1](1-z)-\ln(1-z)\}} D_{h/c}(z,Q_0)$$

Analytic solution to DGLAP evolution

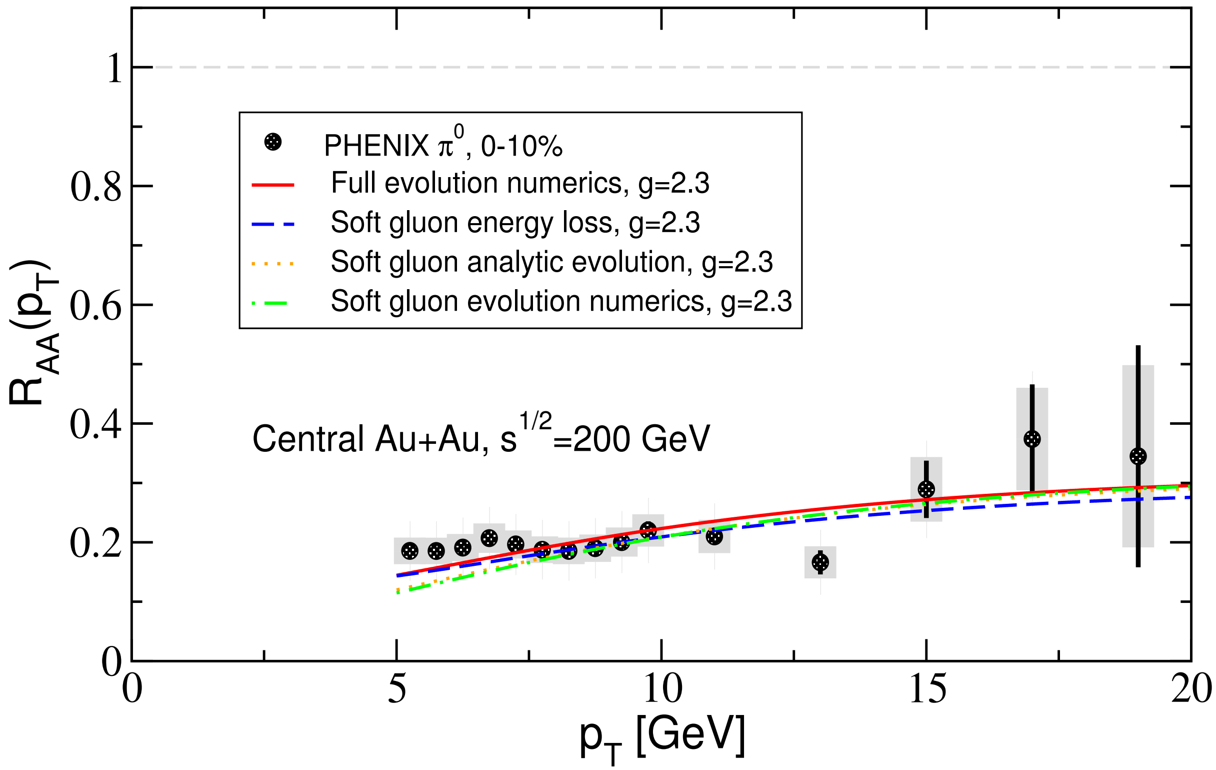
$$\times e^{-[n(z)-1]\left\{\int_0^{1-z} {\rm d}z'\, z' \int_{Q_0}^{Q} dQ' \frac{dN}{dz'dQ'}(z',Q')\right\} - \int_{1-z}^{1} {\rm d}z' \int_{Q_0}^{Q} dQ' \frac{dN}{dz'dQ'}(z',Q')}$$

$$= D_{h/c}(z,Q) e^{-[n(z)-1]\left\langle \frac{\Delta \tilde{E}}{E}\right\rangle_z - \langle \tilde{N^g} \rangle_z} .$$

- *The main result*: direct relation between the evolution and energy loss approaches first established here

Comparison of energy loss and QCD evolution approaches

- The in-medium QCD evolution approach works over a wide variety of energies. This is, of course, expected because we have an analytic proof of the relation between QCD evolution and energy loss

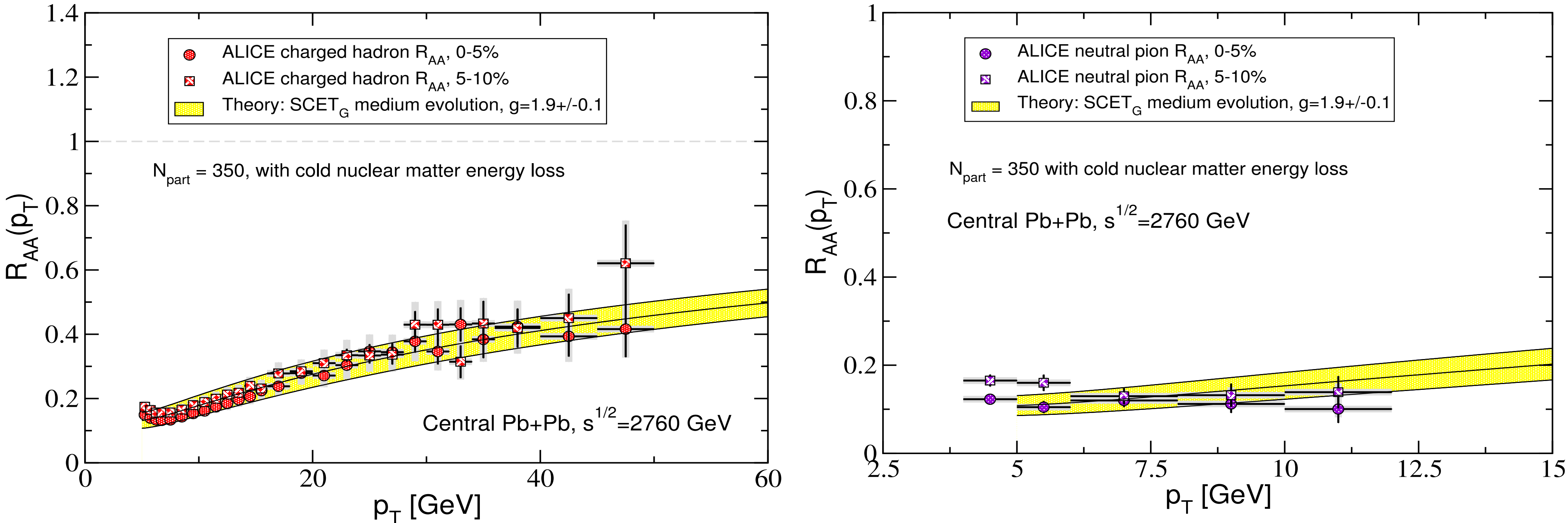


Z. Kang et al. (2014)

- 4 calculations compared
- Eliminates the uncertainty associated with the application of the e-loss.
- Can constrains $\Delta g/g=5\%$
- Validates more than a decade of hadron suppression phenomenology

Results at the LHC 2.76 TeV

- Good description of the p_T dependence if inclusive particle quenching, including high transverse momenta, centrality dependence, particle species

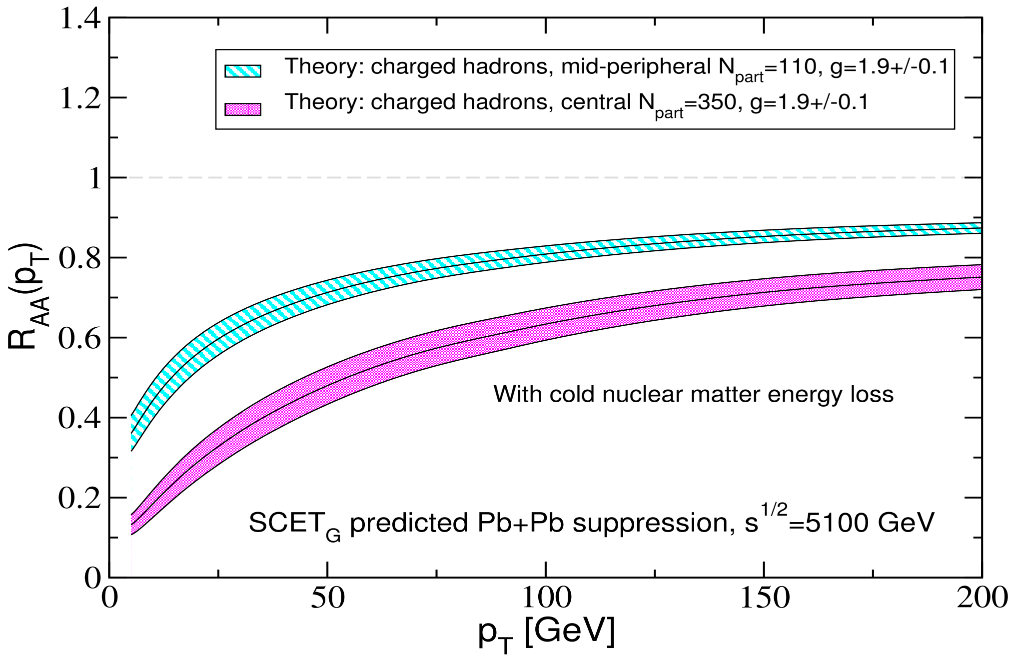


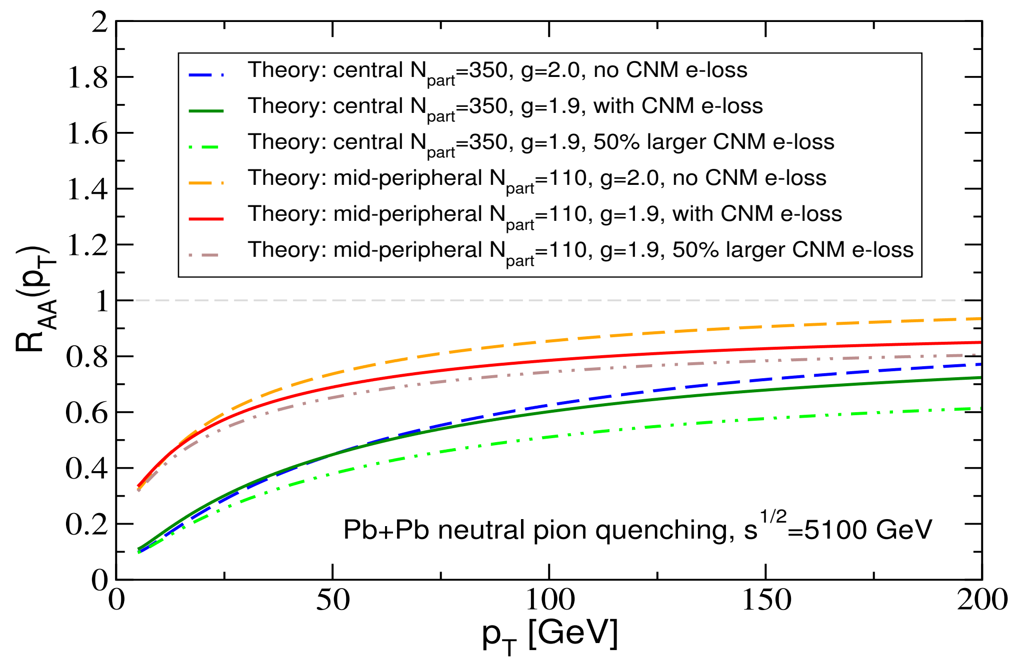
A. Emerman et al. (2015)

- Some discrepancy in the p_T trend of neutral pion suppression (magnitude appears well described). Important to check at higher CM energies.

Predictions for the LHC 5.1 TeV run

- We find results very similar to the 2.76 TeV run, within 10% of the known R_{AA}, accounting for the perturbative spectra, < 10% increase in the medium density, CNM effect



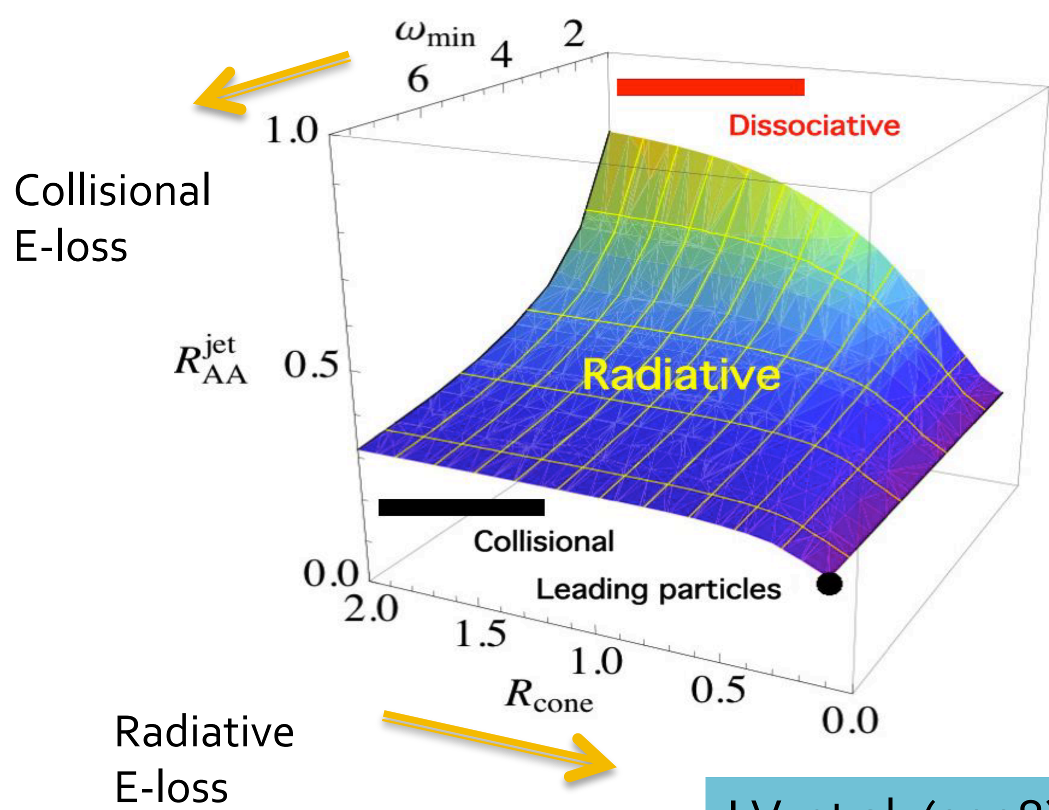


- At the highest transverse momenta there is sensitivity to CNM especially CNM energy loss
- Further results and predictions are available in the preprint arXiv:

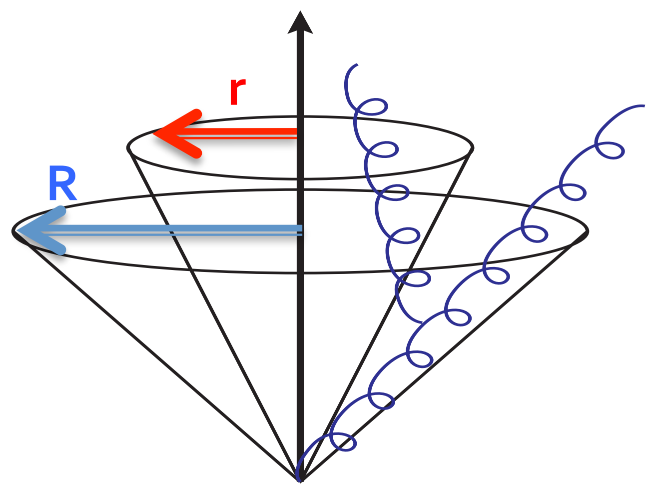
Applications of $SCET_G$ to jet shapes and jet cross sections

- Jet cross sections reflect the total amount of energy retained in the jet cone

- Jet shapes reflect the energy density inside the jet and the structure of the parton shower



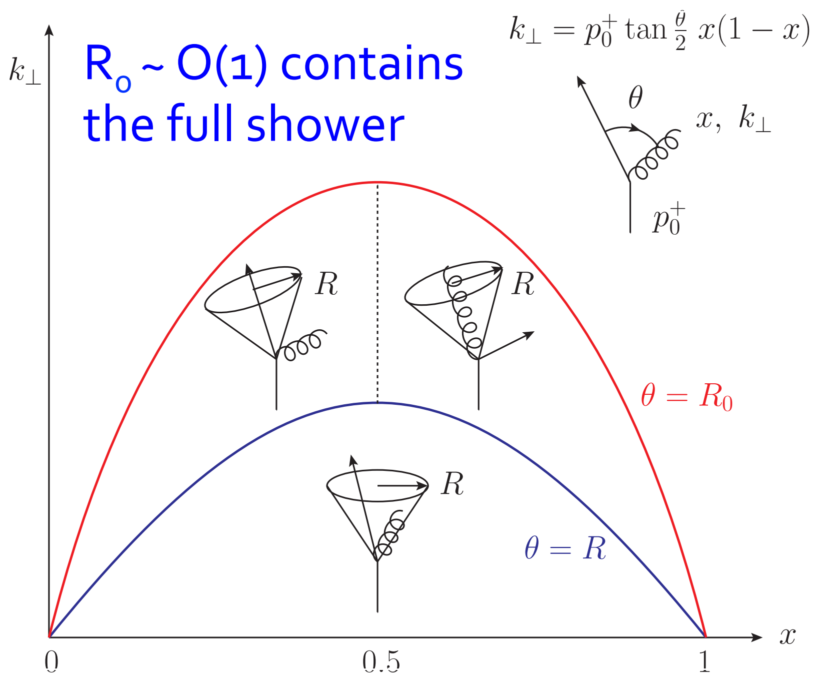
I.V. et al. (2008)



$$\Psi_{\rm int}(r;R) = \frac{\sum_i (E_T)_i \Theta(r-(R_{\rm jet})_i)}{\sum_i (E_T)_i \Theta(R-(R_{\rm jet})_i)} ,$$

$$\psi(r;R) = \frac{d\Psi_{\rm int}(r;R)}{dr} .$$

Generalizing the concept of energy loss to jets



- In contrast to haron production, the jet definition allows to generalize the concept of energy loss beyond the soft gluon approximation

Y.-T. Chien et al. (2015)

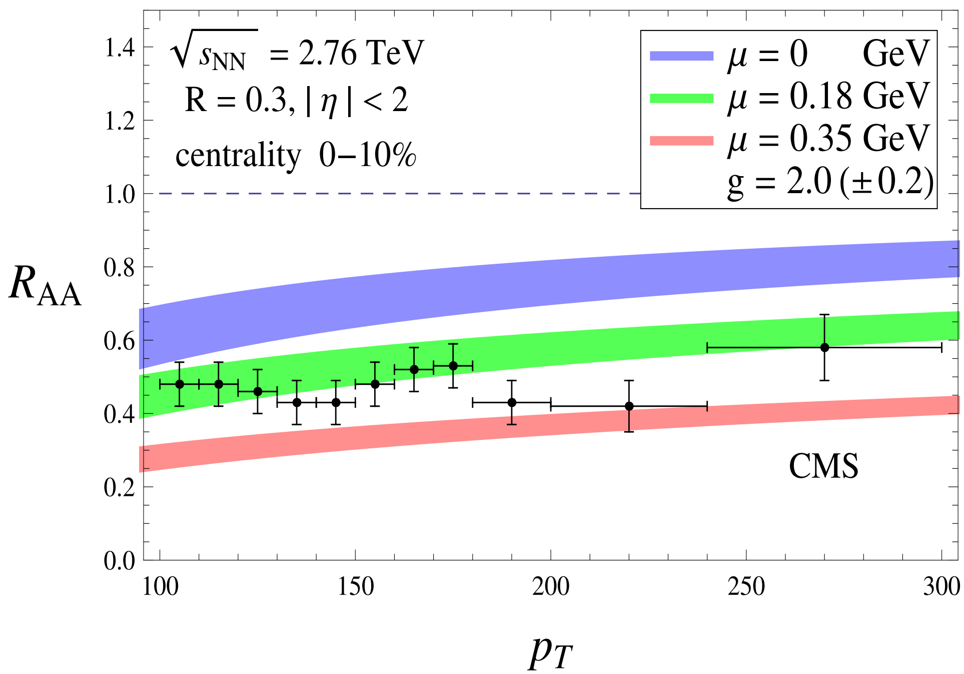
- The universal quantities – in-medium parton splitting functions come into play

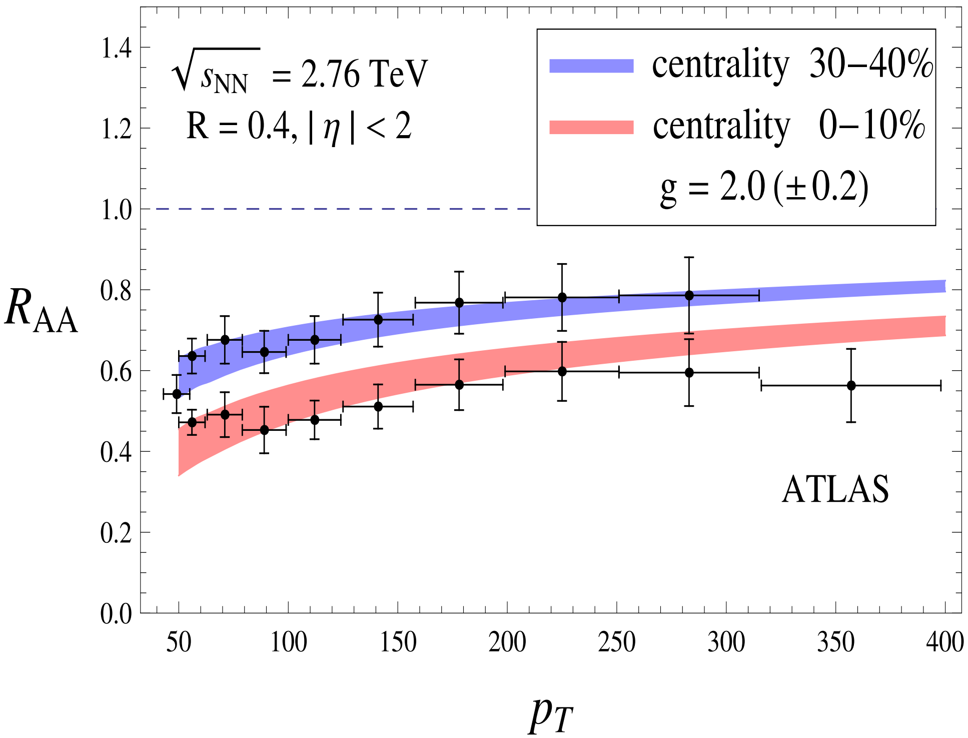
$$\epsilon_q = \frac{2}{\omega}\Big[\int_0^{\frac{1}{2}} dx k^0 + \int_{\frac{1}{2}}^{1} dx (p^0 - k^0)\Big] \int_{\omega x(1-x)\tan\frac{R}{2}}^{\omega x(1-x)\tan\frac{R_0}{2}} dk_\perp \frac{1}{2}\Big[\mathcal{P}^{med}_{q\to qg}(x,k_\perp) + \mathcal{P}^{med}_{q\to gq}(x,k_\perp)\Big]$$

$$\epsilon_g = \frac{2}{\omega}\Big[\int_0^{\frac{1}{2}} dx k^0 + \int_{\frac{1}{2}}^{1} dx (p^0 - k^0)\Big] \int_{\omega x(1-x)\tan\frac{R}{2} x(1-x)}^{\omega x(1-x)\tan\frac{R_0}{2}} dk_\perp \frac{1}{2}\Big[\mathcal{P}^{med}_{g\to gg}(x,k_\perp) + \sum_{q,\bar{q}} \mathcal{P}^{med}_{g\to q\bar{q}}(x,k_\perp)\Big]$$

Fractional energy loss outside of the jet beyond the soft gluon approximation

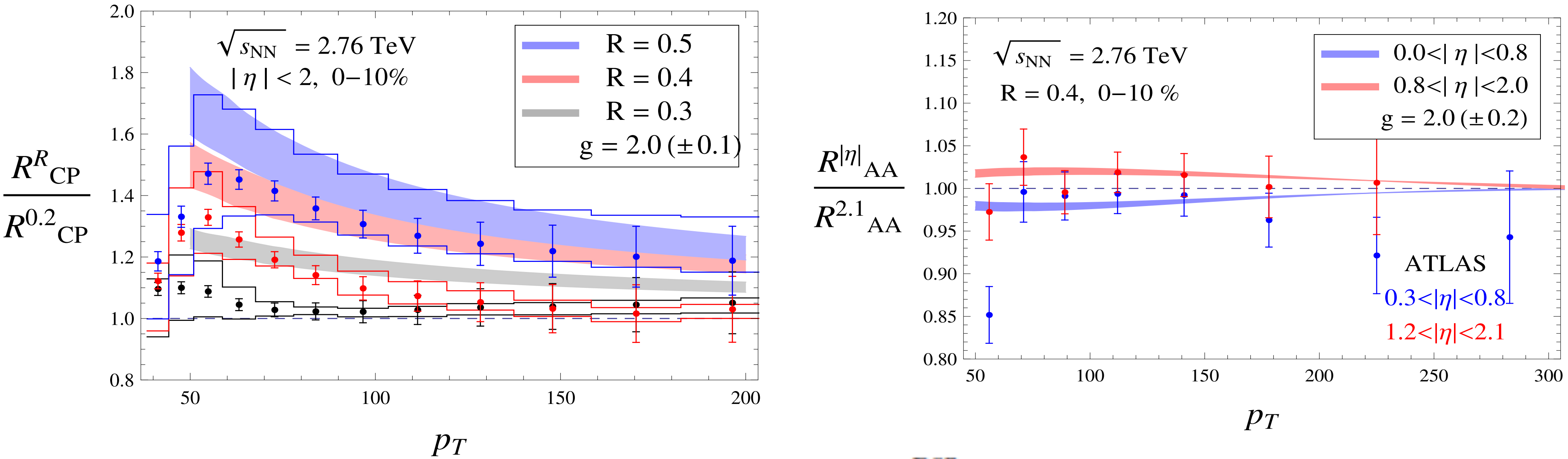
Suppression of reconstructed jets at the LHC, Pb+Pb at 2.76 TeV





- Cold nuclear matter effects contribute toward the inclusive jet suppression at high p_T. Approximately ½ of the effect
- Describes well the centrality dependence of the inclusive jet suppression
- There is some p_T dependence remaining to RAA. Important to investigate soft function effects, collisional energy loss

Jet radius and rapidity dependence of inclusive jet suppression



$$R^R_{cp}(p_T) = \langle N^{per}_{bin} \rangle \frac{d\sigma^{cen}_{AA}(p_T, R)}{dyd^2p_T} \Big/ \langle N^{cen}_{bin} \rangle \frac{d\sigma^{per}_{AA}(p_T, R)}{dyd^2p_T} = \frac{R^{cen}_{AA}(p_T, R)}{R^{per}_{AA}(p_T, R)}$$

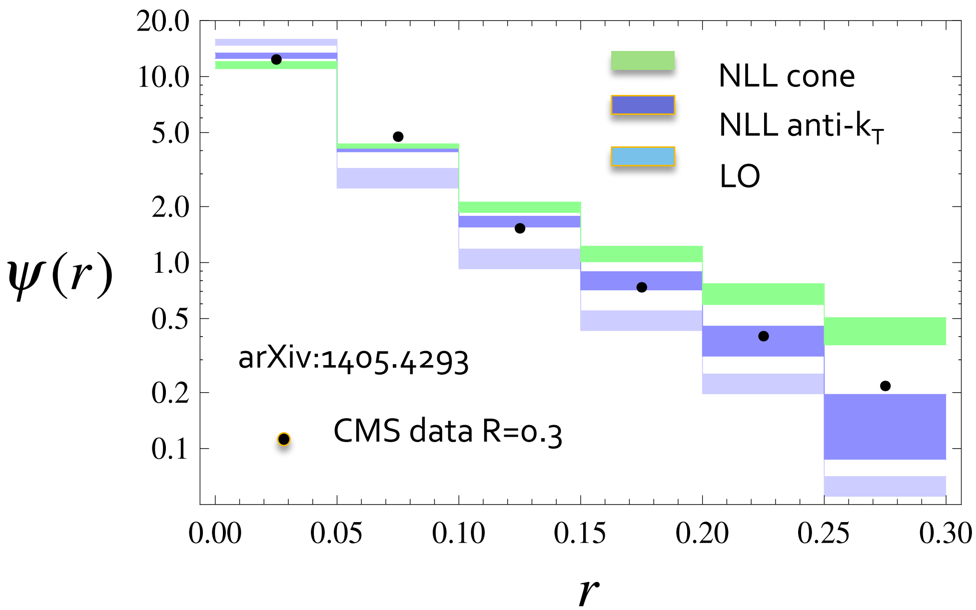
- The radius dependence of inclusive jet quenching versus p_T and R captured. For small radii the calculation over predicts the differences
- Rapidity dependence is consistent with 1. The trend is captured by the theoretical calculation

I.V. et al. (2008)

NLL calculation of jet shapes

- The jet shape is defined by the ratio of two jet energy functions

$$\Psi_\omega(r) = \frac{\langle E_r\rangle_\omega}{\langle E_R\rangle_\omega} = \frac{J_\omega^{E_r}(\mu)/J_\omega(\mu)}{J_\omega^{E_R}(\mu)/J_\omega(\mu)} = \frac{J_\omega^{E_r}(\mu)}{J_\omega^{E_R}(\mu)}$$



- To resum the jet shape to NLL accuracy we use SCET RG evolution techniques

$$\frac{dJ_\omega^{qE_r}(\mu)}{d\ln\mu} = \left[-C_F\Gamma_{\text{cusp}}(\alpha_s)\ln\frac{\omega^2\tan^2\frac{R}{2}}{\mu^2} - 2\gamma^q(\alpha_s)\right] J_\omega^{qE_r}(\mu)$$

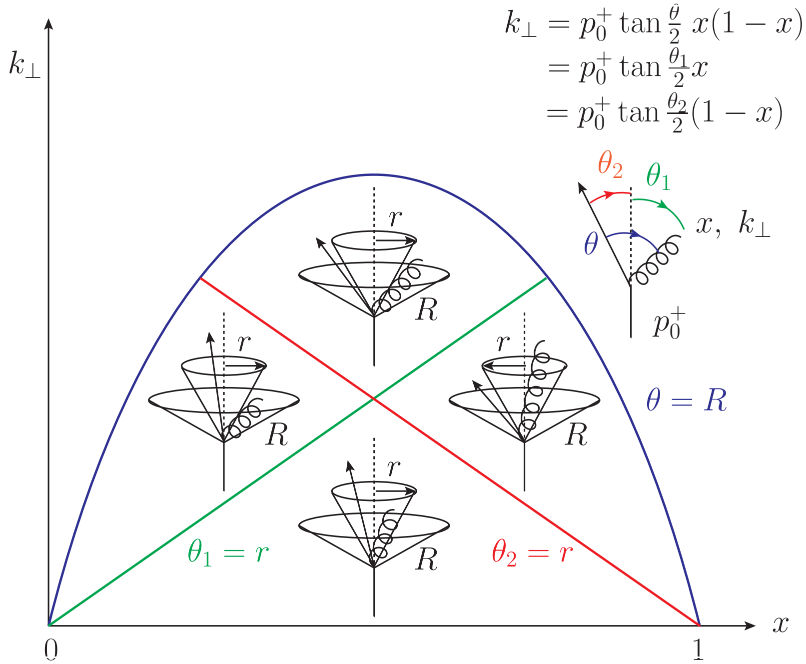
$$\frac{dJ_\omega^{gE_r}(\mu)}{d\ln\mu} = \left[-C_A\Gamma_{\text{cusp}}(\alpha_s)\ln\frac{\omega^2\tan^2\frac{R}{2}}{\mu^2} - 2\gamma^g(\alpha_s)\right] J_\omega^{gE_r}(\mu)$$

NLL	1-loop	2-loop
β	$\beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_F n_f$	$\beta_1 = \frac{34}{3}C_A^2 - \frac{20}{3}C_A T_F n_f - 4C_F T_F n_f$
Γ_{cusp}	$\Gamma_0 = 4$	$\Gamma_1 = 4\left[\left(\frac{67}{9} - \frac{\pi^2}{3}\right)C_A - \frac{20}{9}T_F n_f\right]$
γ	$\gamma_0^q = -3C_F,\ \gamma_0^g = -\beta_0$	

- We derived the algorithm dependence of the jet shapes (anti)k_T vs cone
- Significant improvement over fixed order calculation

Y.-T. Chien et al. (2014)

Medium-modified jet shapes at NLL



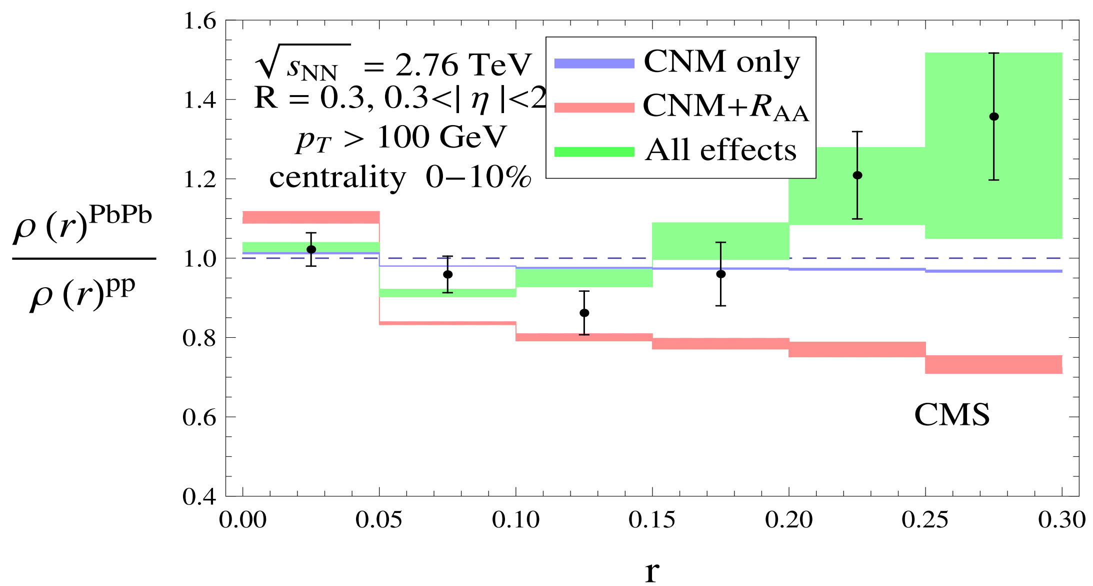
$$E_r(x,k_\perp) = \mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3 + \mathcal{M}_4$$

Measurement operator – tells us how the above configurations contribute energy to J (jet function)

- One can evaluate the jet energy functions from the splitting functions

$$J^i_{\omega,E_r}(\mu) = \sum_{j,k} \int_{PS} dx dk_\perp \mathcal{P}_{i\to jk}(x,k_\perp) E_r(x,k_\perp)$$

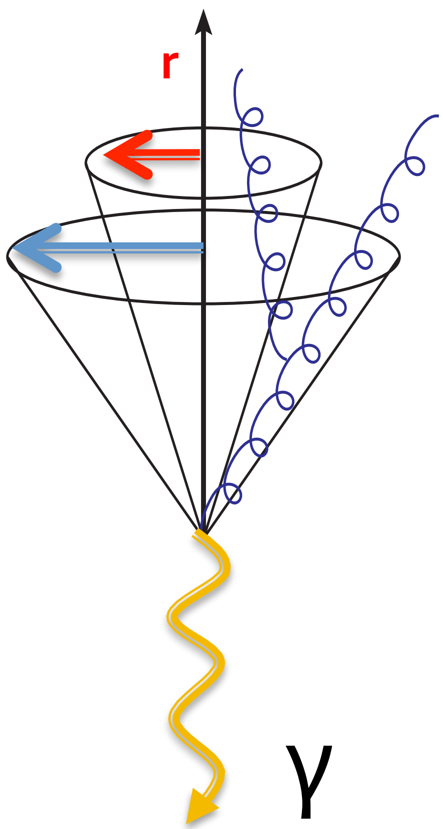
$$J_{\omega,E_r}(\mu) = J^{vac}_{\omega,E_r}(\mu) + J^{med}_{\omega,E_r}(\mu).$$



- First quantitative pQCD/SCET description of jet shapes in HI

Predictions for the 5.1 GeV Pb+Pb run at the LHC

- We extend predictions for other observables – photon tagged jets and photon tagged jet shapes

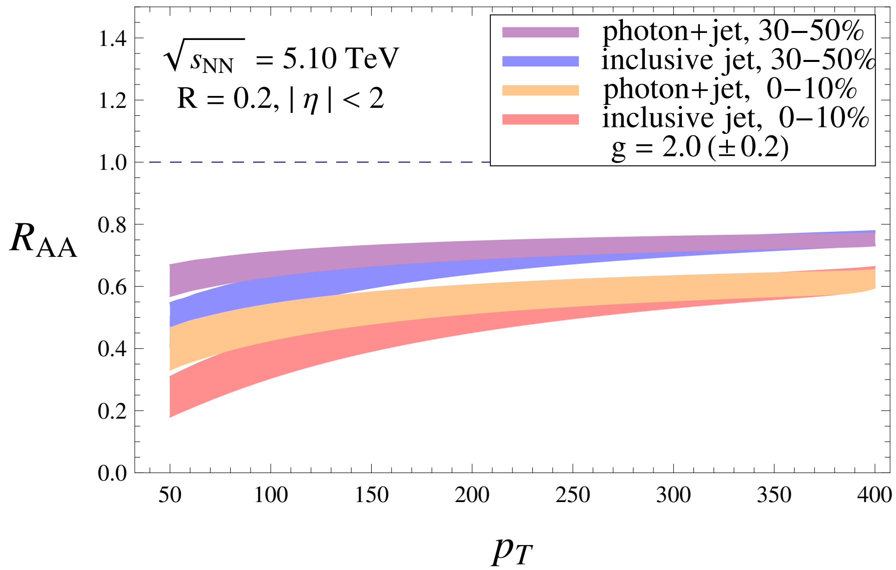


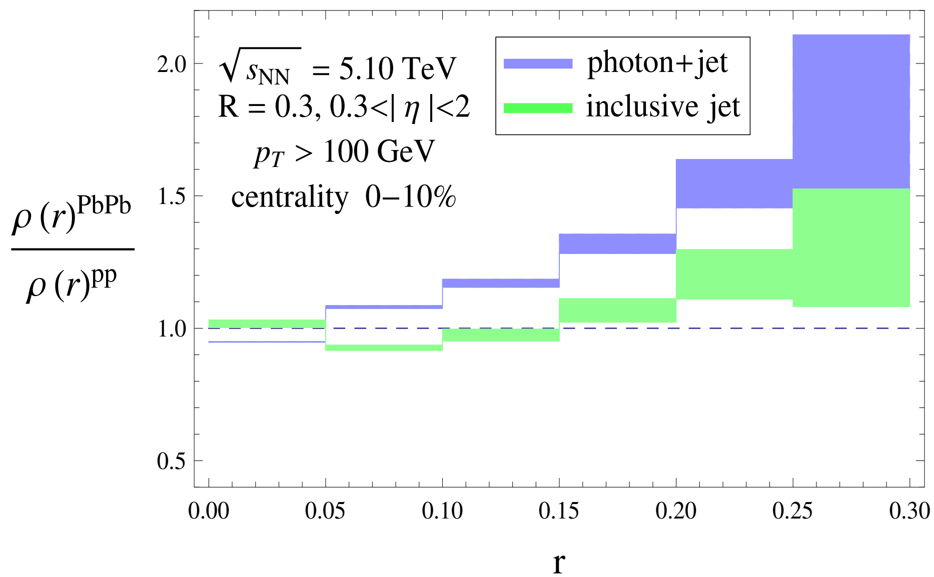
Photon tagging allows to alter/control the recoil jet composition

Measurable differences are predicted in the jet suppression at low p_T

Significant differences are expected in the jet shapes

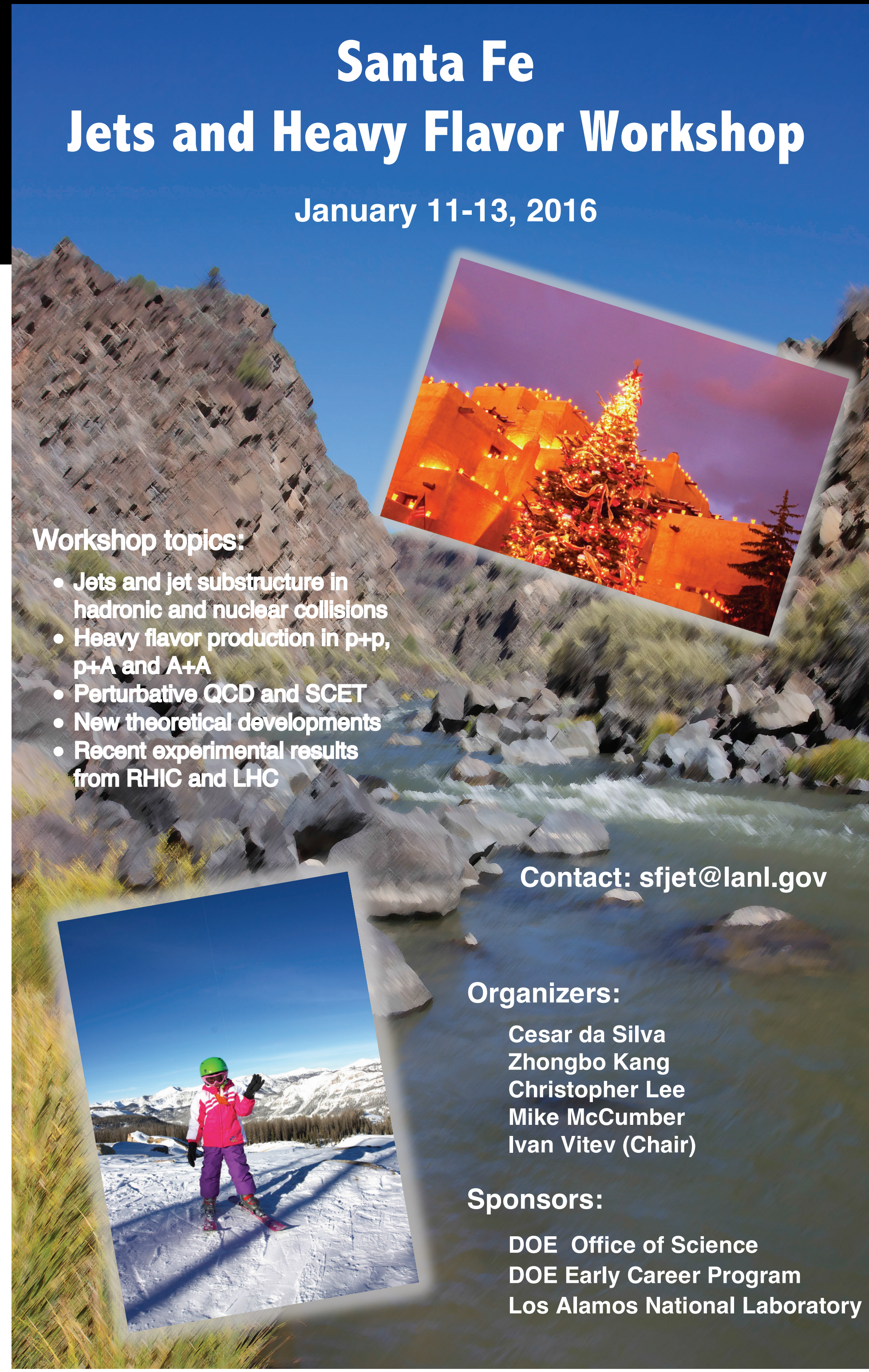
Y.-T. Chien et al. (2015)





- For more details and results on inclusive and tagged jet cross sections and shapes see poster by Yang-Ting Chien

- First in a series of workshops to bring the NP and HEP communities working on jets and heavy flavor, with emphasis on QCD and SCET

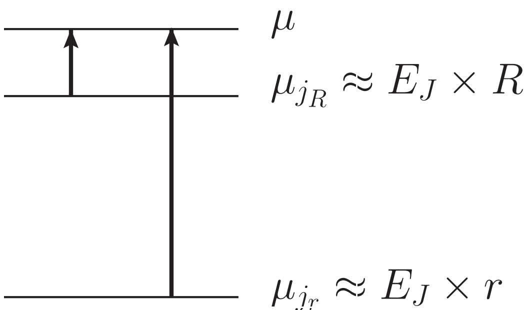


Conclusions

- An effective theory of jet propagation in matter $SCET_G$ was constructed (collinear sector). All medium-induced parton splittings derived, factorization and gauge invariance proven
- The connection between the traditional energy loss phenomenology and the QCD evolution/parton shower approach to jet quenching now established. Very good description of inclusive hadron suppression, predictions for the 5.1 TeV run
- First SCET calculation of jet shapes performed to NNL accuracy. Improved predictions for p+p collisions. More work needed to understand heavy flavor
- Calculations of jet cross sections and jet shapes are now available beyond the energy loss approach. Comparable description of inclusive jet suppression to the energy loss approach. Much improved description of jet shape modifiction
- We will look in the future at heavy flavor, soft functions and collisional interactions of the parton shower in the medium

NLL calculation of jet shapes

- We use SCET resummation techniques and SCET$_G$.



We start form the natural scales that eliminate all large logarithms in the fixed order calculation and evolve to a common scale [resumming ln(r/R)]

$$\frac{dJ_\omega^{qE_r}(\mu)}{d\ln\mu} = \left[-C_F\Gamma_{\rm cusp}(\alpha_s)\ln\frac{\omega^2\tan^2\frac{R}{2}}{\mu^2} - 2\gamma^q(\alpha_s)\right] J_\omega^{qE_r}(\mu)$$

$$\frac{dJ_\omega^{gE_r}(\mu)}{d\ln\mu} = \left[-C_A\Gamma_{\rm cusp}(\alpha_s)\ln\frac{\omega^2\tan^2\frac{R}{2}}{\mu^2} - 2\gamma^g(\alpha_s)\right] J_\omega^{gE_r}(\mu)$$

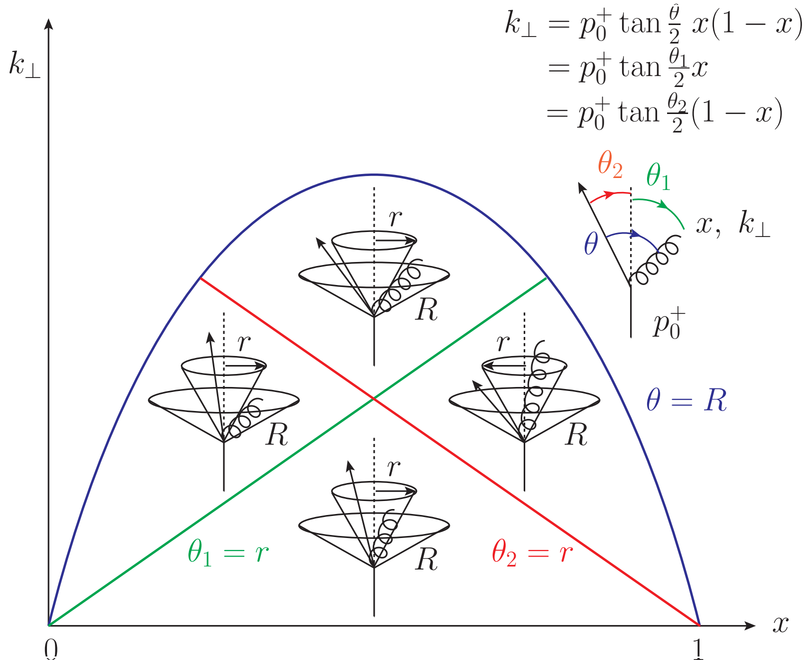
- To resum the jet shape to NLL accuracy

$$\Gamma_{\rm cusp}(\alpha_s) = \left(\frac{\alpha_s}{4\pi}\right)\Gamma_0 + \left(\frac{\alpha_s}{4\pi}\right)^2\Gamma_1 + \cdots ,$$
$$\gamma(\alpha_s) = \left(\frac{\alpha_s}{4\pi}\right)\gamma_0 + \left(\frac{\alpha_s}{4\pi}\right)^2\gamma_1 + \cdots .$$

Order	$\Gamma_{\rm cusp}$	γ	β
NLL	2-loop	1-loop	2-loop

NLL	1-loop	2-loop
β	$\beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_F n_f$	$\beta_1 = \frac{34}{3}C_A^2 - \frac{20}{3}C_A T_F n_f - 4C_F T_F n_f$
$\Gamma_{\rm cusp}$	$\Gamma_0 = 4$	$\Gamma_1 = 4\left[\left(\frac{67}{9} - \frac{\pi^2}{3}\right)C_A - \frac{20}{9}T_F n_f\right]$
γ	$\gamma_0^q = -3C_F,\ \gamma_0^g = -\beta_0$	

Phase space for the jet energy distribution



- To first non-trivial order, the phase space for the jet shape contributions is tractable

Y.-T. Chien et al. (2014)

- Define a jet energy function

$$J_\omega(E_r,\mu) = \sum_{X_c} \langle 0|\bar{\chi}_\omega(0)|X_c\rangle\langle X_c|\chi_\omega(0)|0\rangle \delta(E_r - \hat{E}^{<r}(X_c))$$

- Need the distribution of the average energy

$$J_\omega^{E_r}(\mu) = \int dE_r E_r\ J_\omega(E_r,\mu)$$

$$\Psi_\omega(r) = \frac{\langle E_r\rangle_\omega}{\langle E_R\rangle_\omega} = \frac{J_\omega^{E_r}(\mu)/J_\omega(\mu)}{J_\omega^{E_R}(\mu)/J_\omega(\mu)} = \frac{J_\omega^{E_r}(\mu)}{J_\omega^{E_R}(\mu)}$$

- Integral jet function

$$\frac{2}{\omega} J_\omega^{qE_r}(\mu) = \alpha_s \left[a \ln^2 \frac{\omega^2 \tan^2\frac{r}{2}}{\mu^2} + b \ln \frac{\omega^2 \tan^2\frac{r}{2}}{\mu^2} + \text{finite} \right]$$

Solution to the resummed jet shape

- Define

$$S(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha \frac{\Gamma_{\rm cusp}(\alpha)}{\beta(\alpha)} \int_{\alpha_s(\nu)}^{\alpha} \frac{d\alpha'}{\beta(\alpha')} \,,$$

$$A_i(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha \frac{\gamma^i(\alpha)}{\beta(\alpha)} \,, \; A_\Gamma(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha \frac{\Gamma_{\rm cusp}(\alpha)}{\beta(\alpha)} \,,$$

- Jet function

$$J_\omega^{iE_r}(\mu) = J_\omega^{iE_r}(\mu_{j_r}) \exp\left[-2C_i S(\mu_{j_r},\mu) + 2A_i(\mu_{j_r},\mu)\right] \left(\frac{\omega^2 \tan^2 \frac{R}{2}}{\mu_{j_r}^2}\right)^{C_i A_\Gamma(\mu_{j_r},\mu)}$$

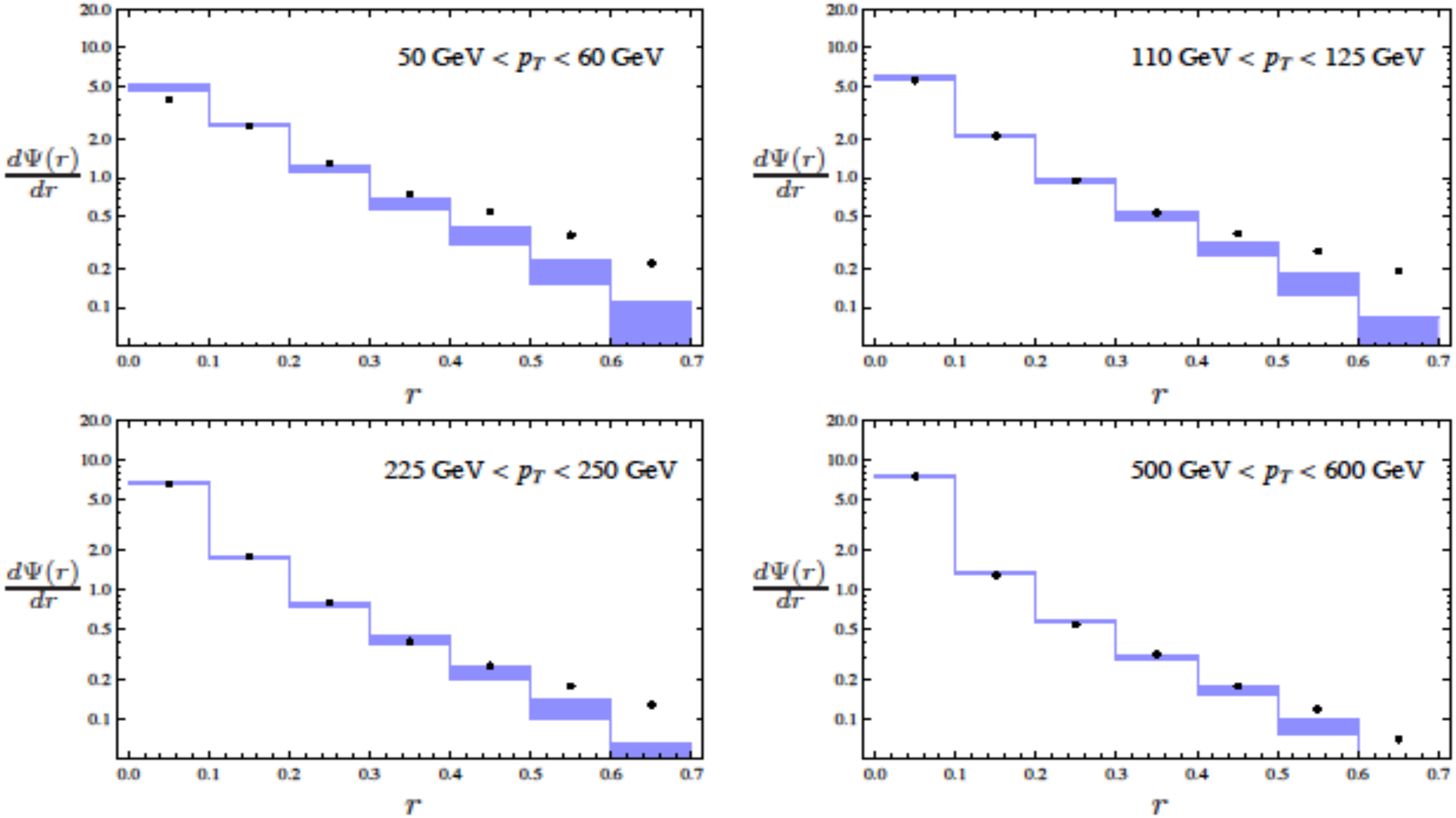
- Integral jet shape

$$\Psi_\omega^i(r) = \frac{J_\omega^{iE_r}(\mu)}{J_\omega^{iE_R}(\mu)} = \frac{J_\omega^{iE_r}(\mu_{j_r})}{J_\omega^{iE_R}(\mu_{jR})} \exp[-2C_i S(\mu_{j_r},\mu_{jR}) + 2A_i(\mu_{j_r},\mu_{jR})] \times \left(\frac{\mu_{j_r}^2}{\omega^2 \tan^2 \frac{R}{2}}\right)^{C_i A_\Gamma(\mu_{jR},\mu_{j_r})}$$

Y.-T.Chien et al. (2014)

We start form the natural scales that eliminate all large logarithms in the fixed order calculation and evolve to a common scale [resumming ln(r/R)]

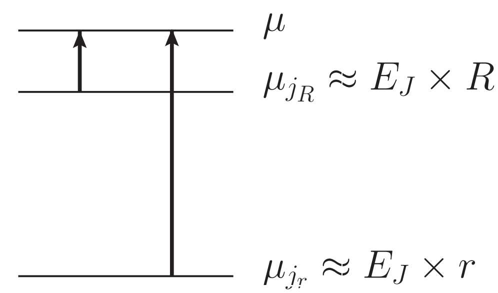
Limitations of the calculation



- Deviations at large R and/or small energy

NLL calculation of jet shapes

- We use SCET resummation techniques and $SCET_{G.}$



$$\Psi_\omega(r) = \frac{\langle E_r \rangle_\omega}{\langle E_R \rangle_\omega} = \frac{J_\omega^{E_r}(\mu)/J_\omega(\mu)}{J_\omega^{E_R}(\mu)/J_\omega(\mu)} = \frac{J_\omega^{E_r}(\mu)}{J_\omega^{E_R}(\mu)}$$

The jet shape is a a ratio of 2 jet energy functions . The measured jet energy functions are obtained at 1 loop. We start form the natural scales that eliminate all large logarithms in the fixed order calculation and evolve to a common scale [resumming ln(r/R)]

$$S(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha \frac{\Gamma_{\text{cusp}}(\alpha)}{\beta(\alpha)} \int_{\alpha_s(\nu)}^{\alpha} \frac{d\alpha'}{\beta(\alpha')} ,$$

- Result

$$A_i(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha \frac{\gamma^i(\alpha)}{\beta(\alpha)} , \quad A_\Gamma(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha \frac{\Gamma_{\text{cusp}}(\alpha)}{\beta(\alpha)} ,$$

Y.-T.Chien et al. (2014)

$$\Psi_\omega^i(r) = \frac{J_\omega^{iE_r}(\mu)}{J_\omega^{iE_R}(\mu)} = \frac{J_\omega^{iE_r}(\mu_{j_r})}{J_\omega^{iE_R}(\mu_{j_R})} \exp[-2C_i S(\mu_{j_r},\mu_{j_R}) + 2A_i(\mu_{j_r},\mu_{j_R})] \times \left(\frac{\mu_{j_r}^2}{\omega^2 \tan^2 \frac{R}{2}}\right)^{C_i A_\Gamma(\mu_{j_R},\mu_{j_r})}$$

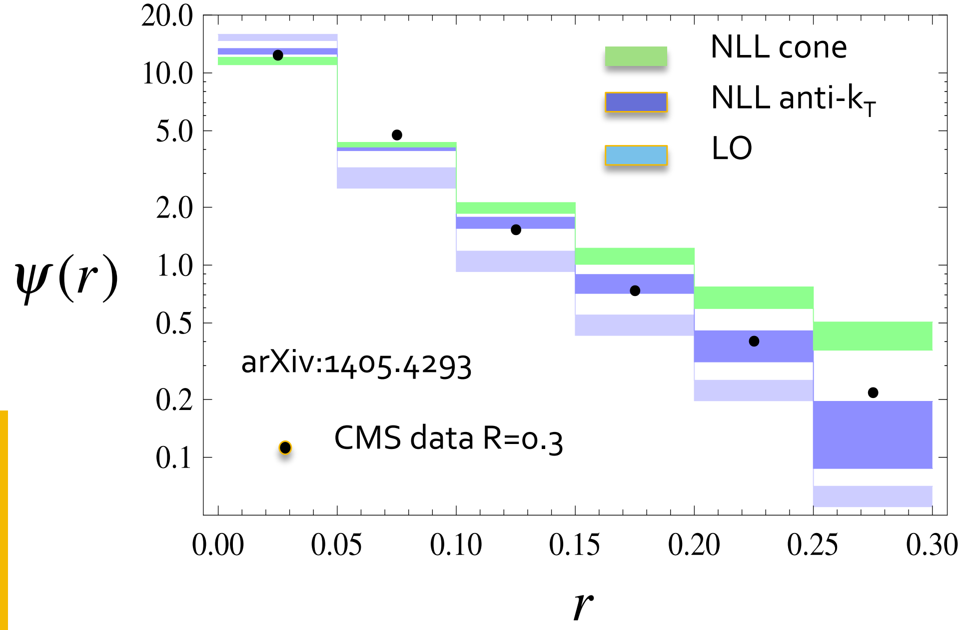
NLL results in p+p collisions

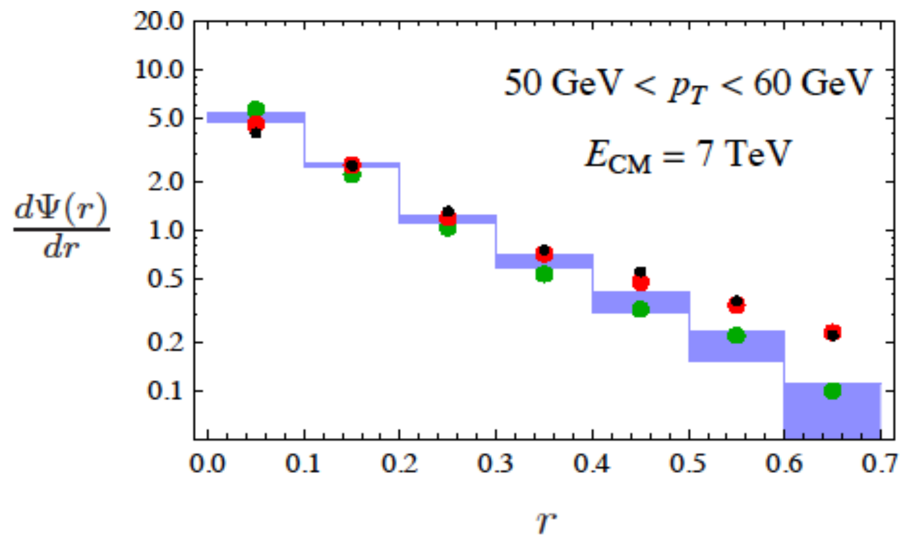
- We derived the algorithm dependence of the jet shapes (anti)k_T/cone
- Significant improvement over fixed order calculation

- The calculation does not include initial-state radiation/beam functions and hadronization effects
- Power suppressed but visible at the tail of the distribution and lower p_T

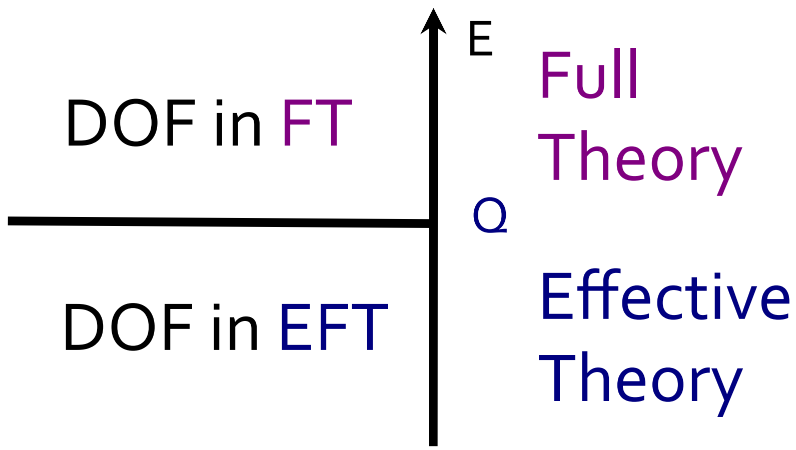
Y.-T.Chien et al. (2014)

See talk by Chien in this conference





Examples of effective field theories [EFTs]



- Simple but powerful idea to concentrate on the significant degrees of freedom [DOF]. Manifest power counting

	Q	power counting	DOF in FT	DOF in EFT
Chiral Perturbation Theory (ChPT)	Λ_{QCD}	p/Λ_{QCD}	q, g	K, π
Heavy Quark Effective Theory (HQET)	m_b	Λ_{QCD}/m_b	ψ, A	h_v, A_s
Soft Collinear Effective Theory (SCET)	Q	$p_\perp/Q$	ψ, A	ξ_n, A_n, A_s

III. In more detail: the jet scattering kinematics

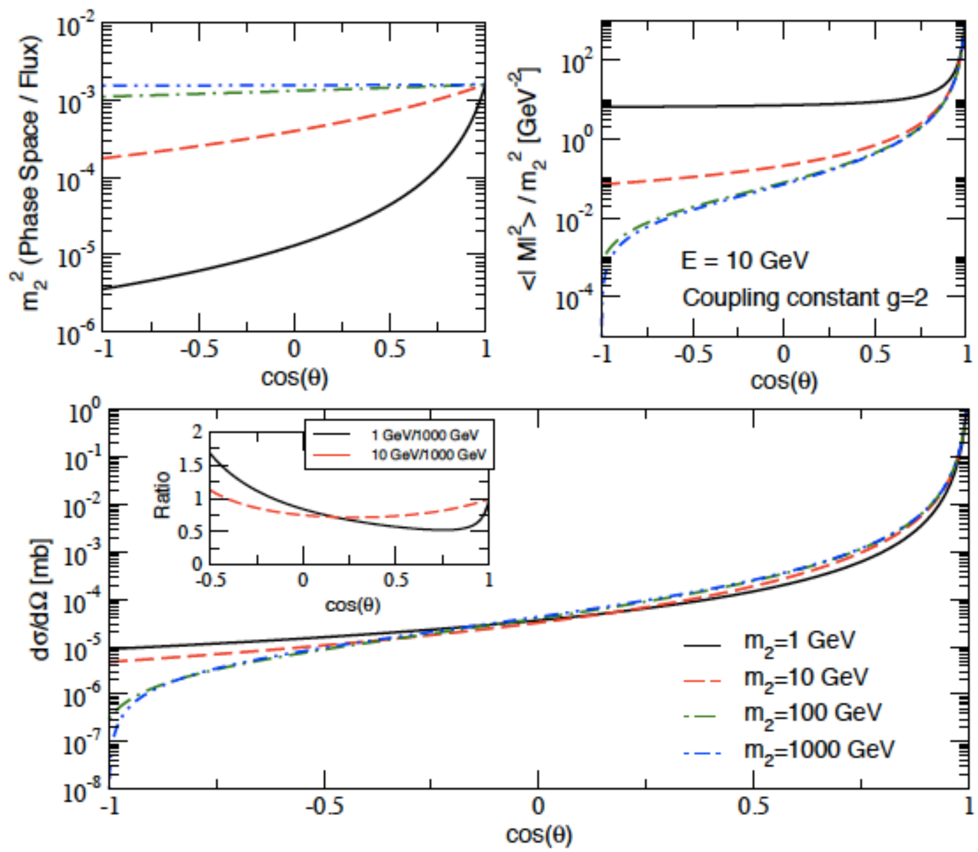
- What is missing in the YM Lagrangian is the interaction between the jet and the medium

G. Ovanesyan et al. (2011)

- Kinematics and channels

t – jet broadening and energy loss
s– isotropisation
u – backward hard scattering

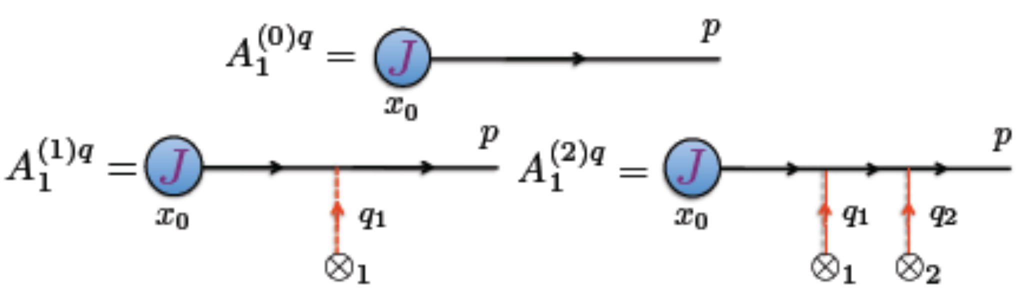
- Fully dynamic medium recoil, cross section reduction (5% – 15%). Completely dominated by forward scattering



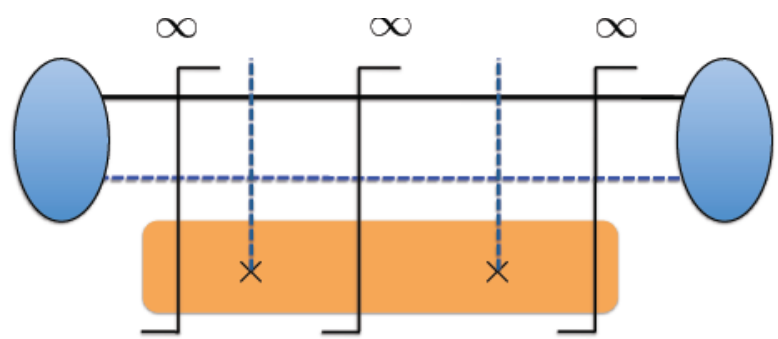
$$\frac{d\sigma}{d\Omega} \to \frac{d\sigma}{d^2\mathbf{q}_\perp} = \frac{C_2(R)C_2(T)}{d_A}\frac{|v(\mathbf{q}_\perp; E, m_1, m_2)|^2}{(2\pi)^2}$$

III. Main results: jet broadening

- Jet broadening and its gauge invariance



M. Gyulassy et al. (2001)



Classes of diagrams (single Born, double Born). Reaction Operator

- General result. Will evaluate the broadening (or lack off) of jets

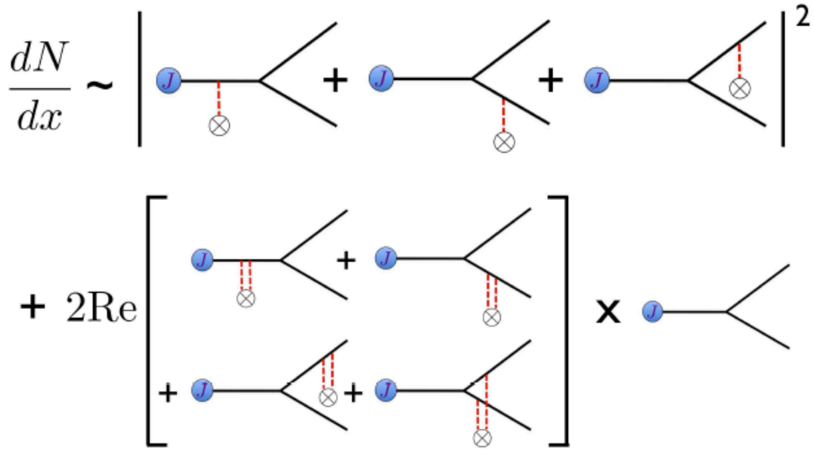
$$\frac{dN^{(n)}(\mathbf{p}_\perp)}{d^2\mathbf{p}_\perp} = \prod_{i=1}^{n} \int_{z_{i-1}}^{L} \frac{dz_i}{\lambda} \int d^2\mathbf{q}_{\perp i} \left[\frac{1}{\sigma_{el}(z_i)} \frac{d\sigma_{el}(z_i)}{d^2\mathbf{q}_{\perp i}} \left(e^{-\mathbf{q}_{\perp i} \cdot \vec{\nabla}_{\mathbf{p}_\perp}} \right) - \delta^2(\mathbf{q}_\perp) \right] \frac{dN^{(0)}(\mathbf{p}_\perp)}{d^2\mathbf{p}_\perp}$$

- In special cases such as constant density and the Gaussian approximation

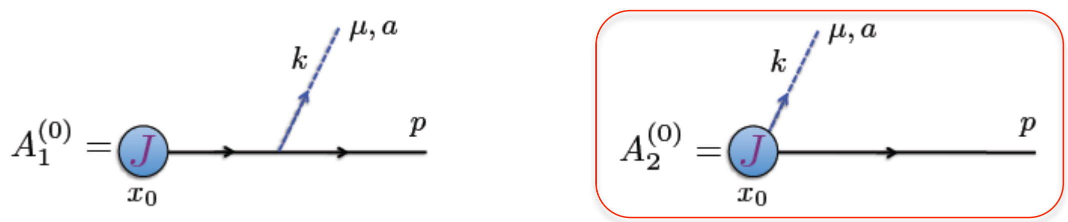
Starting with a collinear beam of quarks/gluons we recover M. Gyulassy et al. (2002)

$$\frac{dN(\mathbf{p}_\perp)}{d^2\mathbf{p}_\perp} = \frac{1}{2\pi} \frac{e^{-\frac{p^2}{2\chi\mu^2\xi}}}{\chi\,\mu^2\,\xi} \qquad \chi = \frac{L}{\lambda}$$

III. Main results: in-medium splitting / parton energy loss



Gluon splitting functions factorize form the hard scattering cross section only for spin averaged processes

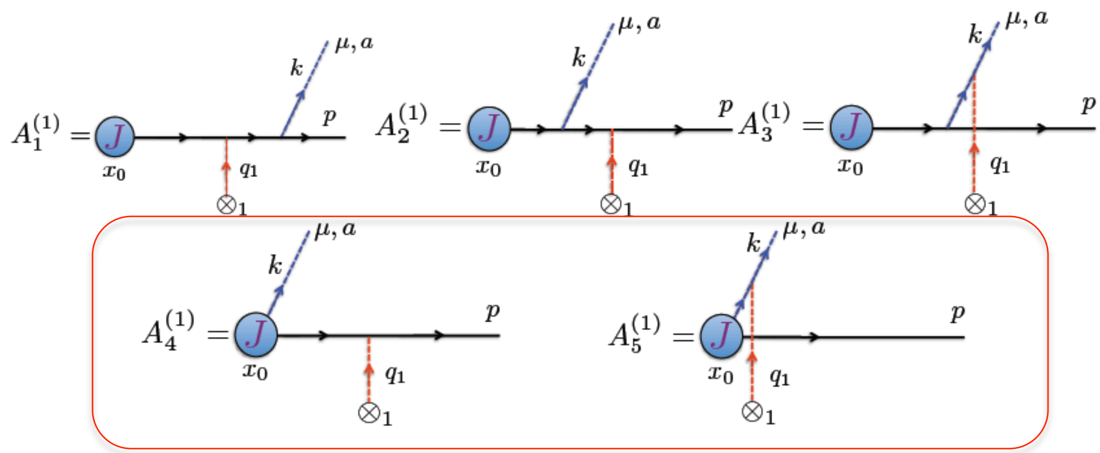


Altarelli-Parisi splitting

G. Altarelli et al. (1978)

$$\Gamma_W^{\alpha,a}(k) = g T_r^a \frac{\bar{n}^\alpha}{k^+ + i\epsilon}$$

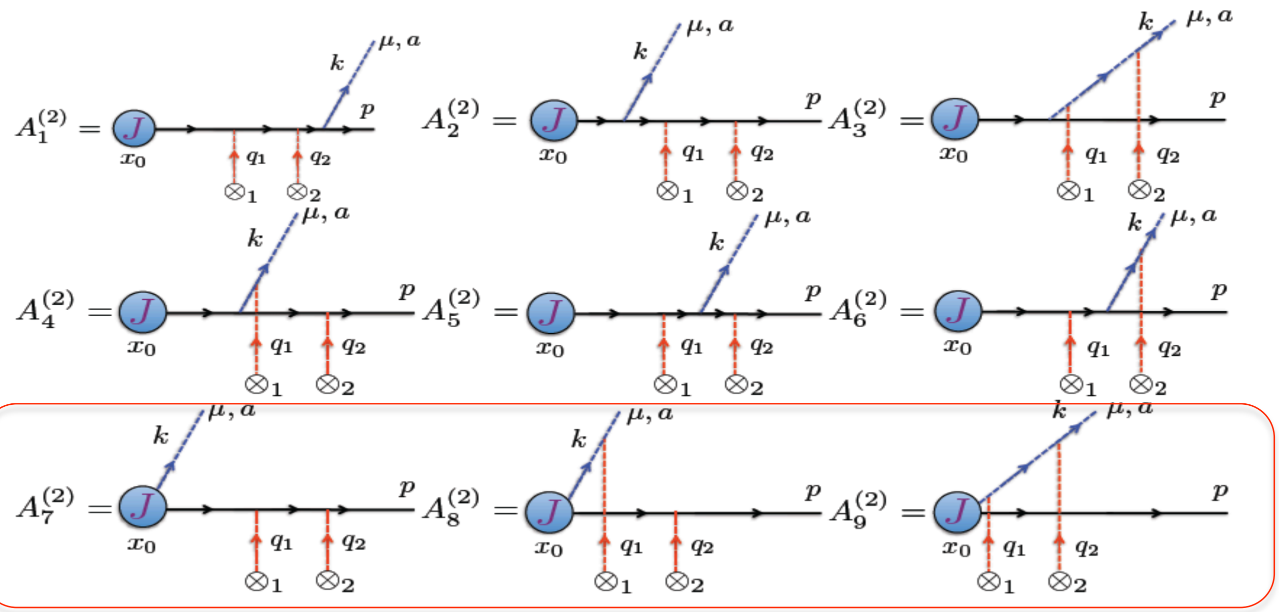
- Note that a collinear Wilson line appears in the R_ξ gauge



Single Born diagrams

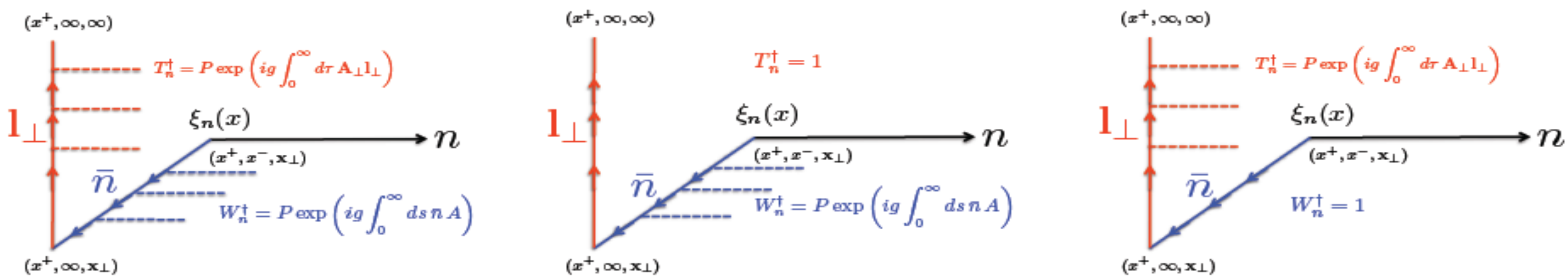
III. Main results: in-medium splitting / parton energy loss

Double Born diagrams

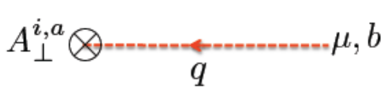


G. Ovanesyan et al. (2011)

- The lightcone gauge



- New Feynman rule

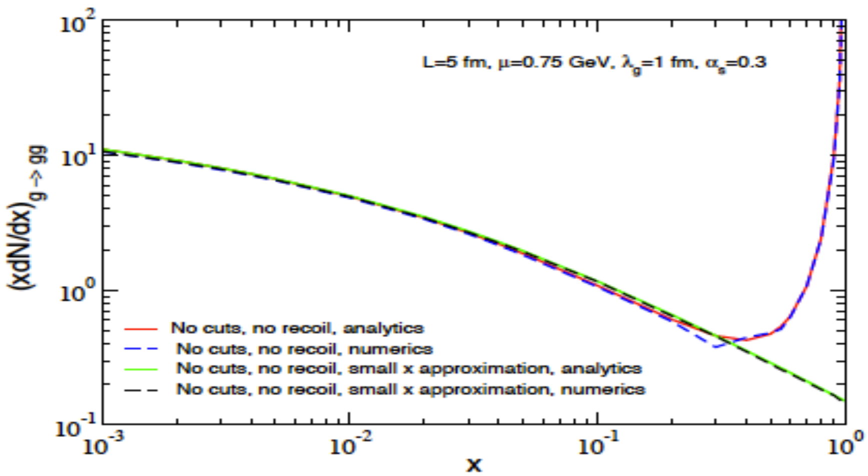


$$= \; i\,\delta^{ab}\,\frac{\bar{n}^\mu q^i}{q^2 + i\varepsilon}\,C_\infty^{(\mathrm{Pres})}\left(\frac{1}{q^+ + i\varepsilon} - \frac{1}{q^+ - i\varepsilon}\right)$$

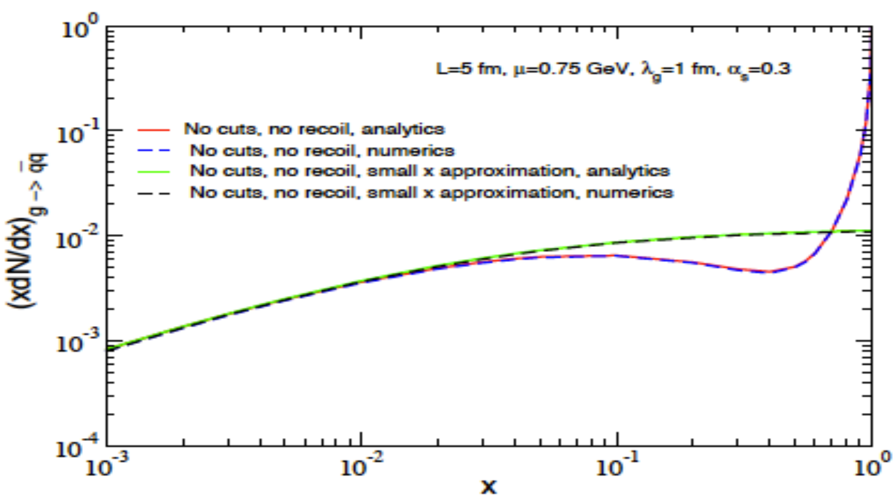
A. Idilbi et al. (2010)

III. Numerical examples

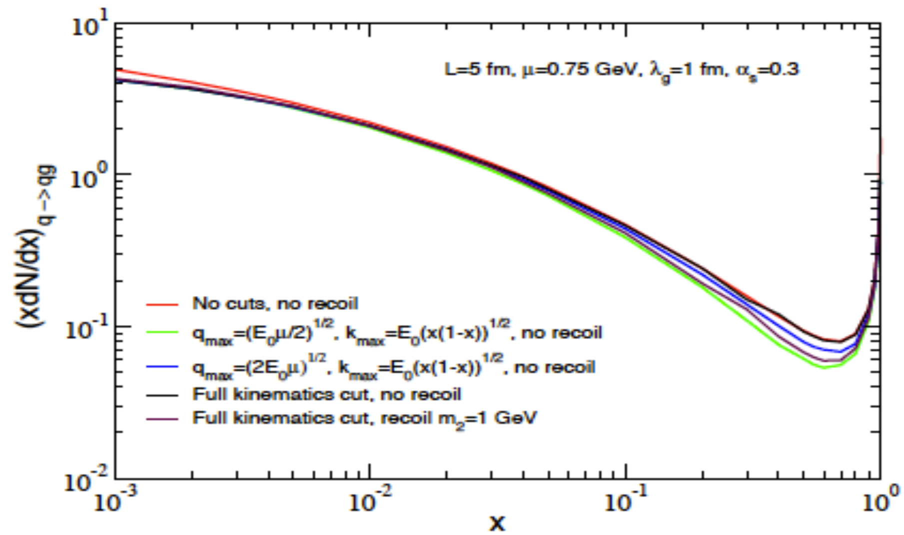
- Leading intensity term



- Sub-leading intensity term



- Kinematic effects



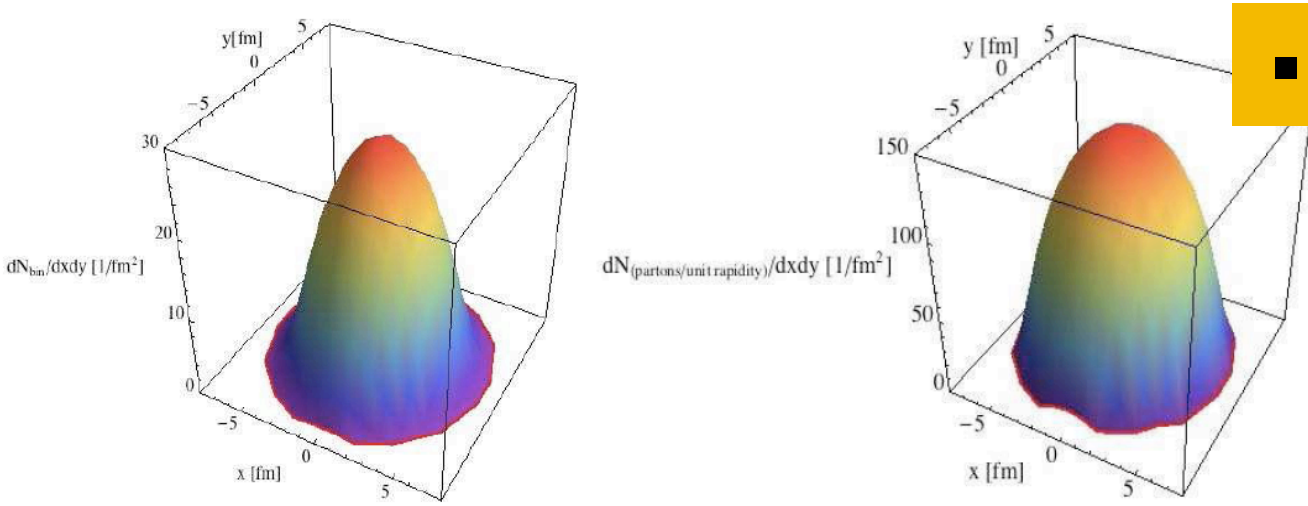
- We have theoretical control over the in-medium splittings
- The large-x and kinematic effects are of the same order
- Will be incorporated in future phenomenological applications

V. Inclusive jet cross sections in A+A reactions

- Jet cross sections with cold nuclear matter and final-state parton energy loss effect are calculated for different R

$$\frac{\sigma^{AA}(R,\omega^{\min})}{d^2E_T dy} = \int_{\epsilon=0}^{1} d\epsilon \sum_{q,g} P_{q,g}(\epsilon) \frac{1}{(1-(1-f_{q,g})\cdot\epsilon)^2} \frac{\sigma^{NN}_{q,g}(R,\omega^{\min})}{d^2E'_T dy}$$

$$|J_i(\epsilon_i)| = 1/\left(1 - [1 - f(R_i, p_{T\,i}^{\min})_{q,g}]\epsilon_i\right)$$



I. Vitev et al (2008)

- Calculate in real time

Fraction of the energy redistributed inside the jet

$$f(R_i, p_{T\,i}^{\min})_{q,g} = \frac{\int_0^{R_i} dr \int_{p_{T\,i}^{\min}}^{E_{T\,i}} d\omega \frac{dI^{rad}_{q,g}(i)}{d\omega dr}}{\int_0^{R_i^\infty} dr \int_0^{E_{T\,i}} d\omega \frac{dI^{rad}_{q,g}(i)}{d\omega dr}}$$

The probability to lose energy due to multiple gluon emission

$$\int_0^1 P_{q,g}(\epsilon_i) d\epsilon_i = 1\,, \quad \int_0^1 \epsilon_i P_{q,g}(\epsilon_i) d\epsilon_i = \frac{\Delta E_{q,g\,i}}{E_i}$$

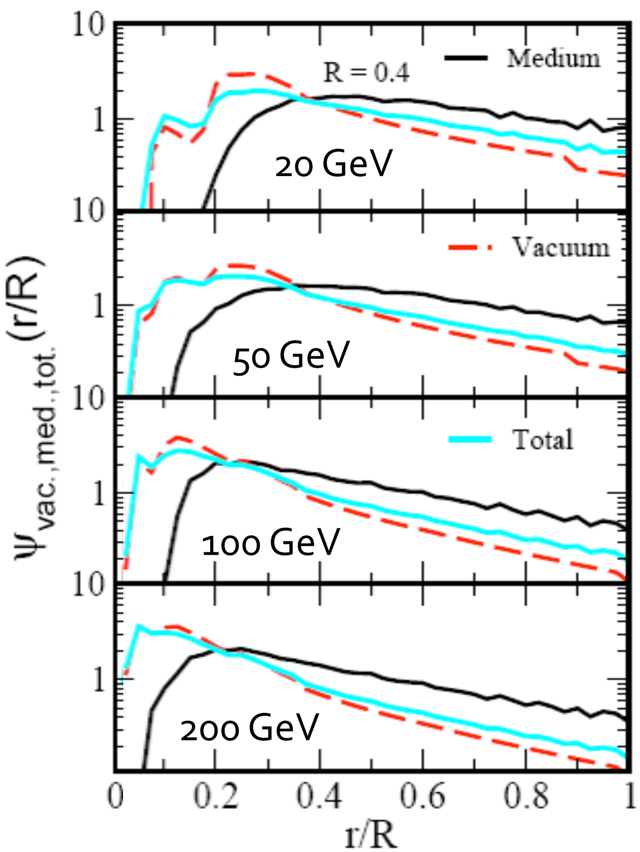
- Obtain

$$R^{jet}_{AA}(E_T; R, p_T^{\min}) = \frac{\frac{d\sigma^{AA}(E_T; R, p_T^{\min})}{dyd^2E_T}}{\langle N_{bin}\rangle \frac{d\sigma^{pp}(E_T; R, p_T^{\min})}{dyd^2E_T}}$$

V. QGP – modified jet shapes

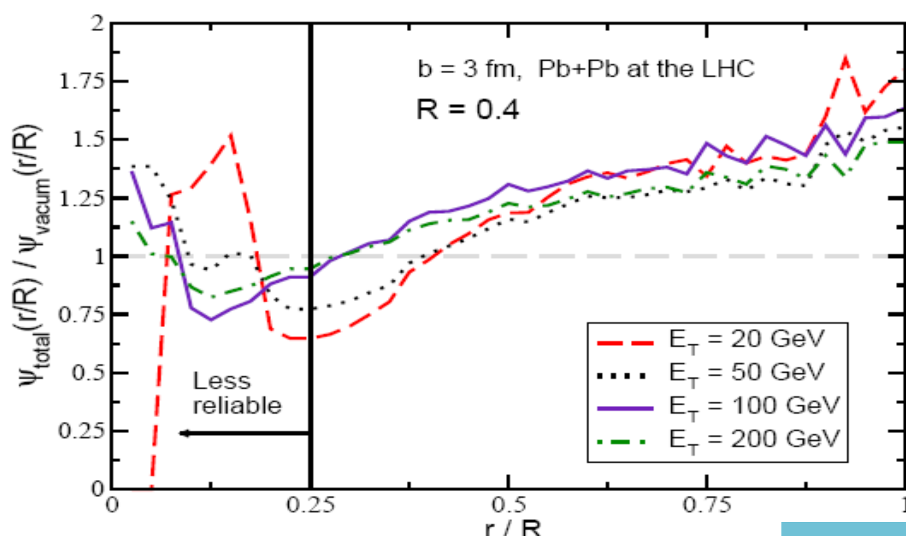
$$\Psi_{\rm int}(r;R) = \frac{\sum_i (E_T)_i \Theta(r-(R_{\rm jet})_i)}{\sum_i (E_T)_i \Theta(R-(R_{\rm jet})_i)}$$

$$\psi(r;R) = \frac{d\Psi_{\rm int}(r;R)}{dr}$$



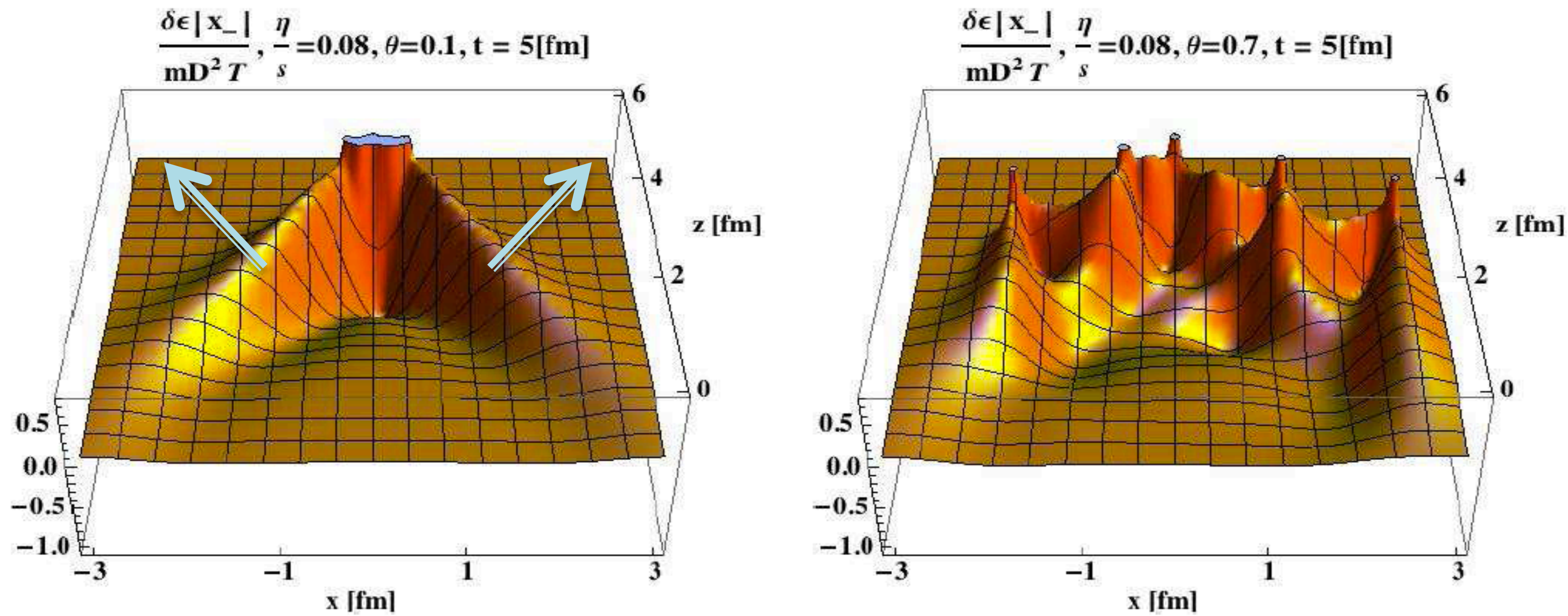
- Surprisingly, there is no big difference between the jet shape in vacuum and the total jet shape in the medium
- Take a ratio of the differential jet shapes

R=0.4	Vacuum	Complete E-loss	Realistic Case
<r/R>,E_T=20GeV	0.41	0.57	0.45
<r/R>,E_T=50GeV	0.35	0.53	0.38
<r/R>,E_T=100GeV	0.28	0.42	0.32
<r/R>,E_T=200GeV	0.25	0.42	0.28



I. Vitev et al. (2008)

III. Why are Mach cones initiated by jets unlikely



- An individual parton (or a collinear system) can produce a Mach cone on an event by event basis. Multiple events will reduce the observable effect

I. Vitev (2005)

- Typical medium-induced shower multiplicities are N^g=4 (quark) and N^g=8 (gluon) and emitted at large angles ~ 0.7 (much larger than in the vcuum)
- Each parton quickly becomes an individual source of excitation and these multiple sources wipe out any conical signature

IV. Resummation, RG equations and Higgs production at the LHC

- SCET is very effective in resumming in large infrared logarithms using Renormalization group equations

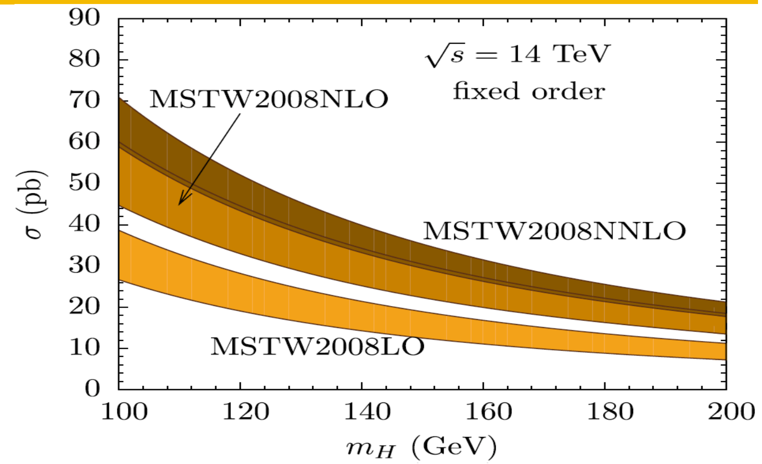
General structure of Sudakov logs

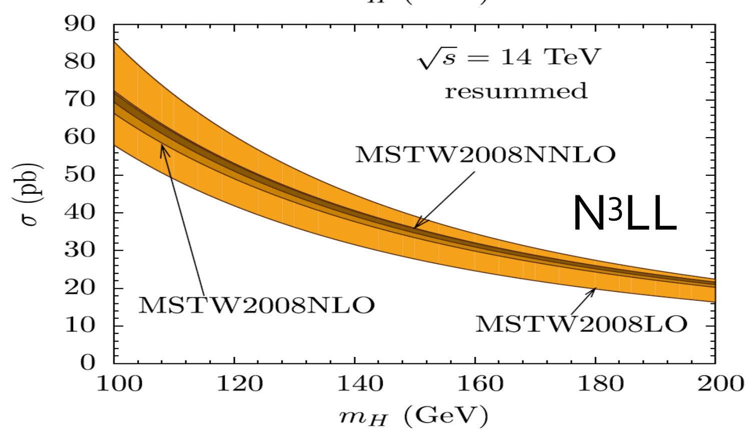
$$C(\bar{n}\cdot p,\mu) = 1 + a_s[L^2 + L + 1] + a_s^2[L^4 + L^3 + L^2 + L + 1] + a_s^3[L^6 + L^5 + L^4 + L^3 + L^2 + L + 1]$$

LL NLL

$$a_s = \frac{\alpha_s}{\pi}$$

$$L = \log\left(\frac{\mu}{\bar{n}\cdot p}\right)$$





- It can improve upon traditional techniques, such as CCS

J. Collins et al. (1985)

V. Ahrens et al. (2009)

IV. Factorization in SCET and angularities

- Factorization theorems have been proven in SCET for a number of observables: event shapes [e^+e^-], Higgs [pp], top [e^+e^-] …

  - Angularity observables: generalization of traditional event shapes

$$\tau_a = \frac{1}{Q}\sum_i |\vec{p}_i^{\,T}|\exp(-\eta_i(1-a))$$

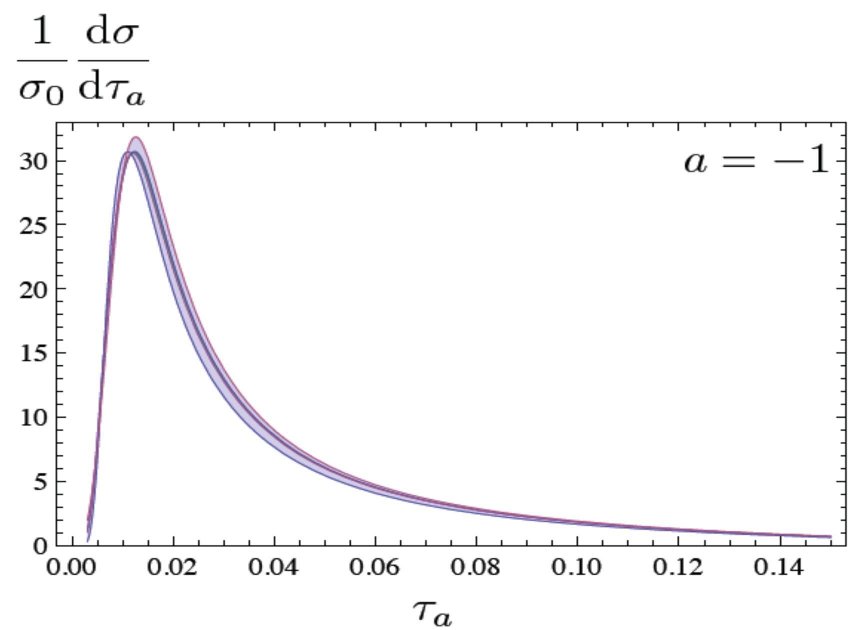
$$-\infty < a < 2$$

C. Berger et al. (2003)

- Factorized in hard function, jet functions and soft function

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}e} = H(Q)\int de_n\, J_n(e_n)\, de_{\bar{n}}\, J_{\bar{n}}(e_{\bar{n}})\, de_s\, S(e_s)\,\delta(e - e_n - e_{\bar{n}} - e_s)$$

C. Bauer et al. (2008)



A. Hornig et al. (2010)

Splitting functions and the parton shower

- Making the connection to the standard LO, NLO, …; LL, NLL … pQCD approach (higher orders and resummation)

- Need to understand the process-dependent contributions

- Need to understand the dependence on the properties of the nuclear medium

→ medium
↓ vacuum

	LO	NLO	NNLO	…
LO	α_s^2	$\alpha_s^2\alpha_s^{med}$	$\alpha_s^2\alpha_s^{2\,med}$	…
NLO	α_s^3	$\alpha_s^3\alpha_s^{med}$	…	
NNLO	α_s^4	…		
…	…			

- In the vacuum $O(\alpha_s^2)$ splitting kernels
- In the medium the full $O(\alpha_s)$ known , now being implemented
- In the medium $O(\alpha_s^2)$ only q→qgg known, computationally demanding

S. Catani et al. (1997)

G. Ovanesyan et al. (2011)

M. Fickinger et al. (2013)

SCET formulation

Energetic quarks and leptons
collinear modes

Include also soft quarks and gluons

C. Bauer et al. (2001)

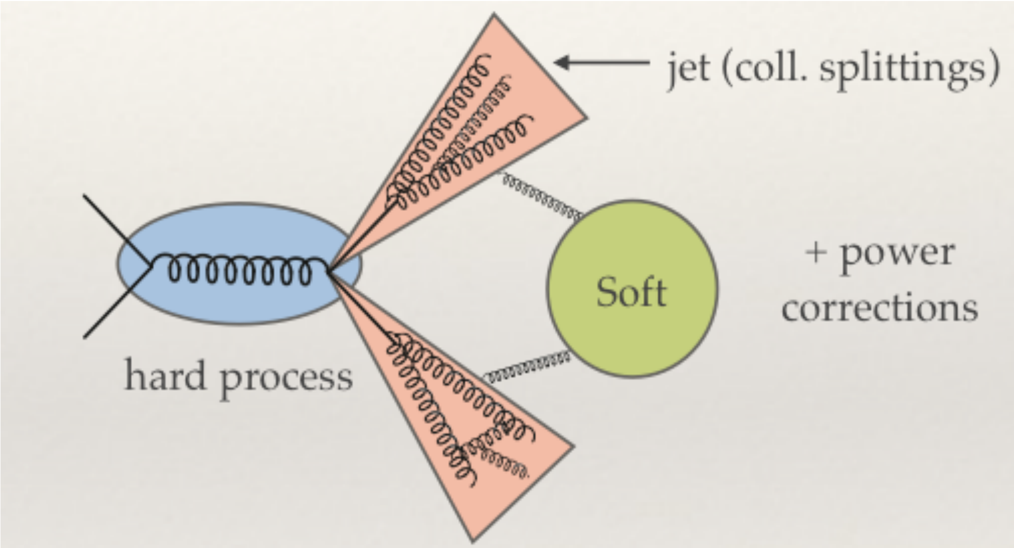
D. Pirol et al. (2004)

$$p_c = (p_+, p_-, p_\perp) \sim (\frac{\Lambda^2}{Q}, Q, \Lambda) = Q(\lambda^2, 1, \lambda)$$

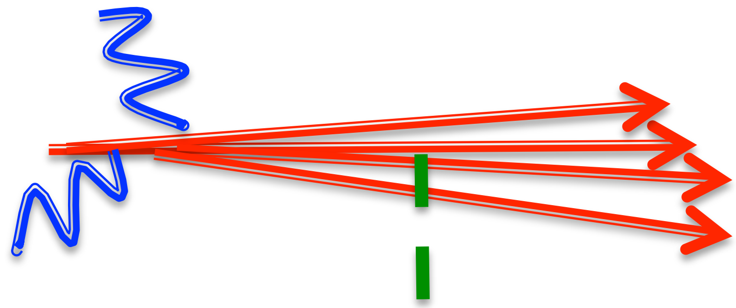
$$p_s = (p_+, p_-, p_\perp) \sim (\Lambda, \Lambda, \Lambda) = Q(\lambda, \lambda, \lambda)$$

Collinear quarks, antiquarks	$\xi_n, \bar{\xi}_n$
Collinear gluons, soft gluons	A_n, A_s

- SCET Lagrangian to all orders in λ [Can expand to LO, NLO,…]



- The missing mode



$q \sim [\lambda^2, \lambda^2, \lambda]$

A new mode – the Glauber gluon

The vacuum splitting kernels

- In the vacuum we have the DGPAL splitting kernels that factorize from the hard scattering cross section and are process independent

Gribov et al. (1972)

G. Altarelli et al. (1977)

Y. Dokshitzer (1977)

$$\left(\frac{dN}{dx\, d^2\mathbf{k}_\perp}\right)_{q\to qg} = \frac{\alpha_s}{2\pi^2} C_F \frac{1+(1-x)^2}{x} \frac{1}{\mathbf{k}_\perp^2}, \quad (\ldots]_+ + A\delta(x))$$

$$\left(\frac{dN}{dx\, d^2\mathbf{k}_\perp}\right)_{g\to gg} = \frac{\alpha_s}{2\pi^2} 2C_A \Big(\frac{1-x}{x} + \frac{x}{1-x} + x(1-x)\Big) \frac{1}{\mathbf{k}_\perp^2}, \quad (\ldots]_+ + B\delta(x))$$

$$\left(\frac{dN}{dx\, d^2\mathbf{k}_\perp}\right)_{g\to q\bar{q}} = \frac{\alpha_s}{2\pi^2} T_R \left(x^2 + (1-x)^2\right) \frac{1}{\mathbf{k}_\perp^2}$$

$$\left(\frac{dN}{dx\, d^2\mathbf{k}_\perp}\right)_{q\to gq} = \left(\frac{dN}{dx\, d^2\mathbf{k}_\perp}\right)_{q\to qg} (x \to 1-x)$$

Reversed convention

- The singular pieces A, B can be obtained form flavor and momentum conservation sum rules
- Can be re-derived using SCET. Use only the collinear sector

$$\int_0^1 P_{qq}(x)\, \mathrm{d}x = 0,$$

$$\int_0^1 \left[P_{qq}(x) + P_{qg}(x)\right] (1-x)\, \mathrm{d}x = 0,$$

$$\int_0^1 \left[2n_f P_{gq}(x) + P_{gg}(x)\right] (1-x)\, \mathrm{d}x = 0.$$

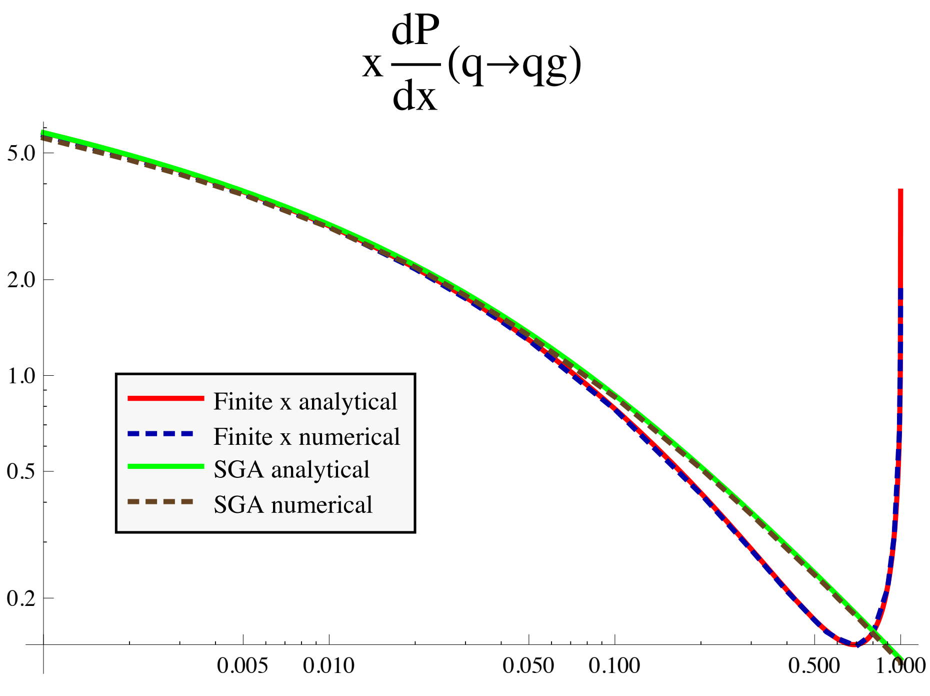
The soft-gluon energy loss limit

Take the small-x soft gluon emission limit

$$x\left(\frac{dN}{dx}\right)_{\left\{\begin{matrix} q\to qg \\ g\to gg \end{matrix}\right\}} = \frac{\alpha_s}{\pi^2}\left\{\begin{matrix} C_F[1+\mathcal{O}(x)] \\ C_A[1+\mathcal{O}(x)] \end{matrix}\right\}\int\frac{d\Delta z}{\lambda_g(z)}\int d^2\mathbf{k}_\perp d^2\mathbf{q}_\perp \frac{1}{\sigma_{el}}\frac{d\sigma_{el}^{\text{medium}}}{d^2\mathbf{q}_\perp}$$

M Gyulassy et al . (2000)

$$\times\frac{2\boldsymbol{k}_\perp\cdot\boldsymbol{q}_\perp}{\boldsymbol{k}_\perp^2(\boldsymbol{k}_\perp-\boldsymbol{q}_\perp)^2}\left[1-\cos\frac{(\boldsymbol{k}_\perp-\boldsymbol{q}_\perp)^2}{xp_0^+}\Delta z\right].$$



The result reduces to the GLV energy-loss differential intensities

- Only 2 medium-induced splittings survive
- There is no flavor mixing
- Results can be interpreted as energy loss

Medium-modified evolution of the fragmentation functions

- Using the same techniques. The vacuum and the medium induced evolution factorize

$$\frac{\mathrm{d}\ln D_{h/c}^{\mathrm{med.}}(z,Q)}{\mathrm{d}\ln Q} = [\cdots]_{\mathrm{vac.}} - [n(z)-1]\left\{\int_0^{1-z} \mathrm{d}z'\, z' Q \frac{dN}{dz'dQ}(z',Q)\right\} - \int_{1-z}^{1} \mathrm{d}z' Q \frac{dN}{dz'dQ}(z',Q)\,.$$

$$D_{h/c}^{\mathrm{med.}}(z,Q) = \mathrm{e}^{-2C_R\frac{\alpha_s}{\pi}\left[\ln\frac{Q}{Q_0}\right]\{[n(z)-1](1-z)-\ln(1-z)\}} D_{h/c}(z,Q_0)$$

$$\times e^{-[n(z)-1]\left\{\int_0^{1-z}\mathrm{d}z'\, z' \int_{Q_0}^{Q} dQ' \frac{dN}{dz'dQ'}(z',Q')\right\} - \int_{1-z}^{1}\mathrm{d}z' \int_{Q_0}^{Q} dQ' \frac{dN}{dz'dQ'}(z',Q')}$$

$$= D_{h/c}(z,Q) e^{-[n(z)-1]\left\langle \frac{\Delta \tilde{E}}{E}\right\rangle_z - \langle \tilde{N^g}\rangle_z}\,.$$

- *The main result*: direct relation between the evolution and energy loss approaches first established here

$$\left\langle \frac{\Delta \tilde{E}}{E}\right\rangle_z = \int_0^{1-z}\mathrm{d}z'\, z' \int_{Q_0}^{Q} dQ' \frac{dN}{dz'dQ'}(z',Q') = \int_0^{1-z}\mathrm{d}z'\, z' \frac{dN}{dz'}(z') \qquad \to_{z\to 0} \left\langle \frac{\Delta E}{E}\right\rangle,$$

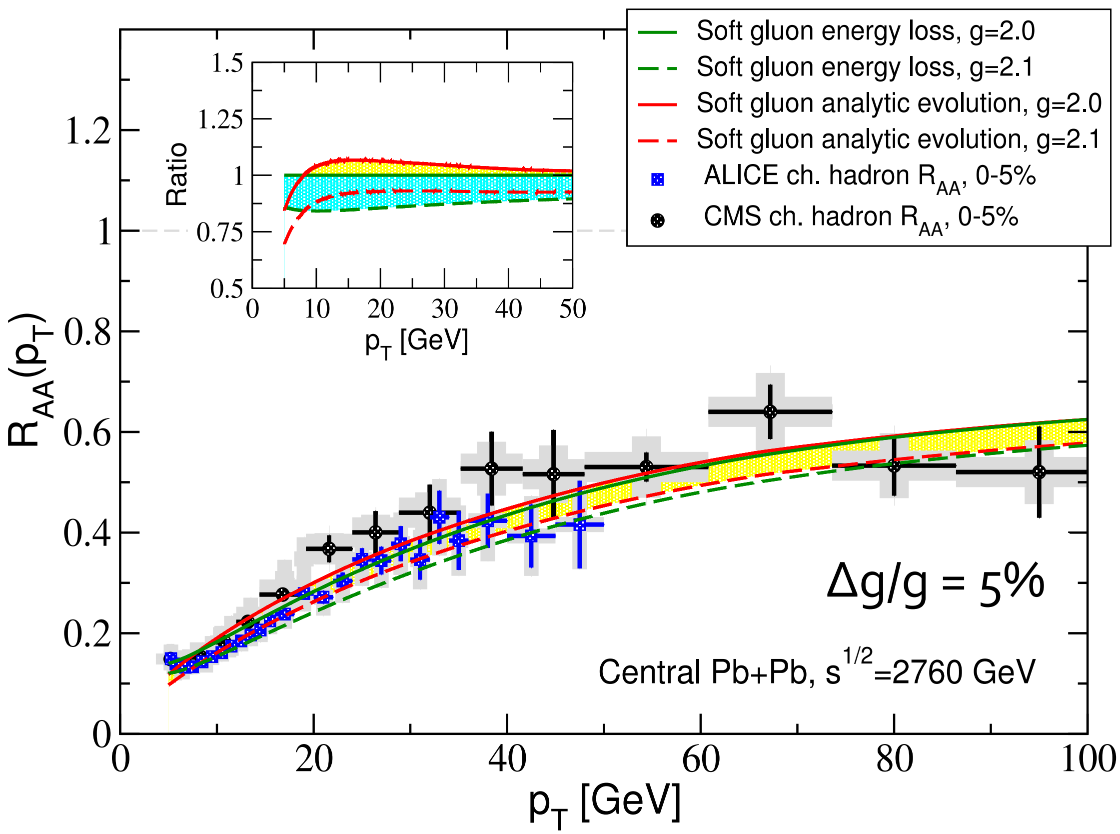
$$\langle \tilde{N^g}\rangle_z = \int_{1-z}^{1}\mathrm{d}z' \int_{Q_0}^{Q} dQ' \frac{dN}{dz'dQ'}(z',Q') = \int_{1-z}^{1}\mathrm{d}z' \frac{dN}{dz'}(z') \qquad \to_{z\to 1} \langle N^g\rangle\,.$$

G. Ovanesyan et al. (2014)

Numerical results: E-loss vs analytic evolution in the soft gluon limit

- The energy loss approach

Note that we have ignored other nuclear matter effects: Cronin effect, cold nuclear matter energy loss, power corrections/shadowing. At the LHC final-state effects dominate

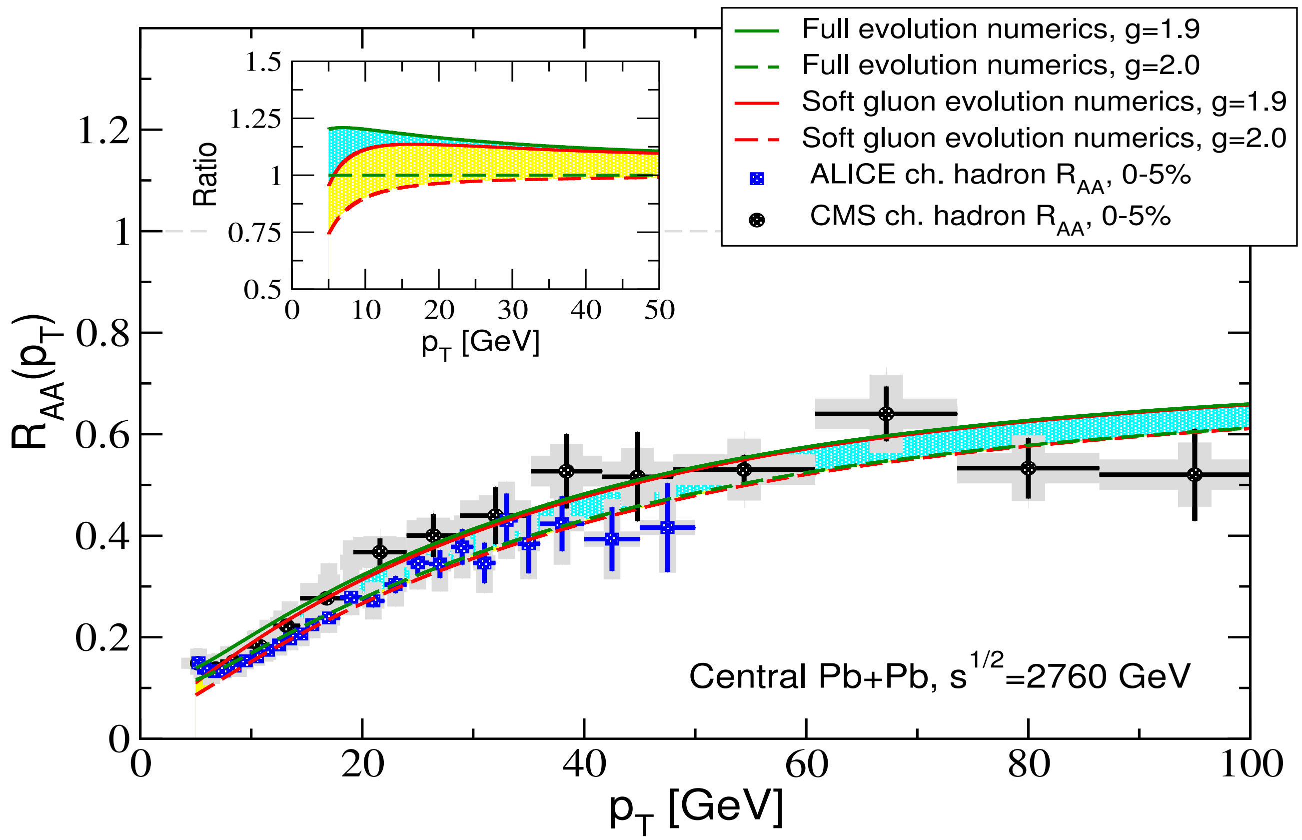


MC code was written by our summer visitor to evaluate the e-loss/medium splittings

The coupling between the jet and the medium can be constrained to 5%, The scattering cross sections/ properties of the medium to 20% (~g^4)

Numerical results: full-x vs small-x evolution

- Implement the fully numerical solution of the DGLAP evolution equations



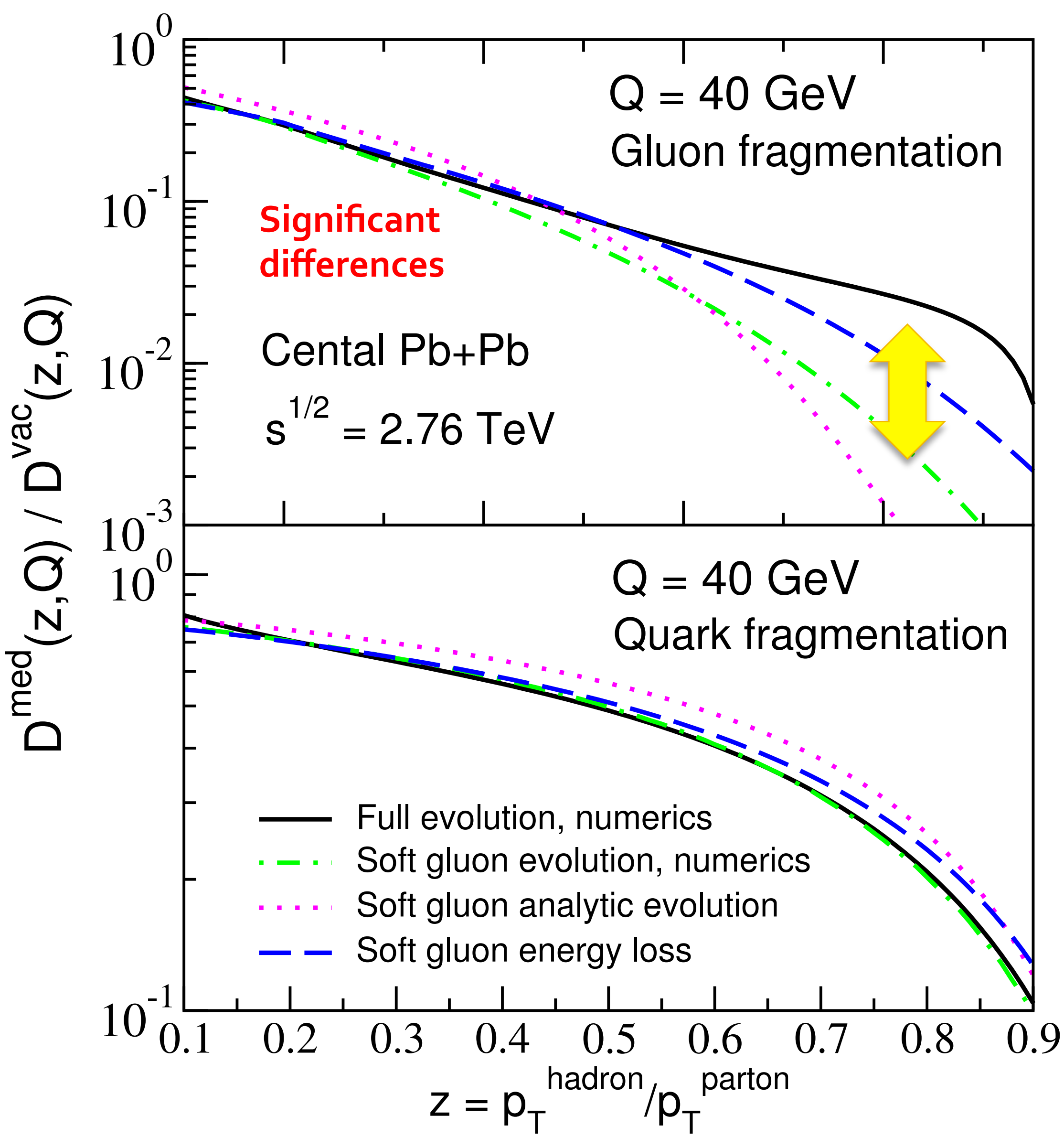
- The coupling between the jet and the medium can be constrained to the same accuracy - 5%

Future directions

"Modified" fragmentation functions

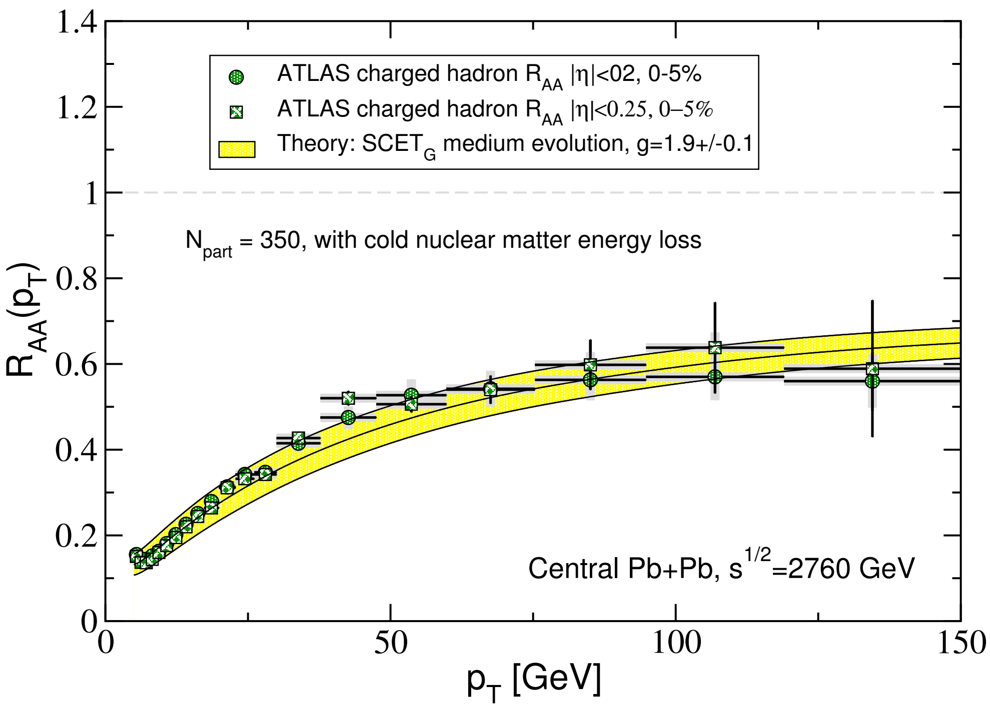
- Dominated by quark fragmentation, small differences, inclusive measurements

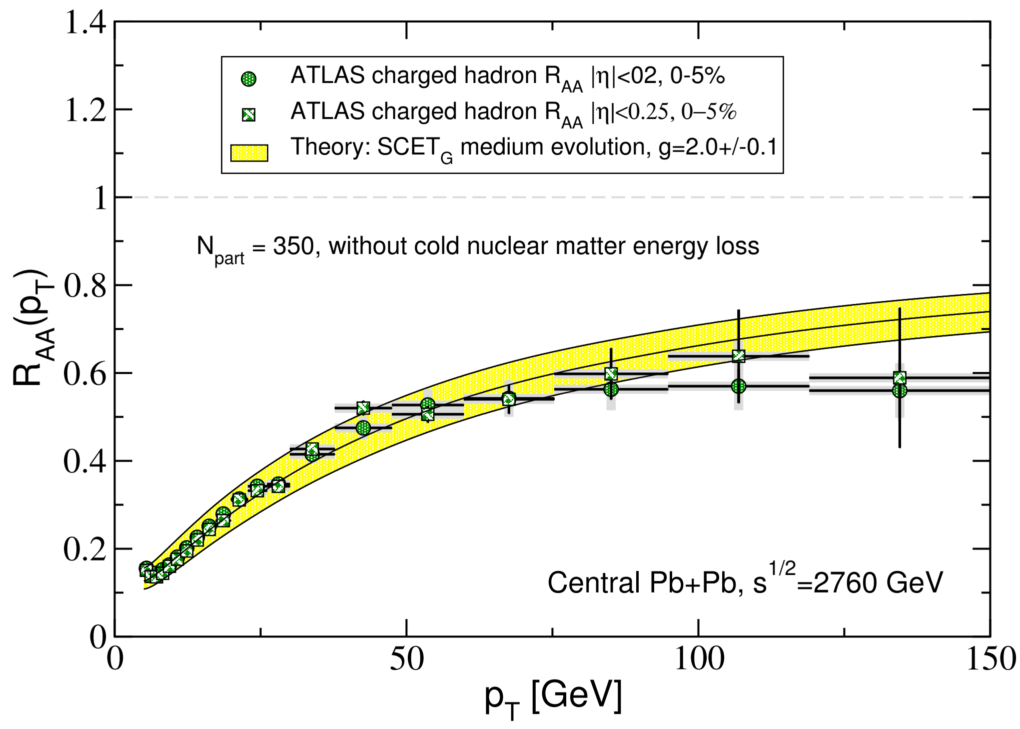
- Points the directions where significant improvement can be expected – longitudinal and transverse structure of jets, tagged jets and dijets



Results at the LHC 2.76 TeV

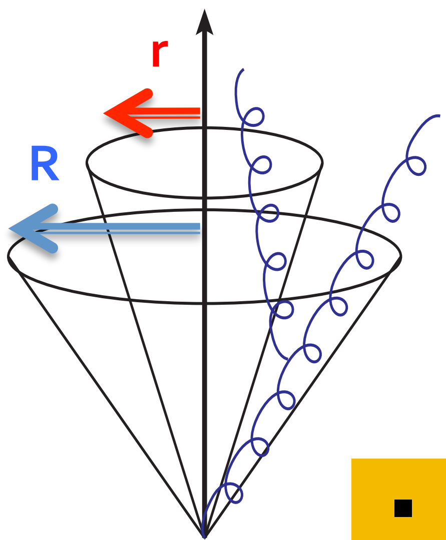
- Very good description of inclusive particle suppression at ALICE, ATLAS and CMS for Pb+Pb run I





- The coupling between the jet and the medium can be constrained to the same accuracy - 5%

Applications of $SCET_G$ to jet shapes



Jet shapes

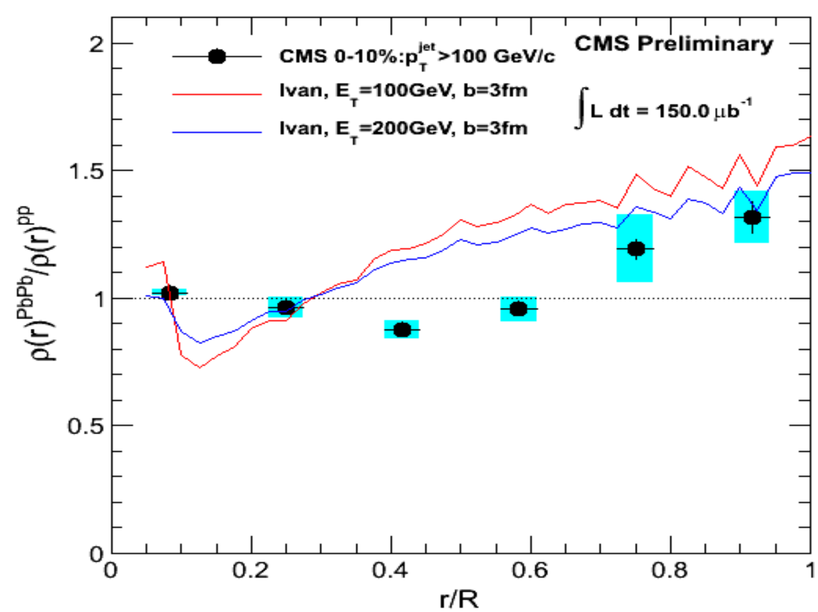
$$\Psi_{\rm int}(r;R) = \frac{\sum_i (E_T)_i \Theta(r-(R_{\rm jet})_i)}{\sum_i (E_T)_i \Theta(R-(R_{\rm jet})_i)} \,,$$

$$\psi(r;R) = \frac{d\Psi_{\rm int}(r;R)}{dr} \,.$$

I.Vitev et al. (2008)

- Jet shapes reflect the energy density inside the jet and the structure of the parton shower

Early prediction and "comparison" to CMS data



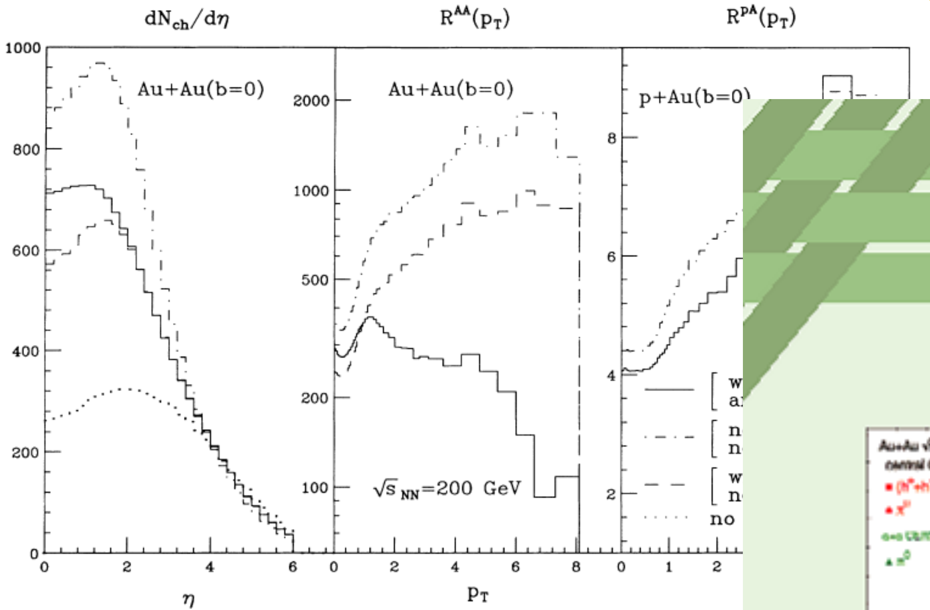
$$C(\bar{n}\cdot p,\mu) = 1 + a_s[L^2 + L + 1] + a_s^2[L^4 + L^3 + L^2 + L + 1] + a_s^3[L^6 + L^5 + L^4 + L^3 + L^2 + L + 1]$$

LL NLL

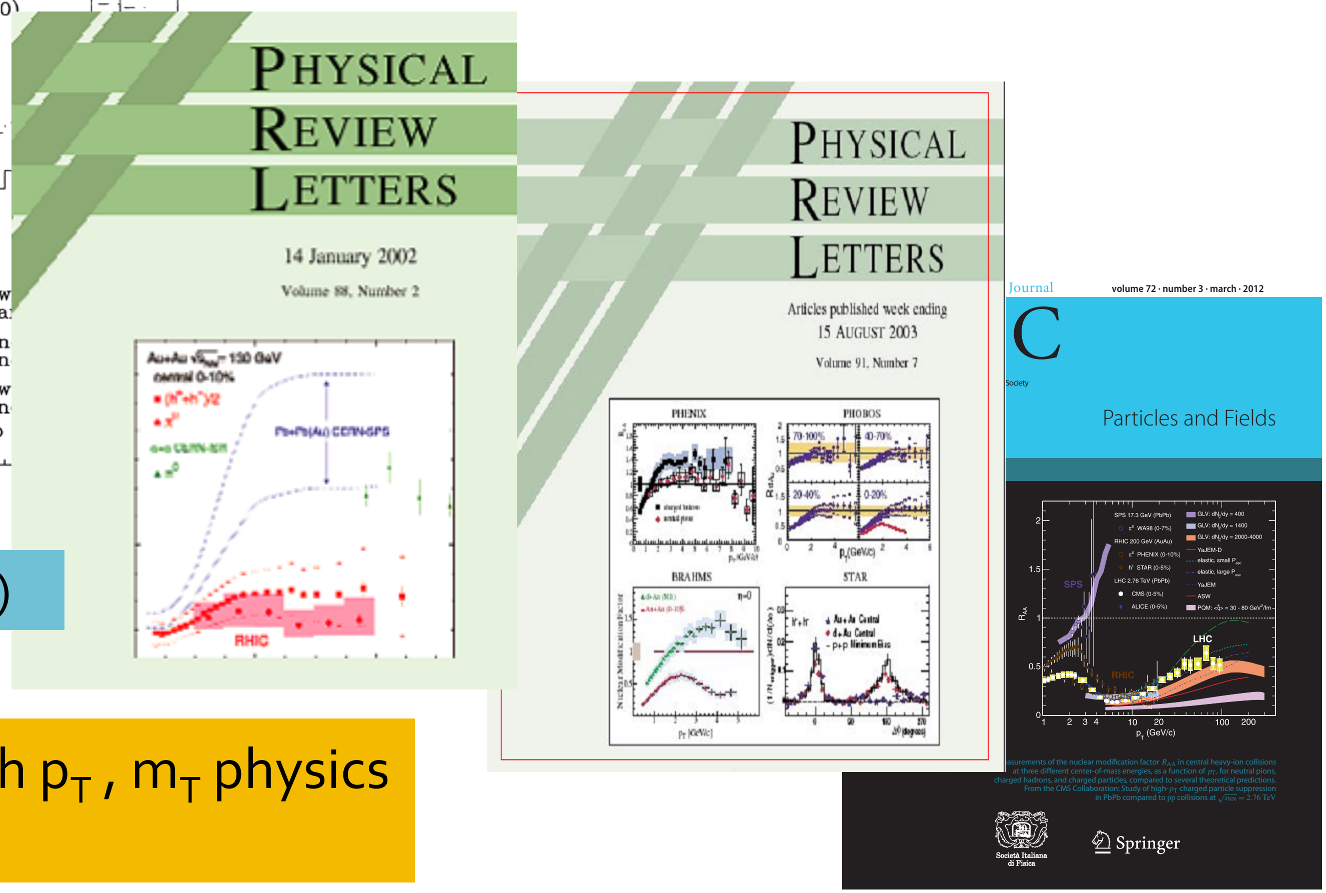
- Small and intermediate r/R - this is the region where we hope we can improve using SCET resummation techniques and full $SCET_G$ medium-induced splittings

Jet quenching

- For my 21th birthday M. Gyulassy and X.N. Wang published their seminal paper on "gluon shadowing and jet quenching



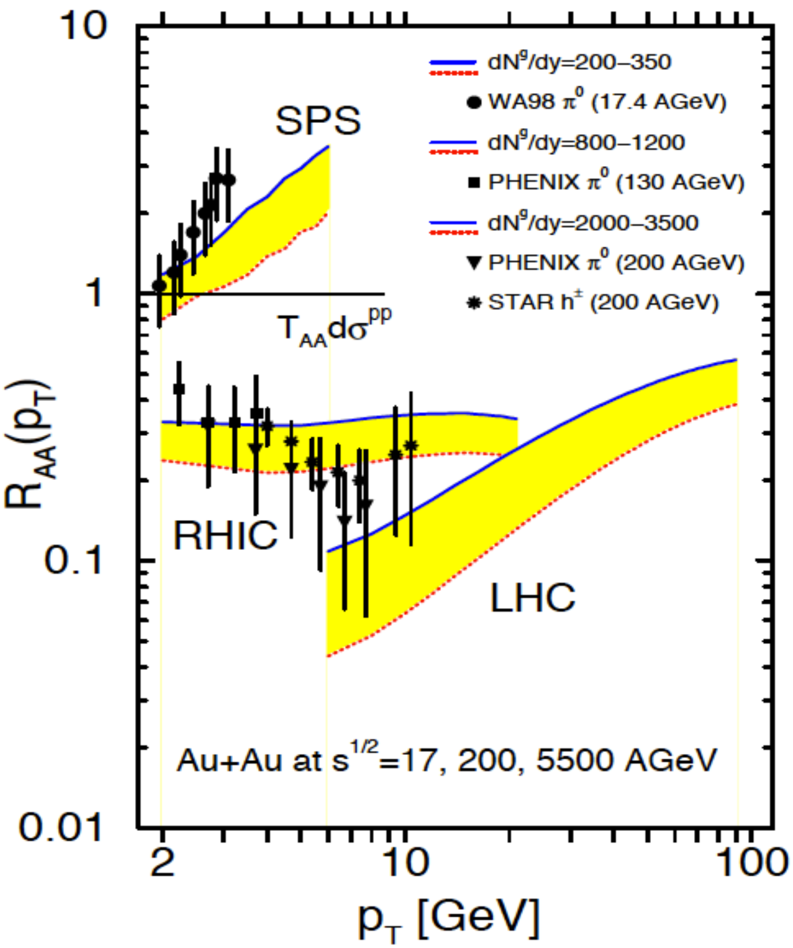
M. Gyulassy, X.N. Wang. (1992)



- Jet quenching defines high p_T , m_T physics in heavy ion collisions

Traditional E-loss – successful but open questions remain

- While still LO, it predicted in 2002, 2006 – the R_{AA} at high p_T for both RHIC and LHC



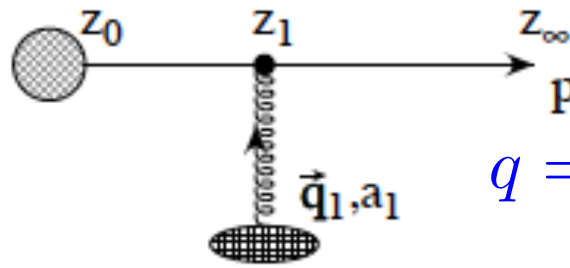
Include the quenched parton and the radiative gluon fragmentation

- Difficult to make connection to the standard LO, NLO, ...; LL, NLL ... pQCD approach (higher orders and resummation)
- There is considerable model dependence and it is difficult to systematically improve this approach

I. V., M. Gyulassy (2002)

The Glauber gluon Lagrangian

- An effective theory of jet propagation in matter



$$q = (\lambda^2, \lambda^2, \lambda)Q$$

$$\mathcal{L}_{\mathrm{G}}\left(\xi_n, A_n, \eta\right) = \sum_{p,p',q} \mathrm{e}^{-i(p-p'+q)x} \left(\bar{\xi}_{n,p'} \Gamma^{\mu,a}_{\mathrm{qqA_G}} \frac{\bar{\not n}}{2} \xi_{n,p} - i \Gamma^{\mu\nu\lambda,abc}_{\mathrm{ggA_G}} \left(A^c_{n,p'}\right)_\lambda \left(A^b_{n,p}\right)_\nu \right) \bar{\eta}\, \Gamma^{\delta,a}_{\mathrm{s}}\, \eta\, \Delta_{\mu\delta}(q)$$

A. Idilbi et al. (2008)

G. Ovanesyan et al. (2011)

Effective potential

- Feynman rules for different sources and gauges

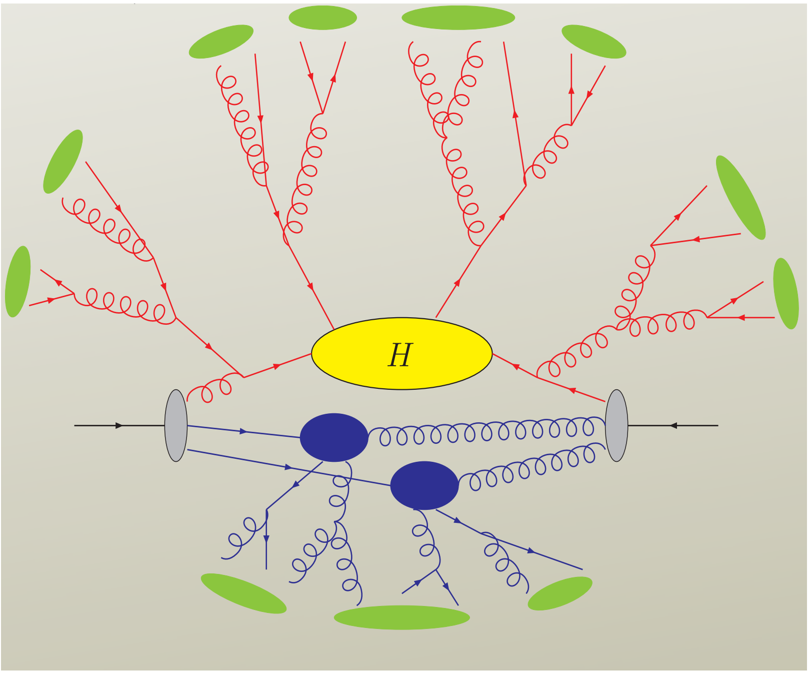
$$\Gamma^{\mu,a}_1 = igT^a n^\mu \frac{\bar{\not n}}{2} \,,$$

$$\Gamma^{\mu,a}_2 = igT^a \frac{\gamma^\mu_\perp \not p_\perp + \not p'_\perp \gamma^\mu_\perp}{\bar{n}\cdot p} \frac{\bar{\not n}}{2} \,,$$

$$\Sigma^{\mu\nu\lambda,abc}_1 = g f^{abc} n^\mu \left[g^{\nu\lambda} \bar{n}\cdot p + \bar{n}^\nu \left(p'^\lambda_\perp - p^\lambda_\perp\right) - \bar{n}^\lambda \left(p'^\nu_\perp - p^\nu_\perp\right) - \frac{1-\frac{1}{\xi}}{2} \left(\bar{n}^\lambda p^\nu + \bar{n}^\nu p'^\lambda\right) \right] \,,$$

$$\Sigma^{\mu\nu\lambda,abc}_2 = g f^{abc} \left[g^{\mu\lambda}_\perp \left(-\frac{n^\nu}{2} p^+ + p^\nu_\perp - 2p'^\nu_\perp \right) + g^{\mu\nu}_\perp \left(-\frac{n^\lambda}{2} p^+ + p'^\lambda_\perp - 2p^\lambda_\perp \right) + g^{\nu\lambda}_\perp \left(n^\mu \bar{n}\cdot p + p^\mu_\perp + p'^\mu_\perp \right) \right] \,,$$

Evolution and resummation of large logarithms



- Collinear splitting kernels

$$\left(\frac{dN}{dx\,d^2\boldsymbol{k}_\perp}\right)_{q\to qg} = \frac{\alpha_s}{2\pi^2} C_F \frac{1+(1-x)^2}{x}\frac{1}{\boldsymbol{k}_\perp^2} \equiv \frac{\alpha_s}{2\pi^2}\frac{1}{\boldsymbol{k}_\perp^2} P_{gq}^{\rm real}(x),$$

$$P_{gg}^{\rm real}(x) = 2C_A\left(\frac{1-x}{x} + \frac{x}{1-x} + x(1-x)\right)$$

$$P_{gq}^{\rm real}(x) = T_R\left(x^2 + (1-x)^2\right),$$

$$P_{qg}^{\rm real}(x) = C_F\frac{1+x^2}{1-x}.$$

Gribov et al. (1972)

G. Altarelli et al. (1977)

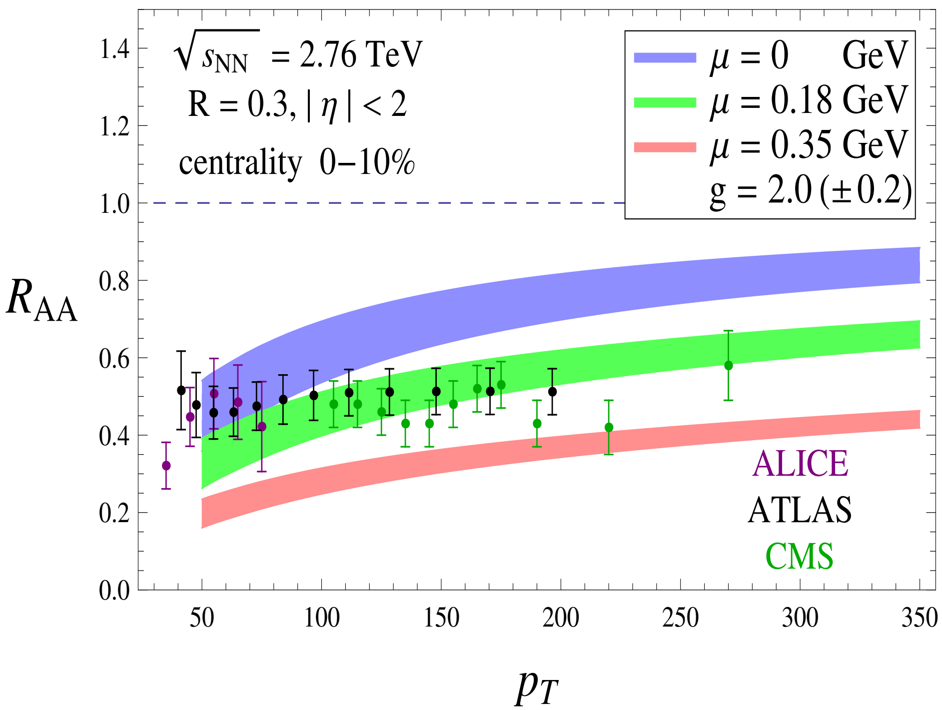
Y. Dokshitzer (1977)

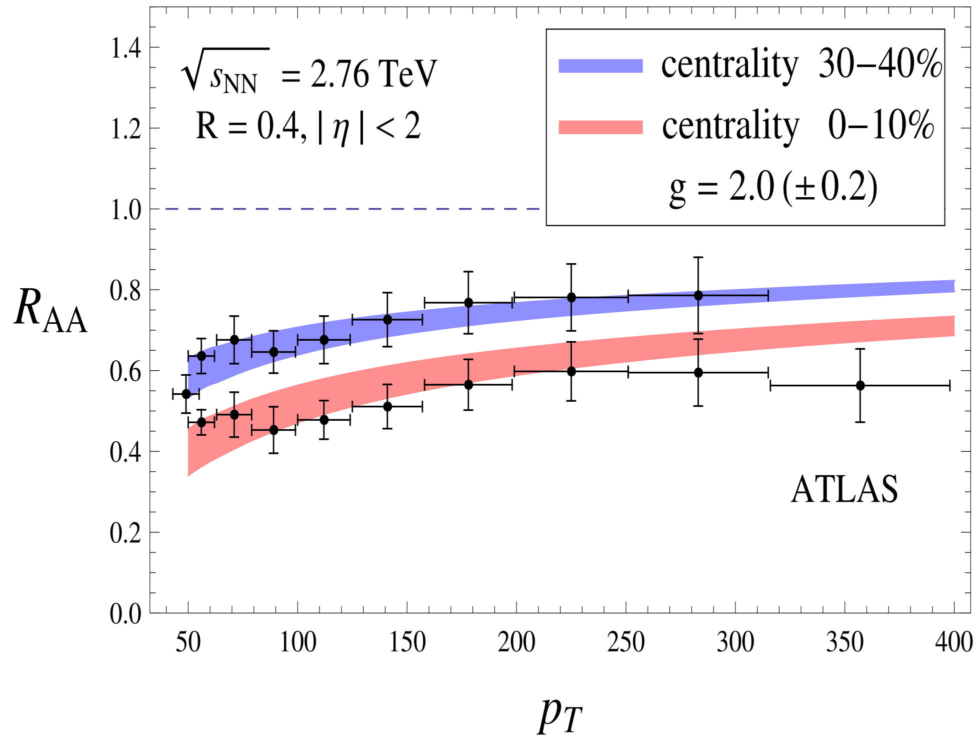
$$\frac{\mathrm{d}D_{h/q}(z,Q)}{\mathrm{d}\ln Q} = \frac{\alpha_s(Q)}{\pi}\int_z^1 \frac{\mathrm{d}z'}{z'}\left[P_{q\to qg}(z')D_{h/q}\left(\frac{z}{z'},Q\right) + P_{q\to gq}(z')D_{h/g}\left(\frac{z}{z'},Q\right)\right]$$

$$\frac{\mathrm{d}D_{h/g}(z,Q)}{\mathrm{d}\ln Q} = \frac{\alpha_s(Q)}{\pi}\int_z^1 \frac{\mathrm{d}z'}{z'}\left[P_{g\to gg}(z')D_{h/g}\left(\frac{z}{z'},Q\right) + P_{g\to q\bar{q}}(z')\sum_q D_{h/q}\left(\frac{z}{z'},Q\right)\right]$$

- Yield LLA or MLLA (LL')

Suppression of reconstructed jets at the LHC, Pb+Pb at 2.76 TeV

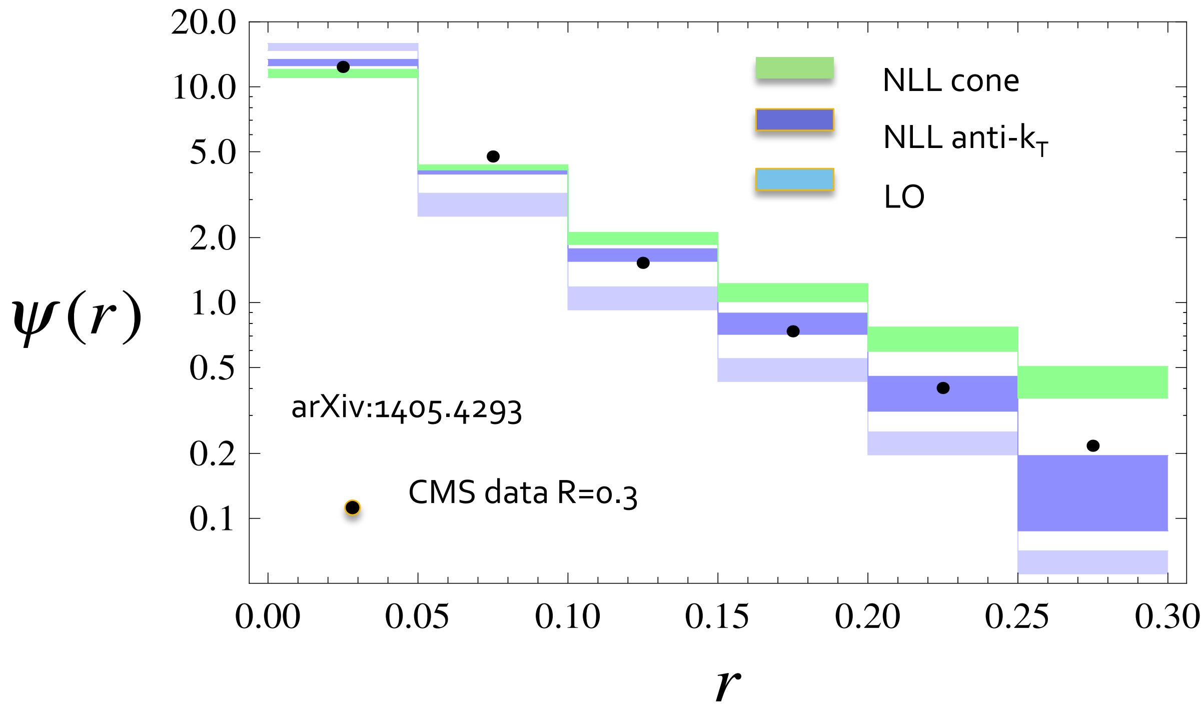




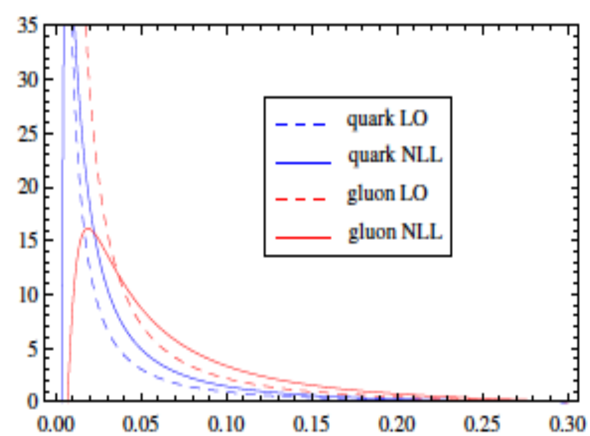
- Cold nuclear matter effects contribute toward the inclusive jet suppression at high p_T. Approximately ½ of the effect
- Describes well the centrality dependence of the inclusive jet suppression
- There is some p_T dependence remaining to RAA. Interesting to investigate soft function effects, collisional energy loss

Numerical NLL results in p+p collisions

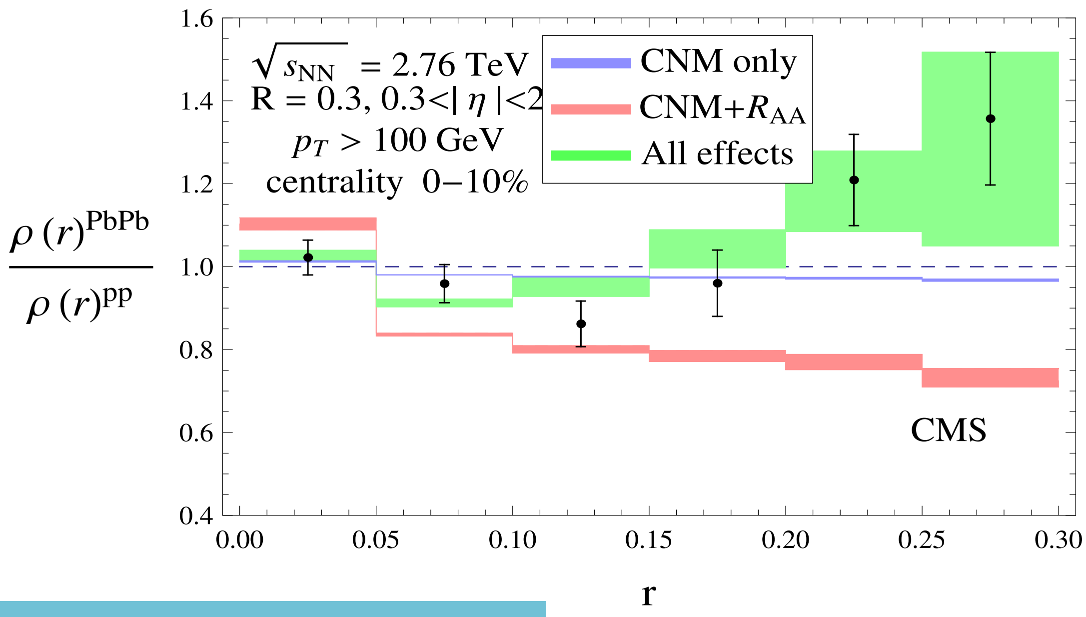
- We derived the algorithm dependence of the jet shapes (anti)k_T vs cone
- Significant improvement over fixed order calculation
- For large radii (0.7) works less well (hadronization et al.) also for smaller energies



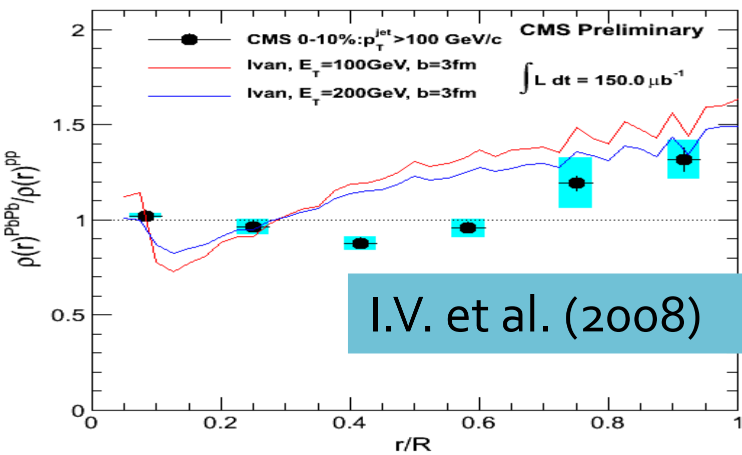
Just t show that the fixed order is divergent



Jet shape modification in heavy ion collisions



Y.-T. Chien et al. (2015)



I.V. et al. (2008)

- Improvements relative to the traditional E-loss approach are indeed significant
- CNM effects do not play a role in intrajet (or jet correlation observables)
- Jet quenching alone leads to narrowing of the shape.
- The broad parton shower leads to enhancement in the periphery