Causal hydrodynamic fluctuation in Bjorken expansion

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Introduction

Hydrodynamic fluctuation!

Hadron gas

QGP fluid

Initial collision
Thermal equilibrium state = Maximum entropy state state

The effect of causal hydrodynamic fluctuation on entropy production on an event by event basis.

⟹ Using the Bjorken expansion model
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Hydrodynamic equations in Bjorken expansion

Bjorken’s equation of energy density evolution

\[ \frac{de}{d\tau} = -\left( \frac{e + P}{\tau} \right) \left( 1 - \frac{\pi}{sT} + \frac{\Pi}{sT} \right) \]

Correction from viscosities

- \( e \): Energy density
- \( P \): Hydrostatic pressure
- \( s \): Entropy density
- \( \pi \): Shear stress tensor
- \( \Pi \): Bulk pressure
Constitutive equation

First order constitutive equations for 3D and Bjorken expansion

<table>
<thead>
<tr>
<th>3-dimensional expansion</th>
<th>Shear stress $\pi^{\mu\nu}$</th>
<th>Bulk pressure $\Pi$</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>$\pi^{\mu\nu} = 2\eta \nabla \langle \mu u^\nu \rangle$</td>
<td>$\Pi = -\zeta \partial_\mu u^\mu$</td>
</tr>
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</table>

| Bjorken expansion       | $\pi = \frac{4\eta}{3\tau}$                              | $\Pi = -\frac{\zeta}{\tau}$ |

$\pi = \pi^{00} - \pi^{33}$, $\nabla^\mu = \Delta^{\mu\nu} \partial_\nu$

$\langle \ \rangle$: Symmetric, traceless and perp. to flow velocity

$\eta$: Shear viscosity, $\zeta$: Bulk viscosity
Stochastic constitutive equations

For shear stress

\[
\tau_\pi \frac{d\pi}{d\tau} + \pi = \frac{4\eta}{3\tau} + \xi_\pi
\]

(\xi_\pi = \xi^{00} - \xi^{33})

For bulk pressure

\[
\tau_\Pi \frac{d\Pi}{d\tau} + \Pi = -\frac{\zeta}{\tau} + \xi_\Pi
\]

Relaxation term

\(\tau_\pi, \tau_\Pi: \text{Relaxation time}\)
Fluctuation dissipation relation

\[ \langle \xi(x) \xi(x') \rangle = 2\kappa T \delta^{(4)}(x - x') \]

\(\xi\): Hydrodynamic fluctuation
\(\kappa\): Transport coefficient
\(T\): Temperature
\(\langle \quad \rangle\): Ensemble average

Fluctuation and dissipation are related with each other!
Fluctuation dissipation relation in Bjorken expansion

Fluctuation dissipation relation

Bulk pressure: \[ \langle \xi_\Pi(\tau_i)\xi_\Pi(\tau_j) \rangle = \frac{2T\zeta \delta_{ij}}{\Delta \tau \Delta V} \]

Shear stress: \[ \langle \xi_\pi(\tau_i)\xi_\pi(\tau_j) \rangle = \frac{8\eta T \delta_{ij}}{3\Delta \tau \Delta V} \]

Other properties: \[ \langle \xi_\pi \rangle = \langle \xi_\Pi \rangle = 0, \langle \xi_\pi \xi_\Pi \rangle = 0. \]

\( \Delta V \): Volume of a fluid element \( (\Delta V = \tau \Delta \eta_s \Delta x \Delta y) \)
Model for EoS

Equation of state with crossover

\[ s(T) = d_H \frac{4\pi^2}{90} T^3 \frac{1 - \tanh\left[\frac{T - T_c}{\Gamma}\right]}{2} + d_Q \frac{4\pi^2}{90} T^3 \frac{1 + \tanh\left[\frac{T - T_c}{\Gamma}\right]}{2}, \]

\[ P = \int_0^T dT' s(T'), e = Ts - P. \]


\( d_H = 3, d_Q = 37 \): D.o.f. in QGP and hadronic matter

\( T_c = 0.17 \text{ (GeV)} \): Pseudo critical temperature

\( \Gamma = 0.02T_c \): Width parameter
Models for transport coefficients

Temperature dependence of transport coefficients

\[ \frac{\eta}{s} = \frac{1}{4\pi} \]


\[ \frac{\zeta}{s} = 15 \left( \frac{1}{3} - C_s^2 \right)^2 \]

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Time evolution of temperature

Initial conditions:
\[ T_0 = 0.22 \text{(GeV)}, \pi = \Pi = 0, \tau_0 = 1 \text{fm} \]
Time evolution of entropy

Perfect fluid ($\pi = \Pi = 0$)
→ Entropy conservation
→ $s_\tau = s_0 \tau_0$

Viscous fluid ($\pi \neq 0, \Pi \neq 0$)
→ Increase of entropy

Hydrodynamic fluctuation $\Rightarrow$ final entropy fluctuation

Entropy production rate:

$$\sigma = \frac{dS}{d\tau} = \left(\frac{\pi}{T} - \frac{\Pi}{T}\right) \Delta s \Delta x \Delta y \geq 0$$
Time evolution of shear stress and bulk pressure.

*Entropy production rate:
\[ \sigma = \frac{dS}{d\tau} = \left( \frac{\pi}{T} - \frac{\Pi}{T} \right) \Delta \eta_s \Delta x \Delta y \geq 0 \]

\( \pi \) and \( \Pi \) take positive or negative value in the case of fluctuation.

Fluctuations of \( \Pi \) and \( \pi \) closely relate with entropy fluctuation.
Final entropy in hydrodynamic fluctuation is distributed around that of the viscous fluid.

- Final entropy can be smaller than initial entropy in a small number of events.
- The smaller volume, the larger variance of entropy.

\[
\frac{\langle (s \tau - \bar{s} \tau)^2 \rangle}{(s_0 \tau_0)^2} : \text{Normalized variance}
\]
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Summary

• Causal hydrodynamic fluctuation in Bjorken expansion model
• Final entropy fluctuates around the result of viscous fluid.
• Fluctuation effect would be important in small systems.
Backup
Space-time rapidity:

$$\eta_s = \frac{1}{2} \ln \frac{t+z}{t-z}$$

Proper time:

$$\tau = \sqrt{t^2 - z^2}$$

Flow velocity:

$$u^\mu_{\text{Bj}} = \frac{t}{\tau} \left(1, 0, 0, \frac{z}{t} \right)$$

$$= (\cosh \eta_s, 0, 0, \sinh \eta_s)$$
Parameter dependence of final entropy

\[ \eta/s \text{ dependence}* \]

*This simulation was done without bulk pressure

\[ \text{Relaxation time dependence}** \]

**\( \eta/s = 1/4\pi \), With bulk pressure
Introduction

Entropy

Dissipation

Fluctuation

Thermal equilibrium state
= Maximum entropy state

the effect of causal hydrodynamic fluctuation on entropy production on an event by event basis.

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