Conserved Charge Fluctuations and Correlations from Lattice QCD and the Beam Energy Scan

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Conclusion

the preliminary STAR data on cumulant ratios of conserved net-charge distributions for

$$\sqrt{s_{NN}} \geq 27 \text{ GeV}$$

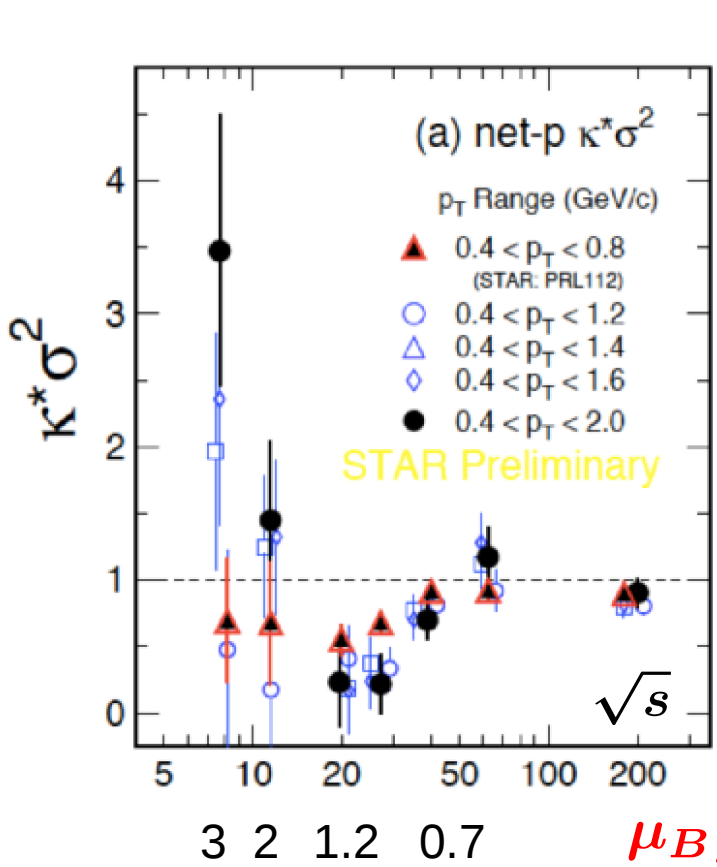
can be understood in terms of a next-to-leading order expansion of net-baryon number fluctuations calculated in QCD equilibrium thermodynamics

Bielefeld-BNL-CCNU Collaboration

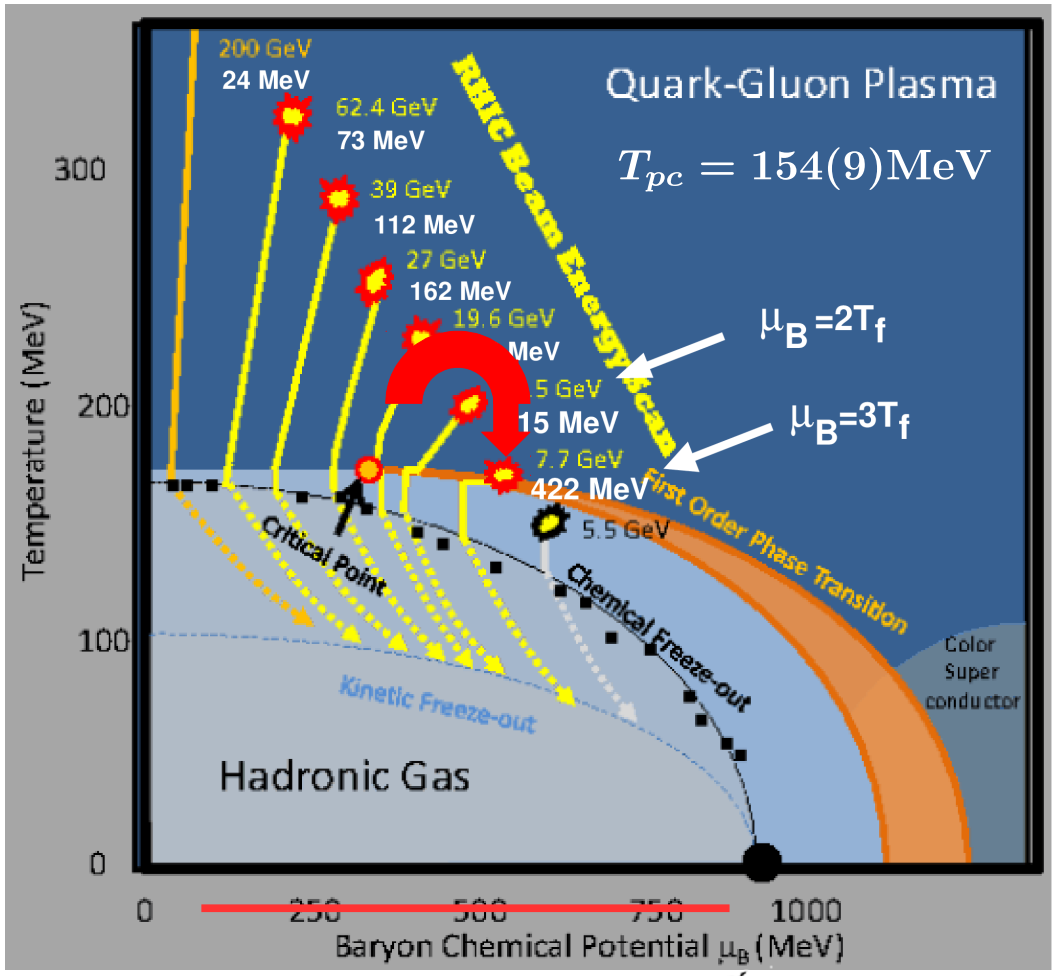
A. Bazavov, H.-T. Ding, P. Hegde, O. Kaczmarek, FK, E. Laermann, S. Mukherjee, H. Ohno, P. Petreczky, C.Schmidt, S. Sharma, W. Soeldner and M.Wagner,

arXiv:1509.05786 and preliminary results

Exploring the QCD phase diagram



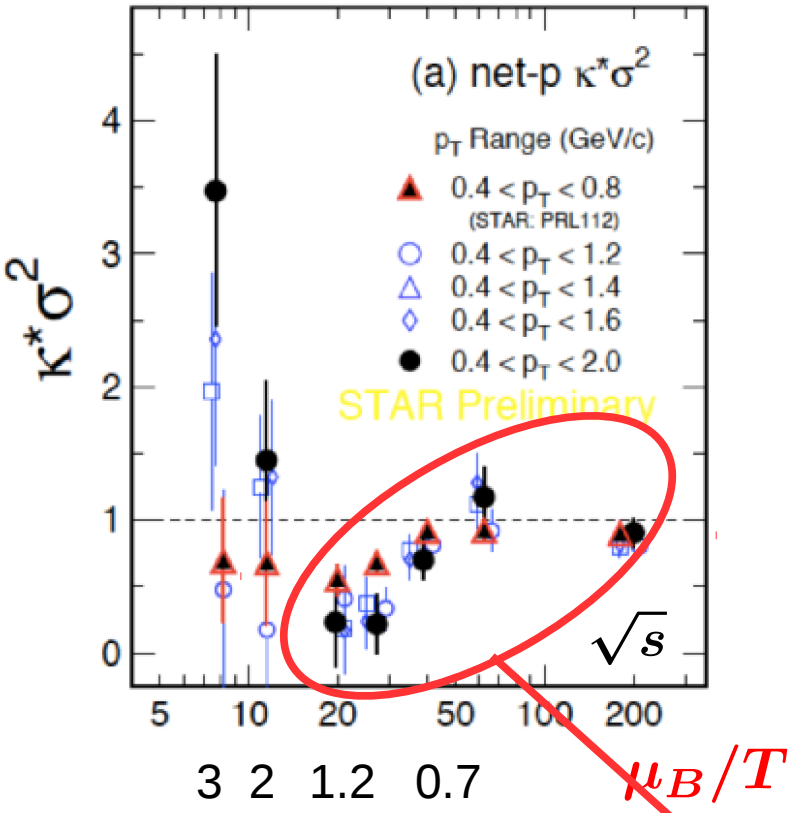
X. Luo (STAR Collaboration), PoS CPOD2014 (2014) 019



equilibrium thermodynamics:
conserved charge fluctuation
controlled by T_f, μ_B, μ_S

Where is the critical point?

Exploring the QCD phase diagram



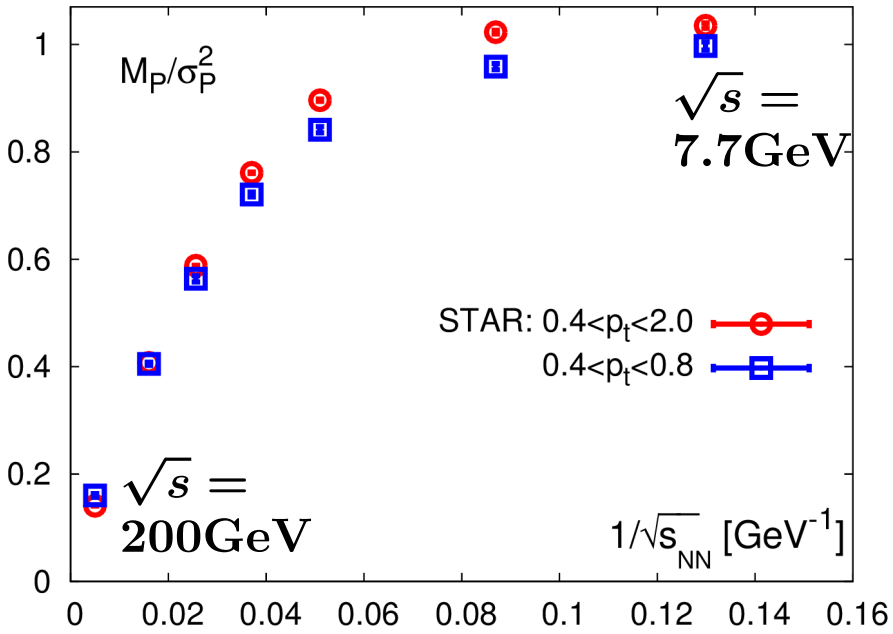
?

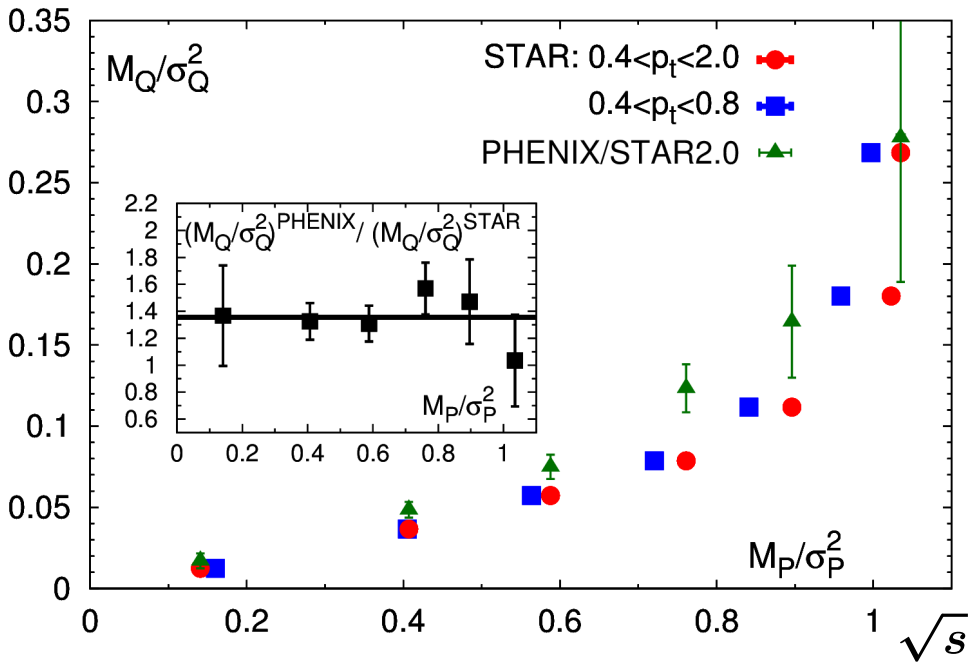
More moderate questions:

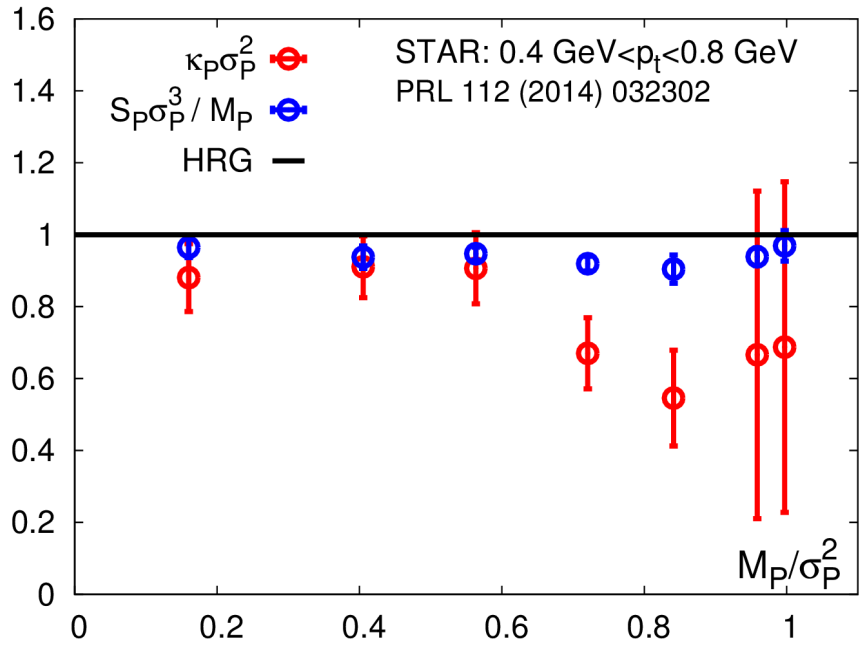
- Can we understand the systematics seen in cumulants of charge fluctuations in terms of QCD thermodynamics ?
- How far do we get with low order Taylor expansions of **QCD** in explaining the obvious deviations from HRG model behavior ?

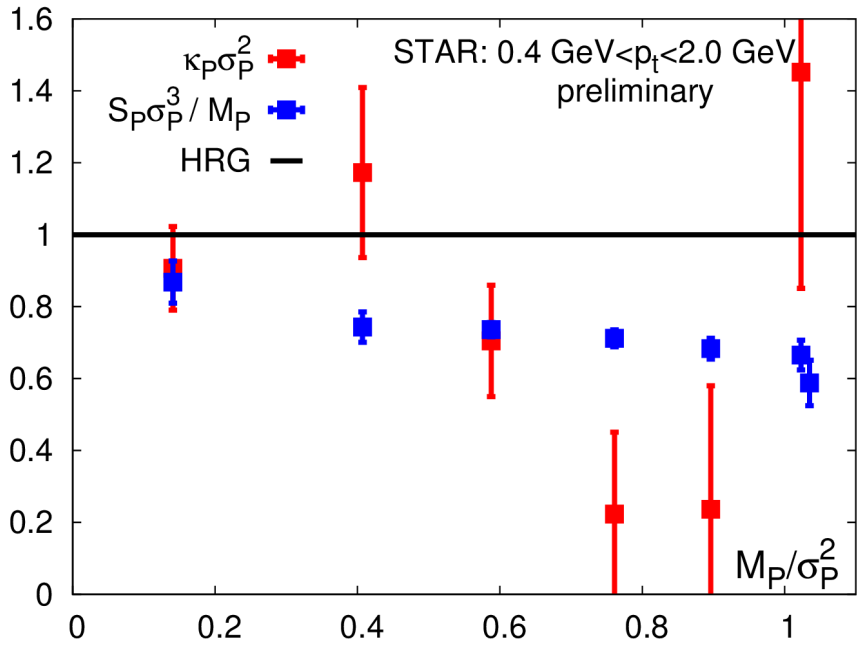
- For $\sqrt{s} \geq 27$ GeV : Structure of net-electric charge and net-proton cumulants is inconsistent with HRG thermodynamics, but can eventually be understood in terms of **QCD thermodynamics in a next-to-leading order Taylor expansion**

STAR and PHENIX data on cumulant ratios of net-proton number and net-electric charge fluctuations









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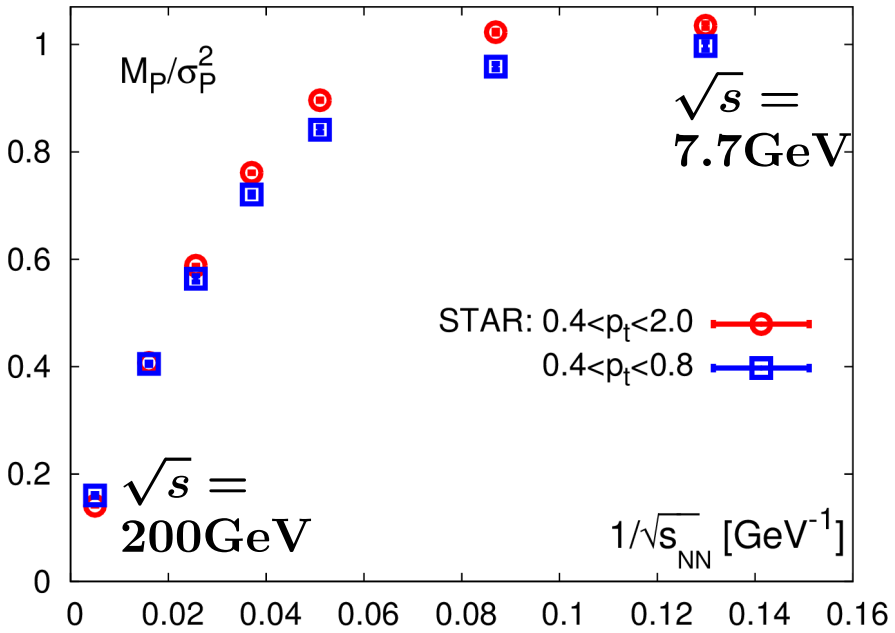

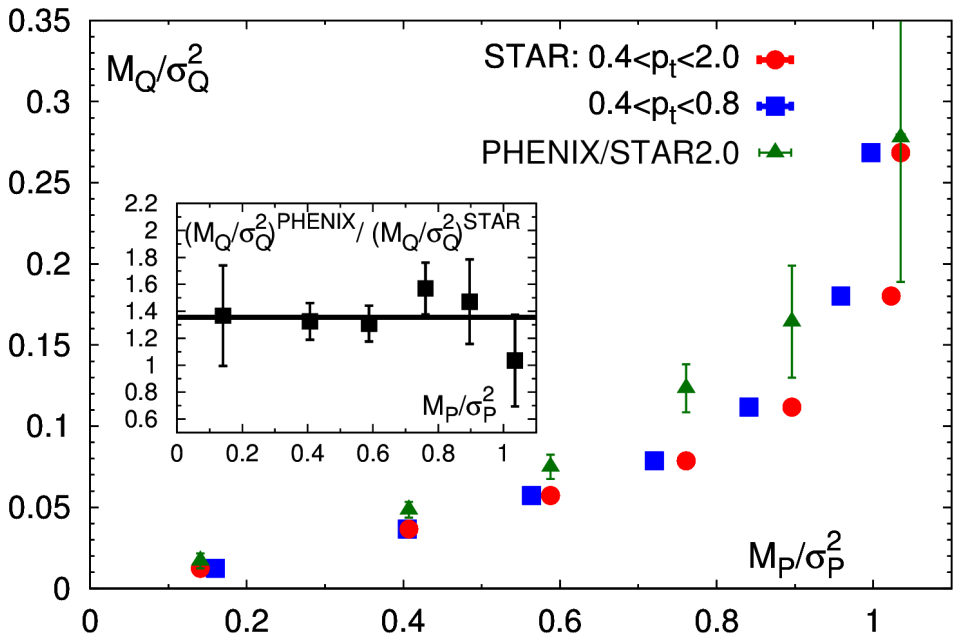

$R_{12}^P \equiv \frac{M_P}{\sigma_P^2}$ and $R_{12}^Q \equiv \frac{M_Q}{\sigma_Q^2}$ are monotonic functions of $\sqrt{s_{NN}}$

→ and hence also of μ_B

(this is not trivial; It will not hold close to a critical point !)

→ replace μ_B/T in favor of R_{12}^B, e.g.

$$\frac{\mu_B}{T} = m_1^B R_{12}^B + m_3^B \left(R_{12}^B\right)^3 + \mathcal{O}\left((R_{12}^B)^5\right), \text{ e.g. } m_1^B = \frac{1}{R_{12}^{B,1}}$$

Conserved charge fluctuations and freeze-out
mean and variance

for simplicity: $\mu_S = \mu_Q = 0$

e.g. $M_S \equiv \chi_1^B(\mu) = 0$

$M_Q/M_B \equiv \chi_1^Q(\mu)/\chi_1^B(\mu) = r$

$\Rightarrow \mu_{Q/S} \equiv \mu_{Q/S}(\mu_B)$

ratio of cumulants on "a line" in the (T, μ_B) plane (NLO Taylor expansion)

$$\frac{M_B}{\sigma_B^2} = \frac{\mu_B}{T} \frac{1 + \frac{1}{6}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}{1 + \frac{1}{2}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}$$

$$\frac{M_Q}{\sigma_Q^2} = \frac{\mu_B}{T} \frac{\chi_{11}^{BQ}}{\chi_2^Q} \frac{1 + \frac{1}{6}\frac{\chi_{31}^{BQ}}{\chi_{11}^{BQ}}\left(\frac{\mu_B}{T}\right)^2}{1 + \frac{1}{2}\frac{\chi_{22}^{BQ}}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}$$

will not discuss here:

– non-equilibrium effects
(S. Mukherjee et al., arXiv:1506.00645)
– proton vs. baryon number distributions
(M. Kitazawa et al, arXiv:1205.3292, arXiv:1303.3338)
– acceptance and pt-cuts
(P. Garg et al, arXiv:1304.7133,
FK, K. Morita, K. Redlich, arXiv:1508.02614
A. Bzdak and V. Koch, arXiv:12064286)

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$$\frac{\mu_B}{T} = m_1^B R_{12}^B + m_3^B \left(R_{12}^B\right)^3$$

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$$\frac{\mu_B}{T} = m_1^B R_{12}^B + m_3^B \left(R_{12}^B\right)^3$$

freeze-out line

$$T_f(\mu_B) = T_{f,0}\left(1 - \kappa_2^f \left(\frac{\mu_B}{T}\right)^2\right)$$

$$\frac{M_Q}{\sigma_Q^2} = \frac{\mu_B}{T} \frac{\chi_{11}^{BQ}}{\chi_2^Q} \frac{1 + \frac{1}{6}\frac{\chi_{31}^{BQ}}{\chi_{11}^{BQ}}\left(\frac{\mu_B}{T}\right)^2}{1 + \frac{1}{2}\frac{\chi_{22}^{BQ}}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}$$

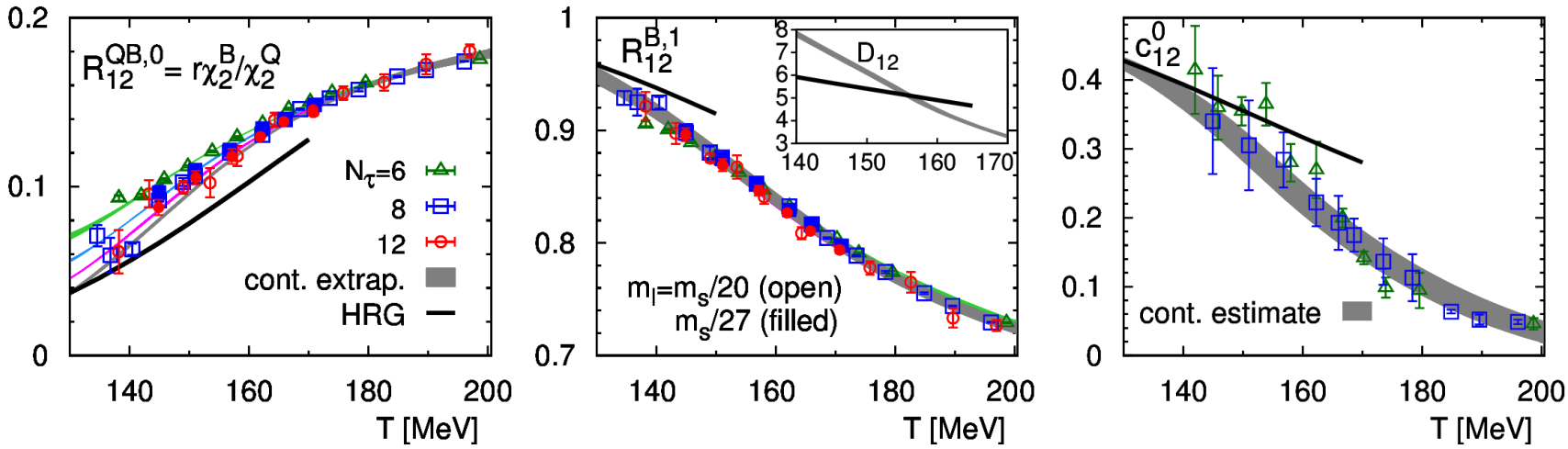
$$\left(\frac{\chi_{11}^{BQ}}{\chi_2^Q}\right)_{T_{f,0}} - \kappa_2^f T_{f,0} \left(\frac{\chi_{11}^{BQ}}{\chi_2^Q}\right)' \left(\frac{\mu_B}{T}\right)^2$$

all $\chi_n \equiv \chi_n(T)$ eventually need to be expanded in T

Curvature of the freeze-out line

$$T_f(\mu_B) = T_{f,0}\left(1 - \kappa_2^f \hat{\mu}_B^2\right)$$

compare lattice QCD calculation of R_{12}^{QB} with experimental data for R_{12}^{QP}



BI-BNL-CCNU, arXiv:1509:05786

$$R_{12}^{QB} \equiv \frac{M_Q/\sigma_Q^2}{M_B/\sigma_B^2} = a_{12}\left(1 + c_{12}\left(R_{12}^B\right)^2\right)$$

$$c_{12}(T, \kappa_2^f) \equiv c_{12}^0(T) - \kappa_2^f D_{12}(T)$$

intercept fixes freeze-out temperature

$$R_{12}^{QB,0}(T) = r\frac{\chi_2^B(T)}{\chi_2^Q(T)}$$

slope fixes curvature coefficient

$$r = N_p/(N_p + N_n) = 0.4$$

Curvature of the freeze-out line

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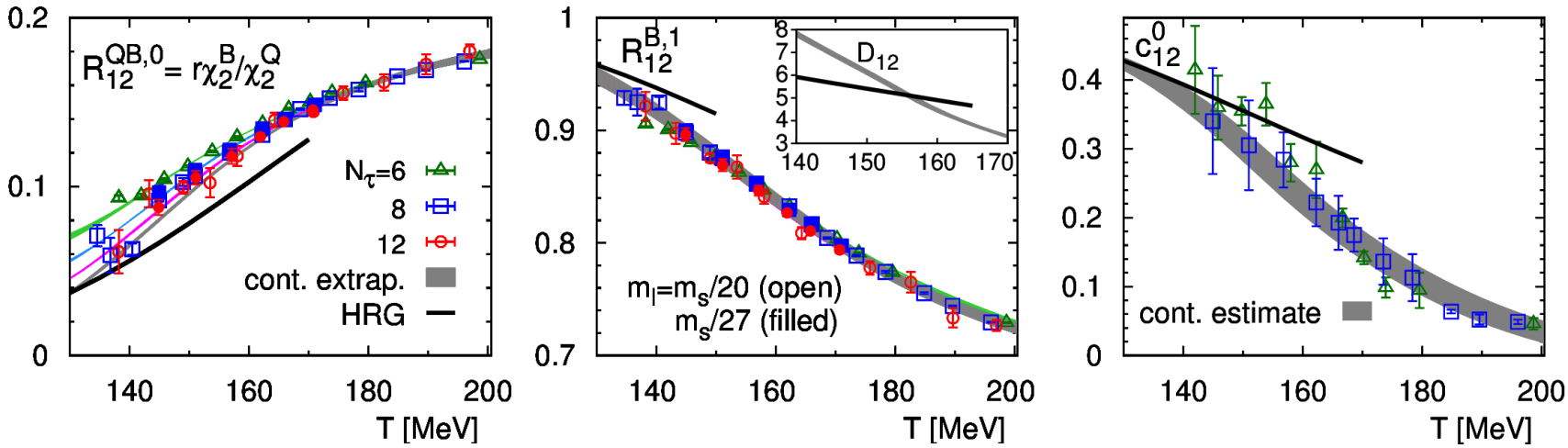

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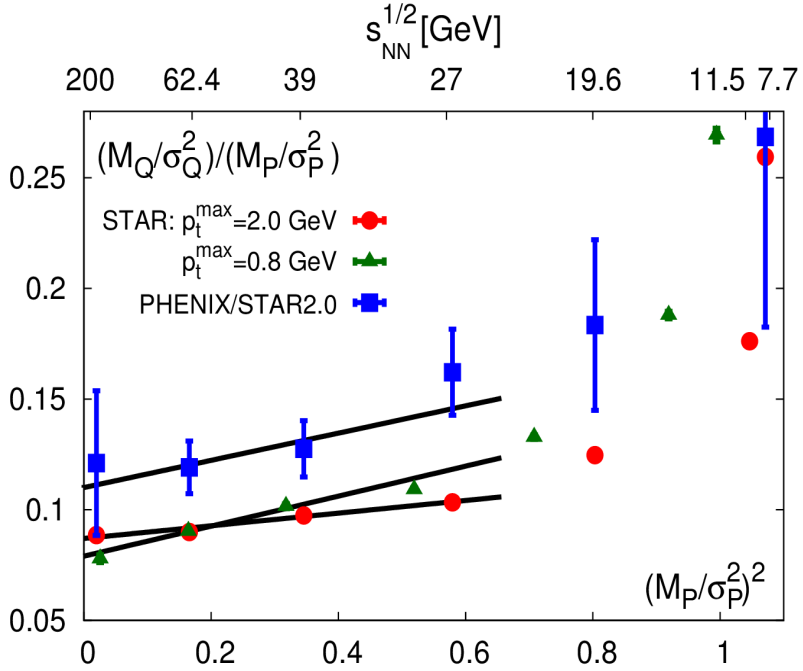
fits to data yield

a_{12}, c_{12}



$T_{f,0}$

155 MeV

147 MeV

145 MeV

– comparing intercept with $R_{12}^{QB,0}$ yields $T_{f,0}$

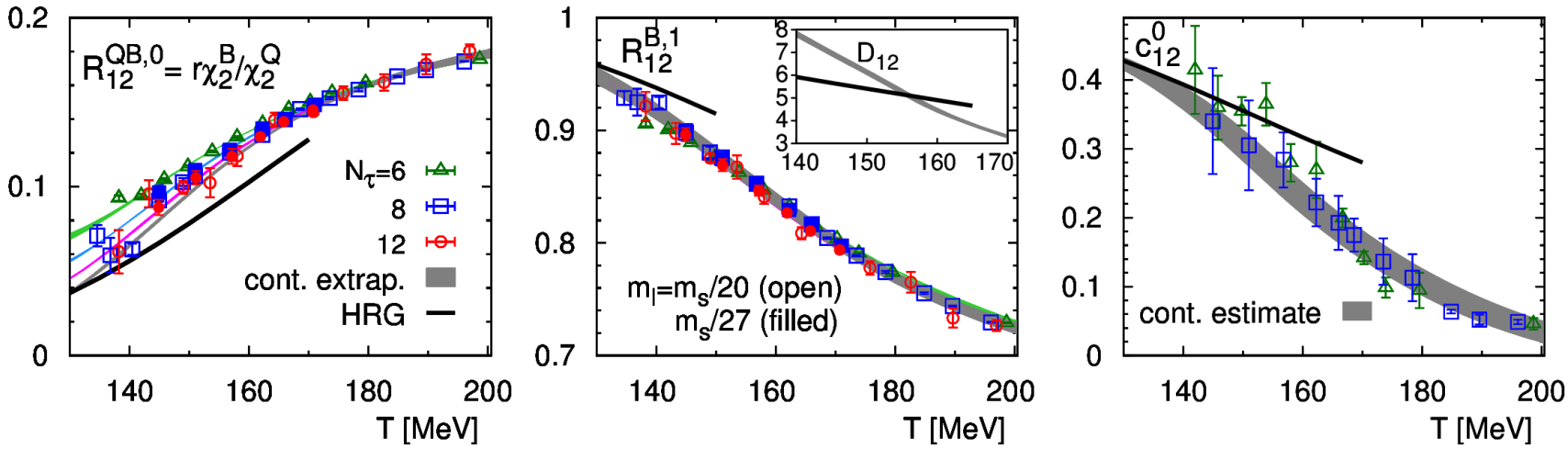
– once $T_{f,0}$ is fixed, the coefficients $c_{12}^0(T)$ and $D_{12}(T)$ are fixed and the slope of the data determines κ_2^f

$$c_{12} \equiv c_{12}^0 - \kappa_2^f D_{12}$$

Curvature of the freeze-out line

$$T_f(\mu_B) = T_{f,0}\left(1 - \kappa_2^f \hat{\mu}_B^2\right)$$

compare lattice QCD calculation of R_{12}^{QB} with experimental data for R_{12}^{QP}



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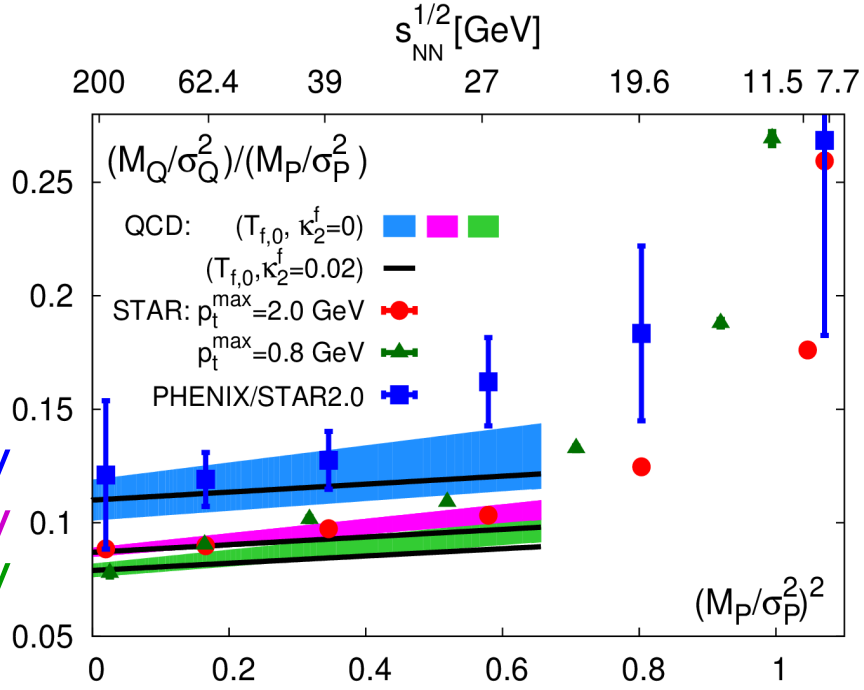


$T_{f,0}$

155 MeV

147 MeV

145 MeV

BI-BNL-CCNU, arXiv:1509:05786

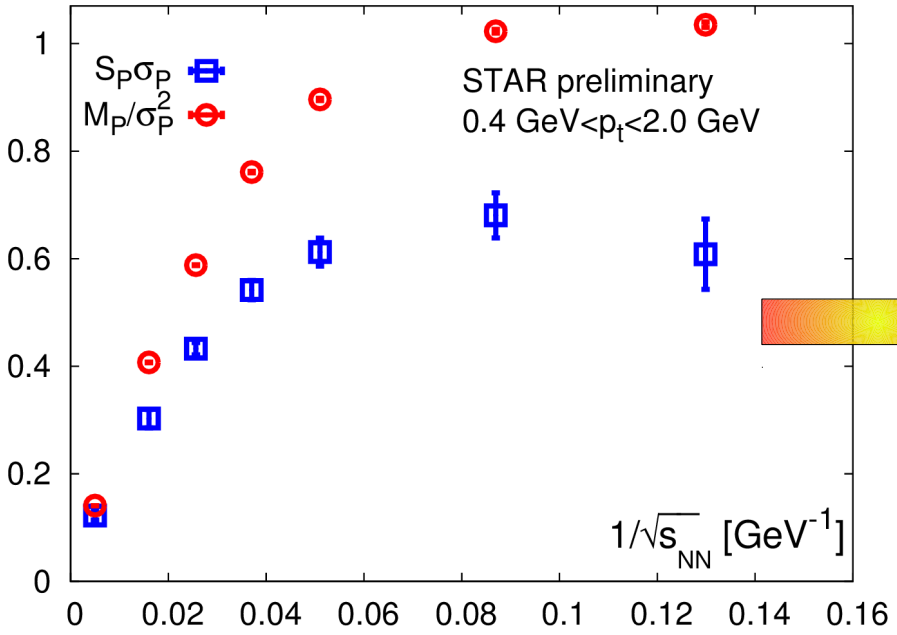
– published STAR data are too steep, yield a negative curvature

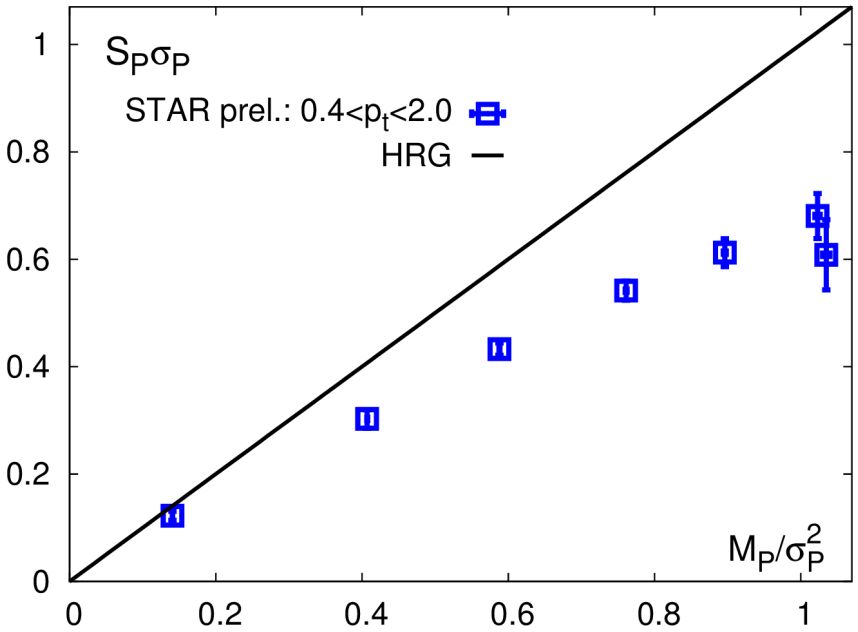
– prelim. STAR data favor a small curvature coefficient for the freeze-out curve

$$\kappa_2^f < 0.011$$

Conserved charge fluctuations and freeze-out

mean, variance and skewness





huge deviations from HRG at all $\sqrt{s_{NN}}$

do we understand this in QCD? – NLO Taylor expansion

What does the curvature tell us?

for simplicity: $\mu_S = \mu_Q = 0$

$$S_B\sigma_B = \frac{\chi_4^B}{\chi_2^B}\frac{M_B}{\sigma_B^2} + \frac{1}{6}\left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right)\left(\frac{M_B}{\sigma_B^2}\right)^3 + \ldots$$

Conserved charge fluctuations and freeze-out

mean, variance and skewness

NLO Taylor expansion

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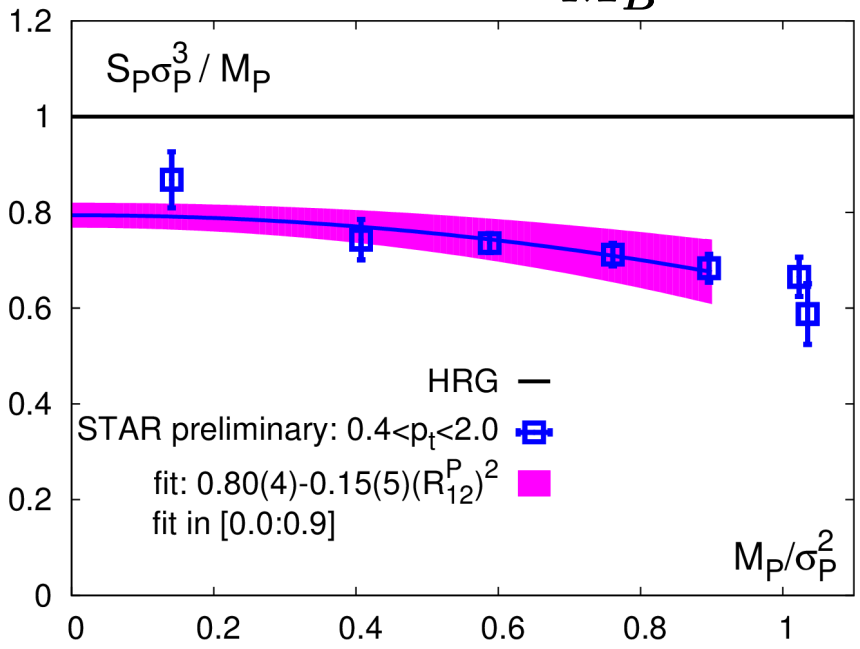
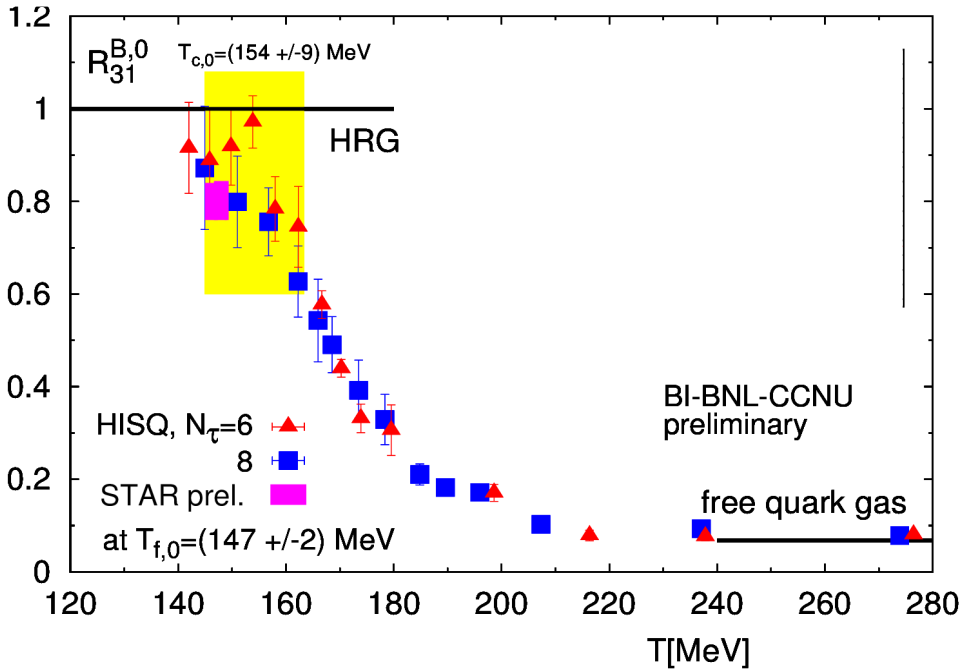
$$\Leftrightarrow \quad R_{31}^B \equiv \frac{S_B\sigma_B^3}{M_B} = R_{31}^{B,0} + R_{31}^{B,2}\left(R_{12}^B\right)^2$$





used $T_{f,0}$ determined from fit to $(M_Q/\sigma_Q^2)/(M_P/\sigma_P^2)$

Conserved charge fluctuations and freeze-out

mean, variance and skewness

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$$S_B\sigma_B = \frac{\chi_4^B}{\chi_2^B}\frac{M_B}{\sigma_B^2} + \frac{1}{6}\left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right)\left(\frac{M_B}{\sigma_B^2}\right)^3 + \ldots$$

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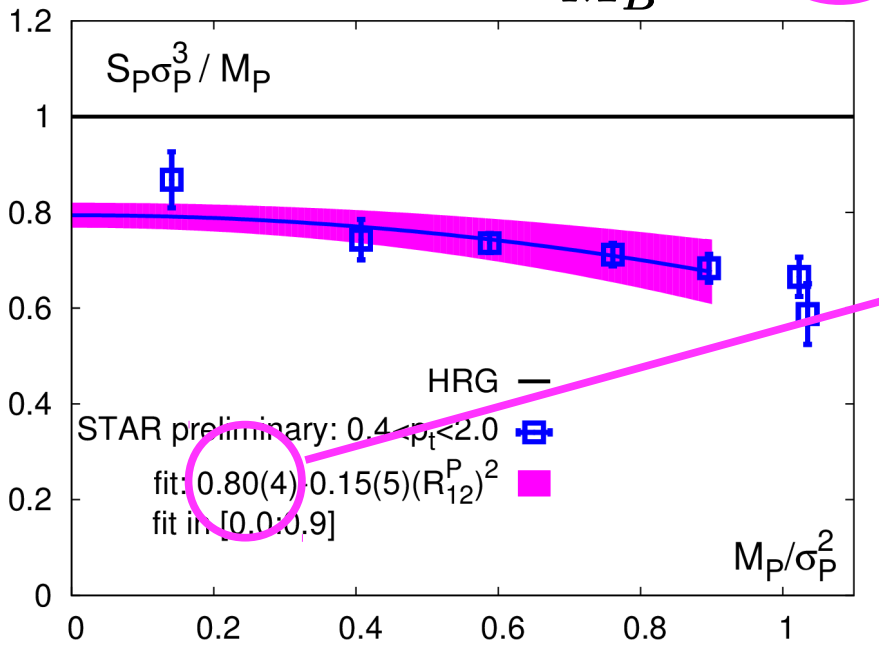

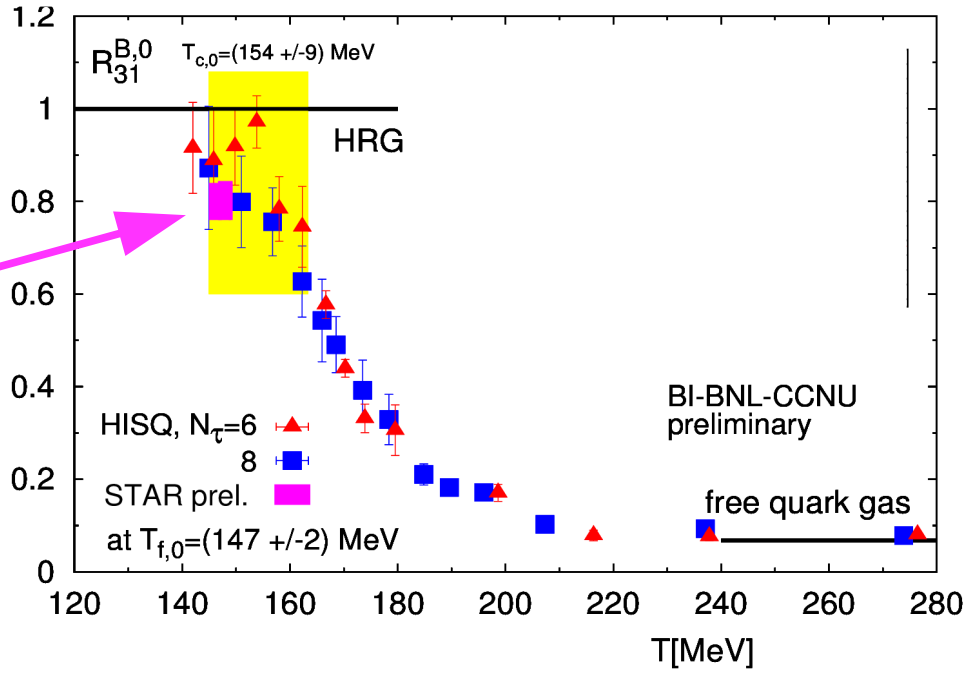

intercept consistent with QCD result,

– used $T_{f,0}$ determined from fit to $(M_Q/\sigma_Q^2)/(M_P/\sigma_P^2)$

Conserved charge fluctuations and freeze-out
mean, variance and skewness

NLO Taylor expansion

$$S_B\sigma_B = \frac{\chi_4^B}{\chi_2^B}\frac{M_B}{\sigma_B^2} + \frac{1}{6}\left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right)\left(\frac{M_B}{\sigma_B^2}\right)^3 + \ldots$$

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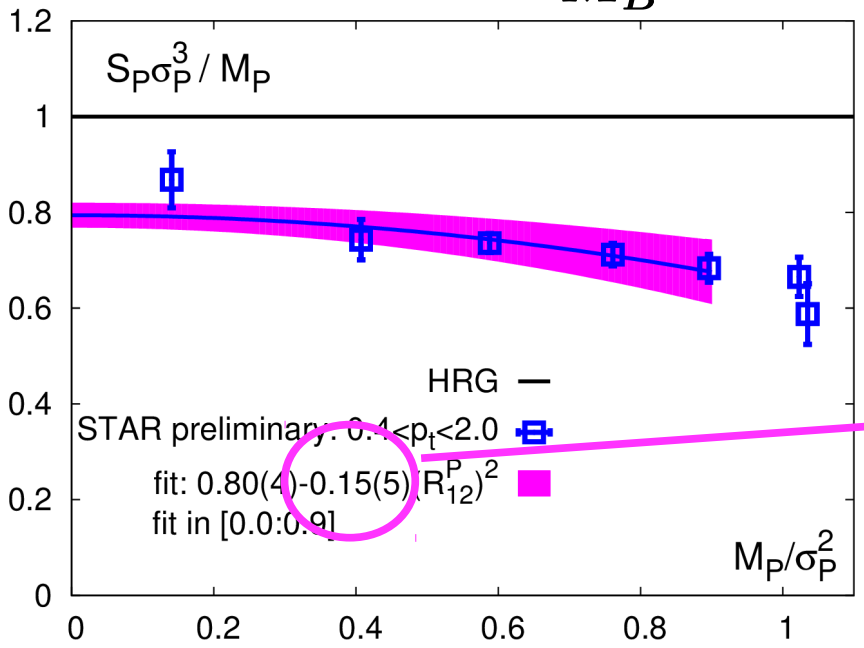
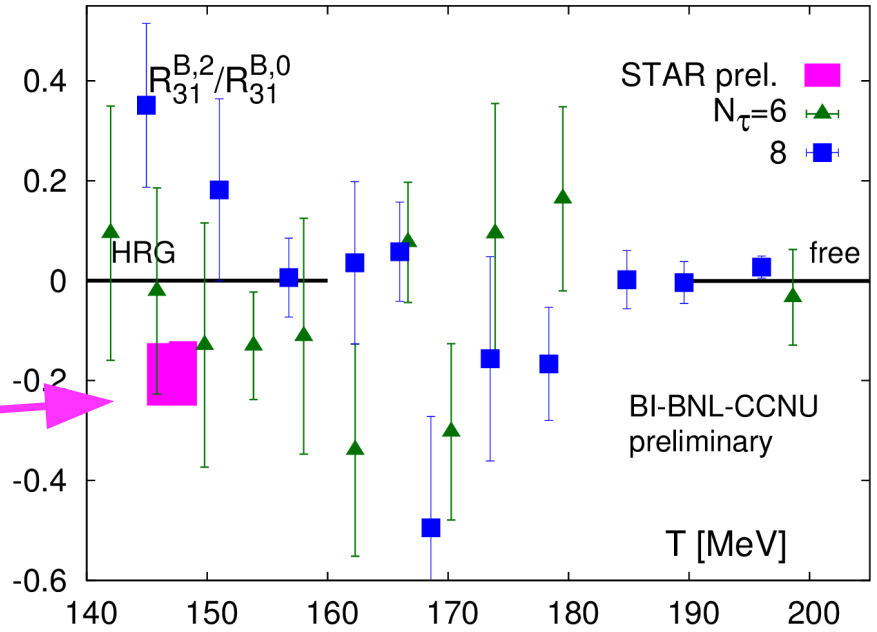
lattice QCD calculation involves 6th order cumulants





curvature consistent with (still preliminary) QCD result (noisy, coarse lattice)

used $T_{f,0}$ determined from fit to $(M_Q/\sigma_Q^2)/(M_P/\sigma_P^2)$

Conserved charge fluctuations and freeze-out
mean, variance, skewness and kurtosis

in a NLO Taylor expansion $R_{31}^B \equiv S_B \sigma_B^3 / M_B$

$R_{42}^B \equiv \kappa_B \sigma_B^2$

} **are closely related**

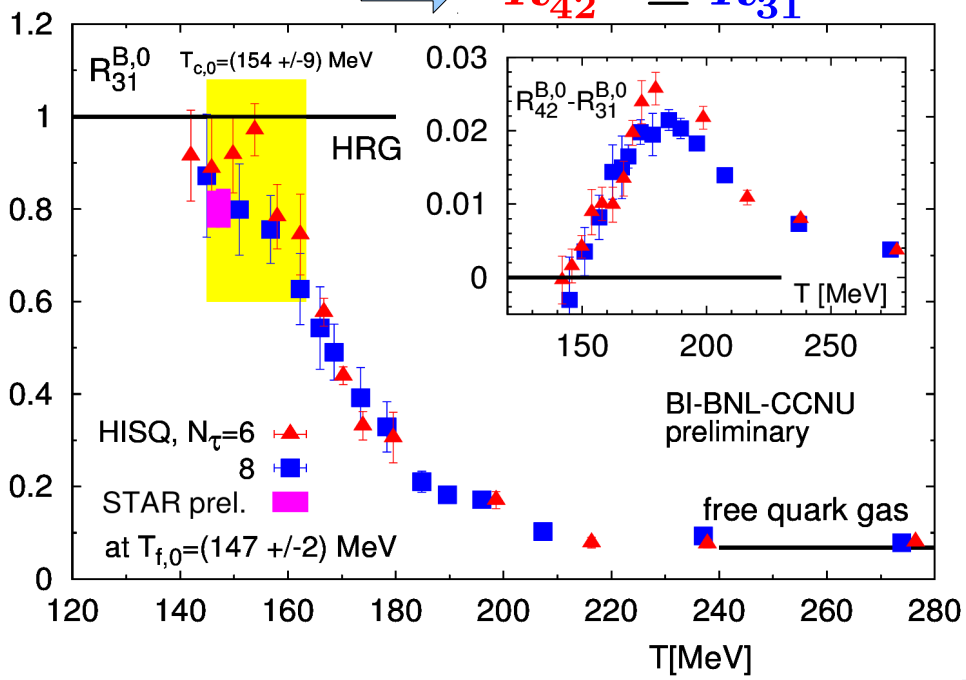
$$R_{31}^B = R_{31}^{B,0} + R_{31}^{B,2} \left(R_{12}^B\right)^2$$

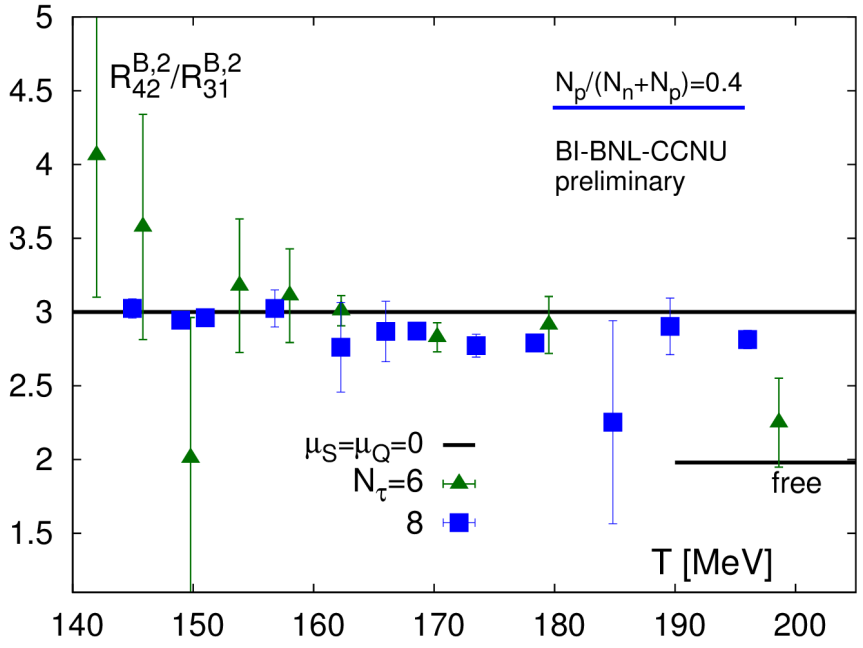
$$R_{42}^B = R_{42}^{B,0} + R_{42}^{B,2} \left(R_{12}^B\right)^2$$

$\Rightarrow \quad R_{42}^{B,0} \simeq R_{31}^{B,0}$

$\underline{\mu_S = \mu_Q = 0:}$

$$R_{42}^{B,2} = 3R_{31}^{B,2} = \frac{1}{2}\left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right)$$





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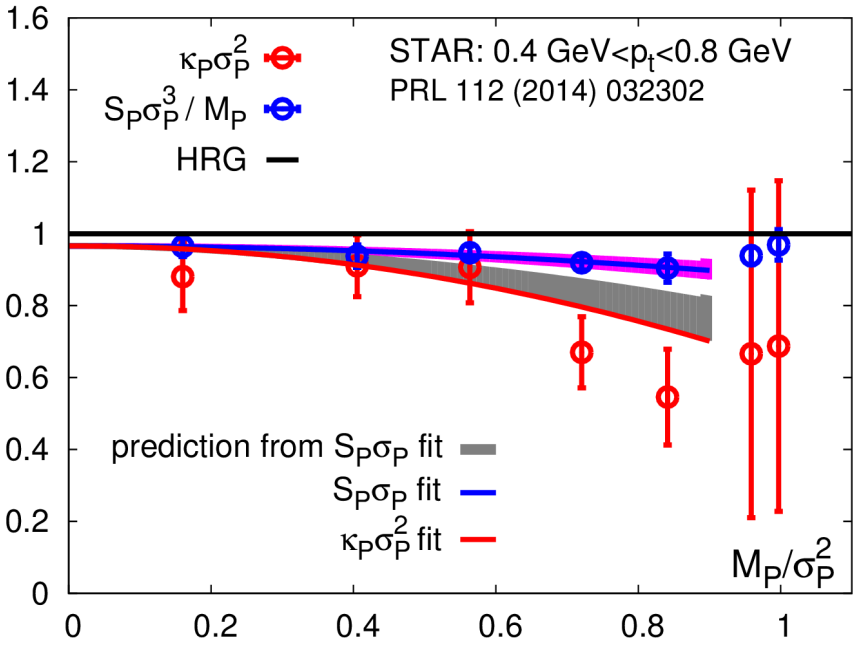
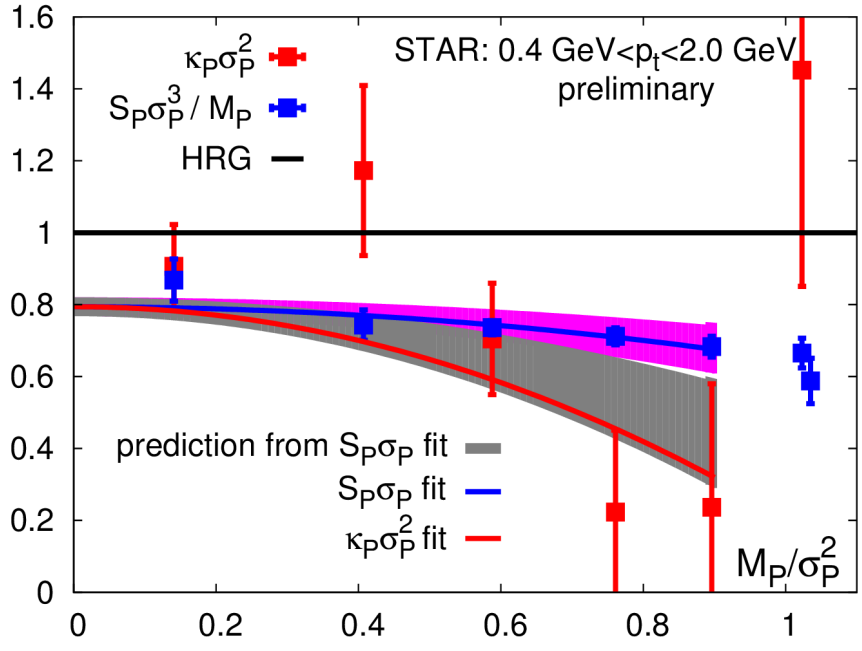
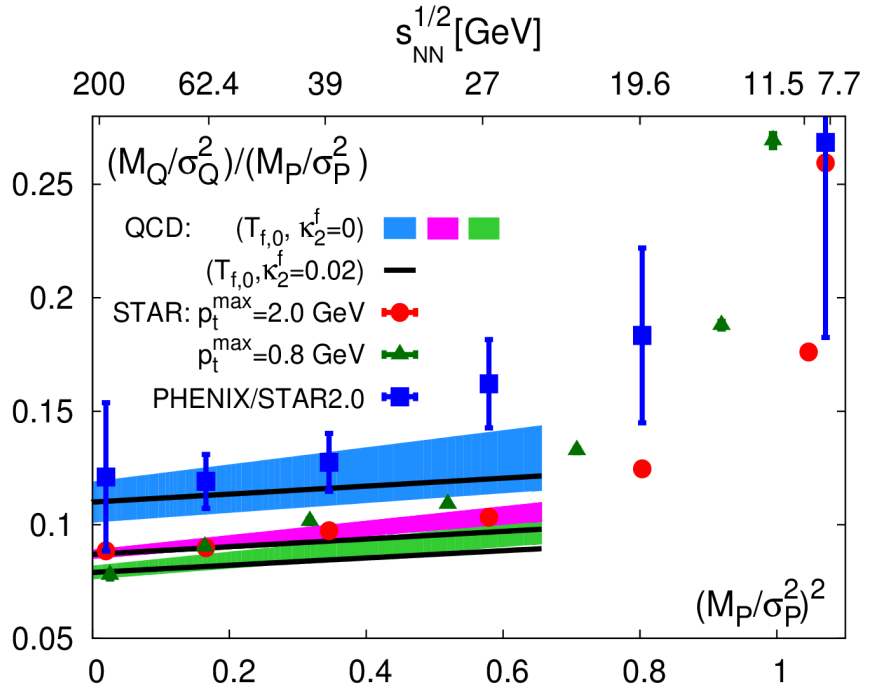




Conclusion

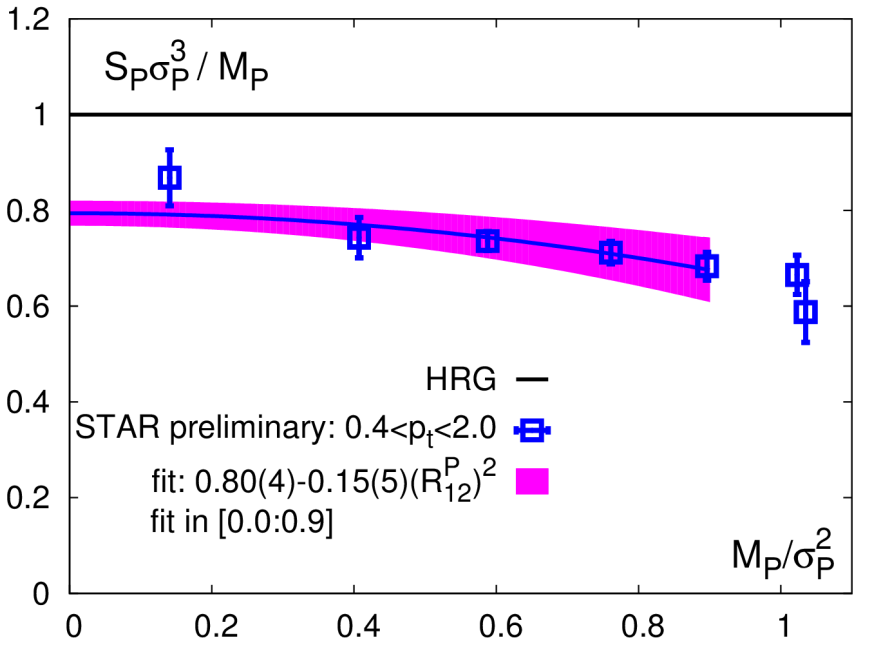


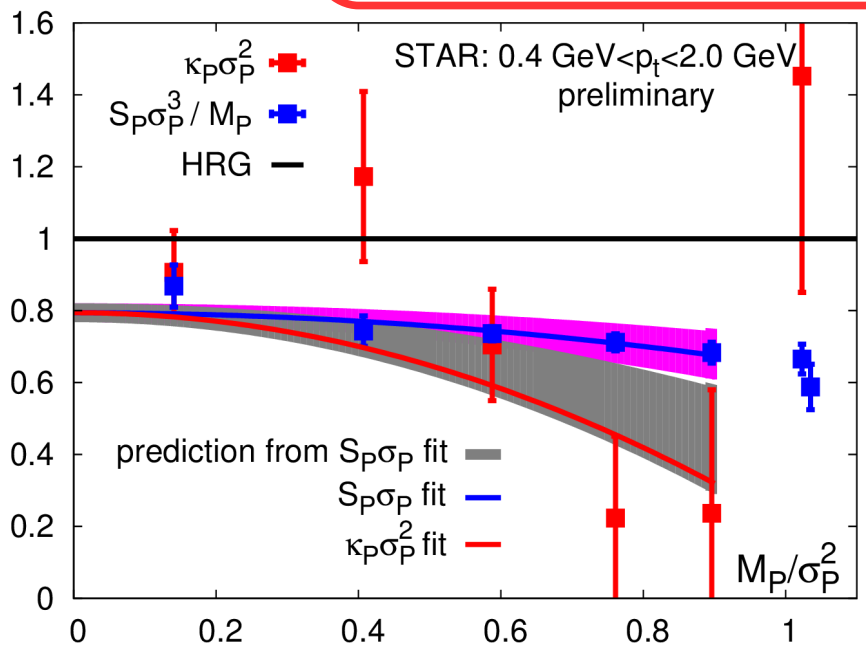
a comparison of the STAR data on net-proton number fluctuations with **QCD thermodynamics** suggests

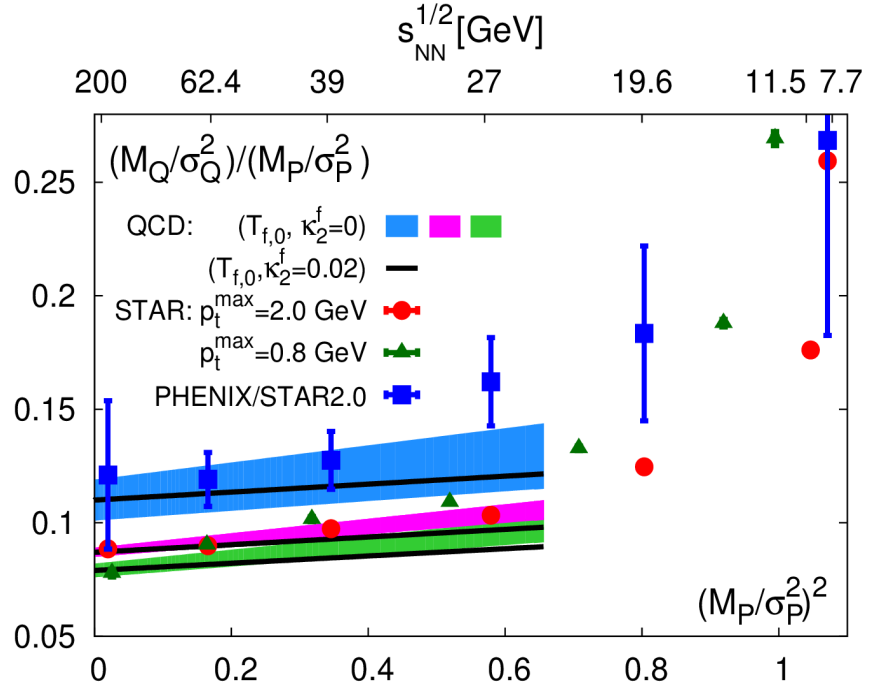
- curvature of the freeze-out line is compatible with zero
- the freeze-out temperature is below that for the crossover transition
- the kurtosis ratio is smaller than the skewness ratio

$\sqrt{s_{NN}} \geq 27$ GeV :

$$\kappa_2^f < 0.011$$
$$T_{f,0} = (147 \pm 2)\ \text{MeV}$$
$$\kappa_P \sigma_P^2 < S_P \sigma_P$$





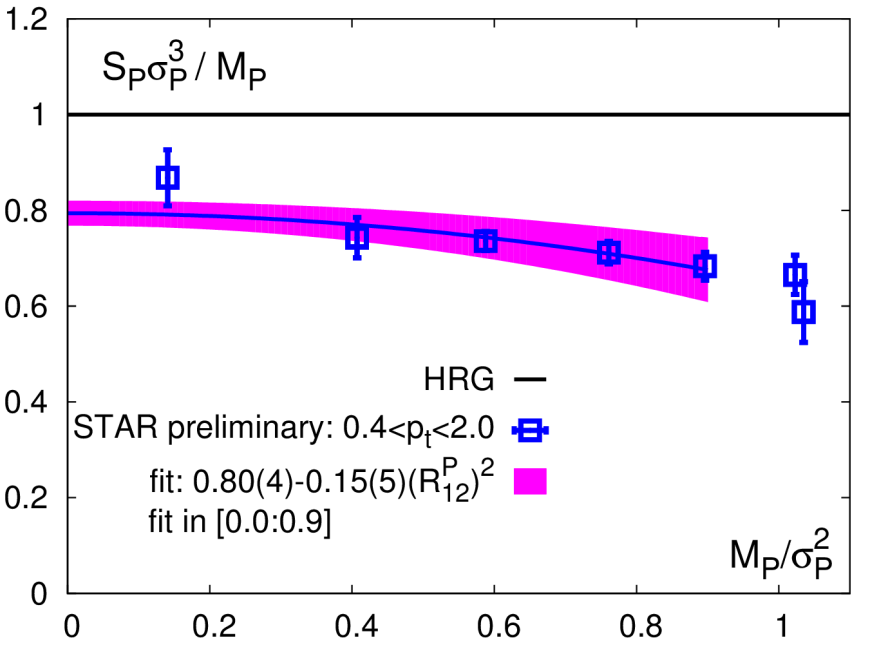


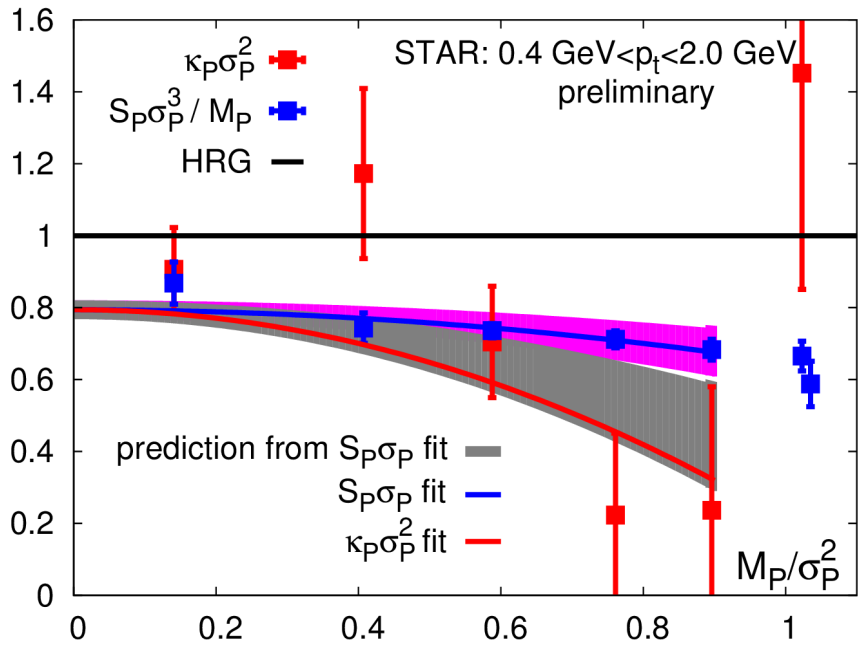
Conclusion

the preliminary STAR data on cumulant ratios of conserved net-charge distributions for

$$\sqrt{s_{NN}} \geq 27 \text{ GeV}$$

can be understood in terms of a next-to-leading order expansion of net-baryon number fluctuations calculated in QCD equilibrium thermodynamics





Taylor expansion of the pressure

$$\frac{p}{T^4} = \frac{1}{VT^3}\ln Z(V,T,\mu_B,\mu_S,\mu_Q)$$

$$= \sum_{i,j,k}\frac{1}{i!j!k!}\chi^{BQS}_{ijk}\left(\frac{\mu_B}{T}\right)^i\left(\frac{\mu_Q}{T}\right)^j\left(\frac{\mu_S}{T}\right)^k$$

generalized susceptibilities: $$\chi^{BQS}_{ijk} = \left.\frac{\partial^{i+j+k}p/T^4}{\partial\hat{\mu}_B^i\partial\hat{\mu}_Q^j\partial\hat{\mu}_S^k}\right|_{\mu=0}$$

conserved charge fluctuations: $$\chi^X_n(T,\mu_B,...) = \frac{\partial^n P/T^4}{\partial(\mu_X/T)^n}$$

$X = B,\ Q,\ S$

$$\frac{M_X}{\sigma_X^2} = \frac{\chi_1^X(T,\mu)}{\chi_2^X(T,\mu)}\ ,\quad S_X\sigma_X = \frac{\chi_3^X(T,\mu)}{\chi_2^X(T,\mu)}\ ,\quad \kappa_X\sigma_X^2 = \frac{\chi_4^X(T,\mu)}{\chi_2^X(T,\mu)}$$

$\mu \equiv (\mu_B,\mu_Q,\mu_S)$