Multipole anisotropies in hydrodynamics
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Hydrodynamics
- Collective dynamics observed at RHIC
- Exact, analytic solutions: important to determine initial and final state
  Famous 1+1D solutions: Landau, Hwa, Bjorken
  Many new 1+1D solutions, few 1+3D, with spherical/axial/ellipsoidal symmetry
- First 3D relativistic solution
- Describes hadron data
- Describes photon data
- Even compatible with thermal dileptons
  Aligned by

Higher order anisotropies
- Finite number of nucleons → anisotropy!

Effect of the speed of sound
- Smaller $c_s^2$ (larger $\kappa$) ⇒ slower anisotropy change
- Freeze-out earlier, final anisotropies may be larger
- See for example time evolution of flow pattern:
  ![Flow pattern](image1)

Effect of viscosity on pressure
- Investigations in novel framework
- Delayed disappearance of pressure anisotropy

Effect of viscosity on the flow field
- Opposite effect in flow field anisotropies!
- Larger anisotropies develop
- Quantify this with the time evolution of the anisotropies

Time evolution of the anisotropies
- Good hydro description of the final state anisotropies
- Elliptical/multipole relativistic solutions so far: no pressure gradient!
- Method: extending analytic solutions numerically
- Multi stage predictor-corrected G FORCE method
  E. F. Toso et al., 2000, J. Comp. Phys

Numerical method tested on analytic solutions
- Quantify anisotropies: $e_n = \langle \cos(n\phi) \rangle_{\rho,u}$
- Their time evolution is influenced by the transport coefficients
- Thermodynamic anisotropies $e_n \Rightarrow$ scale variable anisotropies $\varepsilon_n$
  - First approximation: $\varepsilon_n = -e_n/2$
  - Third order approximation:
    $$e_1 \approx \frac{(e_2 + e_3)}{2 + \sum \varepsilon_i}, \quad e_2 \approx \frac{-e_2 + e_3}{2 + \sum \varepsilon_i}, \quad e_3 \approx \frac{-e_3}{2 + \sum \varepsilon_i}, \quad e_4 \approx \frac{-e_4 + \frac{1}{2} e_3}{2 + \sum \varepsilon_i}$$

Multipole velocity field
- Analytic solutions: 3D Hubble flow,
  $u^\mu = \gamma(1, H_{x,y,z}, H_{x,y,z})$
- Derivable from $\Phi = (H_{x}r_x^2 + H_{y}r_y^2 + H_{z}r_z^2)/2$
- Another type of generalization, à la Buda-Lund:
  add multipole flow
- Generalization: $\Phi = \frac{1}{2} [H_{x}r_x^2 (1 + \sum_\alpha \chi_{\alpha} \cos(n_\alpha \phi)) + H_{z}r_z^2]$
- Flow pattern in the final state:
  $z_\alpha = \frac{1}{2} \{ e_{1,2,3} \}$

Azimuthal HBT radii
- HBT radii calculated via
  $R_{n,\text{out}}^2 = \langle (r_{\text{out}} - \beta r_t)^2 \rangle$
  $R_{n,\text{side}}^2 = \langle (r_{\text{side}} - \beta r_t)^2 \rangle$
- Higher order oscillations observed!

Amplitudes of the radii
- Oscillating radii parametrized via
  $R_{n,\text{out}}^2 = R_{n,0}^2 + R_{n,2}^2 \cos(2\alpha) + R_{n,4}^2 \cos(4\alpha) + R_{n,6}^2 \cos(6\alpha)$
  $R_{n,\text{side}}^2 = R_{n,0}^2 + R_{n,2}^2 \cos(3\alpha) + R_{n,6}^2 \cos(6\alpha) + R_{n,9}^2 \cos(9\alpha)$
- Higher orders present as well
- Flow and spatial anisotropies mix