Kelvin-Helmholtz instability in relativistic heavy ion collisions

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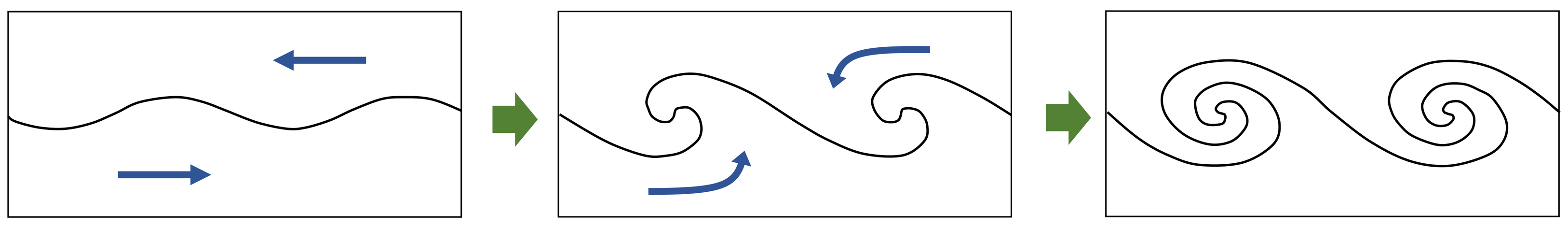
Hydrodynamic instability in Heavy Ion Collisions

Infinitesimal disturbances alter the flow pattern drastically, when there is the hydrodynamic Instability.

Kelvin-Helmholtz instability



In the share flow, small perturbations grow and result in the formation of vortices

There is a possibility that Kelvin-Helmholtz instability exists in relativistic heavy ion collisions relating to the dynamics in the longitudinal direction.

Csernai, Strottman, Anderlik, (2012)

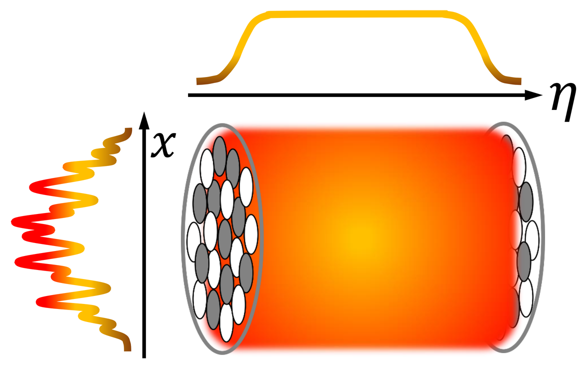
Hydrodynamic simulation and Initial fluctuation

Comparisons between event-by-event hydrodynamic simulations of higher flow harmonics and experimental results are actively performed

→ Constraints on the transport coefficients and initial state models

The argument of the initial longitudinal fluctuation is relatively underdeveloped.

In the longitudinal direction, an uniform state about rapidity (Bjorken flow) is usually assumed.



If Kelvin-Helmholtz instability exists, the longitudinal fluctuation is amplified and affects on the latter hydrodynamic flow remarkably

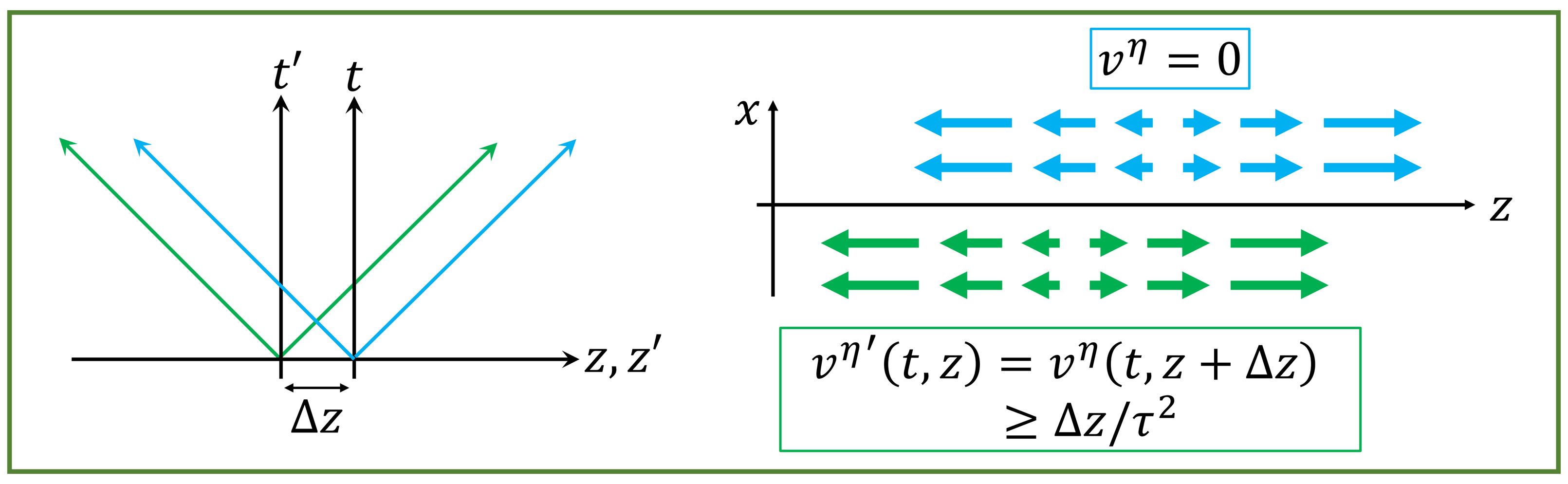
Share flow in Heavy Ion Collisions

A Lorentz contracted nuclei has finite width $\Delta z \sim 1$fm due to the uncertainty principle

→ We assume the center position of Bjorken flow fluctuates

→ Fluid velocity v^η has a large fluctuation around $v^\eta = 0$

We consider following condition



Hydrodynamic simulation

Ideal hydrodynamic simulation

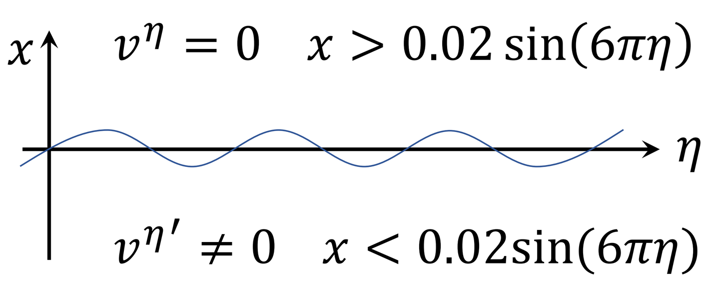
Riemann solver (two shock approximation)

Akamatsu, Inutsuka, Nonaka, Takamoto, (2014)

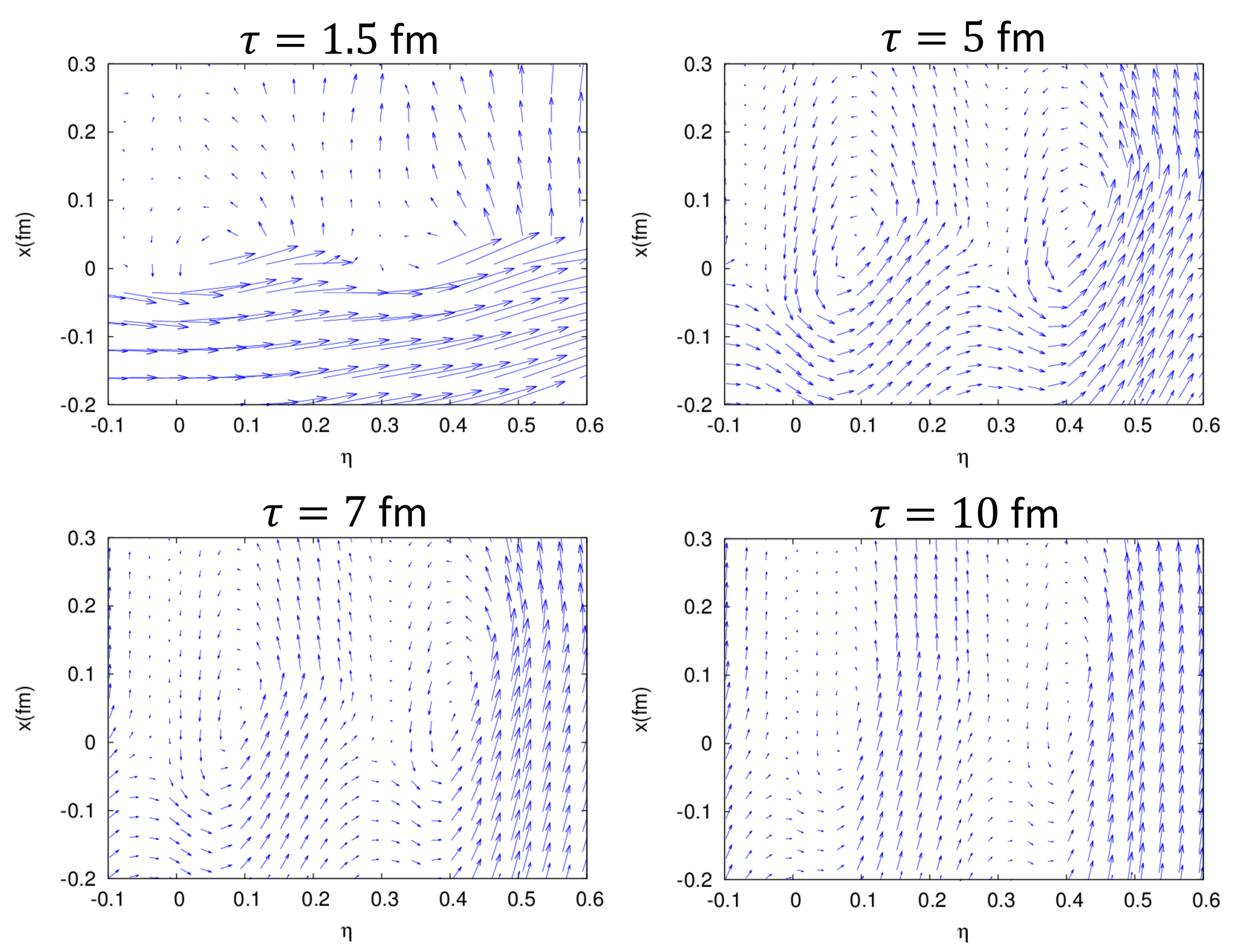
Bjorken coordinates (τ, x, η)

Initial condition

$\tau_0 = 1$ fm



Velocity profile



- Kelvin-Helmholtz instability is observed in expanding system. Vortices are formed, the transvers velocity evolve.
- In the later time, $v^{\eta\prime} \propto 1/\tau^2$ becomes small. → Vortices disappear

Linear analysis

Fluctuations around the Bjorken flow

$$e = e_B + \delta e \quad v^x = \delta v^x \quad v^\eta = \delta v^\eta \quad p = \lambda e \quad e_B = e_0 \left(\frac{\tau_0}{\tau}\right)^{1+\lambda}$$

Linearized Euler equation of fluctuations

$$\partial_\tau \delta e = -(1+\lambda) e_B \partial_i \delta v^i - (1+\lambda)\frac{1}{\tau}\delta e$$

$$\partial_\tau \delta v^x = -\frac{\lambda}{1+\lambda}\frac{1}{e_B}\partial_x \delta e + \lambda \frac{1}{\tau}\delta v^x$$

$$\partial_\tau \delta v^\eta = -\frac{\lambda}{1+\lambda}\frac{1}{\tau^2 e_B}\partial_\eta \delta e + (\lambda - 2)\frac{1}{\tau}\delta v^\eta$$

Concentrating on rapidly growing modes, we ignore the time dependence of coefficients.

Laplace and Fourier transformation

$$\delta e(\tau, x, \eta) = \int \delta\tilde{e}(\sigma, x, k) e^{\sigma\tau + ik\eta} d\sigma dk$$

Analytic solution

$$\delta\tilde{e}(x) = A_1 e^{-mx} \quad m^2 = \left(\sigma - \frac{\lambda}{\tau_0}\right)\left(\sigma + \frac{1+\lambda}{\tau_0}\right)\frac{1}{\lambda} + \frac{\sigma - \frac{\lambda}{\tau_0}}{\sigma - \frac{\lambda - 2}{\tau_0}}\frac{k^2}{\tau_0^2}$$

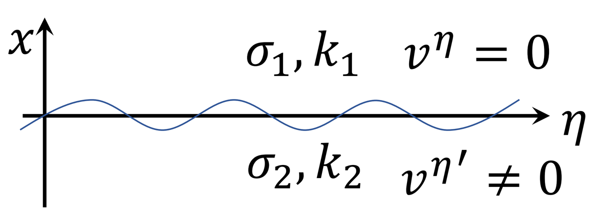
Analytic solution of unstable modes

We obtain the solution with finite $v^\eta (\sim U)$ using coordinate transformation. Wave number is transformed as follows

$$\sigma_2 = W(\sigma_1 - iWUk_1) \quad k_2 = W(i\tau_0^2 U\sigma_1 + Wk_1) \quad U \equiv \Delta z/\tau_0^2$$

Boundary condition between the two fluids (at $x = \xi$)

$$\frac{\partial \xi}{\partial \tau} = v^x \quad p_1 = p_2$$

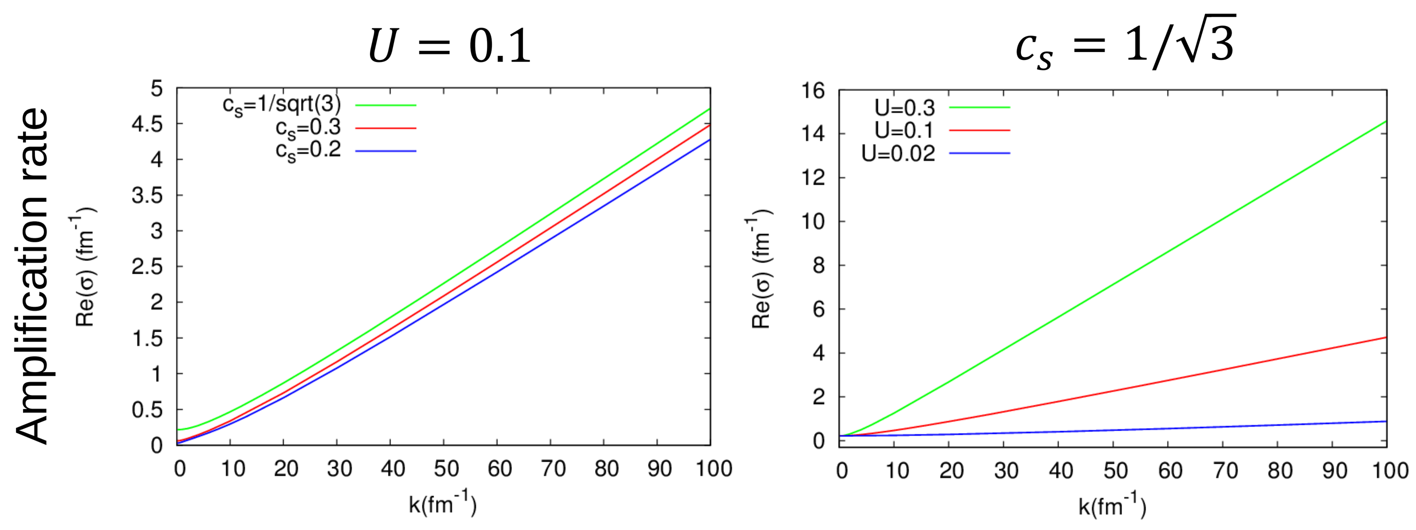


Dispersion relation

$$\frac{\sigma_1(\sigma_1 - \lambda)}{m_1} + \frac{\sigma_2(\sigma_2 - \lambda)}{m_2} = 0$$

If $\mathrm{Re}(\sigma) > 0$, fluid is unstable

→ Unstable modes always exist



- Modes with small wave length grow rapidly.
- Even in a small share flow, there are rapidly growing modes.

Summary

- A share flow expected in heavy ion collisions seems to be enough to cause the Kelvin-Helmholtz instability.
- Small longitudinal fluctuations are amplified due to the Kelvin-Helmholtz instability and have large effects on the latter hydrodynamic flow.

Future works

- Effects on experimental observables
- Effects of viscosity