Bulk evolution of heavy ion collisions in the beam energy scan: New developments and first results

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Introduction

Low energy collisions demand improvements of existing hydrodynamic simulations, including:

- Net-baryon current ✓
- Equation of state at finite baryon chemical potential ✓
- Initial state with fluctuating baryon- and entropy-density ✓
- Fluctuations in all three spatial dimensions ✓

- Baryon diffusion (to do)
- Strangeness and electric currents (to do)

Will show

- momentum and rapidity distributions at different energies
- rapidity dependent flow and the effect of $(\eta/s)(T)$
- two-particle pseudo rapidity correlations (of $h^{+/−}$ and net-baryons)
Hydrodynamics

Use the state of the art 3+1D viscous relativistic hydrodynamics MUSIC with shear and bulk viscosity and all nonlinear terms that couple bulk viscous pressure and shear-stress tensor.

Solve $\partial_\mu T^{\mu\nu} = 0$ and $\partial_\mu J^\mu_B = 0$ along with

$$
\tau_\Pi \ddot{\Pi} + \Pi = -\zeta \theta - \delta_{\Pi\Pi} \Pi \theta + \lambda_{\Pi\pi} \pi^{\mu\nu} \sigma_{\mu\nu}
$$

$$
\tau_\pi \ddot{\pi}^{(\mu\nu)} + \pi^{\mu\nu} = 2\eta \sigma^{\mu\nu} - \delta_{\pi\pi} \pi^{\mu\nu} \theta + \varphi_7 \pi^{(\mu}_\alpha \pi^{\nu)}\alpha - \tau_{\pi\pi} \pi^{(\mu}_\alpha \sigma^{\nu)}\alpha + \lambda_{\pi\Pi} \Pi \sigma^{\mu\nu}
$$

The transport coefficients $\tau_\Pi, \delta_{\Pi\Pi}, \lambda_{\Pi\pi}, \tau_\pi, \delta_{\pi\pi}, \varphi_7, \tau_{\pi\pi}, \lambda_{\pi\Pi}$ are fixed using formulas derived from the Boltzmann equation near the conformal limit.

Viscosities

In the calculations presented here we use:
- shear viscosity (constant or with T dependence to be defined)
- bulk viscosity:

![Graph showing viscosities vs T/Tc]


G. S. Denicol, U. W. Heinz, M. Martinez, J. Noronha and M. Strickland,
Phys. Rev. D 90, 125026 (2014);

**QGP:** F. Karsch, D. Kharzeev and K. Tuchin,

**Hadron Gas:**
J. Noronha-Hostler, J. Noronha and C. Greiner,

To reduce the sensitivity to $\delta f$ corrections at high $p_T$ we have the low T value drop exponentially
Equation of state

Construct EoS at finite $\mu_B$ using Taylor expanded lattice data:

\[
P \left( \frac{1}{T^4} \right) = \frac{P_0}{T^4} + \frac{1}{2} \chi_B^{(2)} \left( \frac{\mu_B}{T} \right)^2 + \frac{1}{4!} \chi_B^{(4)} \left( \frac{\mu_B}{T} \right)^4 + O \left( \left( \frac{\mu_B}{T} \right)^6 \right)
\]

Currently using data for parameters $P_0^{\text{lat}}$ and $\chi_B^{(2)}$ from:


$\chi_B^{(4)}$ from the ratio $\chi_B^{(4)}/\chi_B^{(2)}$ in a HRG and parton gas model
Initial conditions - 3DMC-Glauber with quarks

Introduce a simple extension of the Monte Carlo Glauber model.

We use constituent quarks.

Constituent quark initial positions in the transverse plane are sampled from a 2D exponential distribution around the nucleon center (Nucleons are sampled from Woods-Saxon).

Their rapidities are sampled from nuclear parton distribution functions (in this talk we will use CTEQ10 and EPS09).

Their cross sections can be determined geometrically to reproduce the nucleon-nucleon cross sections.
Event-by-event baryon density

**Transverse distribution:**
Implement black disk and Gaussian wounding to determine wounded quarks

**Longitudinal distribution:**
Implement an MC version of the *Lexus* model
S. Jeon and J. Kapusta, PRC56, 468 (1997)

**Idea:** Rapidity distributions in heavy ion collisions follow via linear extrapolation from p+p collisions
Distribution in p+p collisions is parametrized and fit to data

Probability for a quark with rapidity $y_P$ to get rapidity $y$ after collision with another quark with rapidity $y_T$:

$$Q(y, y_P, y_T) = \lambda \frac{\cosh(y - y_T)}{\sinh(y_P - y_T)} + (1 - \lambda)\delta(y - y_P)$$

where we treat $\lambda$ as a free parameter
(it characterizes the stopping power for quarks)
Event-by-event baryon- and entropy density

Deposit entropy density (fluctuating with NBD) between the collided constituent quarks using a Gaussian profile in the transverse plane and a constant distribution (with Gaussian edges) in rapidity

\[ \sqrt{s} = 200 \text{GeV} \]

energy density

baryon density
Event-by-event baryon- and entropy density

Deposit entropy density (fluctuating with NBD) between the collided constituent quarks using a Gaussian profile in the transverse plane and a constant distribution (with Gaussian edges) in rapidity.

\[ \sqrt{s} = 19.6 \text{GeV} \]

- energy density
- baryon density
Results
Identified particle transverse momentum spectra


Bulk viscosity needed to get mean $p_T$ right
Same as with IP-Glasma initial conditions:
Charged hadron pseudo-rapidity distributions

200 GeV

62.4 GeV

19.6 GeV

Net-baryon rapidity distributions

T dependent $\eta/s$ from rapidity dependence

Numbers (a,b) are the slopes in [GeV$^{-1}$] in:

$$(\eta T/(\varepsilon + P))(T) = 0.08 + a(T_c - T)\theta(T_c - T) + b(T - T_c)\theta(T - T_c)$$

where $T_c(\mu_B)$
$T$ dependent $\eta/s$ from rapidity dependence

Numbers $(a,b)$ are the slopes in $[\text{GeV}^{-1}]$ in:

$$(\eta T/(\varepsilon + P))(T) = 0.08 + a(T_c - T)\theta(T_c - T) + b(T - T_c)\theta(T - T_c)$$

where $T_c(\mu_B)$

$3\text{DMCG+MUSIC} \ \eta/s=0.12$

$3\text{DMCG+MUSIC Tdep 2-20}$

$3\text{DMCG+MUSIC Tdep 20-2}$
T dependent $\eta/s$ from rapidity dependence

Numbers (a,b) are the slopes in [GeV$^{-1}$] in:

$$\frac{\eta T}{(\varepsilon + P)}(T) = 0.08 + a(T_c - T)\theta(T_c - T) + b(T - T_c)\theta(T - T_c)$$

where $T_c(\mu_B)$

![Graph showing dependence of $\eta/s$ on temperature](image)
Data favors large hadronic $\eta/s$. No room for large QGP $\eta/s$. 

Two-particle pseudo-rapidity correlations

\[ C(\eta_1, \eta_2) = \frac{\langle N(\eta_1)N(\eta_2) \rangle}{\langle N(\eta_1) \rangle \langle N(\eta_2) \rangle} \]

to remove the effect of a residual centrality dependence in 5% bin

\[ C_N(\eta_1, \eta_2) = \frac{C(\eta_1, \eta_2)}{C_p(\eta_1)C_p(\eta_2)} \]

with

\[ C_p(\eta_{1/2}) = \frac{1}{2Y} \int_{-Y}^{Y} C(\eta_1, \eta_2) \, d\eta_{2/1} \]

Expand in Legendre polynomials. The coefficients are given by

\[ a_{n,m} = \int C_N(\eta_1, \eta_2) \frac{T_n(\eta_1) T_m(\eta_2) + T_n(\eta_2) T_m(\eta_1)}{2} \, \frac{d\eta_1}{Y} \, \frac{d\eta_2}{Y} \]

see: A. Bzdak, D. Teaney, Phys. Rev. C 87, 024906
Two-particle pseudo-rapidity correlations

A. Monnai, B. Schenke, arXiv:1509.04103

Experimental data: ATLAS-CONF-2015-020

\[
\begin{align*}
(\lambda_{(n,m)})^{1/2} & \\
0.1 & \\
0.01 & \\
0.001 & \\
\langle 1,1 \rangle & \\
\langle 2,2 \rangle & \\
\langle 3,3 \rangle & \\
\langle 4,4 \rangle & \\
\langle 5,5 \rangle & \\
\langle 6,6 \rangle & \\
\langle 1,3 \rangle & \\
\langle 2,4 \rangle & \\
\langle 3,5 \rangle & \\
\langle 4,6 \rangle & \\
\langle 5,7 \rangle & 
\end{align*}
\]

\[\text{Pb+Pb 2760 GeV} \quad p_T>0.5 \text{ GeV}\]

ATLAS prelim.
initial entropy density
\(\eta/s=0, \zeta/s=0\)
\(\eta/s=0.12, (\zeta/s)(T)\)

20-25%

missing short range correlations
Collision energy dependence

A. Monnai, B. Schenke, arXiv:1509.04103

Pb+Pb 2760 GeV $\eta/s=0.12$, $(\zeta/s)(T)$
Au+Au 200 GeV $\eta/s=0.12$, $(\zeta/s)(T)$
Au+Au 19.6 GeV $\eta/s=0.12$, $(\zeta/s)(T)$

$p_T>0.5$ GeV 20-25%

higher energy
Effect of shear and bulk viscosity

Au+Au 200 GeV
\(p_T > 0.5\) GeV

Initial entropy density

\[\eta/s = 0.2 (\zeta/s)(T)\]
\[\eta/s = 0.12 (\zeta/s)(T)\]
\[\eta/s = 0.12, \zeta/s = 0\]

A. Monnai, B. Schenke, arXiv:1509.04103
Effect of the initial number of sources

\[ \langle a_{(n,m)} \rangle^{1/2} \]

initial entropy density - nucleons
initial entropy density - constituent quarks

Au+Au 200 GeV, 20-25%

A. Monnai, B. Schenke, arXiv:1509.04103

more sources
Net baryon pseudo-rapidity correlations

Measure this: Could help our understanding of baryon stopping
Couple to UrQMD: short range correlations matter

G. Denicol, C. Gale, S. Jeon, A. Monnai, S. Ryu, B. Schenke, work in progress

\[ \left| \eta \right| < 2.4 \]

\( p_T > 0.5 \text{ GeV} \)

\[ (1,1)(2,2)(3,3)(4,4)(5,5)(6,6)(1,3)(2,4)(3,5)(4,6)(5,7) \]

ATLAS prelim.
UrQMD \( \eta/s=0.12, (\zeta/s)(T) \)
UrQMD \( \eta/s=0, \zeta/s=0 \)
\( \eta/s=0.12, (\zeta/s)(T) \)
\( \eta/s=0, \zeta/s=0 \)

20-25%

also see P. Bozek, W. Broniowski, A. Olszewski, arXiv:1509.04124
Conclusions

• 3+1D viscous relativistic fluid dynamics with fluctuations of baryon number and entropy density in all three dimensions

• Lattice equation of state at finite $\mu_B$ implemented

• Rapidity and energy dependence of flow harmonics contains information on transport coefficients’ $T$ and $\mu_B$ dependence

• Two particle rapidity correlations contain information on the number of sources; are sensitive to short range correlations

• Net-baryon rapidity correlations can shed more light on baryon stopping: Measure them!
Backup
Constructing the equation of state (EoS)

Taylor Expansion

Cannot deal with complex Fermion determinants on lattice, so Taylor expand around zero baryon chemical potential

\[
\frac{P}{T^4} = \frac{P_0}{T^4} + \frac{1}{2} \chi_B^{(2)} \left( \frac{\mu_B}{T} \right)^2 + \frac{1}{4!} \chi_B^{(4)} \left( \frac{\mu_B}{T} \right)^4 + \mathcal{O} \left[ \left( \frac{\mu_B}{T} \right)^6 \right]
\]

because of matter-anti-matter symmetry only even powers appear similarly for energy density and entropy density

For net-baryon density we have

\[
\frac{n_B}{T^3} = 0 + \chi_B^{(2)} \frac{\mu_B}{T} + \frac{1}{3!} \chi_B^{(4)} \left( \frac{\mu_B}{T} \right)^3 + \mathcal{O} \left[ \left( \frac{\mu_B}{T} \right)^5 \right]
\]
Constructing the equation of state (EoS)

**Smooth matching (cross over)**

As a first try, we match the HRG and lattice EoS smoothly

\[
\frac{P}{T^4} = \frac{1}{2} \left[ 1 - \tanh \frac{T - T_C(\mu_B)}{\Delta T_C} \right] \frac{P_{\text{HRG}}(T)}{T^4} + \frac{1}{2} \left[ 1 + \tanh \frac{T - T_C(\mu_B)}{\Delta T_C} \right] \frac{P_{\text{lat}}(T_s)}{T_s^4}
\]

In the future one can introduce a critical point here.

\[T_C: \text{connecting temperature} \]
\[\Delta T_C: \text{width of overlap area} \]
\[T_s: \text{temperature shift} \]
\[T_s = T + d[T_C(0) - T_C(\mu_B)]\]
Constructing the equation of state (EoS)

Smooth matching (cross over)

As a first try, we match the HRG and lattice EoS smoothly

\[
\frac{P}{T^4} = \frac{1}{2} \left[ 1 - \tanh \frac{T - T_C(\mu_B)}{\Delta T_C} \right] \frac{P_{\text{HRG}}(T)}{T^4} + \frac{1}{2} \left[ 1 + \tanh \frac{T - T_C(\mu_B)}{\Delta T_C} \right] \frac{P_{\text{lat}}(T_s)}{T_s^4}
\]

\[
T_C(\mu_B) = 0.166 \text{GeV} - c(0.139 \mu_B^2 + 0.053 \mu_B^4)
\]

based on the chemical freeze-out line (c=1)

Cleymans et al, PRC73, 034905 (2006)

For the connecting line we use c=d=0.4, \( \Delta T_C=0.1 \) \( T_C(0) \)
Constructing the equation of state (EoS)

Smooth matching (cross over)

As a first try, we match the HRG and lattice EoS smoothly

\[
\frac{P}{T^4} = \frac{1}{2} \left[ 1 - \tanh\frac{T - T_C(\mu_B)}{\Delta T_C} \right] \frac{P_{HRG}(T)}{T^4} + \frac{1}{2} \left[ 1 + \tanh\frac{T - T_C(\mu_B)}{\Delta T_C} \right] \frac{P_{lat}(T_s)}{T_s^4}
\]

Parameters \( P_{0}^{\text{lat}} \) and \( \chi_B^{(2)} \) are determined from the lattice:

\[ \chi_B^{(4)} \] is obtained from the ratio \( \chi_B^{(4)}/\chi_B^{(2)} \) in a HRG and parton gas model

Borsanyi et al, JHEP1011, 077 (2010)
δf corrections in the presence of net baryons


Grad’s 14 moment method

\[ \delta f^i = -f_0^i (1 \pm f_0^i) (b_i \varepsilon^{B \mu}_\mu p_i^\mu + \varepsilon_{\mu \nu} p_i^\mu p_i^\nu) \]

particle i’s baryon quantum number

\[ \varepsilon^{B}_\mu \text{ and } \varepsilon_{\mu \nu} \text{ are determined by the self-consistency conditions} \]

\[ \delta T^{\mu \nu} = \sum_i \int \frac{g_i d^3 p}{(2\pi)^3 E_i} p_i^\mu p_i^\nu \delta f^i = -\Pi \Delta^{\mu \nu} + \pi^{\mu \nu} \]

\[ \delta N^\mu_B = \sum_i \int \frac{b_i g_i d^3 p}{(2\pi)^3 E_i} p_i^\mu p_i^\nu \delta f^i = \gamma^\mu_B = 0 \text{ (no baryon diffusion)} \]
δf corrections in the presence of net baryons


Grad’s 14 moment method

\[ \delta f^i = -f_0^i (1 \pm f_0^i) (b_i \varepsilon^B_{\mu} p_i^\mu + \varepsilon_{\mu\nu} p_i^\mu p_i^\nu) \]

After tensor decomposition and one finds

\[ \varepsilon^B_{\mu} = D_{\Pi\Pi} u_\mu \]

\[ \varepsilon_{\mu\nu} = (B_{\Pi\Delta_{\mu\nu}} + \tilde{B}_{\Pi} u_\mu u_\nu) \Pi + B_\pi \pi_{\mu\nu} \]

where the coefficients are computed in kinetic theory

We parametrize them as functions of T and \( \mu_B \)

Note: Results of net baryon density are very sensitive to accuracy of the bulk-δf parametrization
Transverse momentum spectra at 62.4 GeV

\(\frac{1}{2\pi} \frac{dN}{dy} \frac{dp_T}{p_T} [\text{GeV}^{-2}]\)

- STAR \(\pi^+\) 0-5%
- STAR \(K^+\) 0-5%
- 3DMCG+MUSIC 0-3%

RHIC 62.4 GeV
0-5%
$v_2$ vs pseudo-rapidity at different energies

![Graph showing $v_2$ vs pseudo-rapidity at different energies for PHOBOS and 3DMCG+MUSIC at various energies. The graph includes data points and error bars for Au+Au collisions at 0-40% centrality. The y-axis represents $v_2$ and the x-axis represents $\eta_p$. The legend includes symbols for PHOBOS and 3DMCG+MUSIC at 200 GeV, 62.4 GeV, and 19.6 GeV. The $\eta/s=0.12$ is indicated in the plot.]
Pb+Pb 2760 GeV pseudo-rapidity distribution

![Graph showing Pb+Pb 2760 GeV pseudo-rapidity distribution with ALICE 20-30% and 3DMCG+MUSIC data points and curves.](image)
T dependent $\eta/s$ from rapidity dependence

Numbers (a,b) are the slopes in [$\text{GeV}^{-1}$] in:

$$(\eta T/(\varepsilon + P))(T) = 0.08 + a(T_c - T)\theta(T_c - T) + b(T - T_c)\theta(T - T_c)$$

where $T_c(\mu_B)$

![Graph showing $\eta/s$ vs. T[GeV]](image-url)
Pseudo-rapidity dependent flow

Au+Au 200GeV 10-20%  
p_T > 0.15 GeV

3DMCG+MUSIC
STAR TPC
STAR FTPC

Two-particle pseudo-rapidity correlations

A. Monnai, B. Schenke, arXiv:1509.04103

Pb+Pb 2760 GeV
p_T>0.5 GeV

ATLAS prelim.
initial entropy density
\( \eta/s=0.12 \), \( (\zeta/s)(T) \)

65-70%