Energy loss and shear viscosity at NLO in a high-temperature QGP

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ALBERT EINSTEIN CENTER
FOR FUNDAMENTAL PHYSICS

In collaboration with Guy Moore and Derek Teaney

Quark Matter 2015, Kobe, September 28 2015

Overview

- Aim: extend the AMY effective kinetic theory to NLO Arnold Moore Yaffe 2002
- NLO means $O(g)$ effects from the medium
- Relies on cool new light-cone techniques (much more complicated for non-relativistic or mildly relativistic degrees of freedom)

Pedagogical review in JG Teaney **1502.03730**
Gritty details (and quarks) in JG Moore Teaney
1509.07773 OUT TODAY!

Overview

- Applications
  - Jet propagation and quenching in the ~~Q~~GP

    No quarks in this talk

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  - (Isotropic) thermalization (à la Kurkela Lu Moore York, Kurkela Lu (2014)) at NLO JG Kurkela

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    Future Monte Carlo applications
  - (Isotropic) thermalization (à la Kurkela Lu Moore York, Kurkela Lu (2014)) at NLO JG Kurkela
  - Transport coefficients (η,...) at NLO. Theoretically harder, working on an estimate

Motivation

- How reliable is the thermal pQCD when extrapolating to α_s=0.3?
  - For thermodynamical quantities (p, s, ...) either strict expansion in g, QCD (T) + EQCD (gT) + MQCD (g^2T) (Arnold-Zhai, Braaten Nieto, etc) or non-perturbative solution of EQCD (Kajantie Laine etc.) or resummations (HTLpt, Andersen Braaten Strickland etc.)
  - For dynamical quantities? We now have 2 contrasting examples of $O(g)$ corrections: very large for heavy quark diffusion (Caron-Huot Moore (2007)), reasonable (~20%) for e.m. probes (JG *et al.*, Laine, Laine Ghisoiu (2013-14))

Outline

- ✓ Introduction and motivation
- A useful reorganization of the LO kinetic theory
- leading to the NLO extension for jets, with
- effective descriptions in terms of Wilson line operators
- an outlook to the shear viscosity

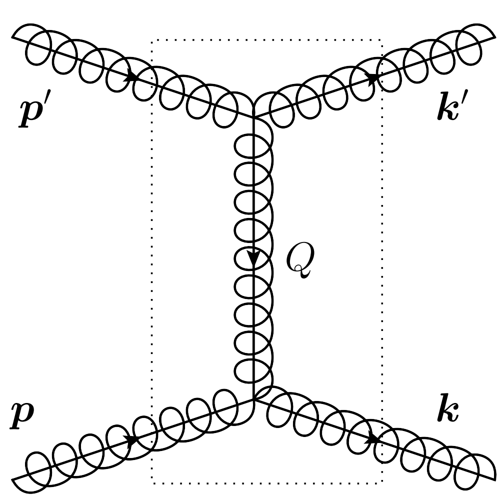
Overview

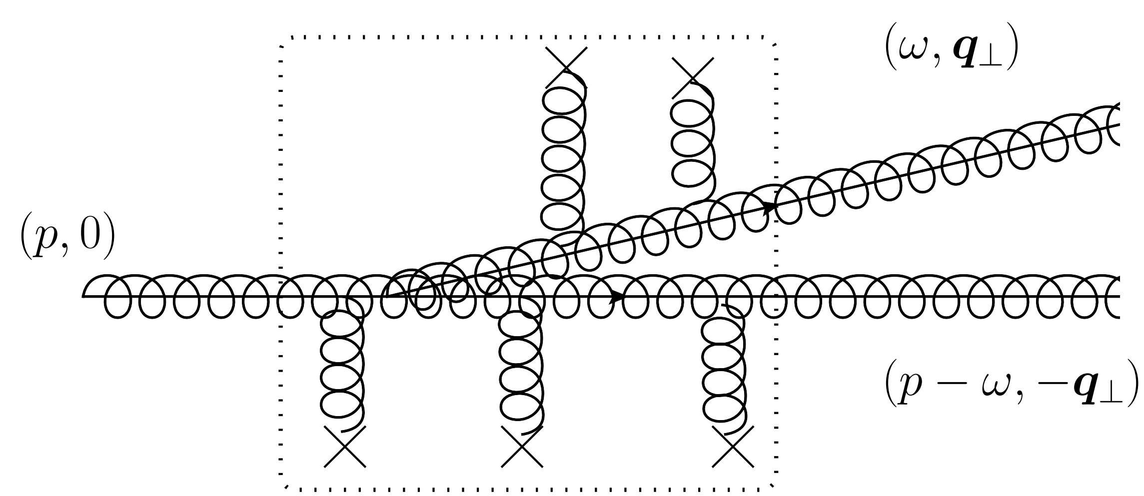
The AMY kinetic approach

- Distribution of dilute high-energy particles in a thermalized medium

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla_{\mathbf{x}}\right) f(\mathbf{p}) = C^{2\leftrightarrow 2} + C^{1\leftrightarrow 2}$$

- At leading order: elastic, number-preserving 2↔2 processes and collinear, number-changing 1↔2 processes (LPM, AMY, all that) AMY (2003)





- Both implemented in MARTINI Schenke Jeon Gale

Reorganization

- Reorganization of the LO collision operator

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla_{\mathbf{x}}\right) f(\mathbf{p}) = C^{\mathrm{large}}[\mu_\perp] + C^{\mathrm{diff}}[\mu_\perp] + C^{\mathrm{coll}}$$

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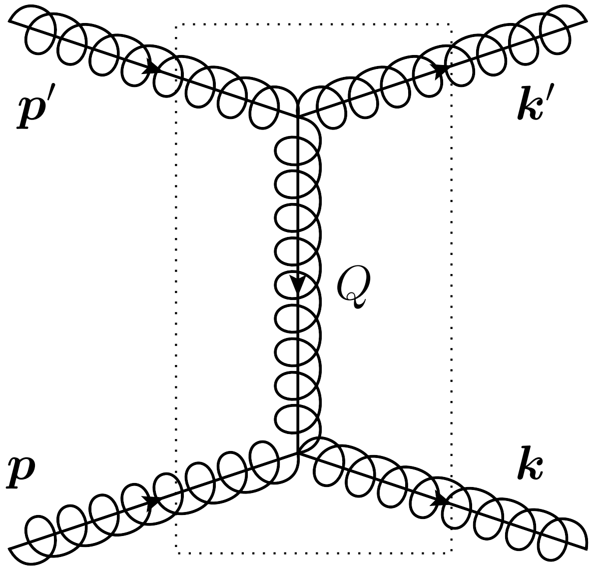
- Large-angle processes: 2↔2 processes with large momentum transfer
- $Q>gT$, $O(1)$ deflection angles
- Need to exclude the IR with a cutoff $\mu_\perp$
- Logarithmic sensitivity to the cutoff
- Can use bare matrix elements



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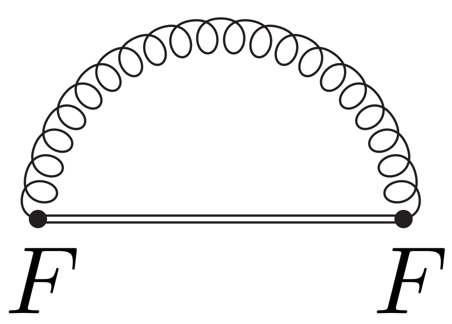
$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla_{\mathbf{x}}\right) f(\mathbf{p}) = C^{\mathrm{large}}[\mu_\perp] + C^{\mathrm{diff}}[\mu_\perp] + C^{\mathrm{coll}}$$

$$C^{\mathrm{diff}}[\mu_\perp] = \frac{\partial}{\partial p^i}\left[\eta_D(p) p^i f(\mathbf{p})\right] + \frac{1}{2}\frac{\partial^2}{\partial p^i \partial p^j}\left[\left(\hat{p}^i\hat{p}^j \hat{q}_L(\mu_\perp) + \frac{1}{2}(\delta^{ij} - \hat{p}^i\hat{p}^j)\hat{q}(\mu_\perp)\right) f(\mathbf{p})\right]$$

- **Diffusion**: Fokker-Planck drag limit for small Q

- Transverse momentum broadening: well known

$$\begin{aligned}\hat{q}(\mu_\perp) =& g^2 C_A \int^{\mu_\perp} \frac{d^2 q_\perp}{(2\pi)^2} \int \frac{dq^+}{2\pi} \langle F^{-\perp}(Q) F^{-}{}_{\perp}\rangle_{q^-=0} \\ =& g^2 C_A T \int^{\mu_\perp} \frac{d^2 q_\perp}{(2\pi)^2} \frac{m_D^2}{q_\perp^2 + m_D^2} = \frac{g^2 C_A T m_D^2}{2\pi} \ln \frac{\mu_\perp}{m_D}\end{aligned}$$



Aurenche Gelis Zaraket **JHEP0205** (2002), Caron-Huot **PRD79** (2009)

Reorganization

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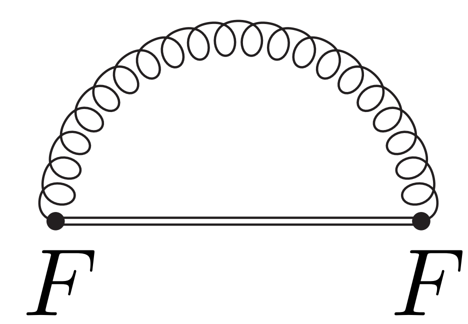
- Longitudinal momentum broadening

$$\begin{aligned}\hat{q}_L(\mu_\perp) =& g^2 C_A \int^{\mu_\perp} \frac{d^2 q_\perp}{(2\pi)^2} \int \frac{dq^+}{2\pi} \langle F^{-z}(Q) F^{-z}\rangle_{q^-=0} \\ =& g^2 C_A T \int^{\mu_\perp} \frac{d^2 q_\perp}{(2\pi)^2} \frac{m_\infty^2}{q_\perp^2 + m_\infty^2} = \frac{g^2 C_A T m_\infty^2}{2\pi} \ln \frac{\mu_\perp}{m_\infty}\end{aligned}$$



Second line: sum rule from analytical properties of amplitudes at space- and light-like separations

Reorganization

- Reorganization of the LO collision operator

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla_{\mathbf{x}}\right) f(\mathbf{p}) = C^{\text{large}}[\mu_\perp] + C^{\text{diff}}[\mu_\perp] + C^{\text{coll}}$$

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- Drag: related by Einstein-like relation to momentum broadening

$$\eta_D(p) = \frac{\hat{q}_L}{2Tp} + \mathcal{O}\left(\frac{1}{p^2}\right)$$

- In the end, cutoff dependence vanishes between diffusion and large-angle scatterings

Reorganization

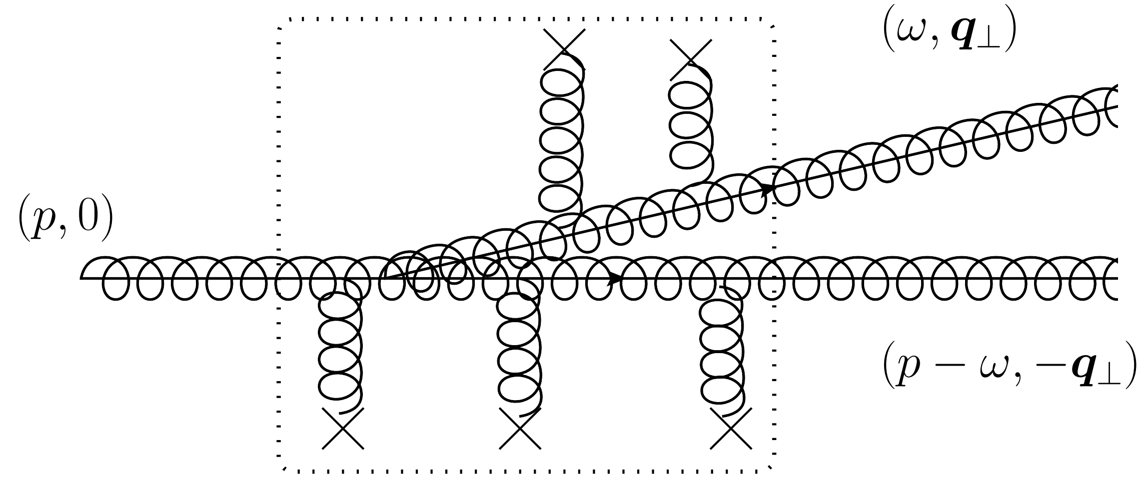
- Reorganization of the LO collision operator

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla_{\mathbf{x}}\right) f(\mathbf{p}) = C^{\text{large}}[\mu_\perp] + C^{\text{diff}}[\mu_\perp] + {\color{red}C^{\text{coll}}}$$

- Collinear processes: 1↔2 processes with strictly collinear kinematics

At LO no change

At NLO remove contaminations from diffusion and semi-collinear



Going to NLO

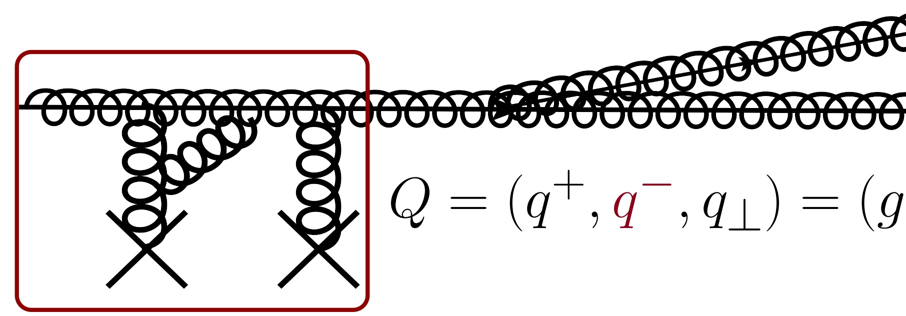
Sources of NLO corrections

- As usual in thermal field theory, the soft scale gT introduces NLO $O(g)$ corrections
- The diffusion and the collinear regions receive $O(g)$ corrections
- There is a new semi-collinear region

Collinear corrections

- The differential eq. for LPM resummation gets correction from NLO $C(q_\perp)$ and from the thermal asymptotic mass at NLO (Caron-Huot 2009)

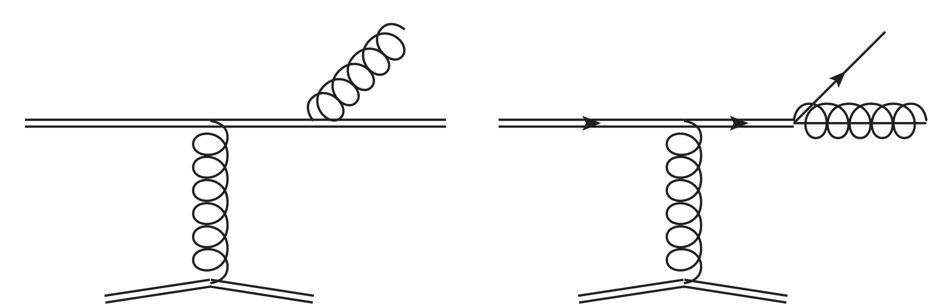
$$\mathcal{C}_{\mathrm{LO}}(q_\perp) = \frac{g^2 C_A T m_D^2}{q_\perp^2 (q_\perp^2 + m_D^2)}$$



$$Q = (q^+, q^-, q_\perp) = (gT,\ g^2T,\ gT)$$

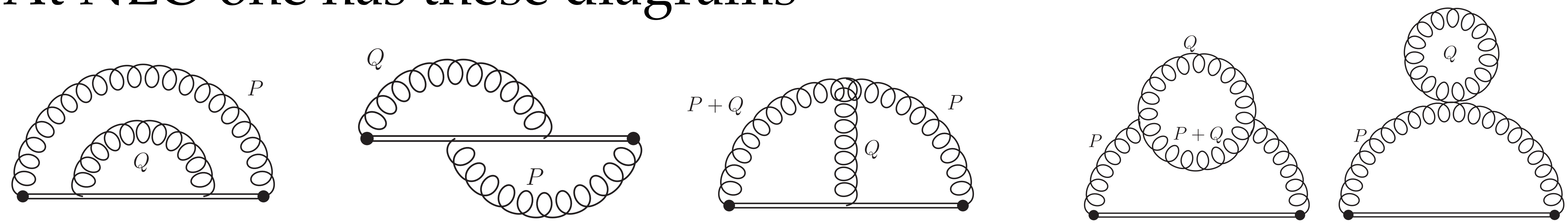
$\mathcal{C}_{\mathrm{NLO}}(q_\perp)$ complicated but analytical (Euclidean tech)
Caron-Huot **PRD79** (2009), Lattice: Panero *et al.* (2013)

Regions of overlap with the diffusion and semi-collinear regions need to be subtracted



Diffusion corrections

- At NLO one has these diagrams



- For transverse: Euclidean calculation Caron-Huot **PRD79** (2009)

$$\hat{q}_{\rm NLO} = \hat{q}_{\rm LO} + \frac{g^4 C_A^2 T^3}{32\pi^2}\frac{m_D}{T}\left(3\pi^2 + 10 - 4\ln 2\right)$$

- For longitudinal:

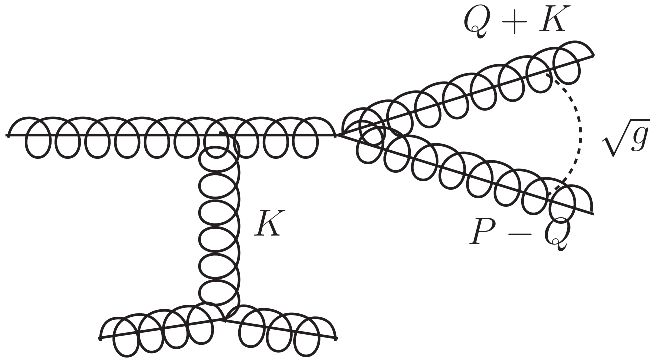
$$\hat{q}_L(\mu_\perp)_{\rm LO} = g^2 C_A T \int \frac{d^2q_\perp}{(2\pi)^2}\frac{m_\infty^2}{q_\perp^2 + m_\infty^2}$$

$$\hat{q}_L(\mu_\perp)_{\rm NLO} = g^2 C_A T \int \frac{d^2q_\perp}{(2\pi)^2}\frac{m_\infty^2 + \delta m_\infty^2}{q_\perp^2 + m_\infty^2 + \delta m_\infty^2} \approx g^2 C_A T \int \frac{d^2q_\perp}{(2\pi)^2}\left[\frac{m_\infty^2}{q_\perp^2 + m_\infty^2} + \boxed{\frac{q_\perp^2\,\delta m_\infty^2}{(q_\perp^2 + m_\infty^2)^2}}\right]$$

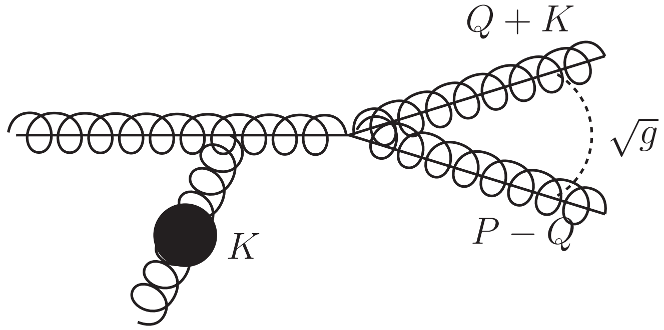
light-cone sum rule still sees only dispersion relation (with $O(g)$ correction). NLO correction UV-log sensitive

Semi-collinear processes

- Seemingly different processes boiling down to wider-angle radiation



K soft cut, spacelike



K soft plasmon, timelike

- Evaluation: introduce "*modified* $\hat{q}$" tracking the changes in the small light-cone component p^- of the gluons. Can be evaluated in EQCD

"*standard*" $$\hat{q} = g^2 C_A \int \frac{d^2 q_\perp}{(2\pi)^2} \int \frac{dq^+}{2\pi} \langle F^{-\perp}(Q) F^{-}{}_{\perp} \rangle_{q^-=0}$$

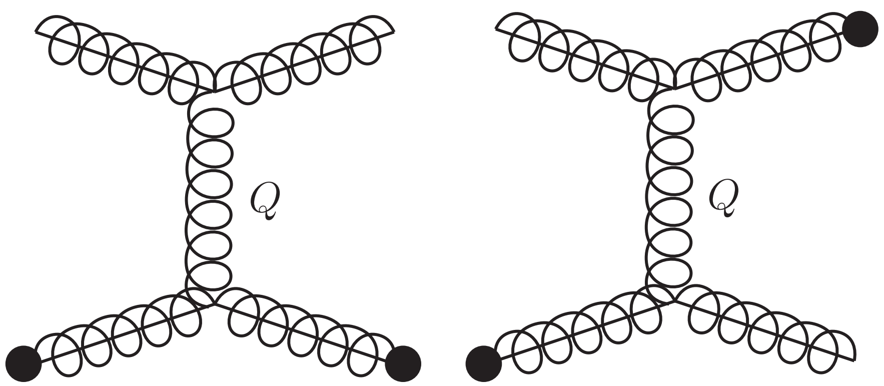
"*modified*" $$\hat{q}(\delta E) = g^2 C_A \int \frac{d^2 q_\perp}{(2\pi)^2} \int \frac{dq^+}{2\pi} \langle F^{-\perp}(Q) F^{-}{}_{\perp} \rangle_{q^-=\delta E}$$

- Rate $\propto$ "*modified* $\hat{q}$" x DGLAP splitting. IR log divergence makes collision operator finite at NLO

Towards NLO shear

- Computing transport coefficients (η) requires knowing how a T^{ij} disturbance induces a second T^{ij} disturbance
- The challenge is again in the soft regions

T^{ij} insertions on the same side, momenta correlated. **Diffusion picture applies**



T^{ij} insertions on opposite sides, momenta uncorrelated. **Diffusion picture does not apply**

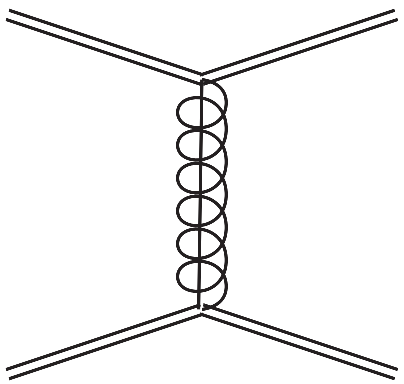
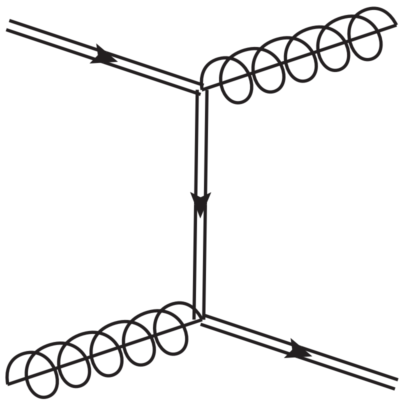
- No diffusion picture = no "easy" light-cone sum rules, only bruteforce HTL. Silver lining: they're finite, so just estimate the number and vary it
- How will NLO η go? Large NLO qhat: $\eta\downarrow$. NLO longitudinal diffusion & semi-collinear: $\eta\uparrow$. Total? Stay tuned!

Conclusions

- Useful reorganization of the kinetic theory with light-front operators that effectively describe soft momentum exchanges
- These operators can be evaluated using new techniques
- The reorganization is valid up to NLO and the operators have all been computed
- Possibility to compute (some of) them on the lattice Panero *et al.*
- Applications are underway
  - implementation in MARTINI
  - thermalization studies
  - estimates of NLO corrections to η

Backup

Elastic processes

Double line: hard (one component O(T) or larger)
Id. specified with curl or arrow when needed

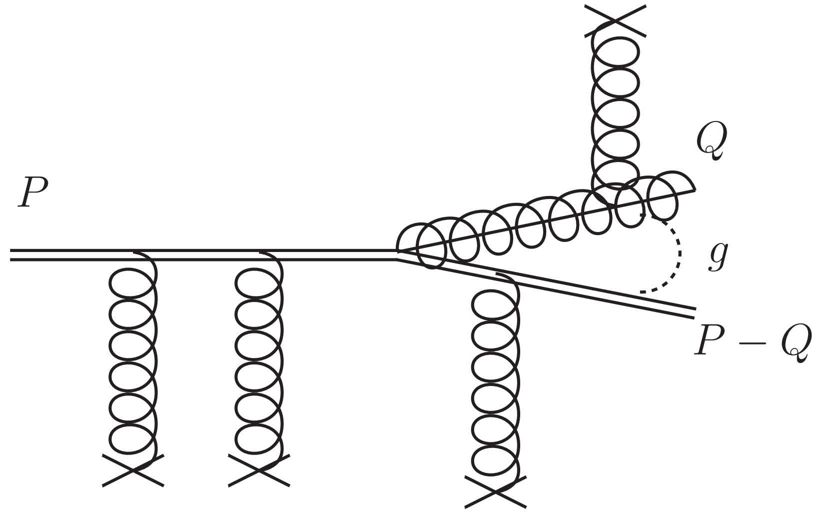
- Boltzmann picture, loss - gain terms

$$C_a^{2\leftrightarrow 2}[P](\boldsymbol{p}) = \frac{1}{4|\boldsymbol{p}|\nu_a}\sum_{bcd}\int_{\boldsymbol{k}\boldsymbol{p}'\boldsymbol{k}'}\left|\mathcal{M}^{ab}_{cd}\right|^2\,(2\pi)^4\,\delta^{(4)}(P+K-P'-K')$$
$$\times\left\{P^a(\boldsymbol{p})\,n^b(k)\,[1\pm n^c(p')]\,[1\pm n^d(k')]-\mathrm{gain}\right\}$$

- Integration with bare matrix elements gives log divergences for soft intermediate states, cured by HTL resummation ⇒ nasty n-dimensional numerics?

Radiative processes

- Effective 1↔2: 1+n↔2+n with LPM suppression, collinear kinematics

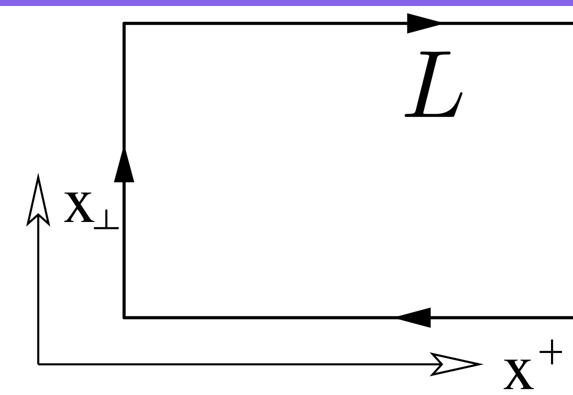


$$C_a^{1\leftrightarrow 2}[P](\boldsymbol{p}) = \frac{(2\pi)^3}{|\boldsymbol{p}|^2\nu_a}\Bigg\{\sum_{bc}\int_0^{p/2} dq\;\gamma^a_{bc}(\boldsymbol{p};(p-q)\hat{\boldsymbol{p}},q\hat{\boldsymbol{p}})\Big\{P^a(\boldsymbol{p})\,[1\pm n^b(p-q)]\,[1\pm n^c(q)] - \text{gain}\Big\}$$
$$+\sum_{bc}\int_0^{\infty} dq\;\gamma^c_{ab}((p+q)\hat{\boldsymbol{p}};\boldsymbol{p},q\,\hat{\boldsymbol{p}})\Big\{P^a(\boldsymbol{p})\,n^b(q)[1\pm n^c(p+q)] - \text{gain}\Big\}\Bigg\}$$

- Rates (gain and loss terms) individually quadratically IR divergent for soft gluon emission/absorption, but gain-loss is finite

- Both processes are implemented in MARTINI Schenke Gale Jeon **PRC80** (2009)

Transverse momentum diffusion



$$\propto e^{\mathcal{C}(x_\perp)L}$$

BDMPS-Z, Wiedemann, Casalderrey-Solana Salgado, D'Eramo Liu Rajagopal, Benzke Brambilla Escobedo Vairo

- All points at spacelike or lightlike separation, only preexisting correlations
- Soft contribution becomes Euclidean! Caron-Huot **PRD79** (2008)
  - Can be "easily" computed in perturbation theory
  - Possible lattice measurements Laine Rothkopf JHEP1307 (2013) Panero Rummukainen Schäfer **1307.5850**

Euclideanization of light-cone soft physics

- For $t/x_z = 0$: equal time Euclidean correlators.

$$G_{rr}(t=0,\mathbf{x}) = \sum\!\!\!\!\!\!\!\int_p G_E(\omega_n, p)e^{i\mathbf{p}\cdot\mathbf{x}}$$

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- Consider the more general case $|t/x^z| < 1$

$$G_{rr}(t,\mathbf{x}) = \int dp^0 dp^z d^2 p_\perp e^{i(p^z x^z + \mathbf{p}_\perp\cdot\mathbf{x}_\perp - p^0 x^0)} \left(\frac{1}{2} + n_{\rm B}(p^0)\right) (G_R(P) - G_A(P))$$

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- Change variables to $\tilde{p}^z = p^z - p^0(t/x^z)$

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- Retarded functions are analytical in the upper plane in any timelike or lightlike variable => G_R analytical in p^0

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$$G_{rr}(t,\mathbf{x}) = T\sum_n \int dp^z d^2p_\perp e^{i(p^z x^z + \mathbf{p}_\perp\cdot\mathbf{x}_\perp)} G_E(\omega_n, p_\perp, p^z + i\omega_n t/x^z)$$

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- Soft physics dominated by $n=0$ (and t-independent) =>EQCD!

Caron-Huot **PRD79** (2009)

Euclideanization of light-cone soft physics

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$$G_{rr}(t,\mathbf{x}) = \int dp^0 dp^z d^2p_\perp e^{i(p^z x^z + \mathbf{p}_\perp\cdot\mathbf{x}_\perp - p^0 x^0)}\left(\frac{1}{2}+n_{\mathrm{B}}(p^0)\right)(G_R(P)-G_A(P))$$

- Change variables to $\tilde{p}^z = p^z - p^0(t/x^z)$

$$G_{rr}(t,\mathbf{x}) = \int dp^0 d\tilde{p}^z d^2p_\perp e^{i(\tilde{p}^z x^z + \mathbf{p}_\perp\cdot\mathbf{x}_\perp)}\left(\frac{1}{2}+n_{\mathrm{B}}(p^0)\right)(G_R(p^0,\mathbf{p}_\perp,\tilde{p}^z+(t/x^z)p^0)-G_A)$$

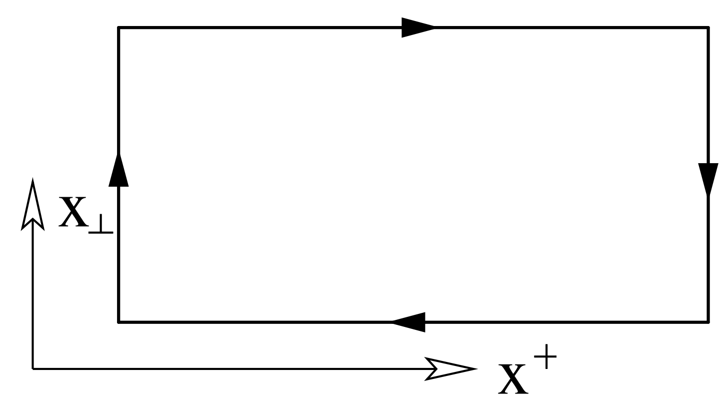
- Retarded functions are analytical in the upper plane in any timelike or lightlike variable => G_R analytical in p^0

$$G_{rr}(t,\mathbf{x})_{\mathrm{soft}} = T\int d^3p\, e^{i\mathbf{p}\cdot\mathbf{x}}\, G_E(\omega_n=0,\mathbf{p})$$

- Soft physics dominated by $n=0$ (and t-independent) =>EQCD!

Caron-Huot **PRD79** (2009)

Euclideanization of light-cone soft physics

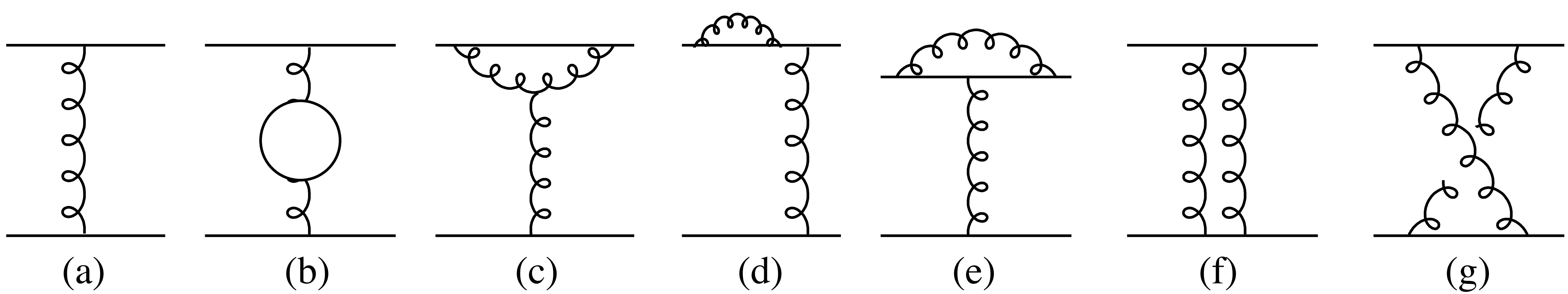


$\propto e^{\mathcal{C}(x_\perp)L}$

- At leading order

$$C(x_\perp) \propto T \int \frac{d^2 q_\perp}{(2\pi)^2} \left(1 - e^{i\mathbf{x}_\perp \cdot \mathbf{q}_\perp}\right) G_E^{++}(\omega_n = 0, q_z = 0, q_\perp) = T \int \frac{d^2 q_\perp}{(2\pi)^2} \left(1 - e^{i\mathbf{x}_\perp \cdot \mathbf{q}_\perp}\right) \left(\frac{1}{q_\perp^2} - \frac{1}{q_\perp^2 + m_D^2}\right)$$

- Agrees with the earlier sum rule in Aurenche Gelis Zaraket **JHEP0205** (2002)
- At NLO: Caron-Huot **PRD79** (2009)



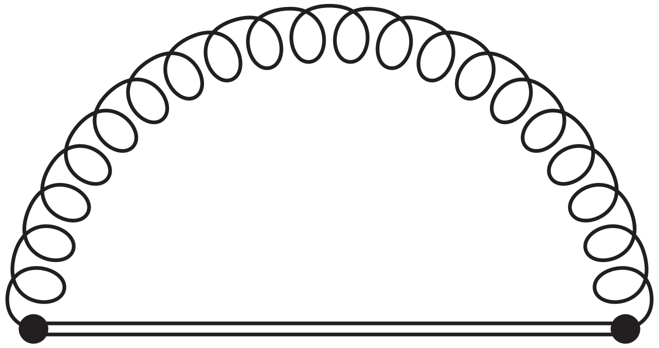
Longitudinal momentum diffusion

- Field-theoretical lightcone definition (justifiable with SCET)

$$\hat{q}_L \equiv \frac{g^2}{d_R}\int_{-\infty}^{+\infty} dx^+ \mathrm{Tr}\left\langle U(-\infty,x^+)F^{+-}(x^+)U(x^+,0)F^{+-}(0)U(0,-\infty)\right\rangle$$

$F^{+-}=E^z$, longitudinal Lorentz force correlator

- At leading order

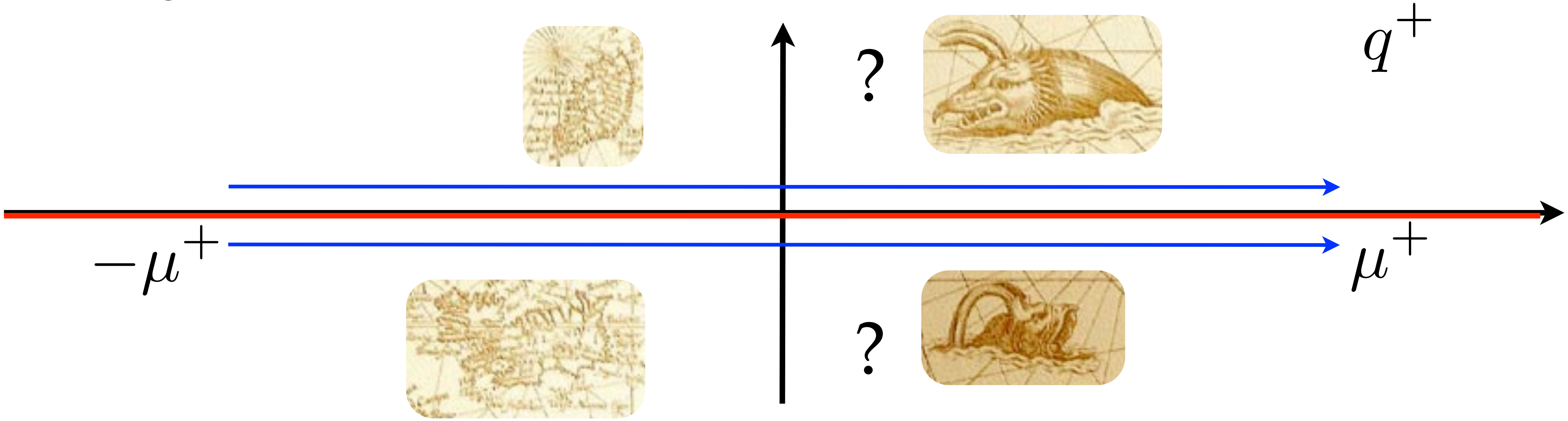


$$\hat{q}_L \propto \int \frac{dq^+ d^2q_\perp}{(2\pi)^3}(q^+)^2 G^{>}_{++}(q^+,q_\perp,0)$$

$$= \int \frac{dq^+ d^2q_\perp}{(2\pi)^3} T q^+ (G^R_{++}(q^+,q_\perp,0) - G^A)$$

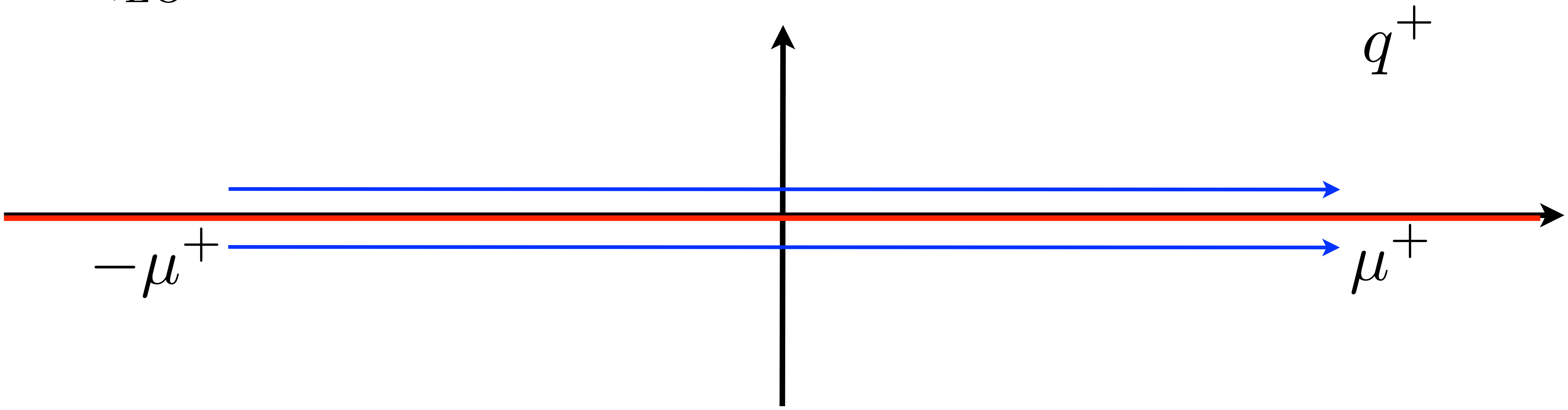
Longitudinal momentum diffusion

$$\hat{q}_L\Big|_{\mathrm{LO}} = g^2 C_R \int \frac{dq^+ d^2 q_\perp}{(2\pi)^3} T q^+ \left(G_R^{--}(q^+, q_\perp) - G_A^{--}(q^+, q_\perp)\right)$$



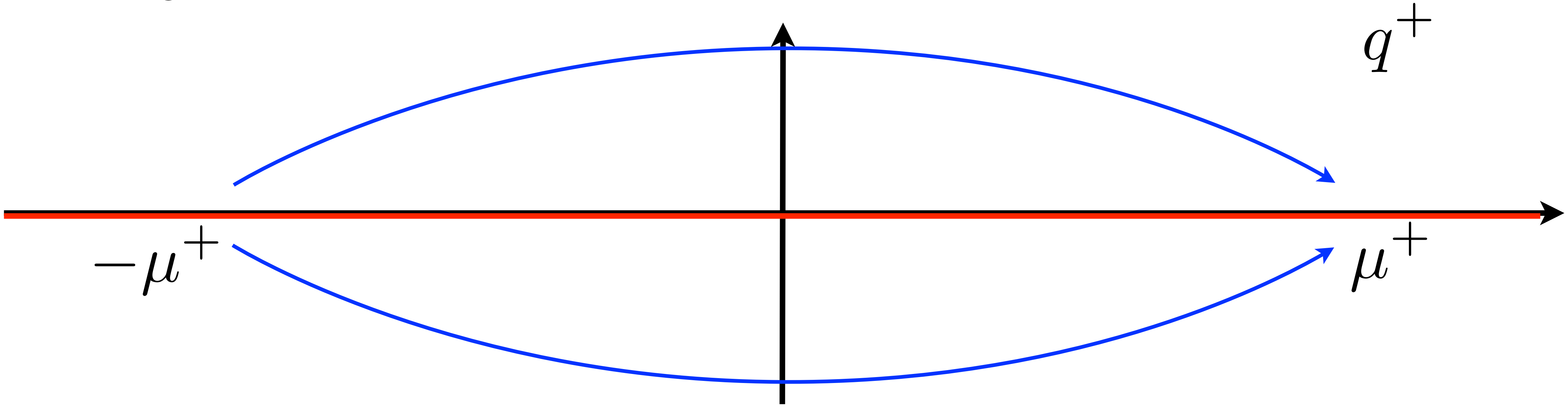
Longitudinal momentum diffusion

$$\hat{q}_L\Big|_{\mathrm{LO}} = g^2 C_R \int \frac{dq^+ d^2q_\perp}{(2\pi)^3} T q^+ (G_R^{--}(q^+, q_\perp) - G_A^{--}(q^+, q_\perp))$$



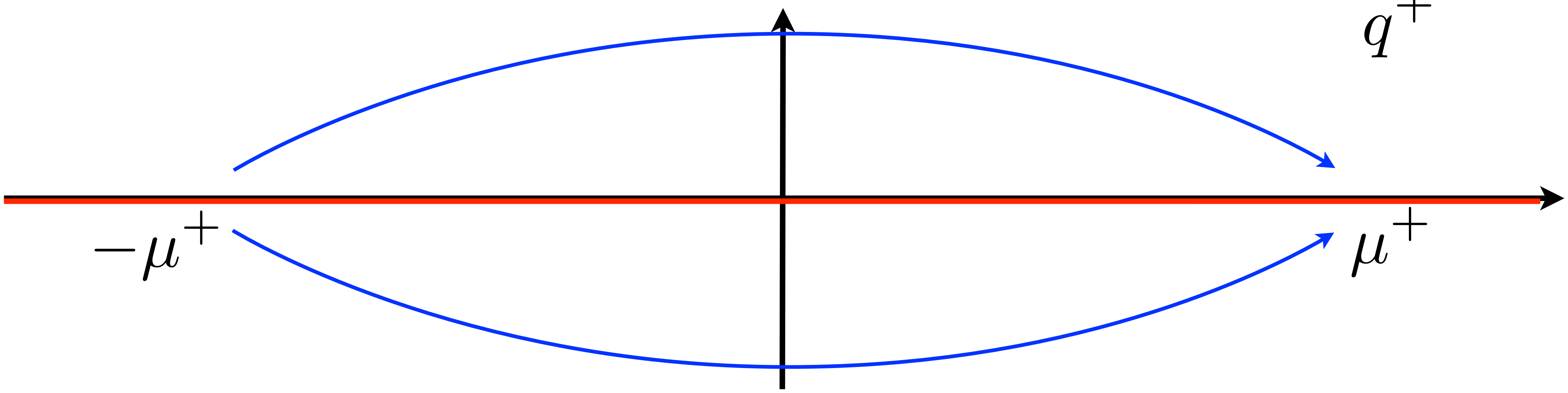
Longitudinal momentum diffusion

$$\hat{q}_L\Big|_{\mathrm{LO}} = g^2 C_R \int \frac{dq^+ d^2 q_\perp}{(2\pi)^3} T q^+ \left(G_R^{--}(q^+, q_\perp) - G_A^{--}(q^+, q_\perp)\right)$$



Longitudinal momentum diffusion

$$\hat{q}_L\Big|_{\mathrm{LO}} = g^2 C_R \int \frac{dq^+ d^2 q_\perp}{(2\pi)^3} T q^+ (G_R^{--}(q^+, q_\perp) - G_A^{--}(q^+, q_\perp))$$



- Use analyticity to deform the contour away from the real axis and keep *1/q*⁺ behaviour

$$\hat{q}_L\Big|_{\mathrm{LO}} = g^2 C_R T \int \frac{d^2 q_\perp}{(2\pi)^2} \frac{M_\infty^2}{q_\perp^2 + M_\infty^2}$$