# A new shock-capturing numerical scheme using an exact Riemann solver

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## 1. Motivation

**The task:** to build a new relativistic hydrodynamics code based on the exact solution of Riemann problem for an arbitrary equation of state suitable for heavy ion collision modelling with a possible source term.

The hydrodynamic description of ultrarelativistic heavy ion collision requires a sophisticated numerical method capable of dealing with complicated initial conditions and shock waves possibly arising from jets propagating in the medium [1-3]. Our particular goal is to simulate such jets using a source term and its effect on the medium.

## 2. Relativistic hydrodynamics

Ideal relativistic hydrodynamic equations express the conservation of energy and momentum and a conserved charge, i.e. the baryon number in heavy-ion collisions. In collisions at highest energies the net baryon density is practically zero, thus we only consider energy and momentum and the equation of state which completes the system of equations is only the relation between pressure and energy density:

$$\partial_\mu T^{\mu\nu} = 0$$
$$T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu - p g^{\mu\nu}$$
$$p = p(\epsilon).$$

## 3. Our approach

- Godunov scheme with linear reconstruction of states: flows of conserved charge are calculated at every cell boundary [4,5]
- Flows at the cell boundary: we use the exact solution of the Riemann problem for an arbitrary equation of state with the presence of tangential velocities [6,7]
- Riemann problem: reconstrucion of wave patterns allows us to calculate energy density, normal and tangential velocities exactly at the interface

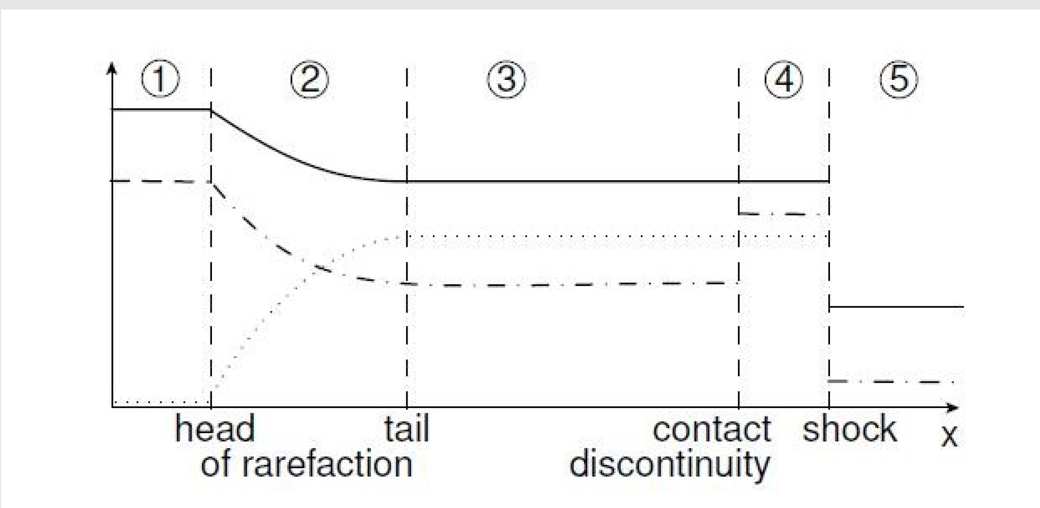


**Fig. 1**: A possible wave pattern at the interface including rarefaction and shock wave and contact discontinuity, figure taken from [5]

## 4. Numerical tests

### 4.a) Sound wave propagation

- We simulate a sound wave over one wavelength in the numerical grid by imposing the following initial conditions:

$$p_{init}(x) = p_0 + \delta p \sin(2\pi x/\lambda), v_{init}(x) = \frac{\delta p}{c_{s0}(e_0 + p_0)} \sin(2\pi x/\lambda)$$

$$p_0 = 10^3 fm^{-4}, \delta p = 10^{-1} fm^{-4}, \lambda = 2 fm$$

- Using the analytic solution for the sound wave we can evaluate the precision of our numerical scheme via L1-norm and its numerical viscosity $\eta_{num}$:

$$L(p(N_{cell}), p_s) = \sum_{i=1}^{N_{cell}} |p(x_i, \lambda/c_s; N_{cell}) - p_s(x_i, \lambda/c_s)| \frac{\lambda}{N_{cell}}$$

$$\eta_{num} = \frac{-3\lambda}{8\pi^2} c_{s0}(e_0 + p_0) \ln[1 - \frac{\pi}{2\lambda\delta p} L(p(N_{cell}, p_s))]$$

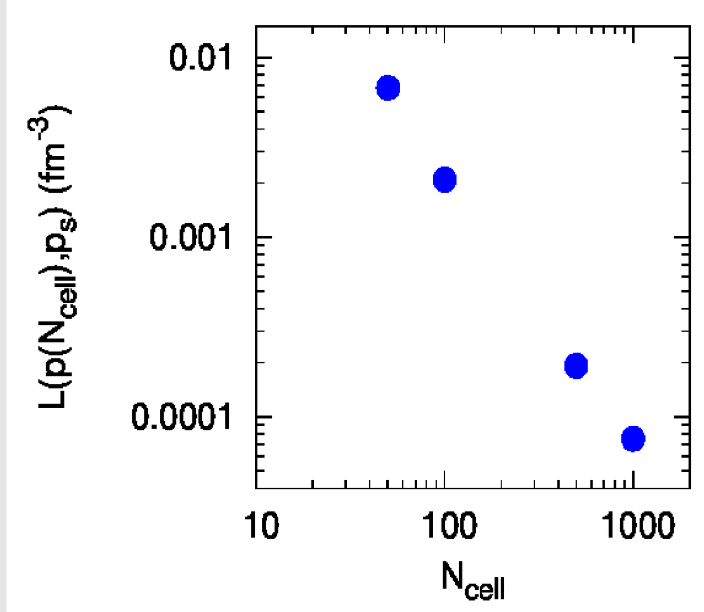


**Fig. 2**: Dependence of L1 norm on the number of cells in the numerical grid

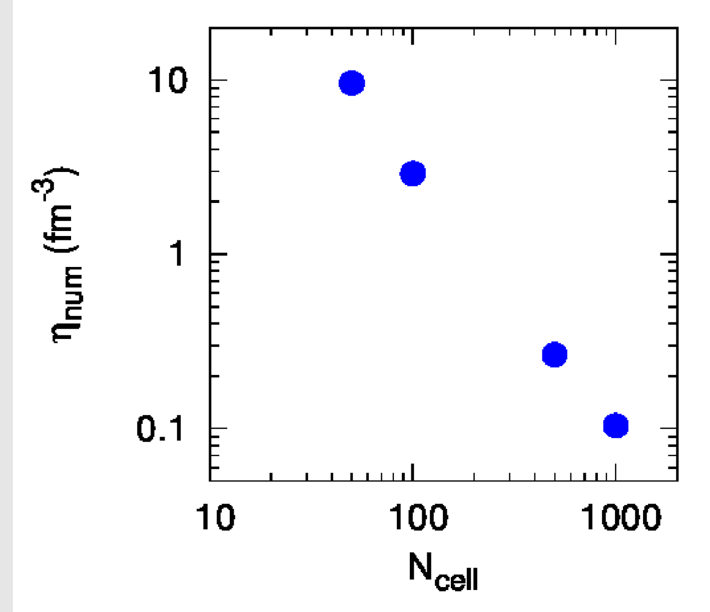


**Fig. 3**: Dependence of num. viscosity $\eta_{num}$ on the number of cells in the numerical grid

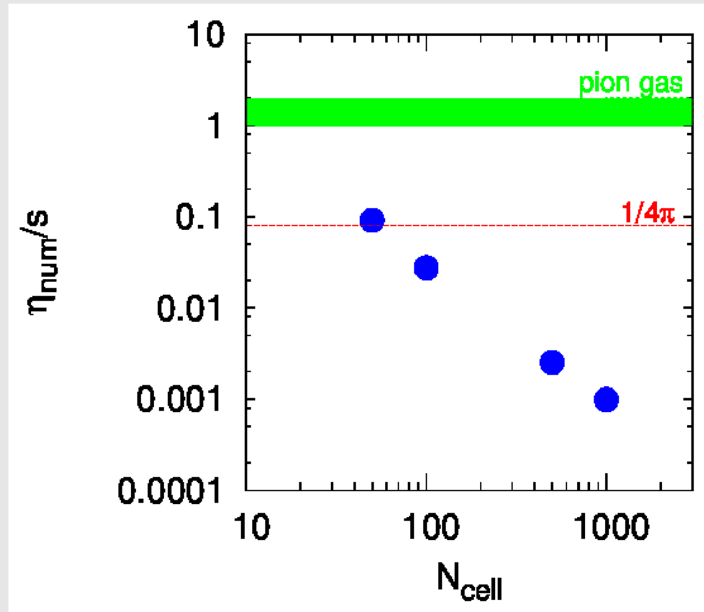


**Fig. 4**: Dependence of the numerical viscosity to entropy density ratio $\eta_{num}$ /s (blue points) compared to the limiting value η/s = 1/4π (in red) and η/s of pion gas [7] (green band).

### 4. b) Shock tube problem:

- We test the capability of the scheme to cope with discontinuities in energy density and tangential velocity
- It consists of imposing two constant states separated by a discontinuity in the initial conditions, we compare the numerical and analytic solution

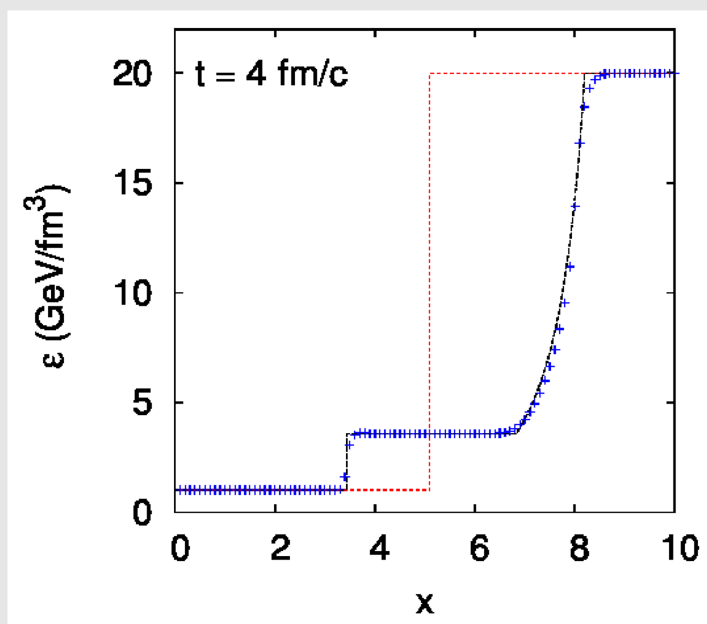


**Fig. 5**: Profile of energy density after 100 time-steps (our scheme in blue, analytic solution in black, initial conditions in red)

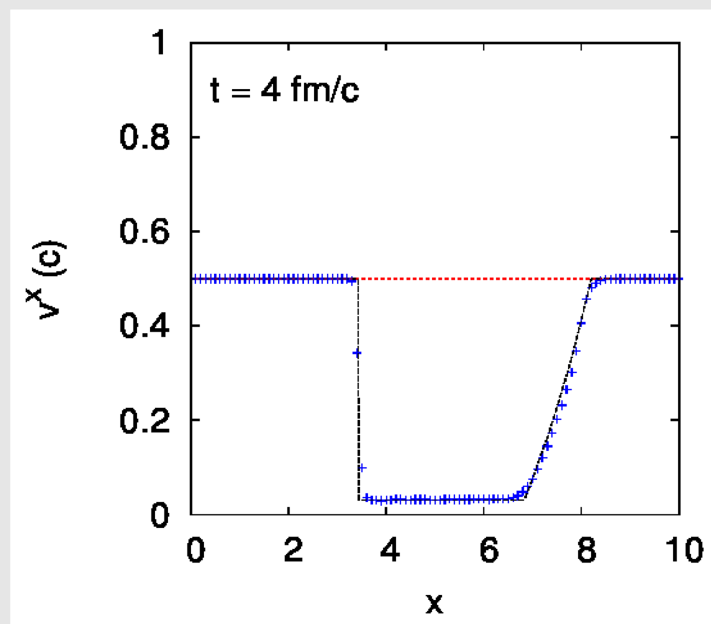


**Fig. 6**: Profile of normal velocity after 100 time-steps (our scheme in blue, analytic solution in black, initial conditions in red)

## 5. Summary

We have built and tested an ideal relativistic hydrodynamic scheme based on the exact solution of the Riemann problem in one spatial dimension with presence of tangential velocity. The sound wave propagation test shows a good precision and low numerical viscosity which will become important when introducing dissipation into the model. The shock tube problem reveals that the scheme is able to capture shock and rarefaction very well. We will extend this scheme to three dimensions and then apply it in description of the flow in ultrarelativistic nuclear collisions.

## References

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