The mixing effects of scalar mesons in a Skyrme model

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1, Abstract:

We construct a skyrme model including two-quark and four-quark scalar mesons as well as the pion, rho meson and omega meson fields within a framework of the hidden local symmetry. We investigate the effects of scalar mesons in the model, we show that the scalar mesons reduce the skyrmion mass.

2, Motivation:

Hadron
- Meson: integer spin
- Baryon: half-integer spin

A theory describe both meson sector and baryon sector in a consistent way?

Scalar meson plays attractive effects

Model	Soliton mass
π,ρ	1054.6
π,ρ,ω	1469.0
π,ρ,ω,χ	1408.3

B.-Y. Park et al. Nuclear Physics A 736 (2004) 129-145

3, The scalar mixing structure(q=u,d):

The 2 quark state and the 4 quark state for scalar meson

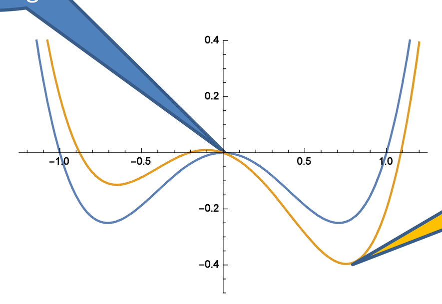
$$M_{(2)} = \frac{1}{2}\xi_L^\dagger \sigma \xi_R\,,\quad M_{(4)} = \frac{1}{2}\phi$$

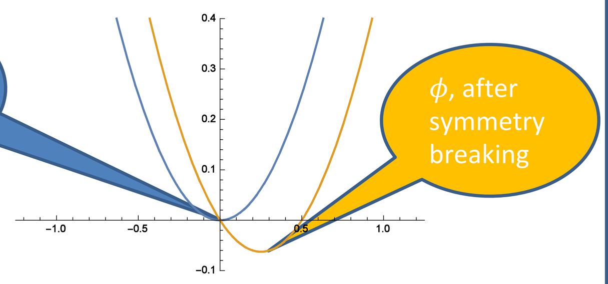
2 quark — 4 quark

The potential

$$V_0 = \frac{1}{8}\lambda\sigma^4 - \frac{1}{2}m_2^2\sigma^2 + \frac{1}{2}m_4^2\phi^2 + \frac{1}{\sqrt{2}}A\sigma^2\phi$$

$$V_{SB} = -\frac{1}{4}M_\pi^2 f_\pi \sigma\, \mathrm{Tr}(U + U^\dagger)$$





4, Modeling:

Symmetry of the model

$SU(2)_L \times SU(2)_R \times U(1)_V$ → $SU(2)_V \times U(1)_V$

Scalar meson mixing structure

$$\begin{pmatrix} f_{500} \\ f_{1370}\end{pmatrix} = \begin{pmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\tilde\sigma \\ \tilde\phi\end{pmatrix}$$

Field	Operator	Physical fields
Pseudoscalar meson	$F(r)$	π
Vector meson	$W(r), G(r)$	ω,ρ
2-quark scalar meson	$\sigma(r) = f_\pi + \tilde\sigma$	$f_{500} = \cos(\theta)\,\tilde\sigma - \sin(\theta)\tilde\phi$
4-quark scalar meson	$\phi(r) = \phi_{vac} + \tilde\phi$	$f_{1370} = \sin(\theta)\,\tilde\sigma + \cos(\theta)\tilde\phi$

- Scalar meson σ,ϕ have vacuum expectation value
- The physical state of scalar mesons are mixed by 2-quark state and 4-quark state

5, The Lagrangian:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \sigma^2\,\mathrm{Tr}(\alpha_{\perp\mu}\alpha_\perp^\mu) + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - (V_0 - \bar V_0) - (V_{SB} - \bar V_{SB})$$
$$+ a_{hls}(s_0\sigma^2 + (1-s_0)f_\pi^2)\,\mathrm{Tr}(\alpha_{\parallel\mu}\alpha_\parallel^\mu) - \frac{1}{2g^2}\mathrm{Tr}(V_{\mu\nu}V^{\mu\nu}) + \mathcal{L}_{anom}$$

- pion
- The ϕ(4-quark state scalar meson) interact with other fields through σ
- Only σ(2-quark state scalar meson) interact with other field directly
- Vector meson
- Vector meson exchange (repulsive force)
- WZW(ω meson)

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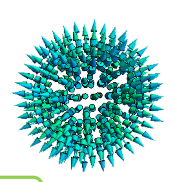
6, For baryon number B=1 case:

The ansatz:

The hedgehog ansatz:



π	J=0, I=1	U	=	$\cos F(r) + i\vec\tau\cdot\hat x \sin F(r)$
ρ	J=1, I=1	$\rho^a_{\mu=i}$	=	$\epsilon^{ika}\hat r^k \frac{G(r)}{g_\rho r}$
ω	J=1, I=0	ω_μ	=	$\delta_{\mu0}W(r)$
σ	J=0, I=0	σ	=	$f_\pi(1+\bar\sigma(r))$
ϕ	J=0, I=0	ϕ	=	$\phi_{vac}(1+\bar\phi(r))$

Parameters:

f_π	92.4 MeV
M_π	139.57 MeV
N_c	3
$c_1 + c_2$	0
$c_1 - c_2$	$-4/3$
c_3	0
g	5.80 ± 0.91
a_{hls}	2.07 ± 0.33

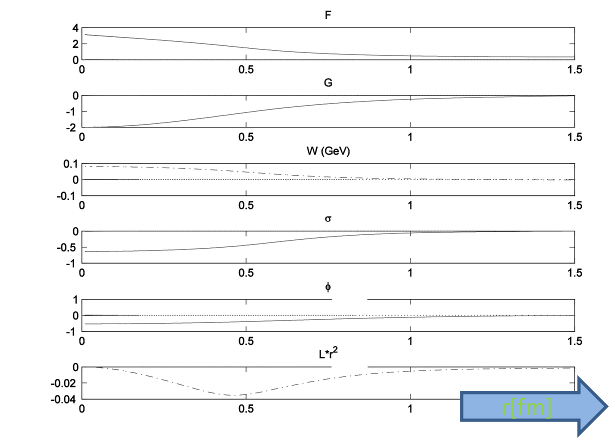
There are several combination for c_1, c_2, c_3, here we choose this combination to reproduce $\omega_\mu B^\mu$.

Baryon number:

$$N_B = 4\pi\left(\frac{4\alpha_3\sin F(\cos F - G - 1)}{3g} - \frac{2r^2W'}{3g} + \frac{\sin(2F)}{8\pi^2} - \frac{F}{4\pi^2}\right)\Bigg|_{r\to0}^{r\to\infty} = 1$$

Boundary Condition

	$r\to0$	$r\to\infty$
$F(r)$	π	0
$F'(r)$	-	-
$G(r)$	-2	0
$G'(r)$	-	-
$W(r)$	-	0
$W'(r)$	0	-
$\bar\sigma(r)$	-	0
$\bar\sigma'(r)$	0	-
$\bar\phi(r)$	-	0
$\bar\phi'(r)$	0	-



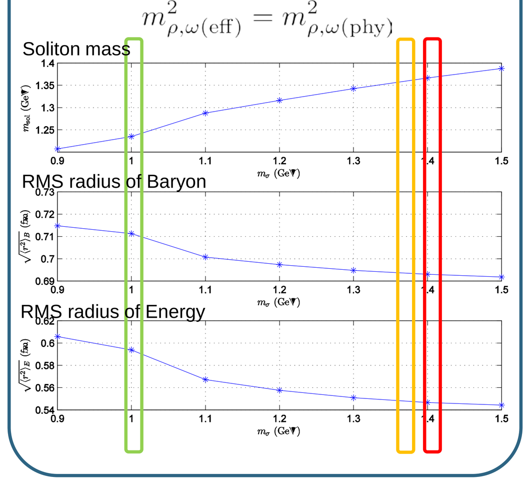
- The boundary condition for π,ρ,ω are taken for baryon number N_B=1
- The boundary condition for $\bar\sigma,\bar\phi$ are taken for no divergence occurs when $r\to0$

7, Numeral results:

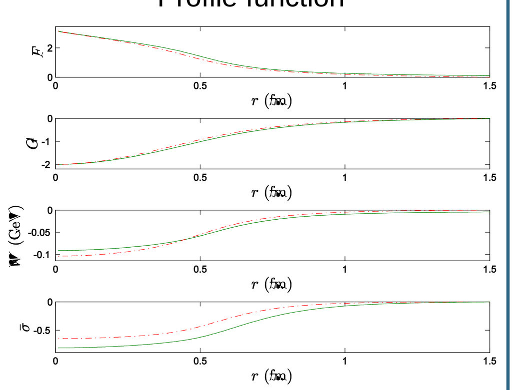
m_σ dependence ($\pi,\rho,\omega+\sigma$ case)

When taking $S_0 = 0$

$m^2_{\rho,\omega(\mathrm{eff})} = m^2_{\rho,\omega(\mathrm{phy})}$



Profile function



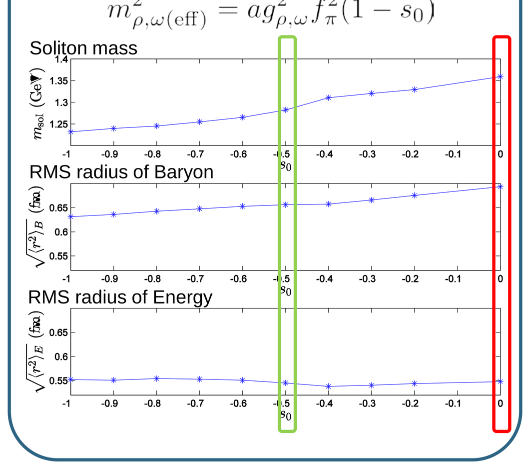
Green curves correspond to m_σ=1 GeV and red dotted curves correspond to m_σ=1.4 GeV.

The pure 2 quark scalar meson will drop about 100 MeV soliton mass, similar with dilaton scalar meson case.

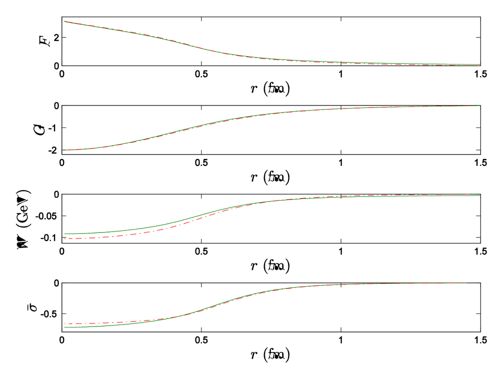
s_0 dependence ($\pi,\rho,\omega+\sigma$ case)

When taking m_σ = 1.37 GeV

$m^2_{\rho,\omega(\mathrm{eff})} = a g^2_{\rho,\omega} f_\pi^2(1-s_0)$



Profile function



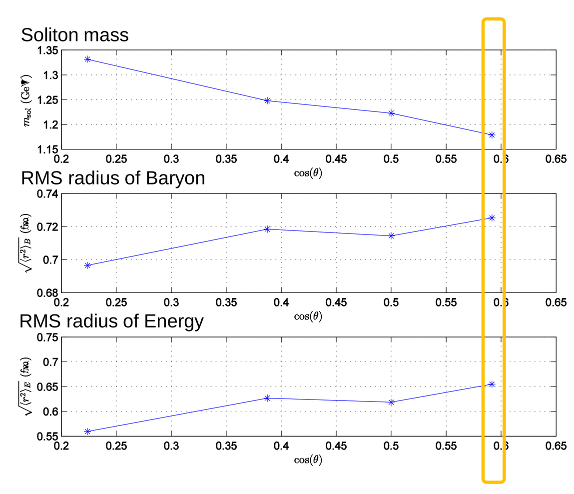
$s_0 = -0.5$ (green line) and $s_0 = 0$ (red dotted line)

$f^0_B \propto -W(1 + s_0(\bar\sigma + 2)\bar\sigma)$

The pure 2 quark scalar meson will drop about 100 MeV soliton mass, similar with dilaton scalar meson case.

The tendency of "charge radius - scalar meson mass" relation, depend on the way how scalar meson is incorporated.

Scalar meson is made by the mixing structure ($\pi,\rho,\omega+\sigma,\phi$ case)



State	$\bar q q$%	$\bar q\bar q qq$%	m (GeV)
$f_0(500)$	40	60	0.742
$f_0(1370)$	60	40	1.493

With q=u,d

$$\begin{pmatrix}\tilde\sigma \\ \tilde\phi\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix} f_{500} \\ f_{1370}\end{pmatrix}$$

$\cos\theta\sim0$	$\cos\theta\sim1$
$\tilde\sigma\sim f_{1370}$	$\tilde\sigma\sim f_{500}$

The mixing structure of 2 quark state and 4 quark state will drop about 180 MeV soliton mass than pure 2 quark case, to get a more physical baryon mass.

Conclusions:

The incorporate of scalar meson will drop the soliton mass.

$\pi,\rho,\omega+\sigma$ case(2-quark state scalar meson)

A lighter 2-quark state scalar meson reduce more soliton mass.

A heavier effective vector meson mass(smaller S_0) reduce more soliton mass

$\pi,\rho,\omega+\sigma,\phi$ case(2-quark state and 4-quark state scalar meson)

the lighter "effective 2 quark scalar meson" mass is, the lighter soliton mass becomes.

The tendency of "charge radius - scalar meson mass(or S_0)" relation, depend on the way how scalar meson is incorporated.

Future works:
Modify the interaction term, Quantization the soliton, Dense media effects(Skyrmion crystal), EOS for neutron starts, ...

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