Yang-Lee Zeros and Phase Boundary from Net-Baryon Number Multiplicity Distribution

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From Multiplicity Distribution to Phase Boundary

- Assuming fluctuations in equilibrium (at freeze-out), net-baryon number probability distribution $P(N)$ is related to the canonical partition function $Z(N)$.

$$P(N;T,V,\mu) = \frac{Z(T,V,N)e^{\mu N/T}}{\mathcal{Z}(T,V,\mu)}$$

- From measured $P(N)$, one can construct $Z(N)$ by making use of charge conjugation symmetry $Z(N) = Z(-N)$. A.Nakamura and K.Nagata, '13
- Canonical approach in lattice QCD provides $Z(N)$ with $N < N_{\max}$.
- Then, the partition function $\mathcal{Z}$ can be obtained as a fugacity series,

$$\mathcal{Z}^{\rm tr}(\mu) = \sum_{N=-N_{\max}}^{N_{\max}} Z(N)\lambda^N$$

Projection onto fixed N states

$$Z(N) = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\theta \cos(N\theta)\mathcal{Z}(\mu=-i\theta T)$$

- One can evaluate moments and Yang-Lee zeros, but effects of truncation? – check with a solvable model.

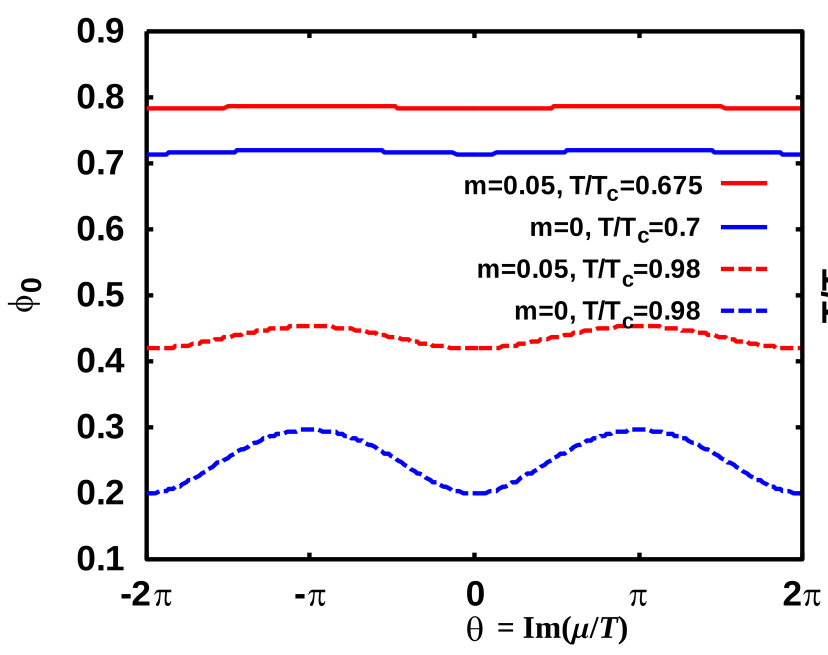
Chiral Random Matrix Model

- An effective model for chiral phase transition in QCD
  A. Halasz et al., Phys. Rev. D58, 096007 ('98)
- Partition function at finite N_s [M. Stephanov, Phys. Rev. D73, 094508 ('06)]

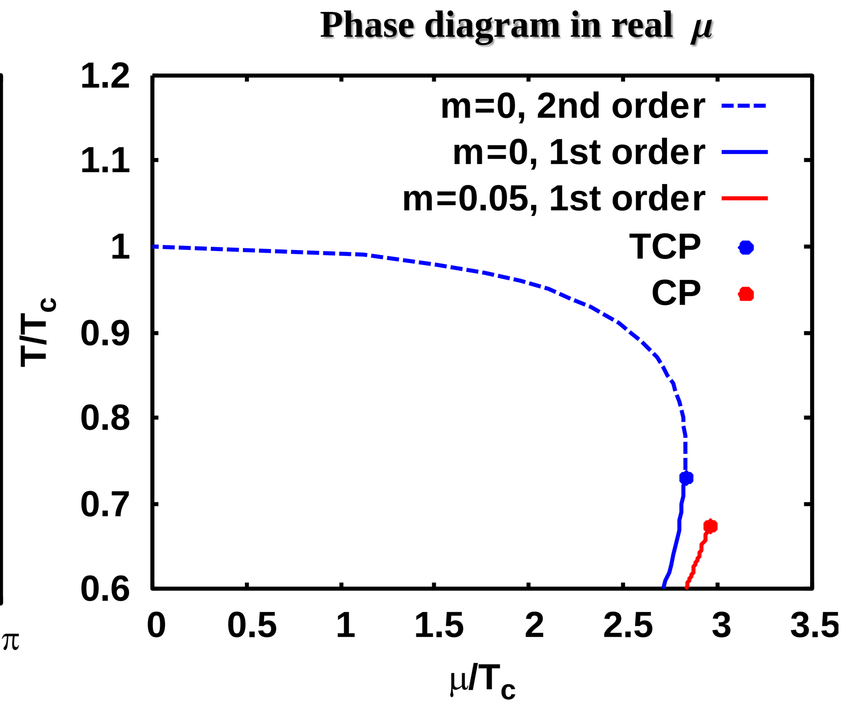
$$\mathcal{Z}_{\rm RM} = \sum_{k_1,k_2=0}^{N_s/2} \binom{N_s/2}{k_1}\binom{N_s/2}{k_2}(N-k_1-k_2)!\,{}_1F_1(k_1+k_2-N_s;1;-m^2N_s)(-N_s\pi^2a^2T^2)^{k_1+k_2}$$
$$\times\left(i+\frac{2b}{a\pi N_c}\sinh\frac{\mu}{2T}\right)^{2k_1}\left(i-\frac{2b}{a\pi N_c}\sinh\frac{\mu}{2T}\right)^{2k_2}$$

- $\sinh(\mu/2T)$ gives correct periodicity $2\pi T$ in imaginary μ, originating from baryon number conservation.
- Phase structure of the model (a=0.2, b=0.13)

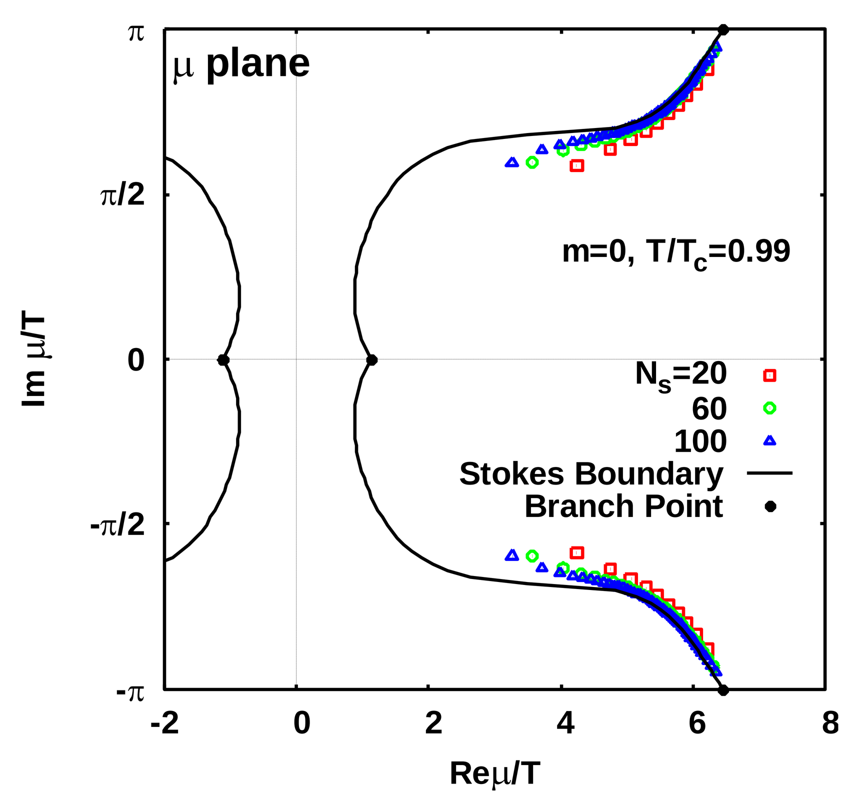
Chiral order parameter in imaginary μ

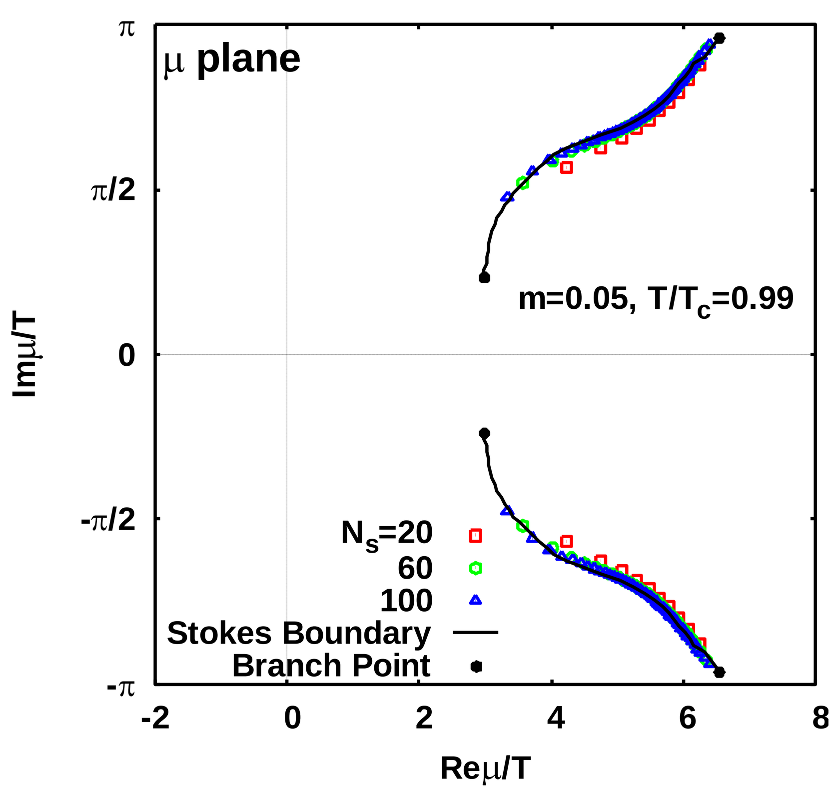


Phase diagram in real μ



- Yang-Lee zeros
  - Obtained by solving $\mathcal{Z}_{\rm RM}(\mu) = 0$ for complex μ
  - Stokes boundary : ${\rm Re}\,p(T,\mu_1) = {\rm Re}\,p(T,\mu_2)$

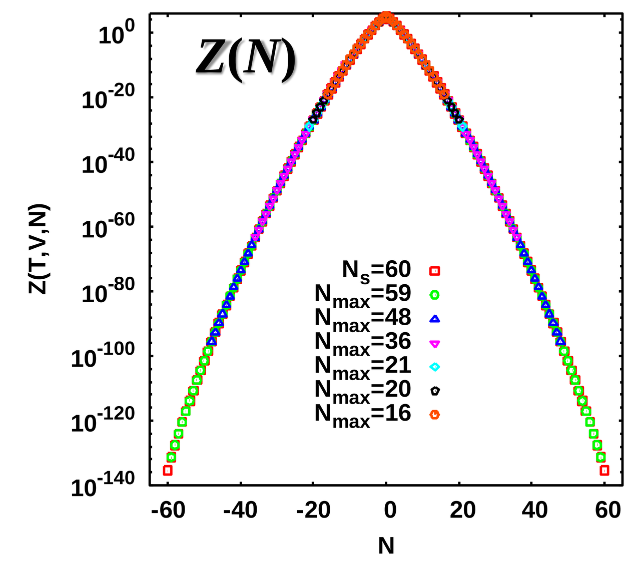


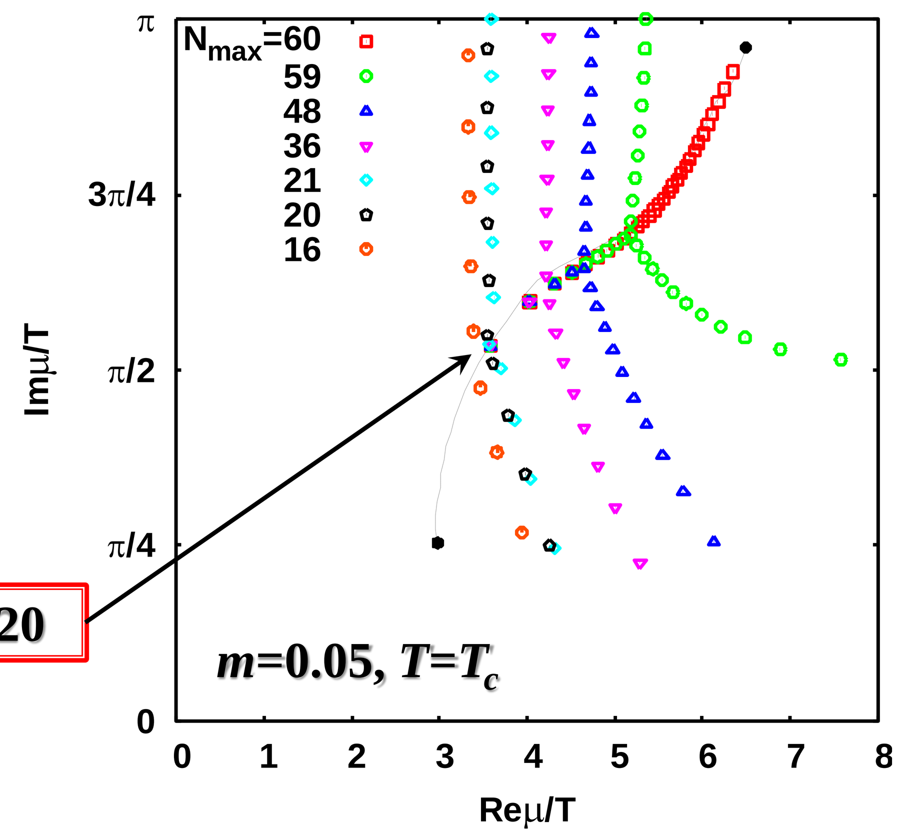


Yang-Lee Zeros from Truncated $\mathcal{Z}(\mu)$

$Z(N)$ at large N cannot be obtained because of statistics / overlap problems.

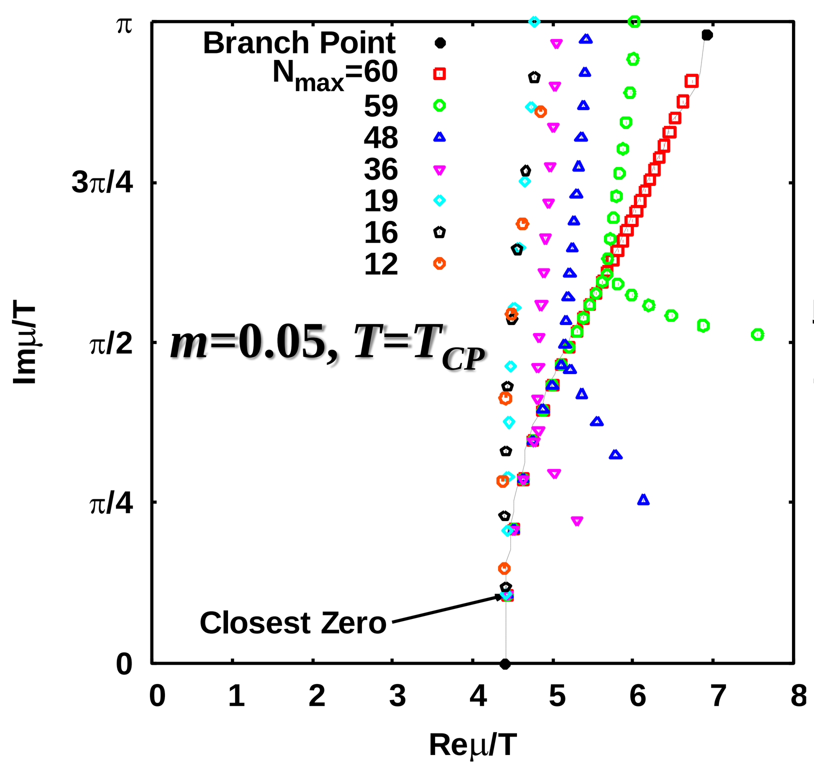
- Random Matrix Model

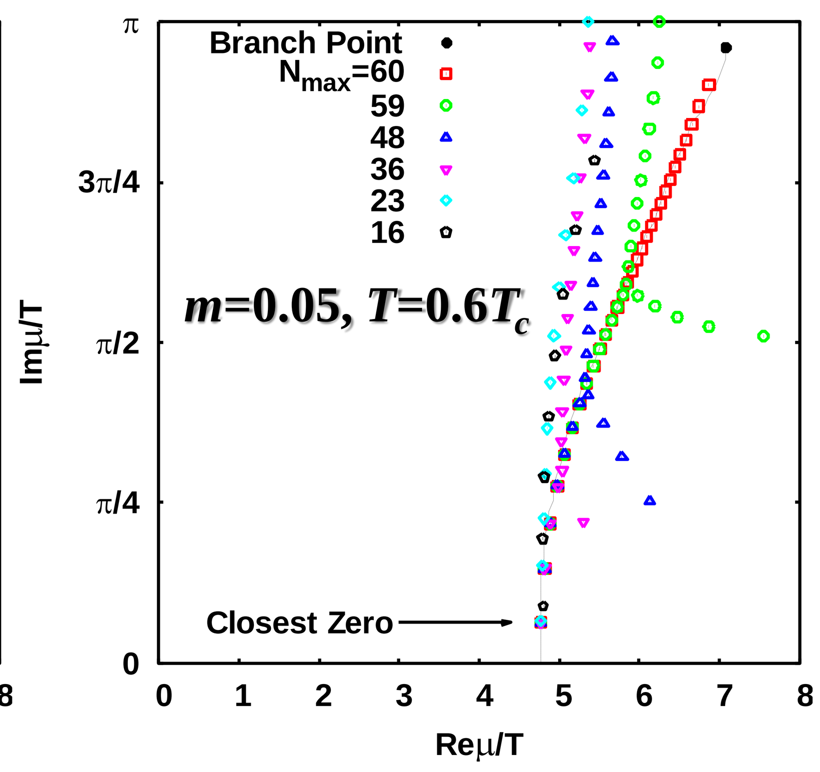




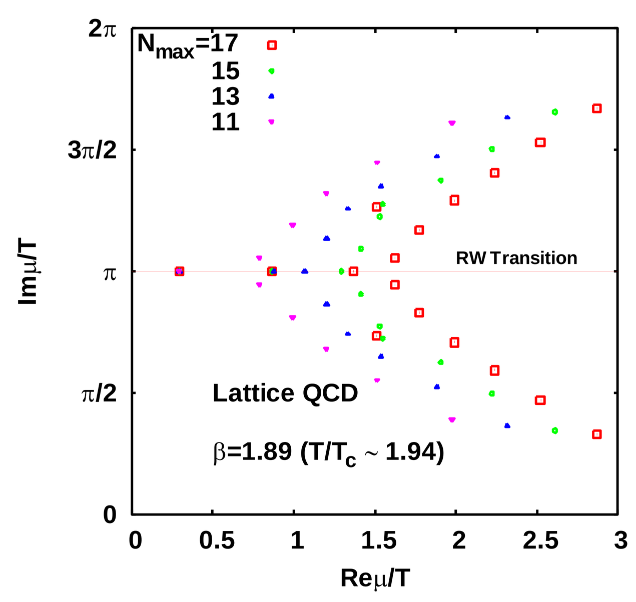
Edge stable for $N_{\max} > 20$

Splitting of distribution even for $N_{\max} = N_s$-1





- Lattice QCD (for Roberge-Weiss transition)



Similar behavior to the RM model

Splitting of distribution can be understood as a truncation effect.

Lattice Data :

Nagata et al., PTEP2012, 01A013 ('12)

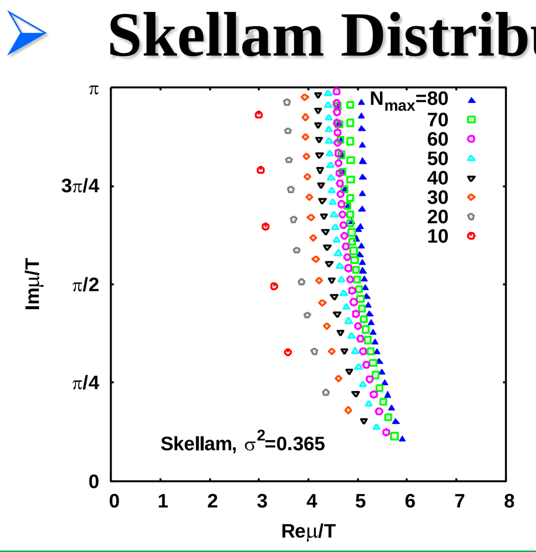
8^3x4, m_{PS}/m_V=0.8

See Nagata et al., Phys.Rev.D91, 094507 ('15) for more detailed calculations.

Observation : Splitting of distribution of the zero and the stable edge closest to the real μ axis with respect to cutting $N_{\max}$ are observed irrespective of temperature and order of the phase transition. This indicates the information on the phase transition is insensitive to $Z(N)$ at very large N, but it is lost at some point ($N_{\max} < 21$ in m=0.05, T=T_c).

Discussion

- Skellam Distribution $Z(N) = I_N(\sigma^2)$, $\mathcal{Z}(\mu) = \exp[\sigma^2\cosh(\mu/T)]$



No singularity in the thermodynamic limit, artificial zeros by truncation.

Different $N_{\max}$ dependence – No stable edge →distinguishable from true singularities

Similar to the RM model with $N_{\max} < 21$.

- How large $N_{\max}$ is needed to see Yang-Lee zeros?
  - N_s=60 RM : $N_{\max}$=6 for χ_6 — 21 for the closest zero
  - 100 : $N_{\max}$=7 for χ_6 — 30 for the closest zero
  - Highly statistic demanding
  - Challenge for experiments and lattice QCD