Mott-Hagedorn resonance gas and lattice QCD thermodynamics

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Introduction

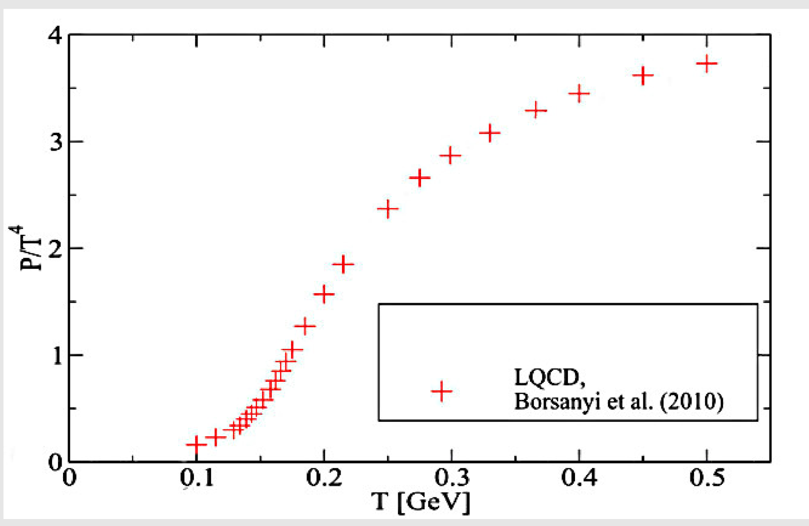
We are investigating

- The hadronic resonance gas at low temperatures
- The hadron dissociation with increasing temperature
- The quark-gluon plasma at high temperatures

Using

- The Mott effect to describe the hadron dissociation
- The Polyakov-loop improved Nambu, Jona-Lasinio (PNJL) model to describe the quark and gluon degrees of freedom

Our objectives



S. Borsanyi *et al.* *"The QCD equation of state with dynamical quarks,"* JHEP **1011**, 077 (2010)

- An effective model description of QCD thermodynamics at finite temperatures which properly accounts for the fact that in the QCD transition region it is dominated by a tower of hadronic resonances
- A generalization of the resonance gas thermodynamics. which includes the finite lifetime of hadronic resonances in a hot and dense medium.
- Join hadron resonance gas with quark-gluon model.

The model

Quark and gluon thermodynamics

The temperature dependencies of the light quark masses $m(T)$ and $m_s(T)$ and of the traced Polyakov loop $\Phi(T)$ are solutions of the coupled gap equations in the generalized mean field approximation. Effects of hadronic resonances are taken here consistently into account.

Mott-Hagedorn hadron resonance gas thermodynamics

- *Hagedorn gas:* clusters may interact with each other and their formation itself shall modify the properties of their constituents.
- *Mott gas:* hadronic states become unbound and their contribution to the thermodynamics as captured in the corresponding phase shift functions
- the improved quasiparticle is described by the temperature dependent mass gap and the contribution of the hadronic correlations has the generalized Beth-Uhlenbeck form

$$P_i(T) = d_i \int \frac{d^3p}{(2\pi)^3} \int_0^\infty \frac{d\omega}{\pi} f_i(\omega)[\delta_i(\omega;T) - \sin\delta_i(\omega;T)\cos\delta_i(\omega;T)] \;;$$

Quarks-hadrons selfconsistency

- Hadron masses become temperature dependent above their Mott temperature, where they hit the two-quark or three thresholds $m_{\rm thr,M}(T)$, respectively. Then, their mass (peak position of the resonance) rises with temperature in the same way as the resonance width $\Gamma_i(T)$, unique for all hadrons

$$M_i(T) = M_i(0) + \Gamma_i(T) \;,$$
$$\Gamma_i(T) = \Theta(T - T_{\rm Mott,i})[a\,(T - T_{\rm Mott,i}) + a^2(T - T_{\rm Mott,i})^2]^{1/2} .$$

- The Mott temperatures $T_{\rm Mott,i}$ is determined from the condition

$$M_i(T_{\rm Mott,i}) = m_{\rm thr,i}(T_{\rm Mott,i})$$

- The temperature dependent continuum thresholds for the two- and three-quark states are determined by the temperature dependence of the quark masses

$$m_{\rm thr,M}(T) = (2 - N_s)m(T) + N_s m_s(T) \;\; ; \quad m_{\rm thr,B}(T) = (3 - N_s)m(T) + N_s m_s(T)$$

$N_s = 0, 1, 2$ for mesons ($i = M$) and $N_s = 0, ..., 3$ for baryons ($i = B$).

- The temperature dependence of the quark masses is obtained using Lattice QCD data for the behavior of the continuum extrapolated chiral condensate $\Delta_{l,s}(T)$. For $m(T), m_s(T)$ - light and strange quark masses respectively

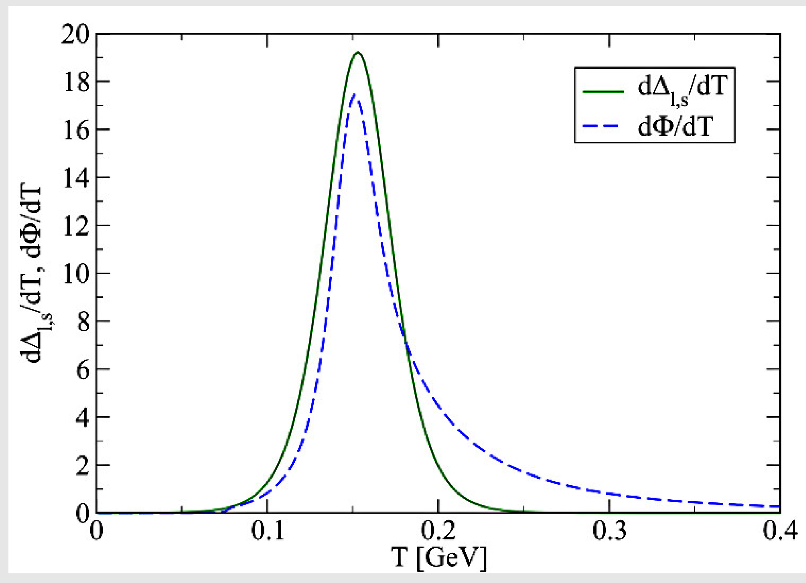
$$m(T) = [m(0) - m_0]\Delta_{l,s}(T) + m_0 \;; \quad m_s(T) = m(T) + m_s - m_0 \,.$$

with $m_s = 100$ MeV.

- The Lattice QCD result for the temperature dependence of the chiral condensate is fitted by

$$\Delta_{l,s}(T) = 0.5 - \tanh\left(\frac{T - T_c}{\delta_T}\right) \;,$$

Corollary



Temperature dependence of the chiral susceptibility (solid line) and the Polyakov-loop susceptibility (dashed line) with almost coincident peak positions at $T_\chi = 153$ MeV and $T_\Phi = 150$ MeV, respectively, defined by the temperature derivatives of the chiral condensate $\Delta_{l,s}(T)$ taken from Lattice QCD data and the traced Polyakov loop Φ obtained here as solution of a PNJL model gap equation.

Mott effect. Temperature dependent hadron phase shifts

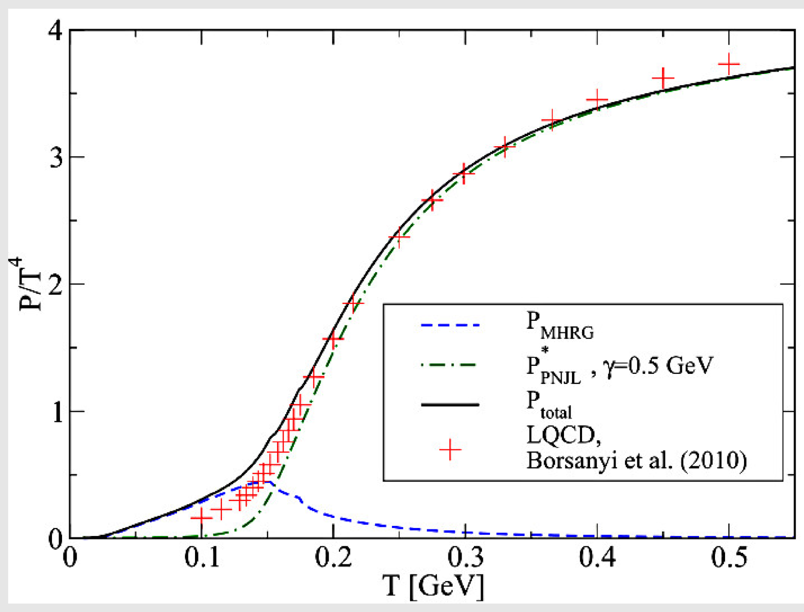
Quarks originating from scattering off hadrons are taken into account by a width parameter γ in the modified Fermi-gas component of the PNJL pressure

$$P^*_{\rm FG}(T) = 4N_c \sum_{q=u,d,s} \int \frac{dp\, p^2}{2\pi^2} \int \frac{d\omega}{\pi} f_\Phi(\omega)\delta_q(\omega;\gamma),$$

The function

$$\delta_q(\omega;\gamma) = \frac{\pi}{2} + \arctan\left[\frac{\omega - \sqrt{p^2 + m_q^2}}{\gamma}\right]$$

plays the role of a quark phase shift due to the scattering off hadrons and the parameter γ stands for the collisional broadening.



Temperature dependence of the total pressure of the present model (solid line) compared to the lattice QCD results (plusses). For comparison, the hadron (MHRG) and quark-gluon (PNJL*) components are shown by dashed and dash-dotted lines, respectively.

Observations

- The lowering of the thresholds for the two- and three-quark scattering state continuous spectrum triggers the transformation of hadronic bound states to resonances in the scattering continuum.
- The lowering of the quark masses in the chiral restoration transition which itself is a result of the behavior of the chiral condensate
- The present schematic model can reproduces basic results of the LQCD in the temperature region from hadron dominance to quark-gluon dominance at $T_{\rm trans} = 156$ MeV. This falls in the range of the pseudocritical transition temperature found LQCD simulations

Conclusions

- An effective model is constructed which is capable of reproducing basic physical characteristics of the hadron resonance gas at low temperatures and embody the crucial effect of hadron dissociation by the Mott effect
- Results are in qualitative agreement with recent ones from the lattice QCD simulations
- The generalized Beth-Uhlenbeck form of the partial pressures is constructed for each hadronic channel. Numerical results show that the simplifying ansatz for the temperature dependence of both, the mass spectrum and the phase shifts of hadronic channels give results in quantitative agreement with recent ones from LQCD
- Presented calculational scheme has been inspired by the $\Phi-$derivable approach of Baym and Kadanoff.
- The presented scheme proves essential to overcome discrepancies in the quantitative modeling of Lattice QCD thermodynamics that existed in a previous version of this model: D. Blaschke, A. Dubinin and L. Turko, *"Mott-hadron resonance gas and lattice QCD thermodynamics,"* Phys. Part. Nucl. **46**, 732 (2015) [arXiv:1501.00485 [hep-ph]]

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