**Motivation**

- Relativistic extension of the Navier-Stokes equation is acausal and unstable
- Second-order hydrodynamics
- QGP is composed of quarks, anti-quarks, and gluons
- Reactive multi-component hydrodynamics

We want to derive the second-order hydrodynamics for multi-component systems by faithfully solving the Boltzmann equation.

**Derivation**

Boltzmann equation

\[ p^k_i \partial_k f_{k,p}(x) = \sum_{\ell=1}^{N} C_{\ell i} f_{j,p}(x) \]

A charge of k-th component associated with A-th current

\[ q^A_i + q^A_j = q^A_i + q^A_j \]

EM tensor

\[ T^{\mu\nu} = \sum_{k=1}^{N} \int dp_i p^k_i p^k_j f_{k,p} \]

Non-diagonal components represent cross-correlation

**Hydrodynamic equation**

\[ \partial_\mu T^{\mu\nu} = 0 \]

\[ T^{\mu\nu} = \varepsilon^{\mu\nu} u^\nu - (P - \Pi) \Delta^{\mu\nu} + \pi^{\mu\nu} \]

\[ \partial_\mu N^A_\mu = 0 \]

\[ N^A_\mu = n_A u^\mu + J^A_\mu \]

**Discussion**

In the Boltzmann theory

\[ s^A = -\sum_{k=1}^{N} \int dp_i p^k_i f_{k,p} \ln f_{k,p} - (1/\alpha_A)(1 + a_k f_{k,p}) \ln (1 + a_k f_{k,p}) \]

**Entropy production rate**

\[ \partial_\mu s^A = \frac{1}{T^2} \Pi^2 + \frac{1}{2 T^2} \pi^{\mu\nu} \pi_{\mu\nu} - \frac{\lambda^2}{T} \sum_{A,B=1}^{M} (\lambda^{-1})_{AB} P^A_\mu J^B_\mu + \cdots \]

Positive definiteness of entropy production rate

**Onsager’s reciprocal relation**

Onsager’s reciprocal relation

\[ \partial_\mu s^A = \sum_i J_i X_i \]

\[ J_i = \sum_j \gamma_{ij} X_j \]

in the linear approximation

\[ \gamma_{ij} = \gamma_{ji} \]

reflecting time-reversal symmetry of scattering process

\[ J_i^A = \lambda_{11} \frac{T^2}{h^2} \nabla \mu_{11}^A + \lambda_{12} \frac{T^2}{h^2} \nabla \mu_{12}^A + \cdots \]

\[ J_i^B = \lambda_{21} \frac{T^2}{h^2} \nabla \mu_{21}^B + \lambda_{22} \frac{T^2}{h^2} \nabla \mu_{22}^B + \cdots \]

Summary and Outlook

- Relativistic second-order hydrodynamic equation for reactive multi-component systems has been derived.
- Microscopic expressions for all the transport coefficients have been obtained.
- Positive definiteness of the entropy production rate and the Onsager’s reciprocal relation are readily satisfied → suggest the validity of the present method!!