Chiral Hall Effect and Chiral Electric Waves in Strongly Coupled Plasmas

Shi Pu*, Shang-Yu Wu**, Di-Lun Yang***

*Institute for Theoretical Physics, Goethe University, Max-von-Laue-Str. 1, 60438 Frankfurt am Main, Germany
**Department of Electrophysics, Yau Shing Tung Center, National Chiao Tung University, Hsinchu 300, Taiwan.
***Theoretical Research Division, Nishina Center, RIKEN, Wako 351-0198, Japan.
Crete Center for Theoretical Physics, Department of Physics University of Crete, 71003 Heraklion, Greece.

Abstract

The electromagnetic-field-induced transport related to the axial chemical potential may play an important role in many chiral systems such as the quark gluon plasma (QGP) created in relativistic heavy ion collisions. It has been found that the presence of both vector and axial chemical potentials could induce an axial current parallel to the applied electric field known as the chiral electric separation effect (CESE), where the axial conductivity is proportional to the product of the small vector and axial chemical potentials in QED and weakly coupled QCD plasmas. By implementing the gauge/gravity duality, we qualitatively obtain the same relation in the strongly coupled scenario. On the other hand, we find that an axial Hall current can also be generated when introducing an electric field and a magnetic field perpendicular to each other with an axial chemical potential, which could be dubbed as the chiral Hall effect (CHE). The fluctuations of chemical potentials will further result in chiral electric waves (CEW) as propagating density waves led by the applied electromagnetic fields. Interestingly, the Hall density waves propagating perpendicular to both applied fields may exist even at zero chemical potentials and become non-dissipative. We investigate the transport coefficients including the Hall conductivity, damping times, wave velocities, and diffusion constants of CEW in a strongly coupled plasma via the AdS/CFT correspondence. We argue that the CHE could lead to nontrivial charge distributions at different rapidity in asymmetric heavy ion collisions. The presentation is based on [1,2].

Chiral Electric Separation Effect in Strong Coupling

In a relativistic system with chiral fermions, a magnetic field will lead to the so-called chiral magnetic effect (CME) in the presence of an axial chemical potential, which generates an electric current along the magnetic field [3,4]. Similarly, the presence of a vector chemical potential will give rise to an axial current parallel to the magnetic field, known as the chiral separation effect (CSE) [4]. Recently, it has been found that the electric field results in an analogous scenario in the chiral system, where the presence of both a vector and axial chemical potentials causes an axial current parallel to the electric field, which is known as the chiral electric separation effect (CESE) [5]. However, unlike the CME and CSE, the CESE does not stem from the axial anomaly.

$$\mathbf{J_v} = \sigma_v\mathbf{E} + \lambda\mu_\mathbf{A}\mathbf{B}, \qquad \mathbf{J_a} = \sigma_a\mathbf{E} + \lambda\mu_\mathbf{V}\mathbf{B}.$$

CESE : $\mathbf{J_a} = \sigma_a\mathbf{E} = \chi_a\mu_V\mu_A\mathbf{E}$, $\mathbf{J_V} = \left(\sigma_0 + \sigma_2(\mu_V^2+\mu_A^2)\right)\mathbf{E}$ for $\mu_{V/A} \ll T$

- Interpretation of Chiral Electric Conductivity :

In a chiral system in the presence of an external electric field E, the right and left handed (R/L) fermions will be dragged by the electric force and two charge currents will be induced. The linear combination of the R/L currents leads to the vector and axial currents.

currents : $\mathbf{J}_v = \frac{1}{2}(\mathbf{J}_R + \mathbf{J}_L) = \sigma_v e\mathbf{E}$, $\mathbf{J}_a = \frac{1}{2}(\mathbf{J}_R - \mathbf{J}_L) = \sigma_a e\mathbf{E}$

conductivity : $\sigma_v = \frac{1}{2}(\sigma_R + \sigma_L)$, $\sigma_a = \frac{1}{2}(\sigma_R - \sigma_L)$

chemical potentials : $\mu_{R/L} = \mu_V \pm \mu_A$

Considering the chiral symmetry $(R \leftrightarrow L)$ and charge conjugation $(e \to -e,\ \mu_{R/L} \to -\mu_{R/L})$ symmetry, up to the leading-order correction of small chemical potentials, we find

$$\sigma_{R/L} = \sigma_0 + c\mu_R^2 + c\mu_L^2 \Longrightarrow \sigma_a = \chi_a\mu_V\mu_A \qquad \sigma_v = \sigma_0 + \chi_v(\mu_V^2 + \mu_A^2)$$

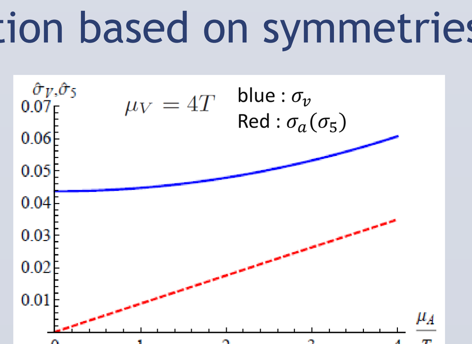
Such relations have been found in QED and weakly coupled (perturbative) QCD [5,6] .

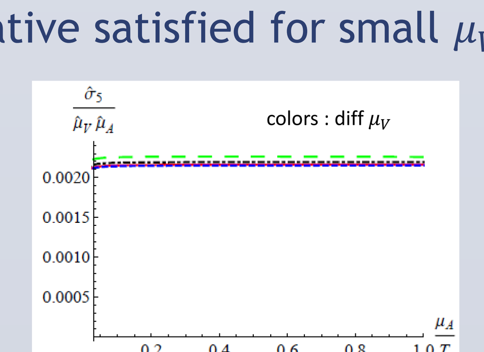
QED: $\sigma_a = \frac{T}{e^3\ln(1/e)}\left(20.499\frac{\mu_V\mu_A}{T^2}\right)$ $\quad \sigma_v = \frac{T}{e^3\ln(1/e)}\left(15.6952 + 7.76052\frac{\mu_V^2+\mu_A^2}{T^2}\right)$

pQCD: $\sigma_a^{2f} = T\frac{\mathrm{Tr}_f Q_e Q_A}{g^4\ln(1/g)}14.5163\frac{\mu_A\mu_V}{T^2}$ $\quad \sigma_v^{2f} = T\frac{\mathrm{Tr}_f Q_e Q_V}{g^4\ln(1/g)}\left(12.9989 + 6.50554\frac{\mu_V^2+\mu_A^2}{T^2}\right)$

We study the CESE in the Sakai-Sugimoto (SS) model at finite temperature [7,8], which corresponds to a non-conformal gauge theory to mimic the QGP at strong coupling. We employ two approaches in holography to compute the conductivity. One stems from the linear response theory, which could be applied to the case with an oscillatory E field in time. Another approach encodes the non-linear effect under strong but constant external fields. Both approaches show that the relations between the conductivity and $\mu_{V/A}$ from the naïve derivation based on symmetries are qualitative satisfied for small $\mu_{V/A}$ and weak E fields.

SS model (sQCD) with oscillatory E :





From the result for an oscillatory E field, we find that the axial current is approximately proportional to $\mu_V\mu_A$. For the case with an external (weak) constant E field, we obtain the analytic result.

SS model (sQCD) with an external E :

$\boldsymbol{E} = E_y$: $(J_v^b)_y = CE_y a^2T^2L^{9/2}\left(1 + \frac{9}{8(a^2T^2L^3)^2}(\mu_V^2+\mu_A^2)\right)$, $\quad (J_a^b)_y = \frac{9CE_y}{4a^2T^2L^{3/2}}\mu_V\mu_A$

$a = 4\pi/3 \quad L^3 = (2M_{KK})^{-1}(g_{YM}^2 N_c l_s^2)$, $\quad C = \frac{T_{D8}V_4L^{3/2}}{g_s} = \frac{N_c}{96\pi^5 l_s^6 L^{3/2}}$

In the SS model, the AdS radius L is associated with the Kalzula Klein (KK) mass M_{KK} related to the mesonic scale of the theory and the typical string length l_s . In the numerical computations, the numerical values of relevant coefficients are

$$2\pi l_s^2 = 1\mathrm{GeV}^{-2},\quad \lambda_t = g_{YM}^2N_c = 17,\quad M_{KK} = 0.94\mathrm{GeV},\quad T = 200\mathrm{MeV} = 0.2\mathrm{GeV},\quad N_c = 3,\quad C = 0.0211\ \mathrm{GeV}^{-15/2}.$$

The values are chosen to fit the ρ −meson mass and pion decay constant. Also, T here corresponds to the average temperature of RHIC. In summary, the leading-order relations between the induced currents and chemical potentials in CESE hold for both weak and strong couplings. Surprisingly, they may approximately hold even for $\mu_{V/A} \approx T$.

Chiral Hall Effect

The presence of an electric field and a magnetic field perpendicular to each other yields a Hall current perpendicular to both fields. Analogously, we find an axial Hall current is induced in the same condition with an axial chemical potential.

normal Hall conductivity : $(\sigma_v)_{zy} = -(\sigma_v)_{yz} = \frac{1}{2}(\sigma_R + \sigma_L)_{zy}$ $\quad$ chiral Hall conductivity : $(\sigma_a)_{zy} = -(\sigma_a)_{yz} = \frac{1}{2}(\sigma_R - \sigma_L)_{zy}$

When $\mu_A \neq 0$, where the R/L charge densities are different, the (Hall) conductivity of the R/L sectors are also different, which thus results in the nonzero chiral Hall conductivity. Since the Hall conductivity is linear to the charge density, it is expected that the chiral Hall conductivity is linear to the (small) μ_A.

$\mathbf{E} = E\hat{y},\ \mathbf{B} = B\hat{x} \Longrightarrow j_i = \sigma_{ij}eE_j$, weak B : $\sigma_{zy} = -n\tau_H^2 eB$ $\quad$ strong B : $\sigma_{zy} = -\frac{n}{eB}$ $\Longrightarrow (\sigma_a)_{zy} \propto \mu_A$

We investigate the CHE in the SS model and find the expected relation. $\boxed{(J_{v/a}^b)_z = -\frac{3CB_xE_y}{a^3T^3L^{3/2}}\mu_{V/A}}$

Moreover, the nonlinear effects are observed for strong E and B fields in the SS model.

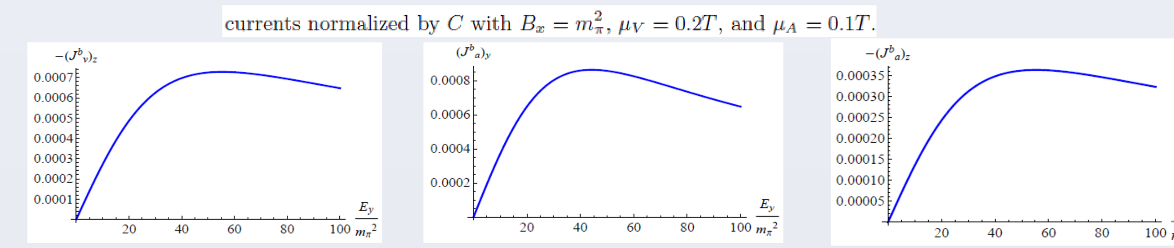
strong E & finite B : $(J_v^b)_y \approx CL^{5/2}E_y^{5/2}$, $\quad (J_a^b)_y \approx \frac{(J_v^b)_t(J_a^b)_t}{CL^{5/2}E_y^{8/3}}$, $\quad (J_{v/a}^b)_z \approx -\frac{B_x(J_{v/a}^b)_t}{E_y}$.

strong B & finite E : $(J_v^b)_y \approx C\frac{U_T^{5/2}E_y}{B_x}$, $\quad (J_a^b)_y \approx \frac{E_y(J_v^b)_t(J_a^b)_t}{CL^3B_x^3}$, $\quad (J_{v/a}^b)_z \approx -\frac{E_y(J_{v/a}^b)_t}{B_x}$.

charge density : $(J_{L/R})_t = \frac{3}{2}U_T^{3/2}\mu_{L/R}$, $\quad T = \frac{3}{4\pi}\left(\frac{U_T}{L^3}\right)^{1/2}$

Numerical results :

currents normalized by C with $B_x = m_\pi^2$, $\mu_V = 0.2T$, and $\mu_A = 0.1T$.



Note: The nonlinear effects become substantial only at a very strong $E \sim 20m_\pi^2$. Similar scenarios are found for a strong B and a finite E.

Chiral Electric Waves

The fluctuations of $\mu_{V/A}$ and the combination of the CME and CSE give rise to propagating density waves known as the chiral magnetic waves (CMW) [9]. In the presence of E and B fields, the CESE and CHE will further results in the chiral electric waves (CEW). We start from the R/L basis and introduce the density $(\mu_{R/L})$ fluctuations. By applying the conservation equation and convert the R/L basis to the V/A basis, we then obtain the dispersion relation of the CEW.

$$\mathbf{j_R} = e\sigma_R(\mu_R)\mathbf{E} = e\sigma_R(j_R^0)\mathbf{E},\quad \mathbf{j_L} = e\sigma_L(\mu_L)\mathbf{E} = e\sigma_L(j_L^0)\mathbf{E}. \xrightarrow{\partial_\mu j^\mu = 0} \partial_0\delta j^0_{R/L} + e\beta_{R/L}\mathbf{E}\cdot\nabla\delta j^0_{R/L} + e\beta_{R/L}n_v\delta j^0_{R/L} + e\sigma_{R/L}\delta j^0_v = 0.$$

$j^0_{R/L} \to n_{R/L} + \delta j^0_{R/L}$, $\quad \delta\mathbf{j_{R/L}} = e\beta_{R/L}\delta j^0_{R/L}\mathbf{E}$, $\quad \boxed{\beta_{R/L} = \left(\frac{\partial\sigma_{R/L}}{\partial j^0_{R/L}}\right)_{j^0_{R/L}\to n_{R/L}}}$

(CEW from only E fields and CESE)

Change to the V/A basis and take the oscillatory form :

$$\delta j^0_{v/a} = C_{v/a}e^{-iwt+i\mathbf{k}\cdot\mathbf{x}} \xrightarrow{\text{Hydro approx.}} \omega_\pm = -ie\left(n_v\beta_+ + \frac{\sigma_v}{2} \pm \sqrt{n_v^2\beta_-^2 + n_v\beta_-\sigma_a + \frac{\sigma_v^2}{4}}\right) + e\left(\beta_+ \pm \frac{\beta_-(2n_v\beta_- + \sigma_a)}{\sqrt{4n_v^2\beta_-^2 + 4n_v\beta_-\sigma_a + \sigma_v^2}}\right)\mathbf{E}\cdot\mathbf{k} \pm \frac{ie\beta_-^2(\sigma_v^2 - \sigma_a^2)(\mathbf{E}\cdot\mathbf{k})^2}{(4n_v^2\beta_-^2 + 4n_v\beta_-\sigma_a + \sigma_v^2)^{3/2}} + \mathcal{O}\left((\mathbf{E}\cdot\mathbf{k})^3\right).$$

damping coefficient

wave velocity

diffusion constant

$\alpha_{R/L} = \left(\frac{\partial\mu_{R/L}}{\partial j^0_{R/L}}\right)_{j^0_{R/L}\to n_{R/L}}$, $\quad \alpha_\pm = \frac{1}{2}(\alpha_R \pm \alpha_L)$, $\quad \beta_\pm = \frac{1}{2}(\beta_R \pm \beta_L)$

$(n_v = 0,\ \sigma_{v/a} = 0) \longrightarrow$

CMW : $\omega_\pm = \lambda(\mathbf{B}\cdot\mathbf{k})(\alpha_- \mp \alpha_+) = -\lambda(\mathbf{B}\cdot\mathbf{k})\alpha_L$ or $\lambda(\mathbf{B}\cdot\mathbf{k})\alpha_R$

CEW : $\omega_\pm = e(\mathbf{E}\cdot\mathbf{k})(\beta_+ \mp \beta_-) = -e(\mathbf{E}\cdot\mathbf{k})\beta_L$ or $e(\mathbf{E}\cdot\mathbf{k})\beta_R$

Analogous to CMW, CEW also have two modes and become non-dissipative in the chargeless case with zero conductivity. Considering the CHE, we further find the Hall electric waves propagating along the direction perpendicular to both E and B fields. $\omega_\pm = -i\tau_\pm^{-1} + (v_\pm)_ik^i - i(D_\pm)_{ij}k^ik^j$,

CEW in the SS model :

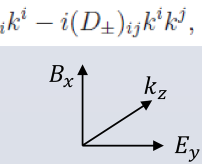
$(\beta_{L/R})_{yy} = \frac{L^{3/2}U_T^{3/2}}{B_x^2L^3 + U_T^3}\frac{(J_{L/R})_t}{\sqrt{(J_{L/R})_t^2 + B_x^2L^3U_T^3 + U_T^5}}$, $\quad (\beta_{L/R})_{zy} = \frac{-B_xL^3}{B_x^2L^3 + U_T^3}$

(here we ignore the axial anomaly & CMW)

damping coefficient: $\tau_\pm^{-1} = \left(n_v(\beta_+)_{yy} + \frac{(\sigma_v)_{yy}}{2} \pm \sqrt{n_v^2(\beta_-)_{yy}^2 + n_v(\beta_-)_{yy}(\sigma_a)_{yy} + \frac{(\sigma_v)_{yy}^2}{4}}\right)$

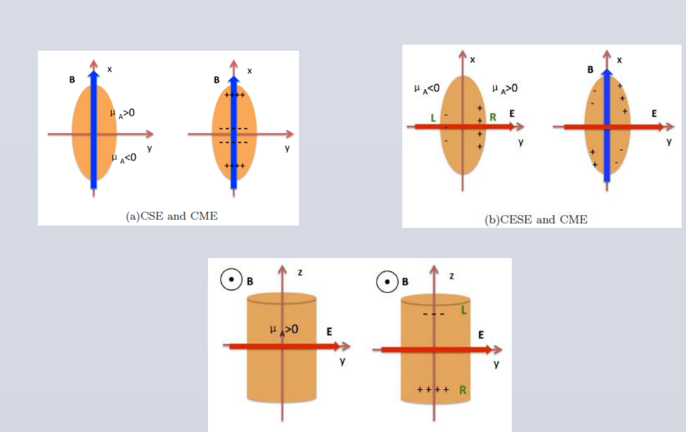
wave velocities: $(v_\pm)_y = \left((\beta_+)_{yy} \pm \frac{(\beta_-)_{yy}(2n_v(\beta_-)_{yy} + (\sigma_a)_{yy})}{\sqrt{4n_v^2(\beta_-)_{yy}^2 + 4n_v(\beta_-)_{yy}(\sigma_a)_{yy} + (\sigma_v)_{yy}^2}}\right)E_y$, $\quad (v_\pm)_z = (\beta_+)_{zy}E_y$

diffusion constants: $(D_\pm)_{yy} = \mp\frac{(\beta_-)_{yy}^2\left((\sigma_v)_{yy}^2 - (\sigma_a)_{yy}^2\right)(E_y^2)}{\left(4n_v^2(\beta_-)_{yy}^2 + 4n_v(\beta_-)_{yy}(\sigma_a)_{yy} + (\sigma_v)_{yy}^2\right)^{3/2}}$, $\quad (D_\pm)_{zz} = (D_\pm)_{zy} = (D_\pm)_{yz} = 0$



When $n_{v/a} = 0((\beta_\pm)_{yy} = 0)$, the "−" mode of the Hall electric waves (HEW) become non-dissipative. Similar to CMW, HEW could exist even when both the vector and axial charge densities vanish. Note that CEW will vanish in such a case. In phenomenology, CESE and CHE (CEW and HEW) may play a role in asymmetric collisions such as Cu+Au collisions, where both strong E and B fields perpendicular to each other are produced in early times.

	Currents	Possible phenomena
Chiral Magnetic Effect	$\mathbf{j}_v = \frac{e}{2\pi^2}\mu_A\mathbf{B}$,	charge separation along $\mathbf{B}$ field
Chiral Separation Effect	$\mathbf{j}_a = \frac{e}{2\pi^2}\mu_V\mathbf{B}$,	chirality separation along $\mathbf{B}$ field
Chiral Electric Separation Effect	$\mathbf{j}_a = \sigma_a\mathbf{E}$,	charge and chirality separation along $\mathbf{E}$ field
Chiral Hall Effect	$j_{v,z} = (\sigma_v)_{zy}E_y$, $j_{a,z} = (\sigma_a)_{zy}E_y$,	charge and chirality separation in rapidity direction
Chiral Magnetic Wave	Evolution equations for currents with CME, CSE	density wave induced by magnetic field and charge separation along $\mathbf{B}$ field
Chiral Electric Wave	Evolution equations for currents with CESE, CHE	density wave induced by electric field, charge separation along $\mathbf{E}$ field and rapidity direction



References

[1] S. Pu, S.-Y.Wu, and D.-L. Yang, Phys.Rev. D89, 085024 (2014), 1401.6972.
[2] S. Pu, S.-Y.Wu, and D.-L. Yang, Phys.Rev. D91, 025011 (2015), 1407.3168.
[3] D. Kharzeev and A. Zhitnitsky, Nucl.Phys. A797, 67 (2007), 0706.1026.
[4] D. E. Kharzeev, L. D. McLerran, and H. J. Warringa, Nucl.Phys. A803, 227 (2008), 0711.0950.
[5] X.-G. Huang and J. Liao, Phys.Rev.Lett. 110, 232302 (2013), 1303.7192.
[6] Y. Jiang, X. G. Huang, and J. Liao, Phys. Rev. D 91, 045001(2015).
[7] T. Sakai and S. Sugimoto, Prog.Theor.Phys. 113, 843 (2005), hep-th/0412141.
[8] T. Sakai and S. Sugimoto, Prog.Theor.Phys. 114, 1083 (2005), hep-th/0507073.
[9] D. E. Kharzeev and H.-U. Yee, Phys.Rev. D83, 085007 (2011), 1012.6026.