Chiral Drag Force

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based on arXiv:1505.07379, by Krishna Rajagopal and AS

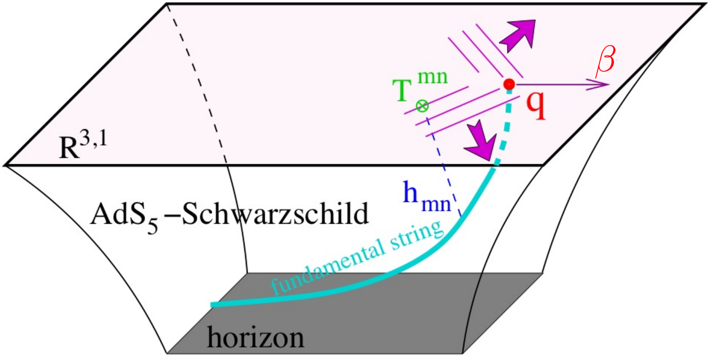
Dragging a Heavy Quark

- As it was argued a long time ago[1] to probe the holographic strongly coupled thermal plasma one could calculate a drag force for a heavy quark, $M \to \infty$, considering a probe string, described by NG action
- To drag a heavy quark with constant velocity $\vec{\beta}$ through the **static, homogeneous, equilibrium, $\mu = 0$** plasma of $\mathcal{N} = 4$ SYM theory at temperature T one has to exert a force:

$$\vec{f} = -\frac{\sqrt{\lambda}}{2\pi}\pi^2 T^2 \vec{\beta} \propto \frac{\vec{p}}{M}$$

  with $\lambda \equiv g^2 N_c$.
- However the fluid of interest is **not static, not homogeneous, not at equilibrium and it may have non-zero chemical potentials (including the axial one)**

[1] L. Yaffe et al, JHEP 0607, 013 (2006); S. Gubser, Phys.Rev.D 74, 126005(2006)



T mn
β
q
R3,1
h mn
AdS5 –Schwarzschild
fundamental string
horizon

Modifications

- To proceed further one can consider **hydrodynamic perturbations of the plasma** and also include **non-zero chemical potential**.
- The perturbations of the fluid correspond on the holographic side to perturbations of the metric and gauge field in the bulk. The solution to the perturbed Einstein-Maxwell equations may be found **order by order in fluid gradient expansion** (as in usual hydro)[2].
- The finite chemical potential (could be considered as a chiral one, say for right quarks) corresponds to **the replacement of the bulk BH by a charged one** on the holographic side. It also should be mentioned that at the finite chemical potential, generally, the gradient corrections are modified by **the bulk CS term which corresponds to the chiral anomaly** on the field theoretical side.

[2] J. Erdmenger et al, JHEP 0901, 055; N. Banerjee et al, JHEP 1101, 094

Effects of Fluid Gradients on Drag

- Some notations:
  Fluid four-velocity is u^μ;
  Heavy quark four-velocity is $w^\mu = \gamma(1, \vec{\beta})$;
  The only zero order Lorentz scalar is $s \equiv u^\mu w_\mu$;
- **The drag force as a double expansion in powers of gradients and μ/T at zero CS coupling**:

$$f^\mu = \left(f^\mu_{(0,0)} + f^\mu_{(0,2)} + \ldots\right) + \left(f^\mu_{1,0} + f^\mu_{(1,2)} + \ldots\right) + \ldots$$

- $f^\mu_{(0)}$ is known[3] and its expansion in powers of μ/T has form

$$f^\mu_{(0,0)} + f^\mu_{(0,2)} = -\frac{\sqrt{\lambda}}{2\pi}\frac{\pi^2 T^2}{\gamma}\left(s w^\mu + u^\mu\right)\left(1 + \frac{1+3s}{6s}\frac{\mu^2}{(\pi T)^2}\right).$$

- One can obtain the analytic answer for $f^\mu_{(1)}$...

[3] C.P. Herzog, JHEP 0609, 032

The first order drag force correction in gradients at $\mu = 0$ has form[4]:

$$f_\mu^{(1,0)} = -\frac{\sqrt{\lambda}}{2\pi}\frac{\pi T}{\gamma}\big(c_1(s)(u_\mu(w\cdot\partial)s - s\partial_\mu s - s(su_\alpha + w_\alpha)\partial^\alpha U_\mu)$$
$$+c_2(s)U_\mu(\partial\cdot u) - \sqrt{-s}(u\cdot\partial)U_\mu\big),$$

where $U^\mu \equiv u^\mu + sw^\mu$, while the contribution of order $(\mu/T)^2$ is[5]

$$f_\mu^{(1,2)} = \frac{\mu^2\sqrt{\lambda}}{48\gamma\pi^2 T}\left(\frac{2c_5(s)}{s}\left((w\partial)\log\frac{\mu}{T}\right)u_\mu + c_3(s)\left(u_\mu(w\partial)s + s\partial_\mu s\right)\right)$$
$$-\frac{\mu^2\sqrt{\lambda}}{48\gamma\pi^2 T}U_\mu\left(c_6(s)(w\partial)\log\frac{\mu}{T} - c_4(s)(\partial u) + c_7(s)(su^\alpha + w^\alpha)\partial_\alpha s + c_{10}(s)(w\partial)s\right)$$
$$-\frac{\mu^2\sqrt{\lambda}}{48\gamma\pi^2 T}\left(c_8(s)(u\partial)U_\mu + c_9(s)(w\partial)U_\mu + 2c_5(s)\partial_\mu\log\frac{\mu}{T}\right).$$

Thus we have an explicit analytic answer and it fits on one slide, if I hide the explicit expressions for $c_i(s)$ functions **;)**

However this is **not the most general answer yet** and we may **turn on the CS coupling**.

[4] M. Lekaveckas, K. Rajagopal, 1311.5577

[5] K. Rajagopal, AS, 1505.07379

Chiral Effects

- **New contributions into the current in the presence of the chiral anomaly**[6]:

$$J_\mu = \xi B_\mu + \xi_\omega \omega_\mu,$$

where $\xi,\ \xi_B$ are **fixed by the chiral anomaly coefficient**. (Note: these contributions also have **counterparts in the entropy and momentum currents**).

- The current is proportional to axial vectors and by symmetries **one expects it to be non-dissipative in equilibrium**.
- On the holographic side there is **anomalous contribution into the bulk metric proportional to the CS coupling κ**.
- The parity could be restored at the end. Introduce two chiralities with μ_R and μ_L and with opposite sign in front of the anomaly $\kappa_R = -\kappa_L$.

[6] J. Erdmenger et al, JHEP 0901, 055; D. T. Son, P. Surowka, PRL 103, 191601

Chiral Drag Force (K. Rajagopal, AS, 1505.07379)

By direct calculation **drag force gains contributions of the anomalous nature: Chiral Vortical Drag Force and Chiral Magnetic Drag Force**, to the lowest nonzero order in μ, for a heavy quark moving through a thermal strongly interacting plasma they are

$$f_\mu^B = -\frac{\sqrt{\lambda}}{2\pi^3\gamma}\left(\frac{\mu}{T}\right)^2 s^2\kappa\, C_B(\pi T\sqrt{-s})\,(B_\mu + (B\cdot w)w_\mu)$$

$$f_\mu^\ell = \frac{\kappa\sqrt{\lambda}\mu^3}{3\gamma\pi^3 T^2}\frac{\ell_\mu + (\ell\cdot w)w_\mu}{s},$$

where μ^2 could be replaced by $\mu_R^2 - \mu_L^2 = 4\mu_A\mu_V$ and μ^3 by $\mu_R^3 - \mu_L^3 = 6\mu_A\mu_V^2 + 2\mu_A^3$, $\ell_\mu \equiv \epsilon_{\mu\nu\alpha\beta}u^\nu\partial^\alpha u^\beta$, $B_\mu \equiv \epsilon_{\mu\nu\alpha\beta}u^\nu\partial^\alpha A^\beta$ and $C_B(r)$ is a known function. **Thus every heavy quark or antiquark (no matter the charge) feels the same force!**

Chiral Effects and Dissipation

- The chiral drag force is **nonzero for a quark at rest** (T^{0i} vanishes in the rest frame):

$$\vec{f} = -\frac{\kappa\sqrt{\lambda}}{2\pi^3}\frac{\mu^2}{T^2}\vec{B} + \text{vortical term.}$$

  Every quark and antiquark feels the same force.
- Comparing with the usual drag force $\vec{f} = \frac{\sqrt{\lambda}}{2\pi}\pi^2 T^2 \vec{v}$ we conclude that there is **a terminal velocity**[7]:

$$\boldsymbol{\vec{v}_{terminal} = \kappa\frac{\mu^2\vec{B}}{(\pi T)^4} + \text{vortical term.}}$$

- Thus in the local fluid rest frame, a heavy quark at rest feels a force while **a quark moving with a terminal velocity feels no force**[7](later confirmed by hydro consideration[8]).

[7] K. Rajagopal, AS, 1505.07379

[8] M. Stephanov, H.-U. Yee, 1508.02396

Let's boost to this "no drag frame" in which a heavy quark at rest:

- **Feels no force.**
- **Sees the charge and momentum flowing around it**, with both $\propto \kappa$, along the $\vec{B}$ or $\vec{\Omega}$ direction.
- Sees no entropy current which appears to be zero in this frame. (Note: it is not the case in the presence of the mixed gauge-gravitational CS[9].)

Thus a heavy ($m \to \infty$) quark **may be considered as a "defect"** in the fluid flow and the absence of the drag force **indicates non-dissipativity of the anomalous transport**.

Open questions:

- How different is the anomalous transport from supercurrent/superflow? From QHE?
- Effects of defects in Dirac semi-metals where CME has been found?
- Other probes of anomalous physics for heavy quarks?

[9] M. Stephanov, H.-U. Yee, 1508.02396

Phenomenological Consequences (KR, AS, 1505.07379)

But first, let's make an estimate of the order of magnitude of the effect of Chiral Magnetic Force on a b or $\bar{b}$ quark in a heavy ion collision. If initially at rest, it accelerates up to the terminal velocity $\vec{v}_{\mathrm{terminal}}$ on the timescale of order of several fm/c and gains characteristic momentum:

$$\vec{p}_{\mathrm{term}} \simeq -3\ \mathrm{MeV}\ \frac{m_b}{4.2\ \mathrm{GeV}}\ \frac{\mu_V}{0.1\ \mathrm{GeV}}\ \frac{\mu_A}{0.1\ \mathrm{GeV}}\ \frac{\vec{B}}{(0.1\ \mathrm{GeV})^2} \left(\frac{0.5\ \mathrm{GeV}}{\pi T}\right)^4$$

This value is small and probably is overestimated. But on the other hand, **at least in principle there is a unique correlation observable:** net out-of-plane momentum *for all D and B mesons in a given event*, *in the direction opposite to the CME current in that event*.

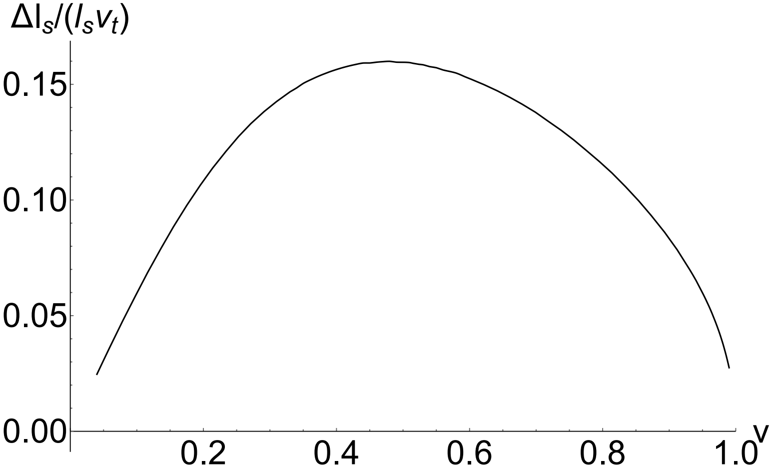
However, it seems that the chiral drag force is principally of theoretical interest, for now...

The Anomalous Wind (AS, Yi Yin, arxiv: 15xx.xxxxx)

On the screening length ℓ_s of color potential

- Another widely known probe of the thermal plasma is a heavy quarkonium.
- The color screening depends on the quarkonium velocity[10]. We know now that **there is an anomalous shift along *B*** of the fluid rest frame definition.
- The direct calculation up to the linear order in $v_{terminal}$ for $v \parallel B$ gives $\boldsymbol{\ell_s(v) = \ell_s^{(0)}(v + f(v)v_{terminal})}$. It manifests the fact that the heavy quarkonium feels the anomalous flow and known function $f(v)$ comes as an averaged effect for its worldsheet.
- The effect is small. However it is **another example of a heavy probe for the anomalous physics**. It also has specific pattern and at least in principle could be observed.

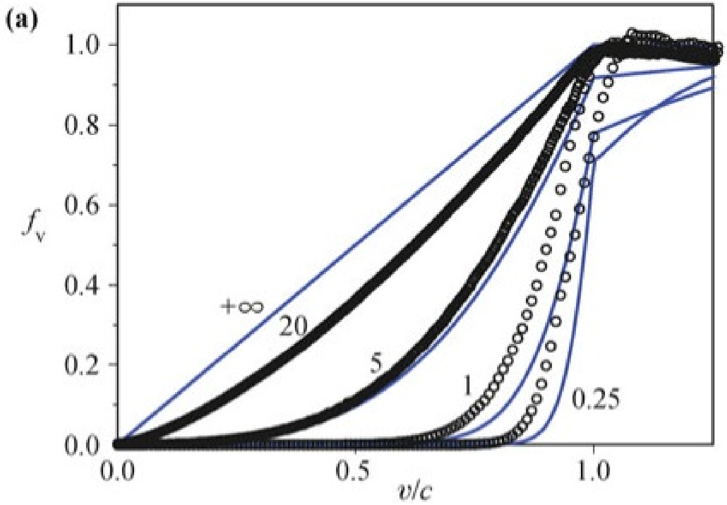
[10] K. Rajagopal et al, PRL 98, 182301



This plot presents $\Delta\ell_s(v)/(v_{terminal}\ell_s(v))$ for $\mu/T = 0.1$ (up to $(\mu/T)^2$).

Limiting Velocity and LLL (AS, Yi Yin in preparation)

- **For a single velocity $v_{terminal}$ a defect in a strongly coupled chiral plasma feels no drag**. (How is it different from a superfluid?)
- Contrast to a weakly coupled superfluid: **a defect will move without dissipation over a range of velocities, with $v < v_{lim}$**.
- In work in progress, we find that a defect in a weakly coupled chiral fluid moves **without dissipation for $v < v_{lim} \to 1$** in the triple limit of zero coupling, infinitely strong magnetic field, and $T = 0$.
- A simple argument: **Landau criteria could be generalized** to a system of chiral fermions with one chirality and for a defect moving along B **LLL doesn't contribute to the drag force** (kinematically forbidden).
- It would be interesting to learn how two pictures of weak and strong couplings turn to each other.



This plot presents superfluid 1d Bose gas drag force as a function of defect velocity (over speed of sound) at different couplings (arXiv:1106.6329v3).

Summary

- The general result for the drag force to first order in gradients second order in μ/T.
- New anomalous manifestations in the drag force in a chiral plasma: Chiral Magnetic Drag Force and Chiral Vortical Drag Force.
- The same force on quarks and antiquarks. The force is nonzero even for quarks at rest in the local rest frame of fluid element.
- Existence of the "no drag frame" defined by the terminal velocity.
- A direct evidence of the dissipationless nature of chiral effects.
- The anomalous effect on the heavy quarkonium screening length.
- Existence of v_{lim} for a defect in chiral plasma below which there is no dissipation for the weak coupling limit.