Non-Abelian Corrections to the Poisson Approximation of Multi-Bremsstrahlung

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Introduction

In the radiative energy loss, for a light quark passing through a medium (Quark Gluon Plasma), the momentum distribution of Multi-gluon Bremsstrahlung has been approximated to be Poisson.
In order to improve this approximation we start with an Abelian theory (QED) in which the distribution is well known then include a Non-Abelian Corrections to this distribution, and we will see how much the non-Abelian will break the Poisson distribution

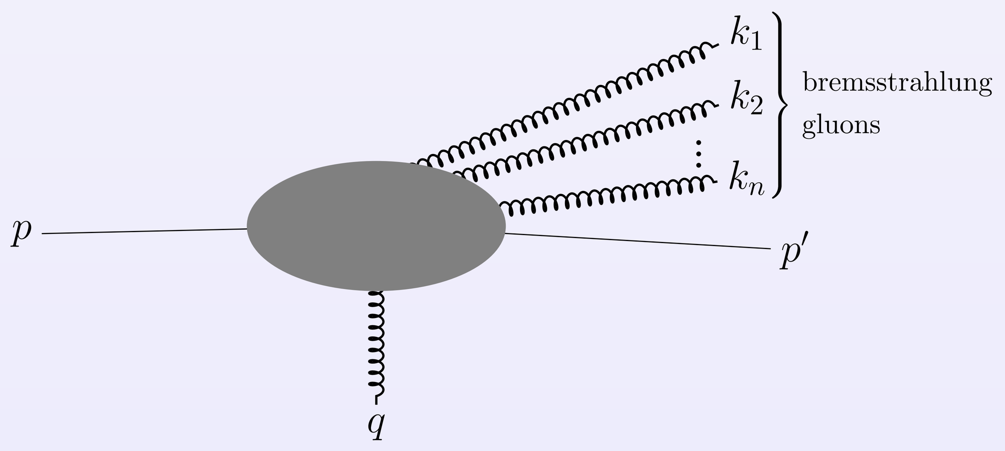

Figure 1 : Bremsstrahlung Gluon Emission

In our calculation, instead of summing Feynmann diagrams, we are using the on-shell method also known as MHV calculation, to compute amplitudes.

Bremsstrahlung Photon in QED

The gauge field of QED is an abelian gauge theory and it has been shown that for the Bremsstrahlung photon the distribution is Poisson

$$P_E(n) = \frac{1}{n!}\lambda^n e^{-\lambda}$$

where $\lambda = \lambda(E)$ and $P_E(n)$ is the probability of emitting n photon with a total energy $E \in [E_-, E_+]$ such that

$$\lambda = \langle n \rangle = \frac{\alpha}{\pi}\log\left(\frac{E_+}{E_-}\right) f_{IR}(q^2),$$

this calculation has been done in Peskin and Schroeder and we can see below the distribution for $\langle n \rangle = 6$

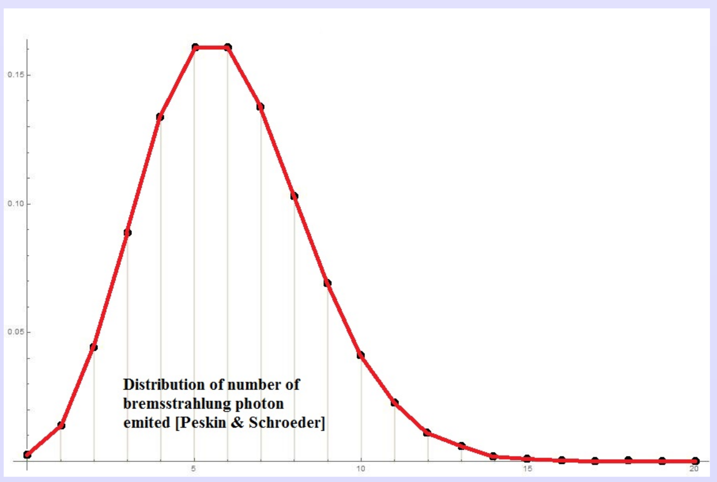

Figure 2 : Bremsstrahlung Photon Distribution

Irreducible Amplitudes

For a given process, an amplitudes $\mathcal{M}$ is a mathematical object that connects the theory into the experimental data where the differential cross section is proportional to the amplitude modulus squared. We defined the irreducible amplitude $\mathcal{M}_{(\alpha)}$as a decomposition of $\mathcal{M}$ such that

$$\mathcal{M} = \sum_\alpha \mathcal{M}_{(\alpha)} \qquad \text{with} \qquad \overline{\mathcal{M}_{(\alpha)}\mathcal{M}^\dagger_{(\beta)}} \propto \delta_{\alpha\beta}.$$

To find those irreducible amplitudes $\mathcal{M}_{(alpha)}$ let us do the following steps:

Step 1: Colour Decomposition

Let us consider the fact that for the emission of multi-gluon, the amplitudes can be factorized as follow

$$\mathcal{M}_n = \sum_{\sigma\in S_n} T_{a_{\sigma(1)}\cdots a_{\sigma(n)}} A_{k_{\sigma(1)}\cdots k_{\sigma(n)}} \qquad (1)$$

where $A_{k_1\cdots k_n} = A_n(p, p', k_1, \cdots, k_n)$ is the partial amplitude which contain the kinematics and $T_{a_{\sigma(1)}\cdots a_{\sigma(n)}}$ is the color part given by

$$T_{a_{\sigma(1)}\cdots a_{\sigma(n)}} = g^n T_{a_1}T_{a_2}\cdots T_{a_n}.$$

Here T_a's are the generators of $SU(N_c)$.

Step 2: Irreducible Representation of S_n

We can decompose the identity into sum of the projector P_α of the irreducible representation of the symmetric group acting on the tensor indices of $T_{a_1\cdots a_2}$ and $A_{k_1\cdots k_n}$

$$1 = \sum_\alpha P_\alpha \quad \text{with} \quad P_\alpha P_\beta = \delta_{\alpha\beta}. \qquad (2)$$

Here P_α projects a tensor into a tensor that have the symmetry of the Young tableau where α is given by the different topology of the Young tableau correspondent, for example

$$\alpha = \left\{ \ \ , \ \ , \ \ \right\} \quad \text{for } S_3$$

Step 3: Combination of Equation (1) and (2)

From step 1 and 2 we can insert two decomposition like (2) one acting in $T_{a_1\cdots a_n}$ and another one in $A_{k_1\cdots k_n}$ and after simplification we obtain

$$\mathcal{M}_{QCD} = C_1\mathcal{M}_{QED} + \sum_{\sigma\in S_n}\sum_{\alpha=2}^{n} C_\alpha(a_\sigma)\mathcal{M}_{(\alpha)}(k_\sigma)$$

in another word the QCD process is the combination of a QED process corrected by some non-abelian effect

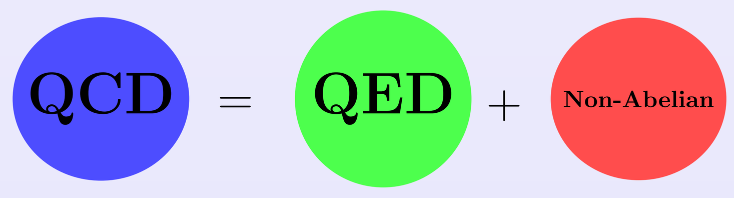

Figure 3 : QCD-effect

NLO Radiative Correction

Consider the 2 bremsstrahlung gluon emitted from a light quark interacting with a medium, using MHV calculation and the irreducible decomposition, the soft factor is given by

$$S(k_1, k_2) = C_1 S_{QED}(k_1, k_2) + \sum_{\sigma\in S_3} C_2(a_\sigma)S_{(2)}(k_\sigma) + C_3(a_1, a_2)S_{(3)}(k_1, k_2)$$

where $k_3 = q$ a transverse momenta from the medium.

- ★ The resummation of $|S_{QED}|^2$ in all order gives us an exponential that lead to the Poisson distribution of QED.
- ★ The purely antisymmetric soft factor $S_{(3)}$ tends to be equal to S_{QED} under the strong ordering $k_1 \ll k_2$
- ★ The mixed symmetry corresponding to the Young tableau is a non-abelian effect, that non vanish when $q \to 0$ in which there is no momentum exchange but just a flip of color of the quark line.

Summary and conclusions

- ★ This method can provide a
- ★ The Poisson approximation of in QCD can be broken by the non-abelian effect in two ways,
  - by the color factor $C_1(a_1, \ldots, a_n)$: composition of T_a's
  - by the non-abelian correction to the QED: $\sum C_\alpha(a_\sigma)\mathcal{M}_{(\alpha)}(k_\sigma)$
- ★ The distribution can be cartoon as bellow where the QCD distribution is the poisson from QED + poisson from color flip + non-poisonnian distribution

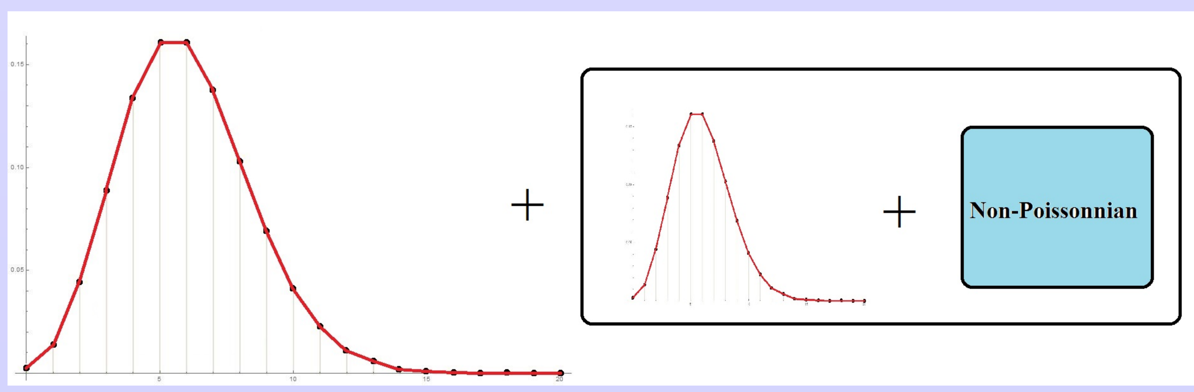

Figure 4 : Bremsstrahlung Gluon Distribution

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References

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