Non-Abelian Corrections to the Poisson Approximation of Multi-Bremsstrahlung

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Introduction

In the radiative energy loss, for a light quark passing through a medium (Quark Gluon Plasma), the momentum distribution of Multi-gluon Bremsstrahlung has been approximated to be Poisson. In order to improve this approximation we start with an Abelian theory (QED) in which the distribution is well known then include a Non-Abelian Corrections to this distribution, and we will see how much the non-Abelian will break the Poisson distribution.

Bremsstrahlung Photon in QED

The gauge field of QED is an abelian gauge theory and it has been shown that for the Bremsstrahlung photon the distribution is Poisson

$$P_\text{\gamma}(n) = \frac{\lambda^n}{n!} e^{-\lambda}$$

where $\lambda = \langle E \rangle$ and $P_\text{\gamma}(n)$ is the probability of emitting $n$ photon with a total energy $E \in [E_-, E_+]$ such that

$$\lambda = \langle n \rangle = \frac{\alpha}{\pi} \log \left( \frac{E_+}{E_-} \right) \frac{f(q)}{f(q)}$$

this calculation has been done in Peskin and Schroeder and we can see below the distribution for $\langle n \rangle = 6$

Irreducible Amplitudes

For a given process, an amplitudes $M$ is a mathematical object that connects the theory into the experimental data where the differential cross section is proportional to the amplitude modulus squared. We defined the irreducible amplitude $M_{a_1 a_2 ... a_n}$ as a decomposition of $M$ such that

$$M = \sum_{a_1 a_2 ... a_n} M_{a_1 a_2 ... a_n} M_{a_1 a_2 ... a_n} \propto \delta_{a_1 a_2}$$

To find those irreducible amplitudes $M_{(alpha)}$ let us do the following steps:

Step 1: Colour Decomposition

Let us consider the fact that for the emission of multi-gluon, the amplitudes can be factorized as follow

$$M_{a_1 a_2 ... a_n} = \sum_{k_1 k_2 ... k_n} T_{a_1 a_2} T_{a_3 a_4} ... T_{a_{n-1} a_n}$$

where $T_{a_1 a_2} = A_{a_1}(p_1 p_2 k_1)$ and $T_{a_{n-1} a_n}$ is the partial amplitude which contain the kinematics and $T_{a_{n-1} a_n}$ is the color part given by

$$T_{a_1 a_2} T_{a_3 a_4} ... T_{a_{n-1} a_n} = q^2 T_1 T_2 ... T_n$$

Here $T_n$'s are the generators of $SU(N_f)$.

Step 2: Irreducible Representation of $S_n$

We can decompose the identity into sum of the projector $P_\alpha$ of the irreducible representation of the symmetric group acting on the tensor indices of $T_{a_1 a_2} a_3 a_4 ... a_n$.

$$1 = \sum P_\alpha \text{ with } P_\alpha P_\beta = \delta_{\alpha \beta}$$

Here $P_\alpha$ projects a tensor into a tensor that have the symmetry of the Young tableau where $\alpha$ is given by the different topology of the Young tableau correspondent, for example

$$\alpha = \left\{ \begin{array}{c} \text{QED} \\ \text{Non-Abelian} \end{array} \right\} \text{ for } S_3$$

Step 3: Combination of Equation (1) and (2)

From step 1 and 2 we can insert two decomposition like (2) one acting in $T_{a_1 a_2}$ and another one in $A_{a_3 a_4}$ and after simplification we obtain

$$M_{\text{QCD}} = C_1 M_{\text{QED}} + \sum_{\alpha \beta} \sum_{a_3 a_4} C_{\alpha \beta}(a_3 a_4) M_{a_1 a_2}(k_3)$$

in another word the QCD process is the combination of a QED process corrected by some non-abelian effect

$$\text{QCD} = \text{QED} + \text{Non-Abelian}$$

NLO Radiative Correction

Consider the 2 bremsstrahlung gluon emitted from a light quark interacting with a medium, using MHV calculation and the irreducible decomposition, the soft factor is given by

$$S(k_1, k_2) = C_S^{\text{QED}}(k_1, k_2) + \sum_{\alpha \beta} C_{\alpha \beta}(a_3 a_4) S_{\alpha \beta}(k_3, k_2)$$

where $k_3$ is a transverse momenta from the medium.

Summary and conclusions

This method can provide

- The Poisson approximation of in QCD can be broken by the non-abelian effect in two ways,
  - by the color factor $C_1(a_1, ..., a_n)$: composition of $T_n$'s
  - by the non-abelian correction to the QED: $\sum_{\alpha \beta} C_{\alpha \beta}(a_3 a_4)$

- The distribution can be cartoon as bellow where the QCD distribution is the Poisson from QED + poisson from color flip + non-poissonian distribution

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References


Figure 1 : Bremsstrahlung Gluon Emission

Figure 2 : Bremsstrahlung Photon Distribution

Figure 3 : QCD-effect

Figure 4 : Bremsstrahlung Gluon Distribution