Chemical potential dependence of the critical quark mass with many flavor approach

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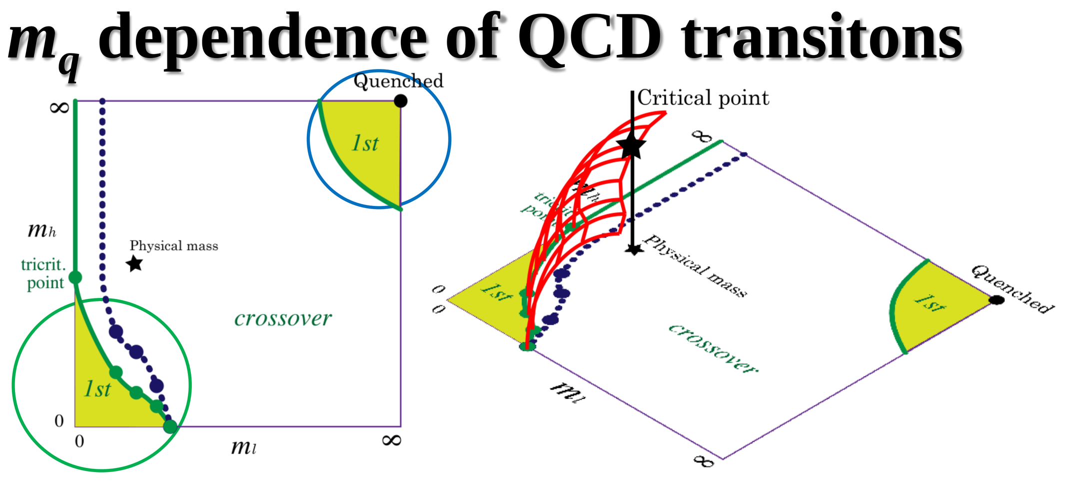
We discuss the QCD critical point at finite density through the study of many-flavor QCD, in which two light flavors and Nf massive flavors exist. Performing simulations of QCD with 2 flavors of improved Wilson fermions, we calculate probability distribution functions in many-flavor QCD at finite temperature and density, where the reweighting technique is used to add the dynamical effect of massive flavors and the chemical potential. From the shape of the distribution functions, we determine the critical surface separating the first order transition and crossover regions in the space spanned by the light and massive quark masses and the chemical potentials.

We found that the critical massive quark mass becomes larger as the chemical potential increases in (2+Nf)-flavor QCD.The indication to the (2+1)-flavor QCD is then discussed.

1. Purpose

m_q dependence of QCD transitons



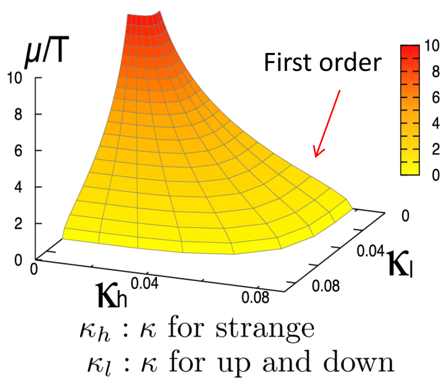
- It is important to determine the boundary of 1st order region.
- Nature of chiral PT in the $m_l \to 0$ is 1st or 2nd ?
- On the line of physical mass, the crossover at low density ➡ 1st order at high density?
- If 1st order region become wider, (yellow region) there exists the critical point **at physical mass**.

Critical surface in heavy quark region

H. Saitoh, et al(WHOT-QCD), Phys. Rev. D89, 034507, (2014)

- Hopping Param. Exp. (HPE)
- Critical surface is controlled by **only 1 parameter**,

$$h \equiv (2\kappa_l^{N_t} + \kappa_h^{N_t}) \cosh(\mu/T)$$



κ_h : κ for strange
κ_l : κ for up and down

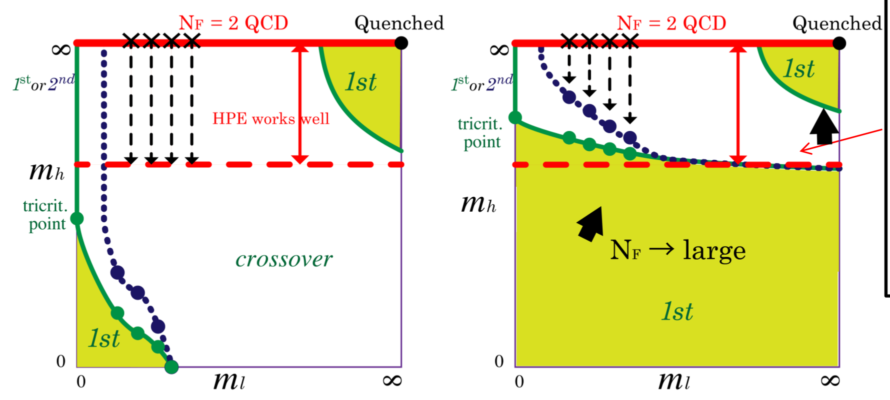
1. In the valid region of HPE, $\kappa_{l,c}$, $\kappa_{h,c}$ satisfy the following eq.

$$(2\kappa_{l,c}^{N_t} + \kappa_{h,c}^{N_t}) = \frac{h_c}{\cosh(\mu/T)} \simeq h_c e^{-\mu/T}$$

$\kappa_{l,c}$, $\kappa_{h,c}$ decrease exponentially as increasing μ/T

We want to apply this method to investigate nature of phase transition in $(2+1)$ QCD

Many flavour approach



As increasing N_f, 1st order region is getting wider. So, we can search for boundary of 1st order region!!

HPE is applicable when the heavy quark is very heavy.

In (2+1) QCD : 2 light quarks and 1 heavy quark exist.

1. Critical surface in heavy quark region
2. Order of chiral transition of massless $N_f=2$ QCD

In $(2+N_f)$ QCD : 2 light quarks and N_f heavy quark exist.

S. Ejiri, N. Yamada, Phys. Rev. Lett. 110, 172001 (2013)

➢ we can investigate the boundary of 1st order region

2. Search the boundary

Histogram method

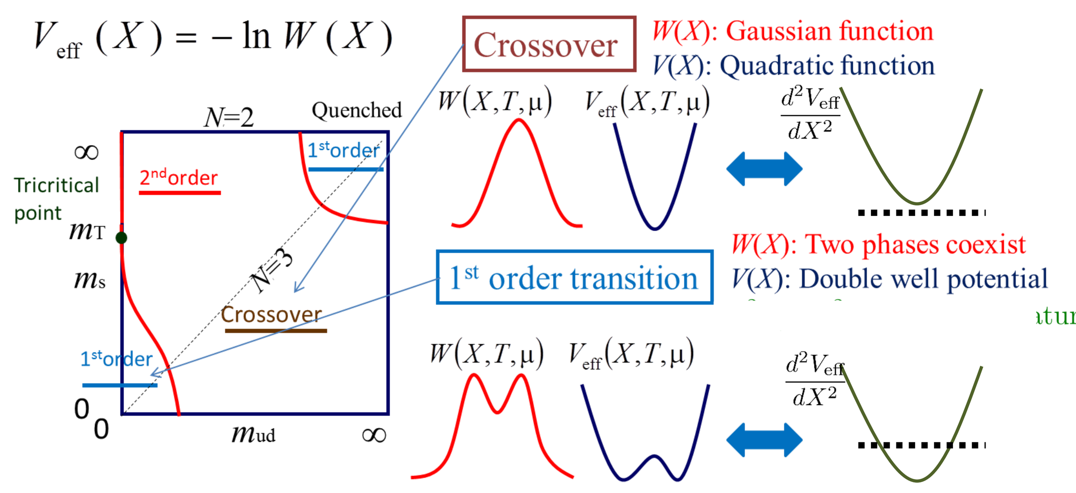
S. Ejiri, PRD77,014508(2008)

Histogram: $W(X;\kappa_h,\mu_h) = \int \mathcal{D}U\delta(X-\hat{X})\prod \det M(\kappa_h,\mu_h) e^{-S_g}$

X : order param., plaquette, $\cdots$ etc.

Effective potetial: $V_{\rm eff}(X) \equiv -\ln W(X;\kappa_h,\mu_h)$

$V_{\rm eff}(X) = -\ln W(X)$



The boundary= "the point where the minimum of the curvature of $V_{\rm eff}$ is equal to 0"

Reweighting technique

This technique is useful for searching this boundary

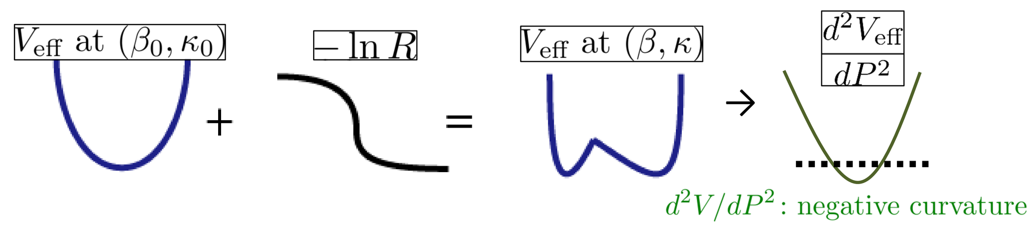
Reweighting factor: $R(P,\beta,\beta_0,\kappa,\kappa_0,\mu,0) = W(P,\beta,\kappa,\mu)/W(P,\beta_0,\kappa_0,0)$

β,κ: any params. β_0,κ_0 : simulation point.

$$R(P,\beta,\beta_0,\kappa,\kappa_0,\mu,0) = \left\langle e^{6N_v(\beta-\beta_0)\hat{P}} \prod \frac{\det M(\kappa,\mu)}{\det M(\kappa_0,0)} \right\rangle_{\text{fixed}: P}$$

We can calculate $V_{\rm eff}(P)$ at $({}^\forall\beta,\kappa)$ using $R(P)$ $V_{\rm eff}(P)$ at (β_0,κ_0)

$$V_{\rm eff}(P,\vec{\beta}) = -\ln\left[R(P,\vec{\beta},\vec{\beta}_0)W(P,\vec{\beta}_0)\right] = -\ln R(P,\vec{\beta},\vec{\beta}_0) + V_{\rm eff}(X,\vec{\beta}_0)$$



Reweighting factor

$$R(P,\beta,\beta_0,\kappa,\kappa_0,\mu,0) = \left\langle e^{6N_v(\beta-\beta_0)\hat{P}} \prod \frac{\det M(\kappa,\mu)}{\det M(\kappa_0,0)} \right\rangle_{\text{fixed}: P}$$

$$e^{6N_v(\beta-\beta_0)\hat{P}} \prod \frac{\det M(\kappa,\mu)}{\det M(\kappa_0,0)} = \left(\frac{\det M(\kappa_l,\mu_l)}{\det M(\kappa_l,0)}\right)^2 \times e^{6N_v(\beta-\beta_0)\hat{P}} \left(\frac{\det M(\kappa_h,\mu_h)}{\det M(0,0)}\right)^{N_f}$$

Taylor expansion of the light quark determinant wrt. μ_l

$$\left(\frac{\det M(\kappa_l,\mu_l)}{\det M(\kappa_l,0)}\right)^2 = 2\sum_{n=1}\left(\frac{\mu_l}{T}\right)^n\left(\frac{\partial^n \ln\det M(\kappa_l,\mu_l)}{\partial(\mu_l/T)^n}\right)$$

$$= 2\left(\frac{\mu_l}{T}\right)\left(\frac{\partial\ln\det M}{\partial(\mu_l/T)}\right) + 2\left(\frac{\mu_l}{T}\right)^2\left(\frac{\partial^2\ln\det M}{\partial(\mu_l/T)^2}\right) + \mathcal{O}\left(\left(\frac{\mu_l}{T}\right)^3\right)$$

At small μ_l/T, evaluate light det$M$ up to $O(\mu_l^2)$

Hopping Param. Exp. (HPE) for heavy quark mass κ_h

$$e^{6N_v\beta\hat{P}}\left(\frac{\det M(\mu_h,\kappa_h)}{\det M(0,0)}\right)^{N_f} = \exp\left[6N_v\beta^*\hat{P} + 6N_s^3 h\left(\hat{L}_R + i\tanh\left(\frac{\mu_h}{T}\right)\hat{L}_I\right)\right]$$

$\beta^* = \beta + 48N_f\kappa_h^4$, $h = 2N_f(2\kappa_h)^{N_t}\cosh\left(\frac{\mu_h}{T}\right)$, $\tanh\left(\frac{\mu_h}{T}\right)$

In the leading order of HPE, there are 3 terms: (P, L_R, L_I)

This factor is controlled by only 3 params. $(\beta^*, h, \tanh(\mu_h/T))$

We change $(\beta^*, h, \tanh(\mu_h/T))$ by reweighting technique.

Especially, complex phase of heavy det$M$ is controlled by $\tanh(\mu_h/T) \le 1$

3. Result μ=0

Lattice Simulations

Iwasaki gauge action + N_f=2 clover-Wilson fermion action, 16^3x4 lattice.

Perform 4 different κ_l simulation (ref. below table)

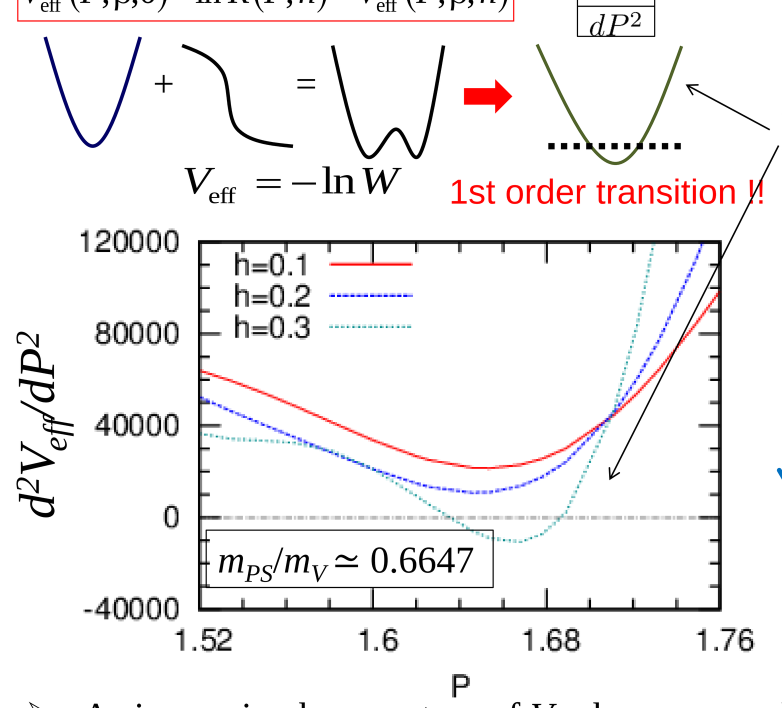
Dynamical heavy quark effect is added by the reweighting method.

det$M$: Hopping parameter expansion

κ_l	C_{sw}	β_{pc}	m_{PS}/m_V
0.1450	1.650	1.778	0.6647
0.1475	1.677	1.737	0.5761
0.1500	1.707	1.691	0.4677
0.1505	1.712	1.681	0.4575

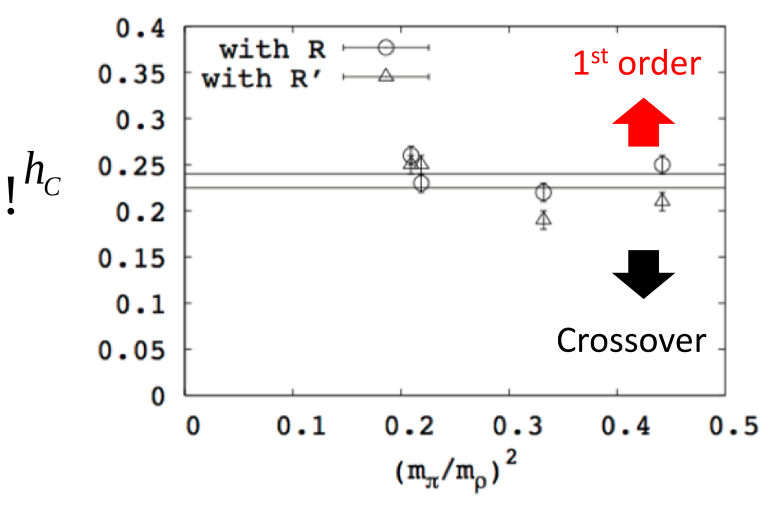
Slope of V_{eff} at finite h

$h = 2N_f(2\kappa_h)^{N_t}$



➢ As increasing h, curvature of $V_{\rm eff}$ become smaller

Dependence of h_c on m_q



Black line: Critical in the m_l=0 limit

➢ We define h_c at the boundary of 1st order region.
➢ The light quark mass dependence of h_c is very small
➢ In the chiral limit($m_l \to 0$), the boundary will not vanish.

In massless N_f=2 QCD, chiral transition should be of 2nd

4. Result μ≠0

Lattice Simulations

Iwasaki gauge action + N_f=2 clover-Wilson fermion action 16^3x4 lattice.

Perform 2 different κ_l simulation

(Since m_l dependence of h_c was small)

Complex phase $\theta \equiv {\rm Im}\ln\det M$

[for heavy quark : HPE,

for light quark : Taylar up to $O(\mu_l^2)$]

For avoiding "sign problem", we use the cumulant exp. method.

H. Saitoh, et al(WHOT-QCD), Phys. Rev. D89, 034507, (2014)

$$\langle e^{i\theta}\rangle = \exp\left[i\langle\theta\rangle_c - \frac{1}{2}\langle\theta^2\rangle_c - \frac{i}{3!}\langle\theta^3\rangle_c + \frac{1}{4!}\langle\theta^4\rangle_c + \cdots\right]$$

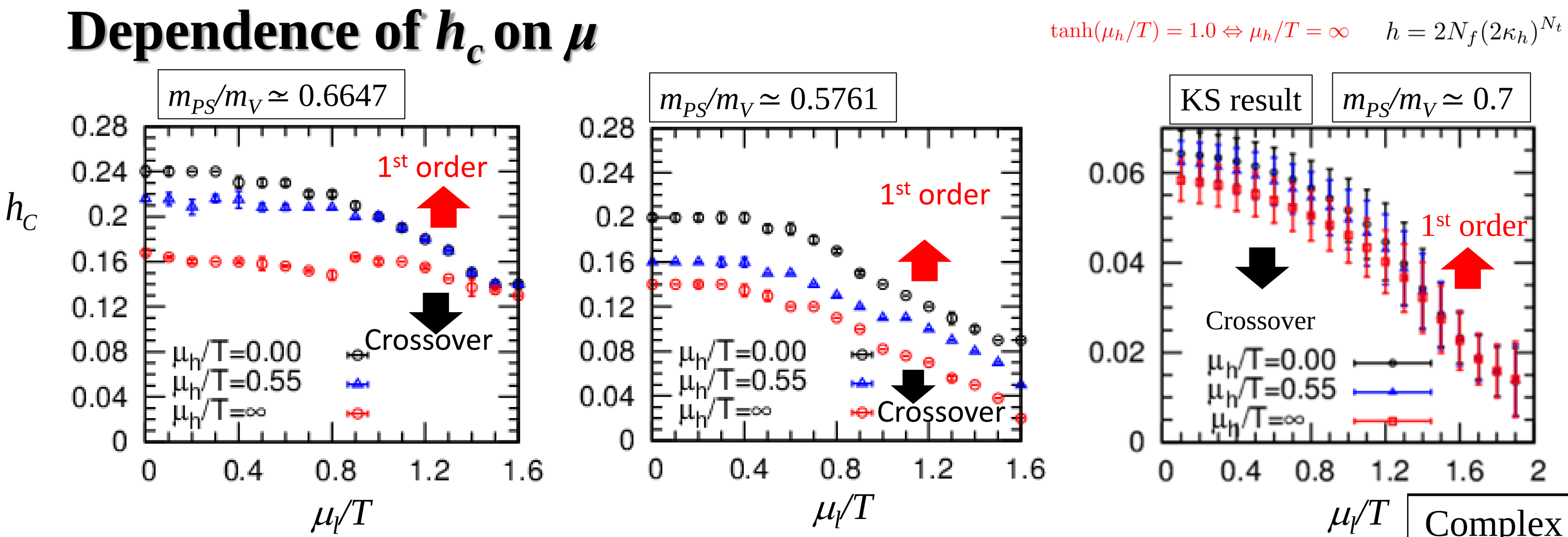
$\langle\theta\rangle_c = \langle\theta\rangle$, $\langle\theta^2\rangle_c = \langle\theta^2\rangle - \langle\theta\rangle^2$, $\langle\theta^3\rangle_c = \langle\theta^3\rangle - 3\langle\theta^2\rangle\langle\theta\rangle + 2\langle\theta\rangle^3$

$\langle e^{i\theta}\rangle \approx \exp\left[-\frac{1}{2}\langle\theta^2\rangle_c\right]$ ➡ No sign probrem !!

κ_l	C_{sw}	β_{pc}	m_{PS}/m_V
0.1450	1.650	1.778	0.6647
0.1475	1.677	1.737	0.5761

Dependence of h_c on μ

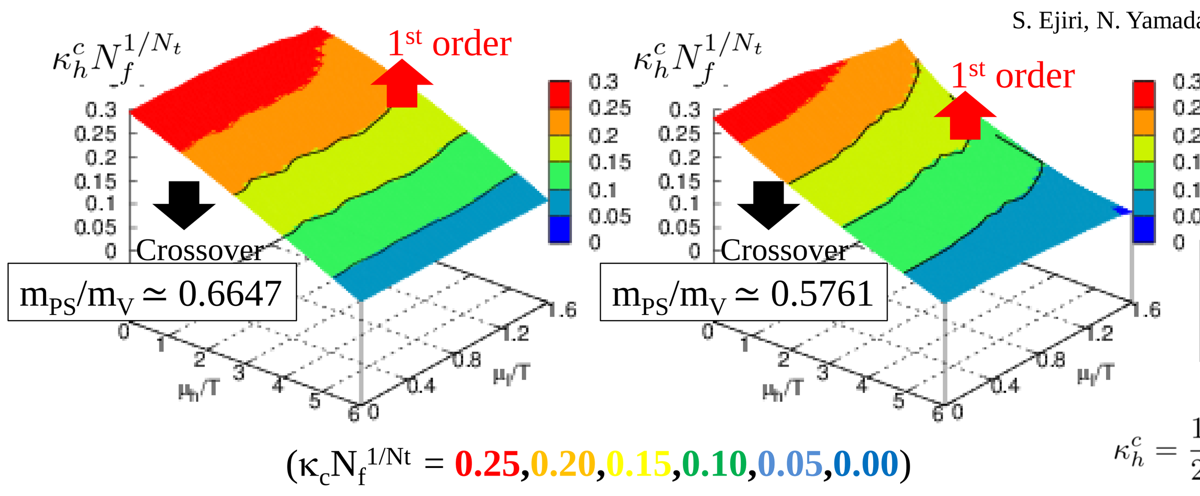
$\tanh(\mu_h/T) = 1.0 \Leftrightarrow \mu_h/T = \infty$ $h = 2N_f(2\kappa_h)^{N_t}\cosh(\mu_h/T)$



➢ As increasing $(\mu_h/T, \mu_l/T)$, h_c decreases. ➡ As increasing $(\mu_h/T, \mu_l/T)$, 1st order region become wider
➢ Compare Wilson-clover results with p4-improved staggered results.

Complex phase of heavy det$M$ has little effect on h_c in both cases

S. Ejiri, N. Yamada, Phys. Rev. Lett. 110, 172001 (2013)



$(\kappa_c N_f^{1/N_t}$ = 0.25, 0.20, 0.15, 0.10, 0.05, 0.00)

$$\kappa_h^c N_f^{1/N_t} = \frac{1}{2}\left(\frac{h_c}{2\cosh(\mu_h/T)}\right)^{1/N_t}$$

➢ At large μ_h/T, κ_c exponentially decreases.

For (2+1) flavor QCD, there is 1st order region in high heavy quark density region ($\mu_h/T > 5.0$)

$$\kappa_h^c = \frac{1}{2}\left(\frac{h_c}{2N_f\cosh(\mu_h/T)}\right)^{1/N_t} \simeq \frac{1}{2}\left(\frac{h_c}{N_f}\right)^{1/N_t} e^{-\mu_h/(N_t T)}$$

4. Conclusion

➢ We investigated the nature of phase transitions in $(2+N_f)$ QCD
➢ For large N_f, we can determine critical point where the 1st order transition terminated relatively easily.
➢ We obtained the following results.

1. The light quark mass dependence of h_c is very smal
2. As increasing $(\mu_h/T, \mu_l/T)$, 1st order region become wider.
3. At large μ_h/T, 1st order region become wider rapidly.
4. In high heavy quark density region ($\mu_h/T > 5.0$), there is 1st order PT in (2+1) flavor QCD.

As increasing $(\mu_h/T, \mu_l/T)$, 1st order region become wider.

we can find the critical point at physical mass!!

5. Work-in-progress

➢ Evaluate hc w/o HPE

We'd like to discuss allowed region of HPE.

- Direct many flavour simulation w/ reweighting
- Evaluate the heavy quark determinant w/ HPE

➢ Calculate hc on finer lattice and/or large volume.

We'd like to discuss finite size effect and lattice discretization error.

- Iwasaki gauge action + N_f=2 clover-Wilson fermion action on12^3x4, 24^3x4, 16^3x6 lattice.

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