We study the finite density phase transition in the lattice QCD at real chemical potential. We adopt a canonical approach and the canonical partition function is constructed for $N_f=2$ QCD. After derivation of the canonical partition function we calculate observables like the pressure, the quark number density, its second cumulant and the chiral condensate as a function of the real chemical potential. We covered a wide range of temperature region starting from the confining low to the deconfining high temperature $0.65\,T_c<T<3.6\,T_c$. We may observe signals for the deconfinement and the chiral restoration phase transition at real chemical potential below $T_c$ starting from the confining phase.

**Conclusion:** Canonical approach is a good choice for finite density QCD. Hopping parameter expansion works more than we expected. We may observe a finite density phase transition.

**Introduction:**
- The canonical partition function is related to the grand canonical one through the fugacity expansion.

**Grand canonical partition function**
$$Z_{G}(\mu, T, V) = \exp \left( \frac{\mu}{T} \left( H - \mu N \right) \right)$$

**Fugacity expansion**
$$F_G = \sum_{n} Z_G(n, T, V) e^{\mu n}$$

**Fugacity**
$$\xi = e^\frac{\mu}{T}$$

**Canonical partition function**
- For fixed quark number $n$
$$Z_{C}(n, T, V) = \sum_{E, n} \exp \left( \frac{H}{T} \right) E, n$$
- positive definite
- No sign problem!

**Our strategy**
- Direct evaluation of the canonical partition function $Z_c(n,T,V)$.
- Reconstruction of the grand canonical partition function with real chemical potential.

**A cheaper method: hopping parameter expansion**
- Perform the hopping parameter expansion
  $$\log \text{Det} D_{W}(\mu, s) = \text{Tr} \log (1 - s Q(\mu)) = -\sum_{n=0}^{\infty} \frac{s^n}{n!} \text{Tr}(Q^n(\mu))$$
- Count a power of the fugacity
  $$\xi = e^\frac{\mu}{T}$$
  - winding number of temporal hopping
  $$\text{Tr} \log D_{W}(\mu, s) = \sum_{n=0}^{\infty} w_n(\mu) e^{\mu n}$$
  - Extract the coefficient $z_n$
  $$z_n(\mu, s) = \sum_{m=0}^{\infty} w_m(\mu) e^{\mu m}$$
  - Use a numerical Fourier transformation
  $$z_n(\mu, s) = \int_{0}^{2\pi} \frac{d\theta}{\pi} e^{-\mu \theta} \sum_{m=0}^{\infty} w_m(\mu) e^{\mu m} = e^{\mu/T}$$

**Numerical simulation**
- Iwasaki gauge action
- Clover fermion $Nf=2$, $cSW=1.1$
- APE stout smeared gauge link
- Box size $8^3 \times 4$
- Temperature $0.65T_c \leq T \leq 3.6T_c$
- Quark mass $0.7 < m_c < 0.9$
- $N_{\text{traj}} = 120$
- $N_{\text{hop}} = 480$

**Re-weighting technique**
- Gauge configuration generated at $\mu=0$
$$Z_c(\mu, T, V) = \int D[U] \exp \left( \frac{\mu}{T} \left( H - \mu N \right) \right)$$

**Grand canonical partition function**
$$Z_G(n, T, V) = \sum_{\{\mu_a\}} \sum_{\{U\}} \left( \prod_{a} \text{Det} D_{W}(\mu_a, U_a) \right)^{n_{a}} e^{-S_U(\mu_a, U_a)}$$

**Canonical partition function**
$$Z_C(n, T, V) = \sum_{\{\mu_a\}} \sum_{\{U\}} \left( \prod_{a} \text{Det} D_{W}(\mu_a, U_a) \right)^{n_{a}} e^{-S_U(\mu_a, U_a)}$$

**Validity of hopping parameter expansion**
- Alembert’s convergence condition
  $$\lim_{n \to \infty} \frac{\left| Z_c(n, \mu, T, V) \right|^2}{Z_C(n, T, V)} < 1$$

**Reconstructed grand canonical partition function**
$$Z_{G}(\mu, T, V) = \sum_{n=0}^{N_{\text{max}}} \left| Z_c(n, T, V) \right| \xi^n$$

**Quark number density**
$$\langle n \rangle = \frac{1}{Z_c(\mu, T, V)} \sum_{n=0}^{N_{\text{max}}} n \left| Z_c(n, T, V) \right| \xi^n$$

**2nd cumulant**
$$\langle n^2 \rangle - \langle n \rangle^2$$

**Chiral condensate**
- No renormalization
- Additive correction is not subtracted
- Poster by A.Suzuki for kurstan

---

**Abstract**
We study the finite density phase transition in the lattice QCD at real chemical potential. We adopt a canonical approach and the canonical partition function is constructed for $N_f=2$ QCD. After derivation of the canonical partition function we calculate observables like the pressure, the quark number density, its second cumulant and the chiral condensate as a function of the real chemical potential. We covered a wide range of temperature region starting from the confining low to the deconfining high temperature $0.65\,T_c<T<3.6\,T_c$. We may observe signals for the deconfinement and the chiral restoration phase transition at real chemical potential below $T_c$ starting from the confining phase.