Baryon number cumulant ratios at finite density in the strong-coupling lattice QCD

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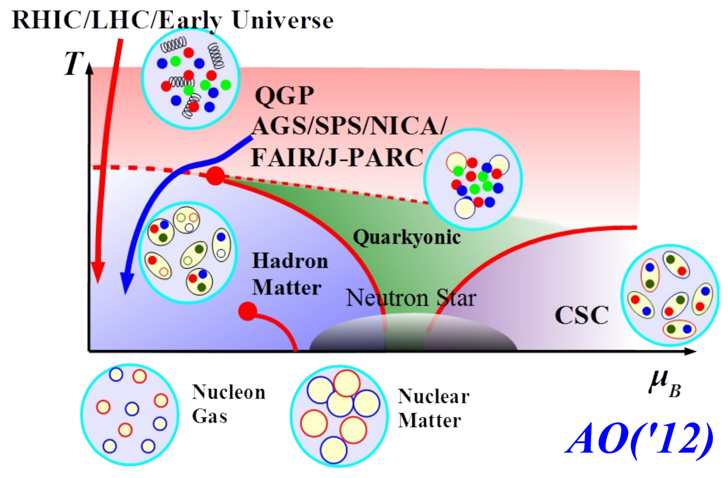
Introduction

- **Critical point in QCD phase diagram connects Crossover trans. at low μ and first order trans. at high μ**
  Asakawa, Yazaki ('89)
  → CP could be accessible in BES.



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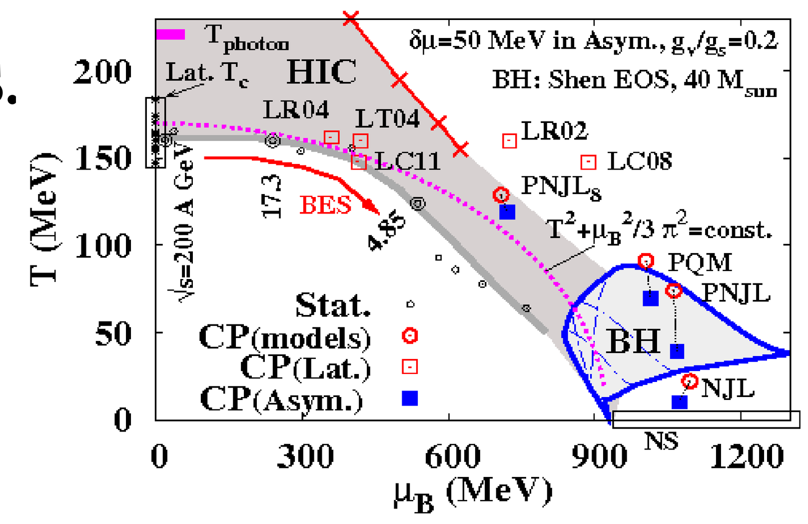
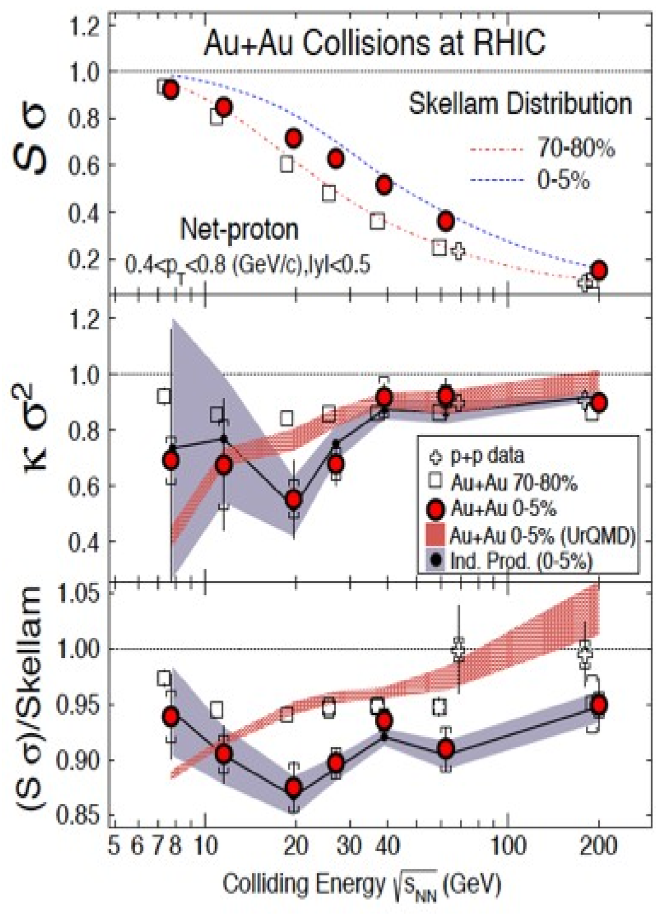


- **Fluctuations of conserved charge = Promising signal of CP**

$$S\sigma = \chi_\mu^{(3)}/\chi_\mu^{(2)} \quad \kappa\sigma^2 = \chi_\mu^{(4)}/\chi_\mu^{(2)}$$

$$\chi_\mu^{(n)} = \frac{1}{VT^3}\frac{\partial^n \log Z}{\partial(N_c\mu/T)^n}$$

  - Contain info. on earlier stage.
  - Remnant of O(4) criticality may cause non-monotonic behavior of the singular part.
    Friman, Karsch, Redlich, Skokov ('11)
  - Larger fluctuation of order par. around CP → Negative κ
    Stephanov ('11) [Z(2)]
  - Non-monotonic behavior of cumulant ratio is observed @ BES → CP signal ?



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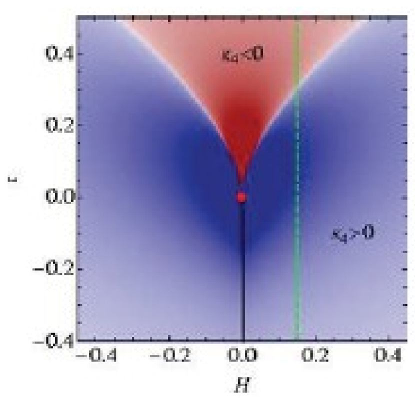
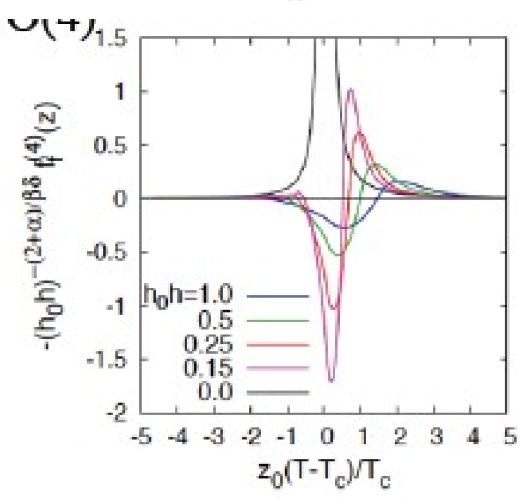




> ***We need calculation including regular part at finite μ !***
> ***This work:***
> ***Net baryon number cumulants at finite T and μ in the strong-coupling and chiral limit of lattice QCD***

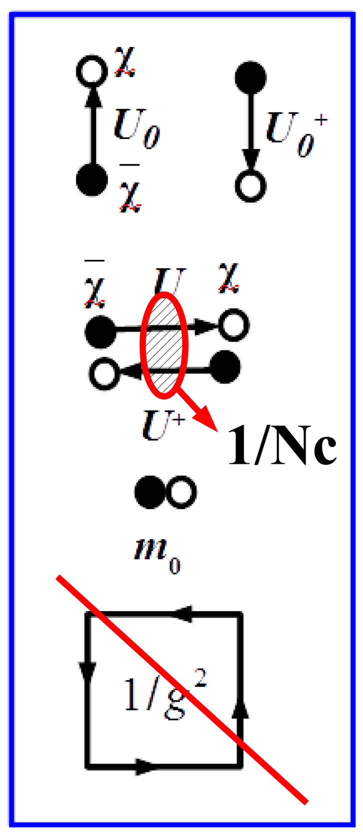
Strong-Coupling Lattice QCD

- **Lattice QCD action (unrooted staggered fermion)**

$$L=\frac{1}{2}\sum_x\left[V_x^+-V_x^-\right]+\frac{1}{2\gamma}\sum_{x,j}\eta_j(x)\left[\bar\chi_x U_j(x)\chi_{x+\hat j}-\bar\chi_{x+\hat j}U_j^\dagger(x)\chi_x\right]+\frac{m_0}{\gamma}\sum_x\bar\chi_x\chi_x+\frac{2N_c}{g^2}S^{\rm plaq}$$

$$V_x^+=\bar\chi_x U_0(x)e^{\mu/\gamma^2}\chi_{x+\hat 0},\quad V_x^-=\bar\chi_{x+\hat 0}U_0^\dagger(x)e^{-\mu/\gamma^2}\chi_x$$

  - $\eta_j(x)=(-1)^{**}(x_0+...+x_{j-1})$
  - $U(1)_L \times U(1)_R$ chiral sym.
    $\chi_x \to \exp[i\,\theta\varepsilon(x)]\,\chi_x$, $\varepsilon(x)=(-1)^{**}(x_0+x_1+x_2+x_3)$
  - Anisotropy parameter γ ($T = \gamma^2/N_\tau$) *Bilic et al. ('92)*



- **Strong coupling limit**
  Damgaard, Kawamoto, Shigemoto ('84), Jolicoeur, Kluberg-Stern, Lev, Morel, Petersson ('84).
  Spatial link integral
  → Fermion action with four-Fermi int. (LO in 1/d expansion)

$$S_{\rm eff}^{\rm (SCL)}=\frac{1}{2}\sum_x\left[V_x^+-V_x^-\right]+\frac{m_0}{\gamma}\sum_x M_x-\frac{1}{4N_c\gamma^2}\sum_{x,j}M_x M_{x+\hat j}\quad (M_x=\bar\chi_x\chi_x)$$

- **Extended Hubbard Stratonovich transf.**
  → Eff. Action of Auxiliary Field

$$\exp(\alpha AB)=\int d\varphi\, d\varphi^*\exp(-\alpha[\varphi^*\varphi+\varphi^* A+\varphi B])$$

$$S_{\rm eff}^{\rm (EHS)}=\frac{1}{2}\sum_x\left[V_x^+-V_x^-\right]+\frac{1}{\gamma}\sum_x m_x M_x+S_{\rm AF}$$

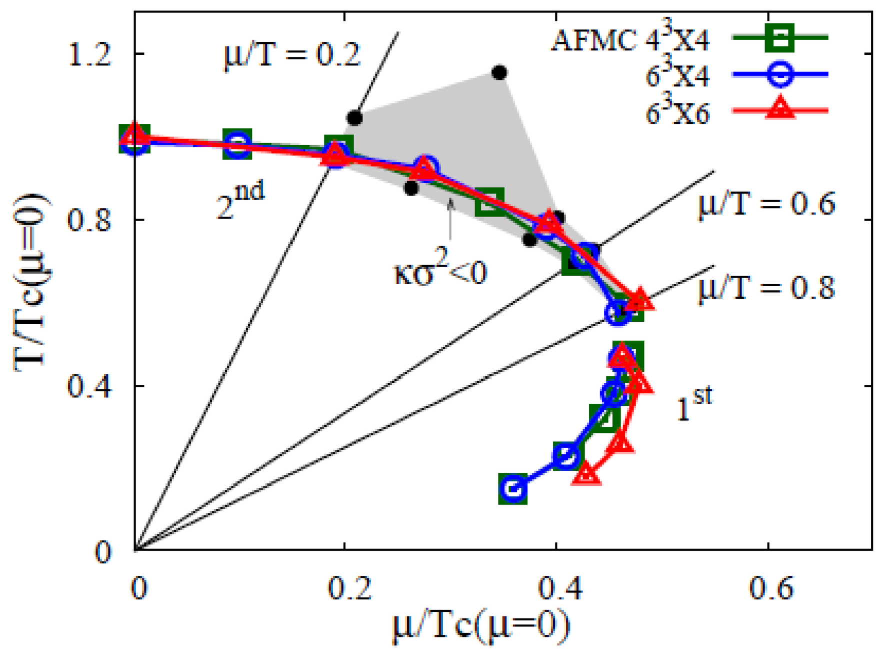
$$S_{\rm AF}=\frac{L^3}{4N_c}\sum_{k\tau,f(k)>0}f(k)\left[\sigma_{k\tau}^*\sigma_{k\tau}+\pi_{k\tau}^*\pi_{k\tau}\right]$$

> ***MC integral over AF → Fluctuation Obsv.***

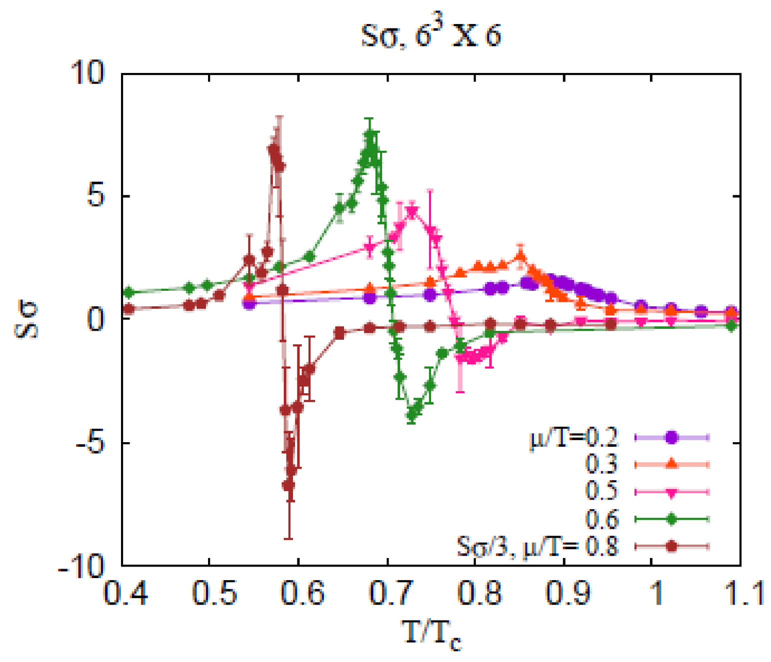
Ichihara, Nakano, AO ('14)

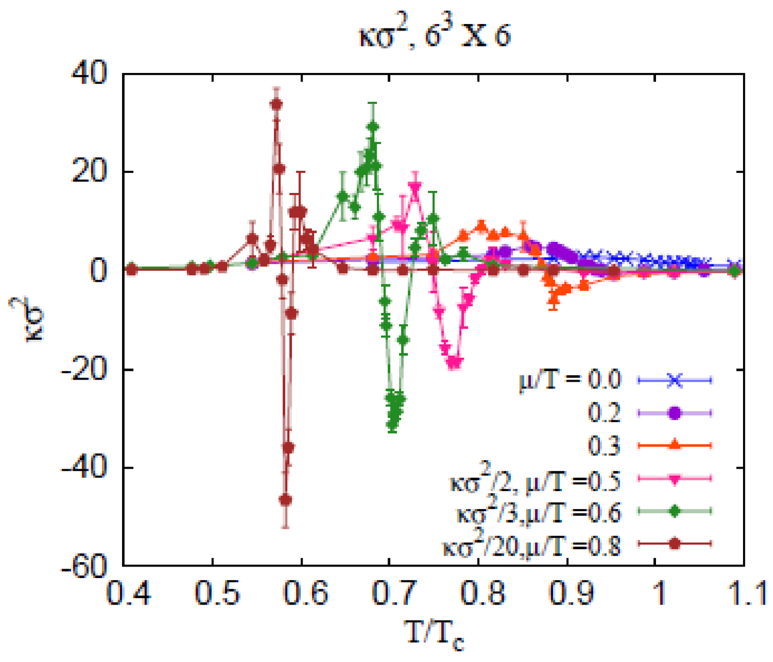
Results

- **Lattice set up**
  - unrooted staggered fermion [O(2) symmetry]
  - Strong-coupling and chiral limit
  - Auxiliary Field MC method on 4^4, $6^3\times4$, 6^4, 8^4 lattices
    Ichihara, Nakano, AO ('14)
  - μ/T = 0.2, 0.3, 0.5, 0.6, 0.8

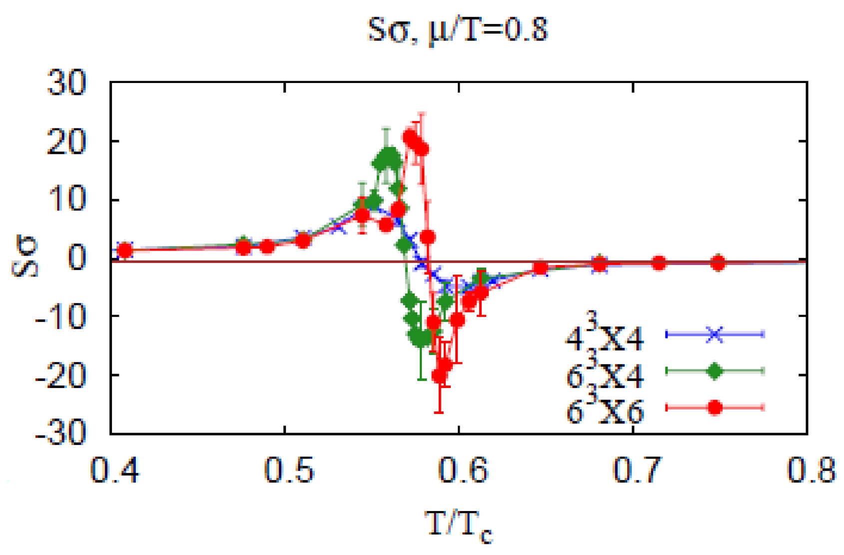


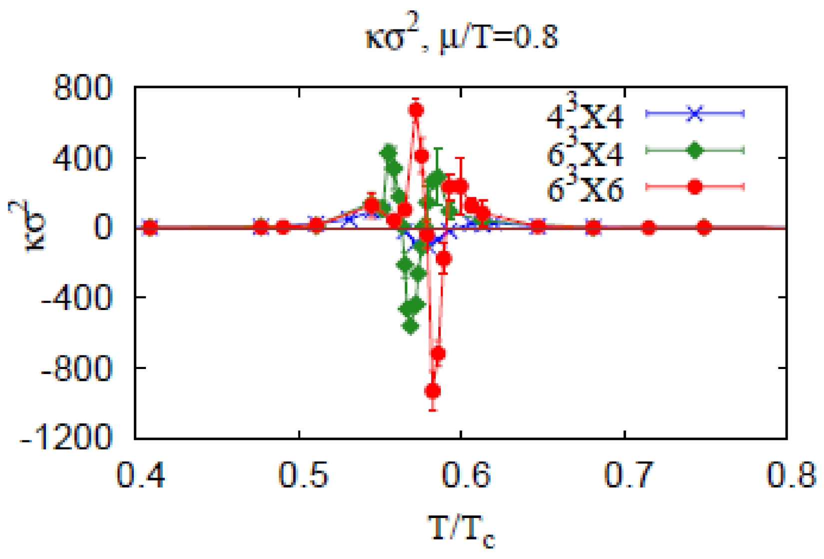
- **(μ, T) dependence**
  - Small μ/T:
    Peak around boundary.
  - Large μ/T:
    Oscillatory behavior (Sσ: +-, κσ² : +-+)
    Larger amplitude towards TCP.





- **Size dependence**
  - Amplitude show divergent behavior and Negative valley in κσ² narrows with increasing lattice size. [Singular part dominance]
  - Consistent with the O(4) scaling analysis
    Friman, Karsch, Redlich, Skokov ('11)
    [Qualitative behavior is similar in O(4) and O(2)]





- **Negative normalized kurtosis region in the QCD phase diagram**
  - Negative kurtosis area exists aound the phase boundary.
  - Expected to shrink in the chiral and thermodynamic limit.

Summary & Discussion

- We investigate normalized skewness (Sσ) and kurtosis (κσ²) in the chiral and strong coupling limit (~6^4 lattice).
- We find oscillatory behavior at large μ/T and negative kurtosis valley due to the finite size effect.
- Peak heights of skewness and kurtosis increases and negative valley of kurtosis shrinks on larger lattices, as suggested in O(4) scaling analysis in the chiral limit.
- Finite lattice size is found to smear the critical behavior of cumulants as the finite quark mass does.
- Important next steps
  - Cumulants on larger lattices [Finite size scaling],
  - Finite mass effects [Negative region should survive on large lattice].

Refs: M. Asakawa, K. Yazaki, NPA504 ('89) 668 / M. A. Stephanov, PRL107 ('11) 052301 / B. Friman, F. Karsch, K. Redlich, V. Skokov, EPJC71 ('11) 1694 / L. Adamczyk, et al. (STAR Collab.), PRL 112 ('14) 032302 / T. Ichihara, A. Ohnishi, T.Z. Nakano, PTEP 2014('14),123D02 / T. Ichihara, K. Morita, A. Ohnishi, PTEP 2015 ('15), in press / A. Ohnishi, PTPS 193 ('12), 1.