# Reaction-diffusion equation for quark-hadron transition in heavy-ion collisions

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# Introduction

- Dynamics of quark hadron transition is one of the most important issues in relativistic heavy-ion collisions, as well as in the universe.
- It used to be believed that the quark-hadron transition is first order even at low chemical potential (as in the early universe)
- Quark nuggets due to the concentration of quarks by moving phase boundaries→ Witten's[1] proposal.
- Lattice result → quark-hadron transition is crossover→ no phase boundary → Witten's proposal is not applicable.
- It seems the presence of moving interfaces is more generic, and not necessarily restricted to the case of first order transitions.

- RD equation[2,3]: $\frac{\partial c}{\partial t} = f + D\nabla^2 c$
- Reaction Diffusion equation with appropriate boundary conditions consists of a traveling front with well-defined profile.
- Field theory in the presence of thermal bath has second order time derivative as well as phenomenological dissipation term (coefficient of first order time derivative).
- Correspondence between field theory equation and RD equation is best established for high dissipation case.
  $\ddot{\phi} - \nabla^2\phi + \eta\dot{\phi} = f$

# RD equation for Chiral Transition

- The field equations for the case of spontaneous chiral symmetry breaking transition for the two flavor case $O(4)$ field $\phi = (\sigma, \pi)$ as the chiral order parameter are[4],

$$\ddot{\phi} - \nabla^2\phi + \eta\dot{\phi} = -4\lambda\phi^3 + m(T)^2\phi + H$$

$$m(T)^2 = \frac{m_\sigma^2}{2}\left(1 - \frac{T^2}{T_c^2}\right)$$

- $\lambda = 4.5$, $m_\sigma = 600 MeV$, $H = (120 MeV)^3$, $T_c = 200$MeV.
- $\phi$ is taken along $\sigma$ direction.
- Our approximations: (1) H= 0, (2) $\eta$ is time independent and very large so that we can neglect $\ddot{\phi}$ term.

- Rescale the variables: $x \to m(T)x$, $\tau \to \frac{m(T)^2}{\eta}\tau$ and $\phi \to 2\frac{\sqrt{\lambda}}{m(T)}\phi$
- Field equations simplify to,

$$\dot{\phi} = \nabla^2\phi - \phi^3 + \phi$$

- Above equation in 1D with $\nabla^2\phi = \frac{d^2\phi}{dx^2}$ is same as RD equation Known as Newell-Whitehead equation.
- The analytic solution with boundary conditions $\phi = 0$ and $\phi = 1$ at $x \to \pm\infty$

$$\phi(Z) = [1 + exp(z\sqrt{2})]^{-1}$$

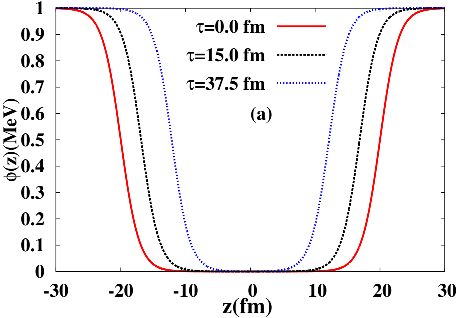
$z = x - v\tau$, $v = 3/\sqrt{2}$.

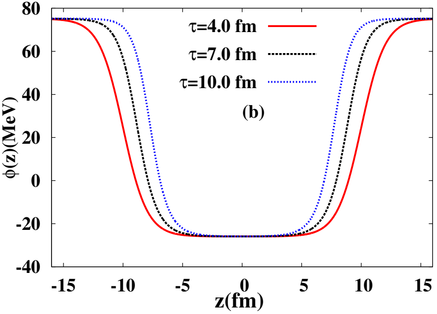
- For the chiral symmetry breaking transition in HIC, boundary conditions are:
  (1) At the center, $T > T_c \Rightarrow$ chiral field takes chirally symmetric value. (2) At large r, $T < T_c \Rightarrow$ chiral field takes symmetry broken value.
- We consider the initial field-profile and evolve it when $T_{cntr}$ reduces to a temperature $T_0$below $T_c$.
- For simplicity, we will assume $T_0$ to be uniform over the range of the profile of $\phi$ with $\phi$ in the center of the plasma having a value $\phi_0$(corresponding to the central maximum of the potential at $T = T_0$). $\phi$ in outer regions of the plasma will take vacuum expectation value $\zeta$ for$T = T_0$.
- Once $T_0$ is reached, we take $T_0$ to be constant over the range of the profile of $\phi$.

- With these BCs the analytic solution is,

$$\phi(z) = \zeta[1 + exp(\frac{m(T)}{\sqrt{2}}(x - v\tau))]^{-1}$$

$\zeta = \frac{m(T)}{2\sqrt{\lambda}}$ is the VEV of $\phi$(for H=0) and $v = \frac{3m(T)}{\eta\sqrt{2}}$. Left plot is the for $H = 0$ and right plot is for H non zero.





Initial profile for non zero *H* case is

$$\phi(z) = -\frac{\zeta - \phi_0}{A_0}[1 + exp(\frac{m(T)(|z| - R_0)}{\sqrt{2}})]{-1} + \zeta$$

$$A_0 = [1 + exp(-\frac{m(T)R_0}{\sqrt{2}})]^{-1} \qquad (1)$$

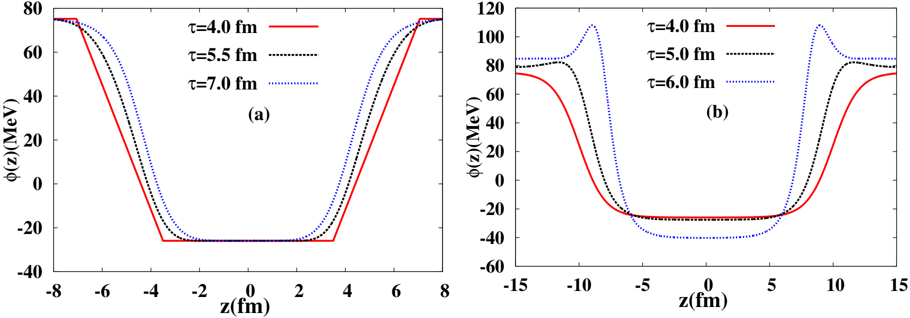


Figure:(Left) Evolution of a profile consisting of linear segments and (right ) Evolution of the profile for time dependent $\eta$ and $T$

# RD equation for Confinement-Deconfinement Transition

- We consider Confinment-Deconfinement Transition during early thermalization stage.
- For RHICE, for very early stages, Bjorken scaling with longitudinally expanding plasma is a very good approximation.
- The dissipation term is very strong during the early stage.
- We study the C-D transition using the expectation value of the Polyakov loop order parameter.
- The Lagrangian density[5],

$$L = \frac{N}{g^2}|\partial_\mu l|^2 T^2 - V(l)$$

$$V(l) = (-\frac{b_2}{2}|l|^2 - \frac{b_3}{6}(l^3 + l^{*3}) + \frac{1}{4}|l|^4)b_4 T^4$$

- We take real l and neglect second order time derivative(for large dissipation case)
- Scale the variables as $x \to gT\sqrt{\frac{b_4}{2N}}x$ and $\tau \to \frac{b_4 g^2 T^2}{2\eta N}\tau$
- The field equation for real l(x), written as $\phi(x)$, is, $\dot{\phi} = \nabla^2\phi + \phi(b_2 + b_3\phi - \phi^2)$→Fitzhugh-Nagumo equation.

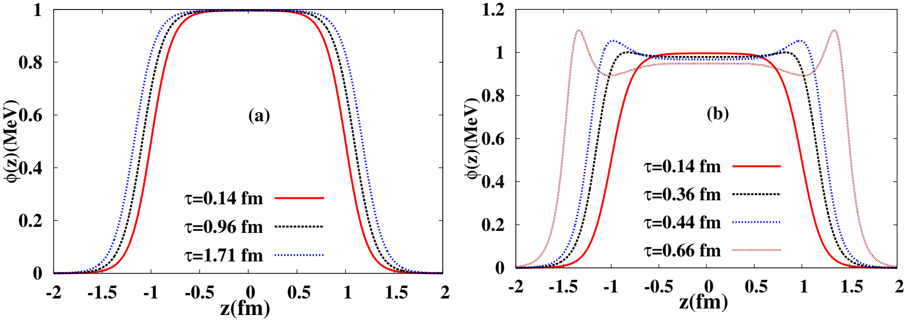


Figure (a) Traveling wave solution for very large dissipation case with constant $\eta = 1/(0.01fm)$.(b) Solution for realistic $\eta = 1/\tau$ and with time dependent $T$.

# Conclusion I

- We conclude by emphasizing that the techniques of reaction-diffusion equation have been used here to show existence of well defined traveling front solutions, which are very similar to phase boundaries for a first order transition case, even though the relevant QCD transitions here are of second order, or a cross-over.
- During the time when dissipation dominates, we see that the transition proceeds by slow moving front, and may take several fm time to complete, leading to long lasting mixed phase stage.
- This will affect calculations of various signals of QGP for RHICE, e.g. production of thermal photons and di-leptons, $J/\psi$ suppression, and especially elliptic flow which develops mostly during early stages.

# Possibility of DCC formation in PP high multiplicity collisions via RD equation

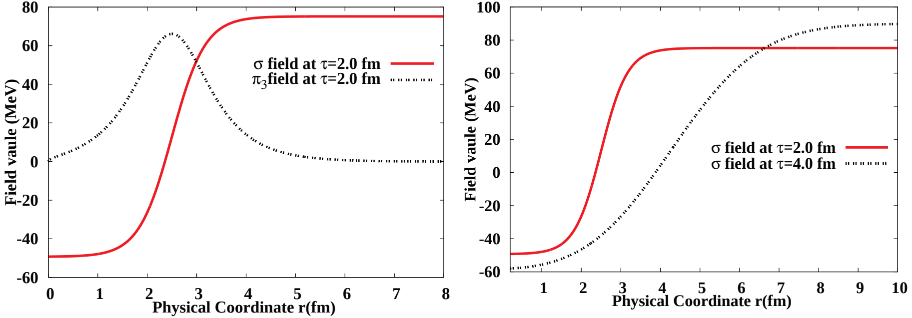
- DCC[6] represents a region of space where the chiral field is disoriented from the true vacuum direction.
- The linear sigma model for chiral symmetry breaking with potential up to one loop,

$$L = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m_\sigma^2}{4}\left(\frac{T^2}{T_c^2} - 1\right)|\phi|^2 + \lambda|\phi|^4 - H\sigma$$

- We consider the case of high multiplicity pp collisions at LHC energy, assuming that the resulting partonic system thermalizes to a chiral symmetry restored phase and undergoes a rapid 3-dimensional expansion. We take the field equations for the chiral field to be

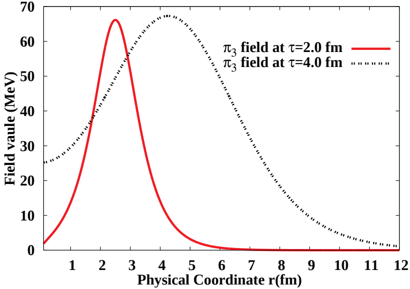
$$\ddot{\phi}_i - \frac{d^2\phi_i}{dr^2} - \frac{2}{r}\frac{d\phi_i}{dr} + (\frac{3}{\tau} + \eta'(T))\dot{\phi}_i = -4\lambda|\phi|^2\phi_i + m(T)^2\phi_i + H\delta_{i4}$$

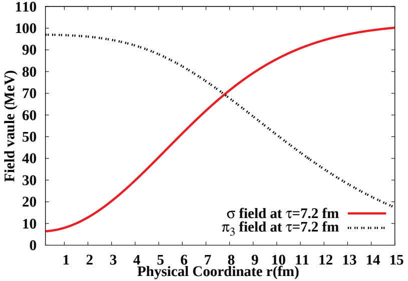
- For 3D expansion case Temp variation: $T(\tau) = T_0 \frac{\tau_0}{\tau}$
- Boundary condition: Outside the parton system $\phi$remains in the true vacuum. Inside the system,$\phi$ is taken to lie (close) to the saddle point, opposite to the true vacuum.
- Initial size of the system is taken to be about 2.5 fm radius at $\tau_0 = 2$ fm.
- System undergoes rapid three-dimensional expansion.
- This leads to stretching of DCC domain, chiral field DOES NOT roll down because of reaction-diffusion front solution.



Fig(a)-Left fig. The initial profile of the chiral field. Solid (red) curve shows the profile of the$\sigma$ field which interpolates between the true vacuum value $\sigma$ = 75.18 MeV and the saddle point opposite to the true vacuum where $\sigma$= âĹŠ49.25 MeV. Corresponding variation of $\pi_3$ , ensuring that the field (approximately) lies on the vacuum manifold, is shown by the dashed (black) curve.

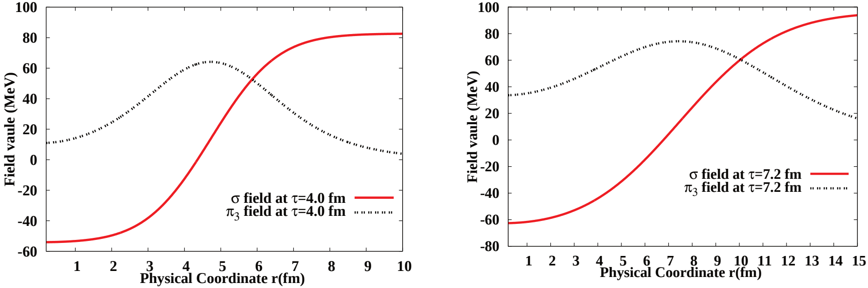
Fig(b)-Right fig. Dashed (black) curve shows the profile of $\sigma$ field after the system has undergone expansion up to $\tau = 4fm$.





Fig(c)-Left fig: Corresponding profile of ÏĂ 3 field at $\tau = 4fm$ is shown by the dashed (black curve), while solid (red) curve shows the initial $\pi_3$ profile.

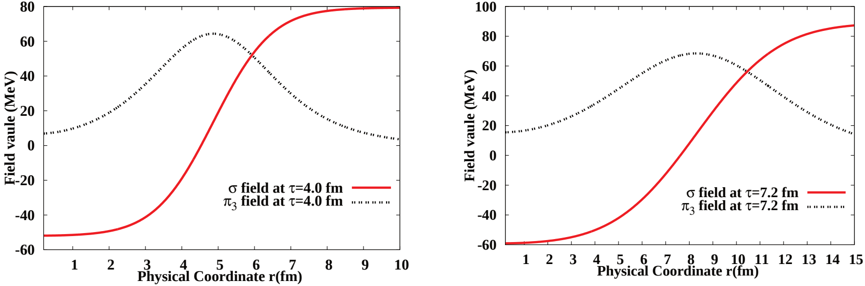
Fig(d)-Right fig:This shows the stage at$\tau = 7.2fm$ when the decay of DCC domain has set in.



Fig(e)-Left fig:show the profiles of the chiral field at the same stages as in Fig.a,c starting from the same initial profile.

Fig(f)-Right fig: shows the profile of the chiral field at the same stage as in Fig(d).

In the above case $\eta'$ is taken as constant for this case with value 20 $fm^{-1}$.



Fig(g)-Left fig: Shows the similar stage as in Fig(e),but with $\eta' = 40 fm^{-1}$.

Fig(h)-Left fig: Shows the similar stage as in Fig(f),but with $\eta' = 40 fm^{-1}$.

We see much larger DCC domain formation.

# Conclusion II

- We have shown how a single large DCC domain can arise in high multiplicity pp collisions.
- Multiple DCC domain problem of heavy-ion collisions is avoided here by starting with a small single DCC domain.
- This domain stretches due to rapid 3-d expansion.
- During stretching, chiral field DOES NOT roll down due to existence of propagating front solution of reaction-diffusion equation.
- High multiplicity pp collision at LHC energy may be the ideal place to look for a clean signal of single DCC formation.

# References

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