Effects of Phase Transition Induced Density Fluctuations on Pulsar Dynamics

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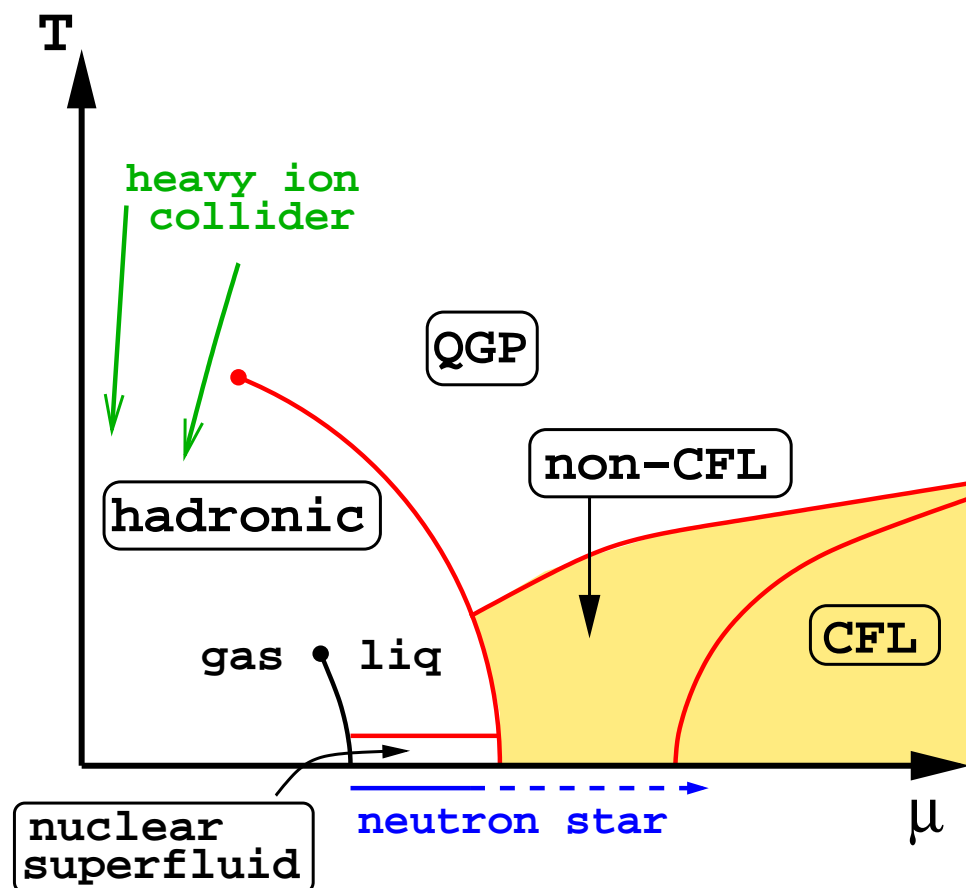
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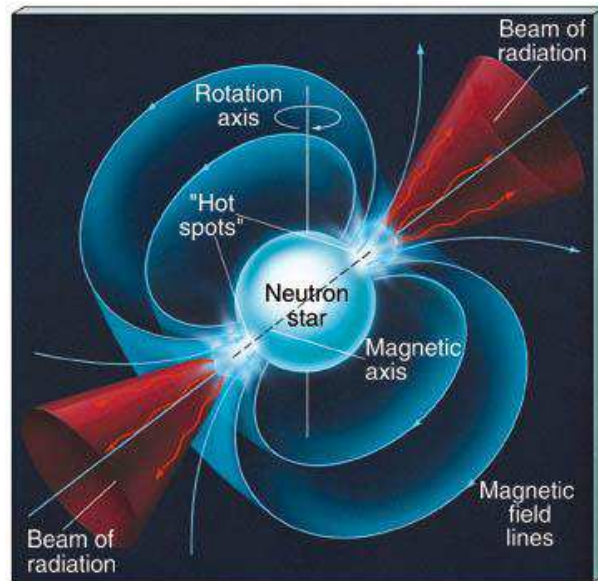Department of Physics, BITS-Pilani, Rajasthan, India.

- Speculation : Existence of Various QCD Phases in Neutron Star Cores [1]
  - Quark Gluon Plasma(QGP)
  - Color Flavor Locked (CFL)
  - 2SC (Color Superconductor) etc.

Introduction : Glitches & Anti-glitches of Pulsar

- Pulsars : Rotating neutron stars, highly magnetized, emits radio pulses

Source : Internet

- Glitches : Sudden increase in rotational frequency

- Relaxation Time: Few Months

- Present Theory : Triggered by crustquakes or transfer of angular momentum of superfluid vortices etc

- Anti-Glitches (relatively recent observations): Sudden slowing down of rotation

- De-pinning of vortices can not account for these

Our Proposals & Predictions

- The core density of a pulsar might change during its evolution by accretion of matter to become supercritical and undergoes phase transition via nucleation of bubbles with macroscopic core size. The rapid phase conversion by expanding bubbles might have observable effects on sudden change of Moment of Inertia (MI) of the Pulsar.

- There are various sources (e.g., bubbles, topological defects etc.) of density fluctuations arising during phase transitions may also alter the MI of the pusar. In fact, these density inhomogeneities might cause wobbling of the pulsar (due to develoment of off-diagonal components of MI tensor).

- Accurate measurements of pulsar timings and intensity modulations may be used to pin down particular phase transition ocurring inside the pulsar.

- The change of MI caused by density change and density inhomogeneities may account for glitches and anti-glitches.

- Rapidly evolving density fluctuations may also produce Quadrupole Moment (QM) and may be a new source of gravitational waves.

- The formation and evolution of topological defects shows universal characteristics and should lead to reasonably model independent predictions for changes in MI & QM and subsequent relaxation.

Case-I: Effects of Density Change During Phase Transitions

- Consider the scenario where the density of the core of a pulsar becomes supercritical by accreting matter [2] (in few million years) and bubbles are nucleated in a macroscopically large core of radius R_0. The subsequent phase conversion is expected to be very fast as is governed by the relativistic speed of bubble walls.

- Assume, the density of the star changes from ρ_1 to higher density ρ_2 inside the core. The fractional change of MI of the pulsar (of radius R $\simeq$ 10 km) is then estimated to be [3] :

$$\frac{\Delta I}{I} \simeq \frac{5}{3}\left(\frac{\rho_2}{\rho_1} - 1\right)\frac{R_0^3}{R^3}$$

- Observation from glitches : $\Delta I/I \leq 10^{-5} \Rightarrow R_0 \leq 0.3 \ km$ (for QCD transition) (considering about 30 % change in density)

- Note : For a superfluid transition, R_0 may be as large as 5 Km with superfluid condensation energy density $\simeq$ 0.1 MeV/fm^3.

● The equation of state for the phases (assuming zero temperature) [4]:

$$P_{nucleon} = \frac{M^4}{6\pi^2}\left(\frac{\mu}{M}\left(\frac{\mu^2}{M^2}-1\right)^{1/2}\left(\frac{\mu^2}{M^2}-\frac{5}{2}\right)+\frac{3}{2}ln\left[\frac{\mu}{M}+\left(\frac{\mu^2}{M^2}-1\right)^{1/2}\right]\right);$$

$$\epsilon_{nucleon} = \frac{2\mu}{3\pi^2}(\mu^2-M^2)^{3/2} - P_{nucleon} \quad ;$$

$$P_{QGP} = \frac{\mu_q^4}{2\pi^2} - B = \frac{1}{3}(\epsilon_{QGP} - 4B).$$

● For quantitative description of nucleation rate at zero temperature we took the model of quantum tunneling mediated by $O(4)$ symmetric instantons, having action
$S = -\frac{1}{2}\pi^2 R^4 \triangle P + 2\pi^2 R^3 S_1$

● Nucleation Rate : $\Gamma = A\frac{S_0^2}{4\pi^2}exp(-S_0)$; $A \simeq R_c^{-4}$; $R_c \Rightarrow$ Critical radius.

$S_0 = \frac{27\pi^2 S_1^4}{2(\Delta P)^3}$ is the Euclidean action of the instanton.

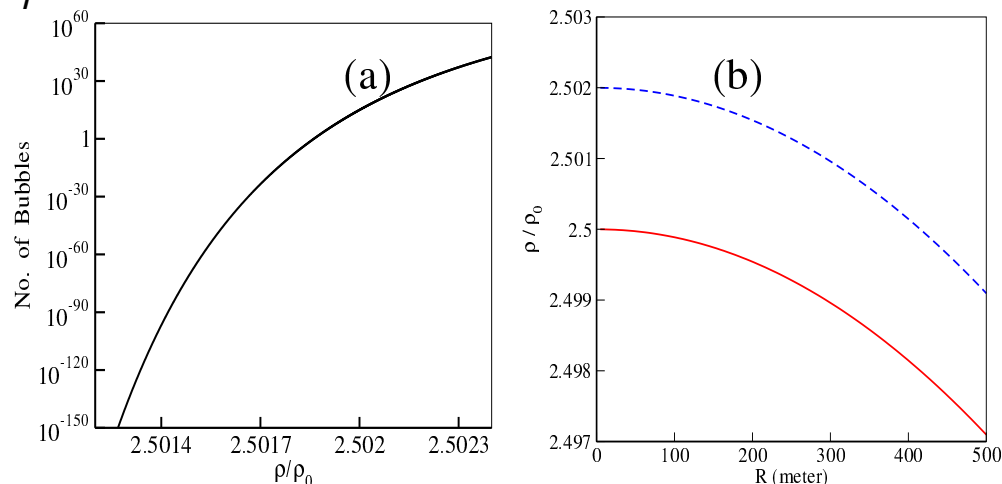The critical density and the critical radius are estimated as :
$\rho_c = 2.500\rho_0$ ($\rho_0 \simeq 0.15m_{nucleon} \Rightarrow$ Nuclear saturation density) and $R_c = 50$ fm ;

The Parameters used : $B^{1/4}$ = 177.9 MeV; $S_1 \equiv \sigma = 0.05$ MeV/fm^2
M = 1087.0 MeV (mean of the nucleon and delta mass)

Density Profile

- Density profile of the neutron star with supercritical core,

$$\frac{1}{\rho}\frac{dP}{dr} = -\frac{Gm}{r^2} \; ; \;\; dm = 4\pi r^2 \rho dr \; ; P = K\rho^\alpha ; \alpha = 2.54; K = 0.021\rho_0^{-1.54}$$

- (a) Number of bubbles nucleated in 300 meter core radius in one million year as a function of core density for a QCD transition, taking the acceration rate of 10^{17} grams/sec.

- (b) Density Profile, $\rho(r)$.

  - Red : $\rho(r)$ with $\rho(r = 0) = \rho_c \Rightarrow$ Neutron star mass, $M_1 = 1.564 M_0$.

  - Blue : When the supercritical core size (with $\rho > \rho_c$) has increased
    to about 300 meter and neutron star, mass $M_2 = 1.567 M_0$.

Case-II : Effects of Density Fluctuations During Phase Transitions

- Density inhomogeneities can be produced by nucleation of large number of bubbles or via formation of Topological defects (string, domain wall etc.).

- Various defects generate different density fluctuation, with specific evolution patterns. High precision measurements of pulsar timings, intensity modulations (wobbling) and its relaxation may be used to identify different sources of fluctuations, thereby pinning down the specific phase transition occurring.

- Random nucleation of spherical bubbles of radius $r_0 = 20 - 50$ meters filling up a spherical core of radius $R_0 = 300$ meters, with QCD transition, $\delta I / I \simeq 4 \times 10^{-8}$.

- Change in Off diagonal component of MI w.r.t initial MI is $10^{-11} - 10^{-10}$.

- Confinement-deconfinement (C-D) transition was studied by taking expectation value of Polyakov loop $l(x)$ as the order parameter for the transition.

$$ l(x) = \frac{1}{3} tr \left(P exp \left(ig \int_0^\beta A_0(x, \tau) d\tau \right) \right); \ \ P \Rightarrow Path \ \ ordering $$

- Note, the expectation value of the Polyakov loop $< l(x) >= \exp(-\beta F)$ measures the free energy F of an external static quark.

$$ \begin{aligned} < l(x) > \quad &\equiv \quad l_0 = 0 \ \ (T < T_c) \\ &\neq \quad 0 \ \ (T > T_c) \end{aligned} $$

- D-C transition can be described by the effective Lagrangian [6],

$$\mathcal{L} = \frac{N}{g^2}|\partial_\mu l|^2 T^2 - V(l).$$

$$V(l) = \left(-\frac{b_2}{2}|l|^2 - \frac{b_3}{6}\left(l^3 + (l^*)^3\right) + \frac{1}{4}\left(|l|^2\right)^2\right) b_4 T^4.$$

b_2, b_3 and b_4 are chosen to fit lattice results for free energy density and pressure.

- $l = |l|e^{i\theta}$ leads to $cos(3\theta)$ in the b_3 term leading to $\mathbf{Z}(3)$ degenerate vacua.

- Domain walls (three of them) interpolate between different vacua. Topological string (QGP string [7]) arises at the junction of three different $\mathbf{Z}(3)$ interfaces.
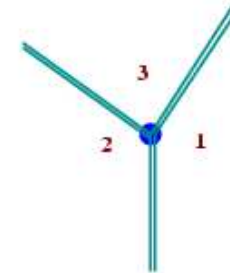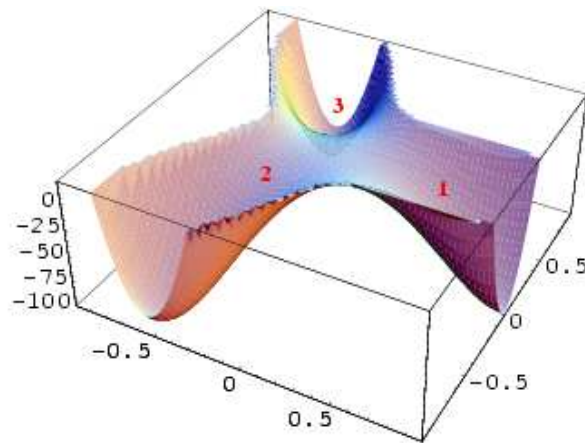
Results

Fractional Change in MI and QM During Phase Transitions :
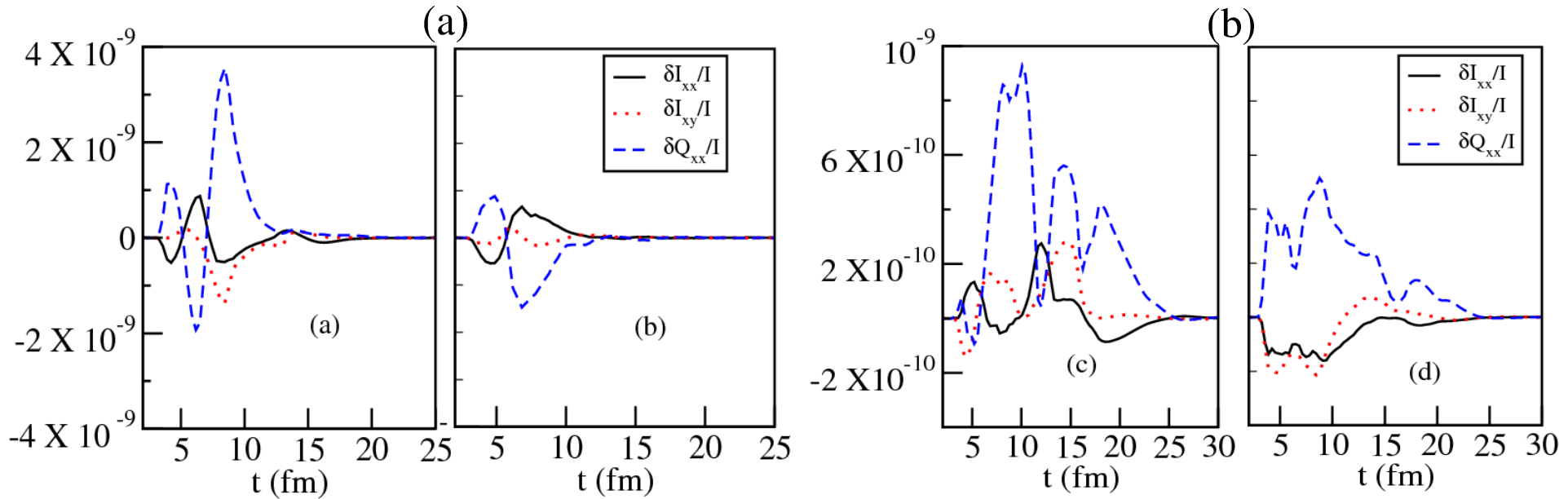
(a),(b) correspond to lattice size $(7.5 \text{ fm})^3$.
(c),(d) correspond to lattice size $(15 \text{ fm})^3$.
(a) and (c) correspond to the C-D phase transition with Z(3) walls and strings.
(b) and (d) correspond to the transition with only string formation as for the CFL phase.
Size of the core : $0.4 \times$ Lattice size .

Studying the Effects by Static Approach

- In the static approach, we model string and wall by forming correlation domains in a cubic lattice, with lattice spacing ξ representing the correlation length [8].

- The mass density of the string and the domain wall tension were taken [9] as 3 GeV/fm and 7 GeV/fm^2, respectively.

- We confine defect network within core radius $R_c = \frac{0.3}{10} R$.

ξ=10 fm	QCD Strings			QCD Walls			Superfluid Strings		
$\frac{R_c}{\xi}$	$\frac{\delta I_{xx}}{I}$	$\frac{\delta I_{xy}}{I}$	$\frac{Q_{xx}}{I}$	$\frac{\delta I_{xx}}{I}$	$\frac{\delta I_{xy}}{I}$	$\frac{Q_{xx}}{I}$	$\frac{\delta I_{xx}}{I}$	$\frac{\delta I_{xy}}{I}$	$\frac{\delta Q_{xx}}{I}$
5	5E-10	-3E-10	-1E-10	2E-8	-1E-8	-8E-10	2E-6	-1E-6	-4E-7
50	5E-12	-2E-12	2E-12	1E-10	-8E-11	-1E-11	2E-8	-7E-9	7E-9
200	1E-13	2E-14	-7E-14	5E-12	-4E-12	-6E-12	5E-10	6E-11	-2E-10
400	-3E-15	-5E-14	-9E-14	3E-12	-2E-12	3E-14	-1E-11	-2E-10	-3E-10

Note: The ratio of quadrupole moment to the MI is much smaller than the quadrupole moment due to deformation of the star. However, power emitted in gravitational waves [$\sim (\dddot{Q})^2$] may not be small due to very short time scale of microseconds.

Gravitational wave generation due to density fluctuation

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$$\frac{dE}{dt} = -\frac{32G}{5c^5}\triangle Q^2 \omega^6 \simeq -(10^{33}J/s)(\frac{\triangle Q/I_0}{10^{-6}})^2(\frac{10^{-3}sec}{\triangle t})^6$$

- For conservative estimates we have taken :

$$\triangle Q/I_0 \simeq 10^{-14} - 10^{-10}$$

$$\triangle t = 10^{-6} - 10^{-5}sec$$

- Expected strain amplitude from a pulsar at a distance r,

$$h = \frac{4\pi^2 G\triangle Q f^2}{c^4 r} \simeq 10^{-24}(\frac{\triangle Q/I_0}{10^{-6}})(\frac{10^{-3sec}}{\triangle t})^2(\frac{1kpc}{r})$$

- With $\triangle Q/I_0 \simeq 10^{-10}$ and $\triangle t = 10^{-6} - 10^{-5}$ sec, strain amplitude comes out to be $h \simeq 10^{-24} - 10^{-22}$ for a pulsar at 1kpc distance.

Concluding Remarks

- The net fractional change in the MI is noted to be dominated by the phase change and is of order 10^{-6} whereas the string induced fractional change in MI is about 3 orders of magnitude smaller, of order 10^{-10}.

- QM and off-diagonal components of MI are also found to be of order 10^{-10}.

- Note that these numbers are not far with the values for a glitch (or anti-glitch). Thus, the change of MI caused by density change and density inhomogeneities may account for glitches and anti-glitches.

- The transient change in the MI decays away when the string system coarsens. Thus one expects a net rapid change in the spinning rate and restoration of only few percent of the original value.

- Ratio of the quadrupole moment to MI is though very small, however note that gravitation power depends on the (square of) third time derivative of the quadrupole moment. Here, phase transition dynamics will lead to changes in density fluctuations occurring in time scales of microseconds. This may more than compensate for the small amplitude and may lead to these density fluctuations as an important source of gravitational wave emission from neutron stars.

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