Speed of Sound at Chiral Phase Transition

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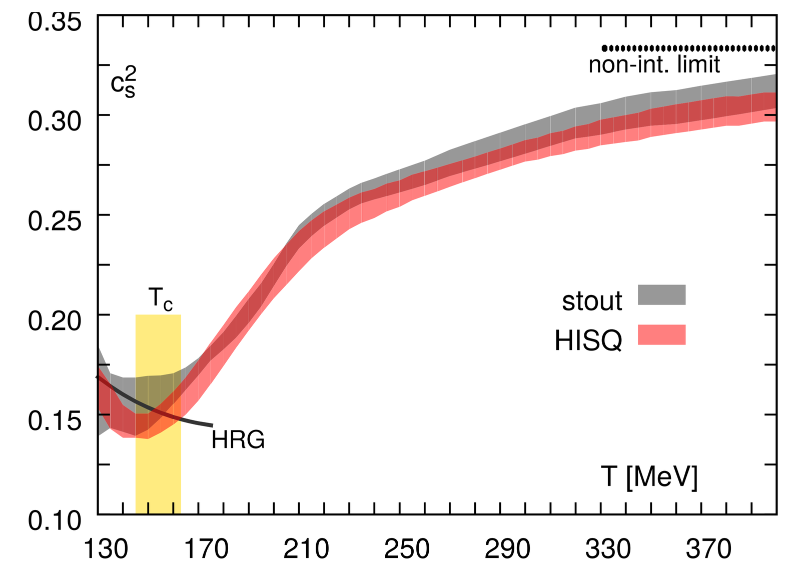
Speed of Sound in QCD

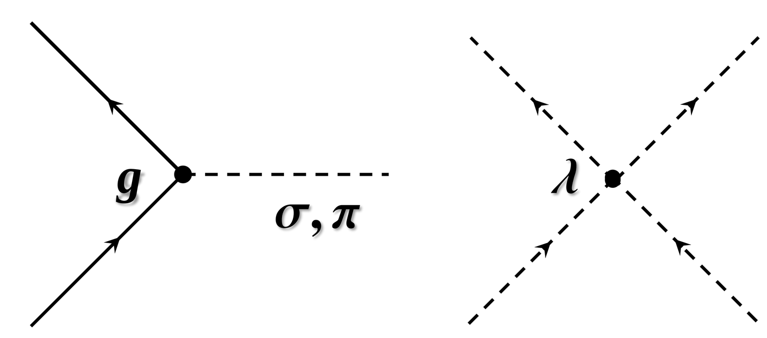
- Speed of Sound $c_s^2 = \left(\frac{\partial p}{\partial \varepsilon}\right)_{\text{adiabatic}}$ characterizes "Stiffness" of a matter.
- Hydrodynamic evolution of the hot and dense matter created in relativistic heavy ion collisions is governed by c_s. In particular, minimum of c_s, "softest point", is expected to affect observables such as flow and correlations.
- The softest point has been measured in lattice QCD as $c_s^2 \simeq 0.15$ at T=145-150 MeV.
- $c_s^2 = \frac{s}{C_V} = \frac{s}{\partial\varepsilon/\partial T}$ at μ=0 (s: entropy density and C_V: specific heat).
  – looks sensitive to deconfinement. Clearly c_s=0 at a 1st order phase transition.
- At a 2nd order transition, the behavior of c_s depends on the critical exponent of the specific heat α, which is dominated by chiral dynamics. c_s vanishes if α>0 (e.g., $Z(2)$ universality class) and stays nonzero for α<0 (e.g., $O(4)$).
- This work : what is the role of the chiral transition in the softening of EoS?



A. Bazavov et al., (HotQCD Collaboration), Phys. Rev. D90, 094503 ('14)

Thermodynamics of a two-flavor chiral quark-meson model

- Lagrangian $\mathscr{L} = \bar{q}[i\gamma_\mu\partial^\mu - g(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})]q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 - U(\sigma,\vec{\pi}),\quad U(\sigma,\vec{\pi}) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\phi^4 - h\sigma,\quad \phi = (\sigma,\vec{\pi})$
  - q, π and σ as effective degrees of freedom. $O(4)$ symmetry broken to $O(3)$ when m^2<0.



- Thermodynamic potential in Mean Field Approximation

1. No-sea approximation : ignoring vacuum fluctuation of quarks.

$$\Omega_{\text{No-Sea MF}}(\langle\sigma\rangle;T,\mu) = U(\langle\sigma\rangle,\vec{\pi}=0) - 12T\int\frac{d^3p}{(2\pi)^3}[\ln(1+e^{-(E_q-\mu)/T}) + \ln(1+e^{-(E_q+\mu)/T})]$$

The order parameter $\langle\sigma\rangle$ is determined by the minimum of the effective potential.
A known problem : artificial 1st order phase transition in the chiral limit.

2. with renormalized vacuum fluctuation of quarks. (V. Skokov et al., Phys.Rev.D82, 034029 '10)

$$\Omega_{\text{MF}} = \Omega_{\text{No-Sea MF}} - \frac{12}{16\pi^2}M_q^2\ln\left(\frac{M_q}{M}\right)$$

M : arbitrary renormalization scale $M_q = g\langle\sigma\rangle$: dynamical quark mass

Including quark vacuum polarization at 1-loop gives a 2nd order phase transition in the chiral limit and significant effects on the phase structure.

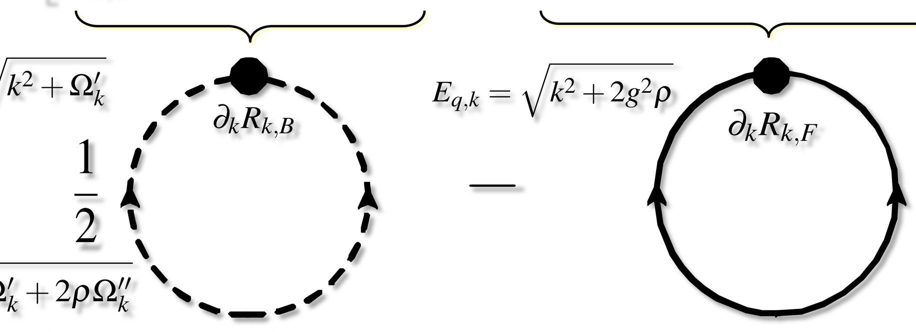
- Functional Renormalization Group (FRG)
  - Provides the evolution equation for a scale-dependent effective potential. (C. Wetterich, Phys.Lett.B301, 90 '93)
  - Systematically includes quantum/thermal fluctuations by integrating from k=Λ to k=0.
  - Critical exponent α can be reproduced at the leading order of the derivative expansion (LPA).

Flow equation for the scale-dependent effective potential

$$\partial_k\Omega_k = \frac{Vk^4}{12\pi^2}\left[\underbrace{\sum_{i=\pi,\sigma}\frac{d_i}{E_{i,k}}[1+2n_B(E_{i,k})]} - \underbrace{\frac{24}{E_{q,k}}[1-n_F(E_{q,k}-\mu)-n_F(E_{q,k}+\mu)]}\right]$$

$$\rho = \frac{\phi^2}{2}\qquad \Omega' \equiv \frac{\partial\Omega(\rho)}{\partial\rho}$$

$$E_{\pi,k} = \sqrt{k^2+\Omega'_k}\qquad E_{\sigma,k} = \sqrt{k^2+\Omega'_k+2\rho\Omega''_k}\qquad E_{q,k} = \sqrt{k^2+2g^2\rho}$$



Mesonic fluctuations smoothen transition near crossover

  - Flow equation was solved by a Taylor expansion around the potential minimum at $\rho = \rho_k$.
  - UV cutoff : Λ=1 GeV.
    – Higher momentum contribution from the ideal quark-gluon gas is added. (J. Braun et al., Phys.Rev.D70, 085016 ('04))

- Common setup

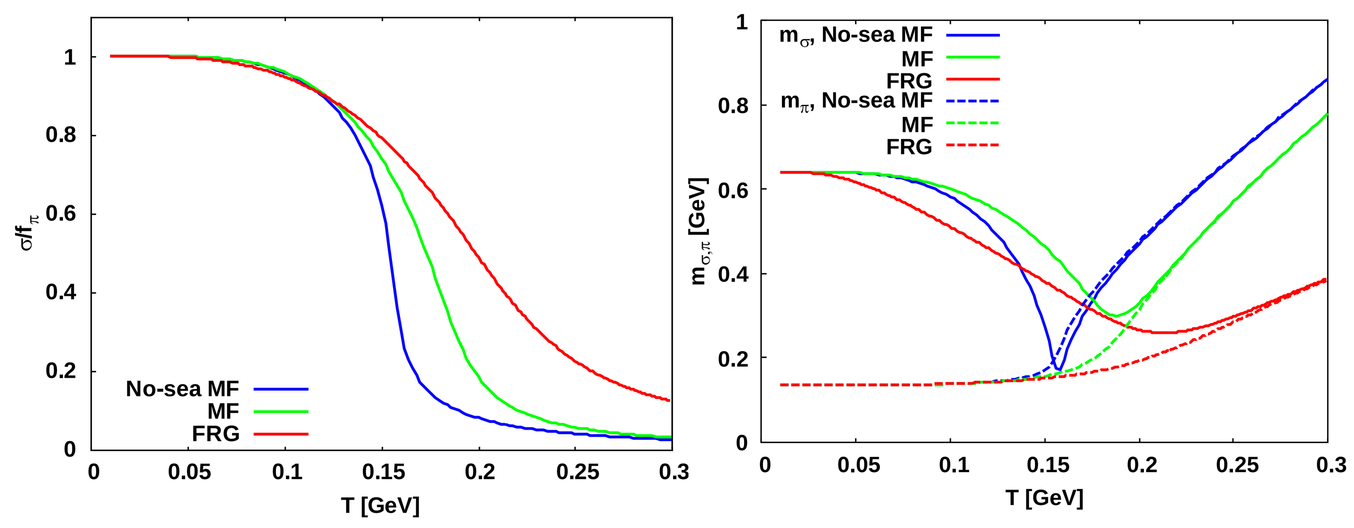
$$h = f_\pi m_\pi^2 = (93\text{MeV})\times(135\text{MeV})^2,\ g = 3.2,\ m_\sigma = 640\text{MeV}$$

  - m^2 and λ are fitted to reproduce the vacuum parameters.

- Formula for Speed of Sound
  - In terms of susceptibilities

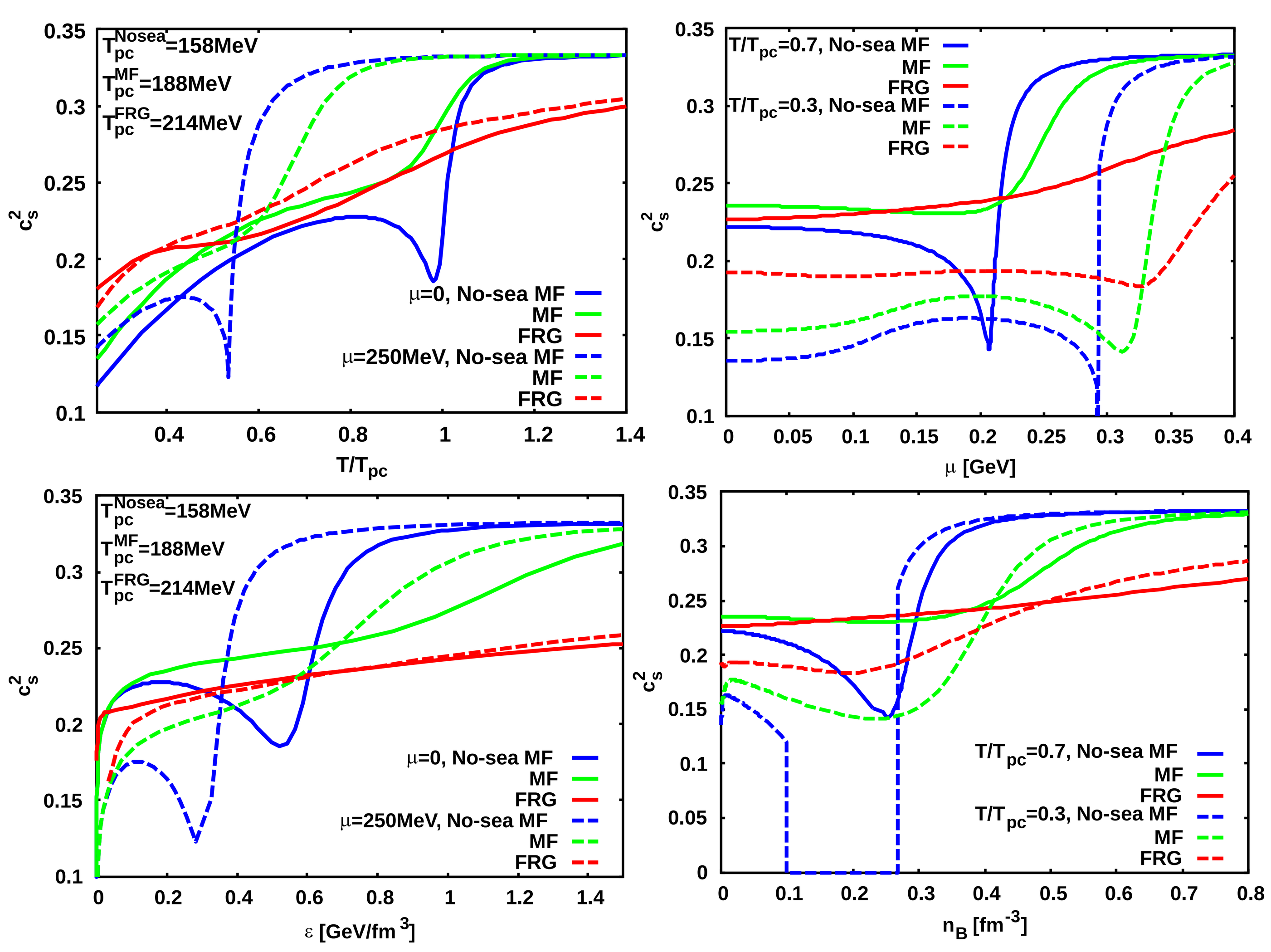
$$c_s^2 = \frac{s^2\chi_{\mu\mu} - 2ns\chi_{\mu T} + n^2\chi_{TT}}{(\varepsilon+p)(\chi_{TT}\chi_{\mu\mu} - \chi_{\mu T}^2)},\qquad \chi_{\alpha\beta} \equiv \frac{\partial^2 p(T,\mu)}{\partial\alpha\partial\beta}$$

Order parameter and meson mass



- Fermionic vacuum fluctuations and mesonic loops significantly modify the behavior around pseudo-critical temperature.

Speed of Sound at finite T and μ



- Significant smearing by fluctuations
  - Dip in "No-Sea" is smeared by fermionic vacuum. (U.S.Gupta et al., Phys.Rev.D85, 014010 ('12))
  - Rapid rise in "MF" is smeared by mesonic fluctuations.
  - FRG does not exhibit softening around chiral crossover and approaches to SB limit very slowly.

Discussion

- Near pseudo-critical temperature of chiral crossover, fermionic and bosonic fluctuations smear strong T and μ dependence seen in mean-field level. As a result, the speed of sound does not exhibit the softest point with the chiral crossover alone. Consequently, the softening of EoS seen in lattice QCD might be solely attributed to deconfinement.
- At low temperature and high baryon density, the stiffness of EoS is a crutical ingredient in the high-mass neutron star puzzle. The difference between MFs and FRG indicates an important role of mesonic fluctuations. If the transition to quark matter is crossover, the similar mechanism might be relevant for a possible resolution of the puzzle.