Effects of Momentum Cuts on Higher-Order Cumulants of Conserved Charges
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Fluctuations of Conserved Charges
- Promising observables to investigate thermodynamic property of the system.
- Couple to order parameters – may exhibit critical behavior in the vicinity of a phase transition.
- Higher order cumulants are more sensitive to the criticality.
- Net-proton (as a proxy of net-baryon), net-electric charge, and net-kaon (as a proxy of net-strangeness) have been measured in BES program at RHIC.
- To confront the data with theory, one needs to understand a non-critical baseline.
- This work: momentum cuts on the baseline for net-electric charge and on the chiral crossover for net-baryon number.

Momentum cuts in net-electric charge fluctuations
- Charge fluctuation in an ideal pion gas
  \[ \chi^2 = \frac{1}{N} \sum \left( \frac{E_{\text{kin}}(k,T) \cos(kq/T) - \left\langle E_{\text{kin}}(k,T) \cos(kq/T) \right\rangle}{\left\langle E_{\text{kin}}(k,T) \cos(kq/T) \right\rangle} \) 
  \[ E_{\text{kin}}(k,T) = \frac{k^2}{2m} + \frac{q^2}{2m} \]
- Bose gas = multicomponent Boltzmann gas with mass \( m \) and charge \( kQ \)
- The leading order, \( k=1 \), corresponds to the Boltzmann approximation, where the fluctuations are given by the Skeltman distribution (Poisson baseline).
- In this case fluctuations do not depend on momentum cuts and \( \chi^2_{\text{cut}} = \chi^2_{\text{uncut}} = 1 \).

Role of higher order terms in \( k \):
- Always larger than unity.
- Larger contribution in higher order cumulants.

Effects of momentum cuts

Cumulant ratios in Hadron Resonance Gas Model

Scenarios for momentum cuts
- A) pT < 0.2 GeV/c
- B) pT < 0.4 GeV/c
- C) A larger pT cut, (0.6 GeV/c) for protons and antiprotons (STAR).

Influence of the collective flow is small.

Scale dependence of \( \langle \sigma \rangle \) and masses

Momentum scale dependence of net baryon number fluctuations
- Negligible effects from quantum statistics due to heavy proton mass.
- What about effects of chiral crossover?
- Use of functional renormalization group to see momentum scale dependence of net-baryon number fluctuations in chiral quark-meson model.
  \[ X = (\rho_{\pi}^{\text{free}} - \rho_{\pi}^{\text{vac}} + \frac{1}{3} \rho_{\pi}^{\text{conf}} - \frac{1}{3} \rho_{\pi}^{\text{str}})^2 - U(\sigma, \varepsilon) \]
  \[ \text{provides the evolution equation for a scale-dependent effective potential.} \]
  \[ \text{Systematically includes quantum/thermal fluctuations by integrating from } kQ \text{ to } kQ. \]
  \( \varepsilon \) critical exponent can be reproduced at the leading order of the derivative expansion (LPA).
  \[ \rho_{\pi}^{\text{free}} = \frac{v^2}{2m^2} \left( \sum \left[ 1 + 2m^2(E_{\pi}(m)) \right] \left( \frac{2m^2}{E_{\pi}(m)} \right) - \frac{2m^2}{E_{\pi}(m)} - v^2 \right) \]
  \[ \rho_{\pi}^{\text{str}} = \frac{v^2}{2m^2} \left( \sum \left[ 1 + 2m^2(E_{\pi}(m)) \right] \left( \frac{2m^2}{E_{\pi}(m)} \right) - \frac{2m^2}{E_{\pi}(m)} - v^2 \right) \]
  \[ \rho_{\pi}^{\text{conf}} = \frac{v^2}{2m^2} \left( \sum \left[ 1 + 2m^2(E_{\pi}(m)) \right] \left( \frac{2m^2}{E_{\pi}(m)} \right) - \frac{2m^2}{E_{\pi}(m)} - v^2 \right) \]
  \[ \text{Flow equation was solved by Taylor expansion around the potential minimum at } \rho_{\pi} = \rho_{\pi}^{\text{opt}}. \]
  \[ \text{UV cutoff: } \Lambda = 1 \text{ GeV}. \]
  - Higher momentum contribution from the ideal quark-ghost gas is added.
  - Potential parameters are fitted to reproduce the vacuum parameters.
  - \( h = f_{\text{opt}} m^2 = (190 \text{MeV}) \times (153 \text{MeV})^3, \varepsilon = 3.2, m_0 = 660 \text{MeV} \)

Scale dependence of \( \langle \sigma \rangle \) and masses

Chiral symmetry breaking at low \( k \) at \( T=0 \)
- Thermal fluctuations prevent \( \Gamma_{\text{had}} \) at \( T_{\text{pc}} \)
- Saturation near \( k-m_0 \)

Scale dependent fluctuations
- Criticality in cumulant ratios (\( \chi^2_{\text{cut}} \) at \( pT=0 \) and \( \chi^2_{\text{cut}} \) at large \( pT \)) disappear when \( k > 2 m_0 \) due to lack of contribution from soft mesonic fluctuations.
- Low \( pT \) particles are presumably more affected by soft modes – cutting low \( pT \) particles may influence the criticality of cumulants.