Effects of Momentum Cuts on Higher-Order Cumulants of Conserved Charges

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Fluctuations of Conserved Charges

- Promising observables to investigate thermodynamic property of the system.
- Couple to order parameters – may exhibit critical behavior in the vicinity of a phase transition.
- Higher order cumulants are more sensitive to the criticality. $\chi_n^i \equiv \frac{\partial^n p(T,\{\mu_i\})/T^4}{\partial(\mu_i/T)^n}$
- Net-proton (as a proxy of net-baryon), net-electric charge, and net-kaon (as a proxy of net-strangeness) have been measured in BES program at RHIC.
- To confront the data with theory, one needs to understand a non-critical baseline.
- This work : momentum cuts on the baseline for net-electric charge and on the chiral crossover for net-baryon number.

Momentum cuts in net-electric charge fluctuations

F.Karsch, KM, K.Redlich, arXiv: 1508.02614

- Charge fluctuation in an ideal pion gas

$$p(T,\mu_Q) = -T\sum_{i=1}^{3}\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\ln(1-e^{-(E_p-\mu_Q Q_i)})$$

$$\chi_n^Q = \begin{cases} \frac{m^2}{\pi^2T^2}\sum_{k=1}^{\infty}k^{n-2}\hat{K}_2(km/T)\cosh(k\mu_Q/T), & n=\text{even} \\ \frac{m^2}{\pi^2T^2}\sum_{k=1}^{\infty}k^{n-2}\hat{K}_2(km/T)\sinh(k\mu_Q/T), & n=\text{odd}\end{cases}$$

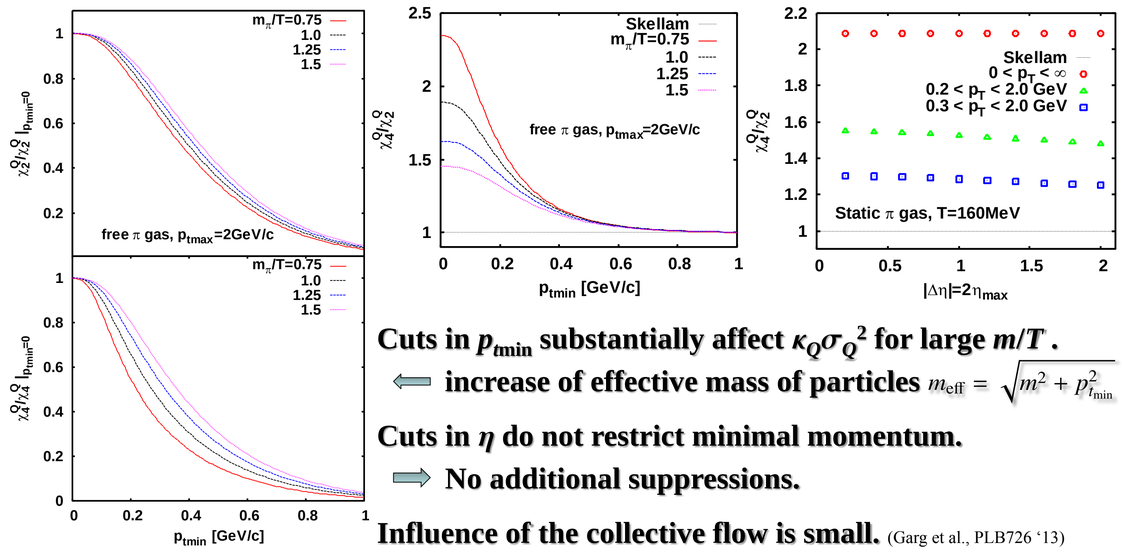
$$\hat{K}_2(km/T) = 2\int_{\eta_{\min}}^{\eta_{\max}}d\eta\int_{p_{t_{\min}}}^{p_{t_{\max}}}dp_t p_t|\boldsymbol{p}|e^{-kE_p/T}$$
$$= K_2(km/T),\ \text{for}\ \eta_{\min}=-\infty,\ \eta_{\max}=\infty,\ p_{t_{\min}}=0,\ \text{and}\ p_{t_{\max}}=\infty$$

⇨ Bose gas = multicomponent Boltzmann gas with mass km and charge kQ

- The leading order, k=1, corresponds to the Boltzmann approximation, where the fluctuations are given by the Skellam distribution (Poisson baseline). In this case fluctuations do not depend on momentum cuts and

$$S_Q\sigma_Q = \frac{\chi_3^Q}{\chi_2^Q} = \tanh(\mu_Q/T),\quad \kappa_Q\sigma_Q^2 = \frac{\chi_4^Q}{\chi_2^Q} = 1.$$

- Role of higher order terms in k:
  - Always larger than unity.
  - Larger contribution in higher order cumulants. ⇨ $\frac{\chi_4^Q}{\chi_2^Q} > 1$
- Effects of momentum cuts



Cuts in p_{tmin} substantially affect $\kappa_Q\sigma_Q^2$ for large m/T .

⇦ increase of effective mass of particles $m_{\rm eff} = \sqrt{m^2+p_{t_{\min}}^2}$.

Cuts in η do not restrict minimal momentum.

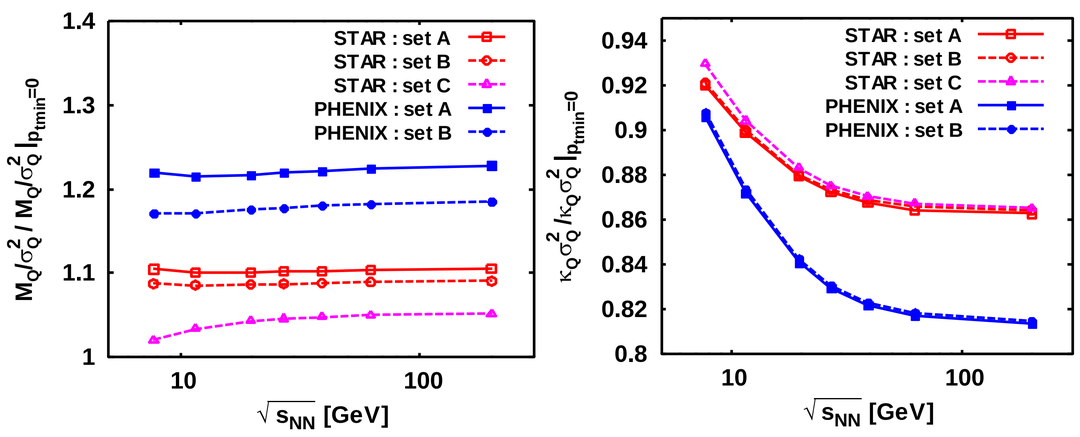
⇨ No additional suppressions.

Influence of the collective flow is small. (Garg et al., PLB726 '13)

Cumulant ratios in Hadron Resonance Gas Model

HRG parameters: F.Karsch and K.Redlich, Phys. Lett. B695, 136 ('11)

- Scenarios for momentum cuts
  - A) p_{tmin} is introduced only for the pion contribution.
  - B) Same momentum cuts for all charged hadrons
  - C) A larger p_{tmin} (0.4GeV) for proton and antiprotons (STAR)



Cuts in p_{tmin} increases M_Q/σ_Q^2, by cutting negative contribution from pions due to negative μ_Q and partly account for difference between STAR (p_{tmin}=0.2GeV) and PHENIX (p_{tmin}=0.3GeV).

$\kappa_Q\sigma_Q^2$ receives weaker (10-20%) reduction than pion gas, owing to additional d.o.f. in HRG.

Momentum scale dependence of net baryon number fluctuations

KM, K.Redlich, PTEP 043D03 '15

- Negligible effects from quantum statistics due to heavy proton mass.
- What about effects of chiral crossover?
- Use of functional renormalization group to see momentum scale dependence of net-baryon number fluctuations in chiral quark-meson model.

$$\mathscr{L} = \bar{q}[i\gamma_\mu\partial^\mu - g(\sigma+i\gamma_5\vec{\tau}\cdot\vec{\pi})]q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 - U(\sigma,\vec{\pi}),$$

- Provides the evolution equation for a scale-dependent effective potential.
- Systematically includes quantum/thermal fluctuations by integrating from k=Λ to k=0.
- Critical exponent α can be reproduced at the leading order of the derivative expansion (LPA).

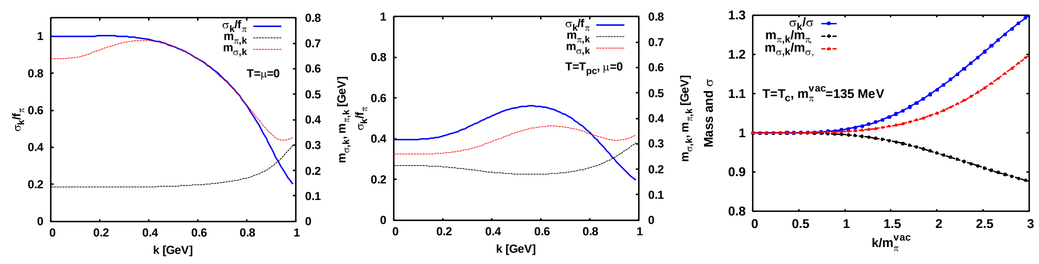
$$\partial_k\Omega_k = \frac{Vk^4}{12\pi^2}\left[\sum_{i=\pi,\sigma}\frac{d_i}{E_{i,k}}[1+2n_B(E_{i,k})] - \frac{24}{E_{q,k}}[1-n_F(E_{q,k}-\mu)-n_F(E_{q,k}+\mu)]\right]$$

$$\rho = \frac{\phi^2}{2}\quad \Omega' \equiv \frac{\partial\Omega(\rho)}{\partial\rho}\quad E_{\pi,k} = \sqrt{k^2+\Omega'_k}\quad E_{\sigma,k} = \sqrt{k^2+\Omega'_k+2\rho\Omega''_k}\quad E_{q,k} = \sqrt{k^2+2g^2\rho}$$

- Flow equation was solved by a Taylor expansion around the potential minimum at $\rho = \rho_k$.
- UV cutoff : Λ=1 GeV.
  - Higher momentum contribution from the ideal quark-gluon gas is added. (J. Braun et al., Phys.Rev.D70, 085016 ('04))
  - Potential parameters are fitted to reproduce the vacuum parameters.

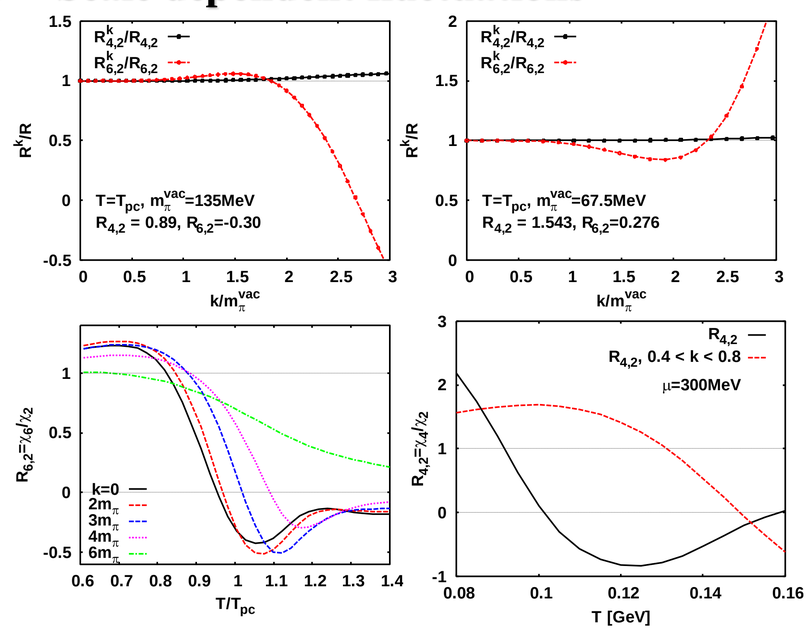
$$h = f_\pi m_\pi^2 = (93\text{MeV})\times(135\text{MeV})^2,\ g=3.2,\ m_\sigma = 640\text{MeV}$$

Scale dependence of $\langle\sigma\rangle$ and masses



- Chiral symmetry breaking at low k at T=0
- Thermal fluctuations prevent χ_{SB} at T_{pc}
- Saturation near k~m_π

Scale dependent fluctuations



- Criticality in cumulant ratios (χ_6/χ_2 <0 at μ=0 and χ_4/χ_2<0 at large μ) disappear when $k > 2m_\pi$, due to lack of contribution from soft mesonic fluctuations.
- Low p_t particles are presumably more affected by soft modes – cutting low p_t particles may influence the criticality of cumulants.