

Parallel session: Collective Dynamics II



Studies on longitudinal fluctuations of anisotropic flow event planes in Pb-Pb and p-Pb collisions at CMS

Maxime Guilbaud

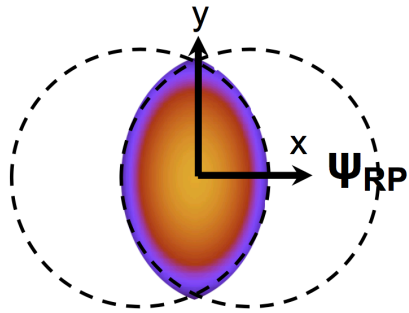
Bonner Laboratory, RICE University, Houston TX

On behalf of CMS collaboration



The XXVth International Conference on Ultrarelativistic Nucleus-Nucleus Collisions

Initial-state: an evolving picture



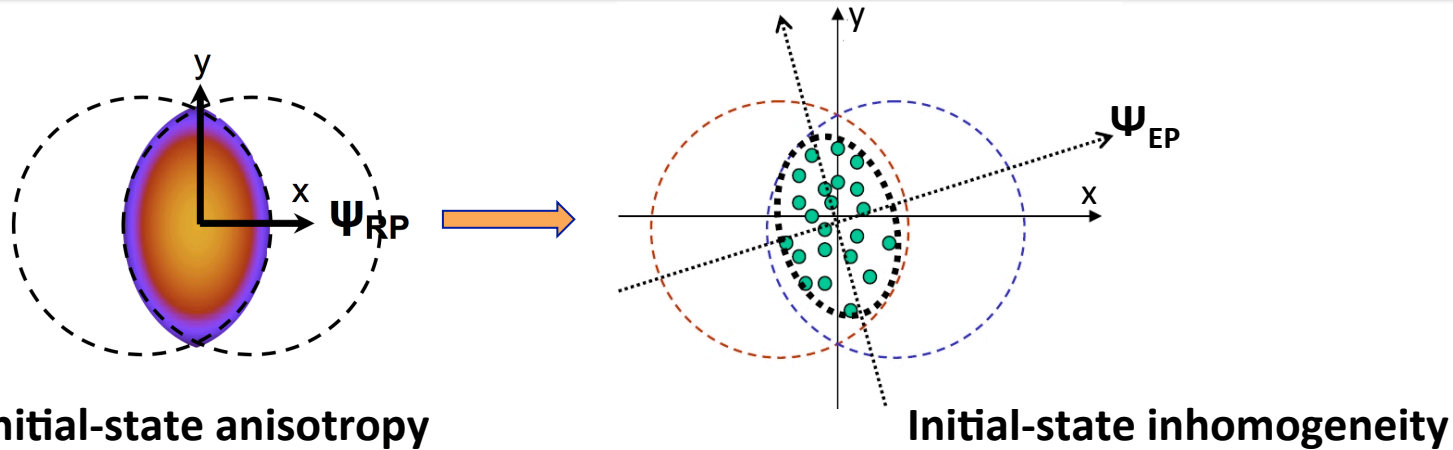
Initial-state anisotropy

Final state:

$$f(p_T, \phi, \eta) \sim 1 + 2v_2(p_T, \eta) \cos [2(\phi - \Psi_{RP})]$$

Elliptic flow

Initial-state: an evolving picture



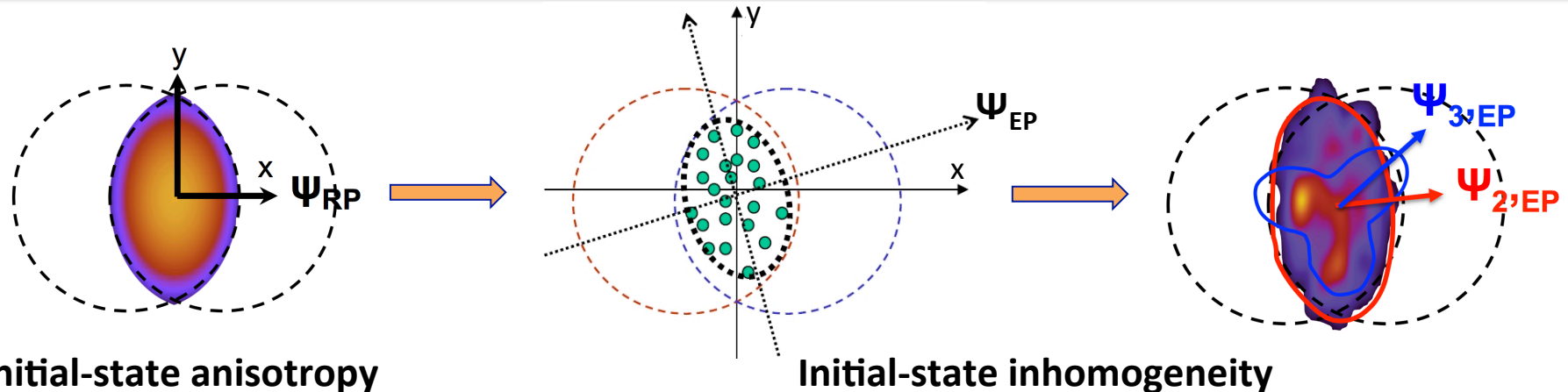
Ψ_{EP} : Direction of maximum particle density

Final state:

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Elliptic flow

Initial-state: an evolving picture



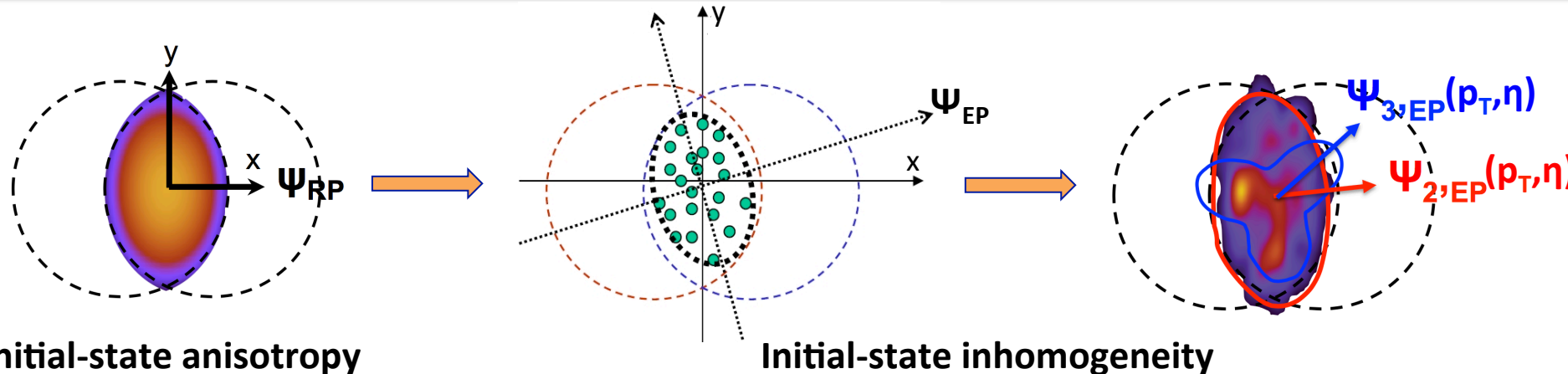
Ψ_{EP} : Direction of maximum particle density

Final state:

$$f(p_T, \phi, \eta) \sim 1 + 2v_2(p_T, \eta) \cos [2(\phi - \Psi_2)] + 2v_3(p_T, \eta) \cos [3(\phi - \Psi_3)] + 2v_4(p_T, \eta) \cos [4(\phi - \Psi_4)] + 2v_5(p_T, \eta) \cos [5(\phi - \Psi_5)] + \dots$$

Elliptic flow
Triangular flow

Initial-state: an evolving picture



Ψ_{EP} : Direction of maximum particle density

Final state:

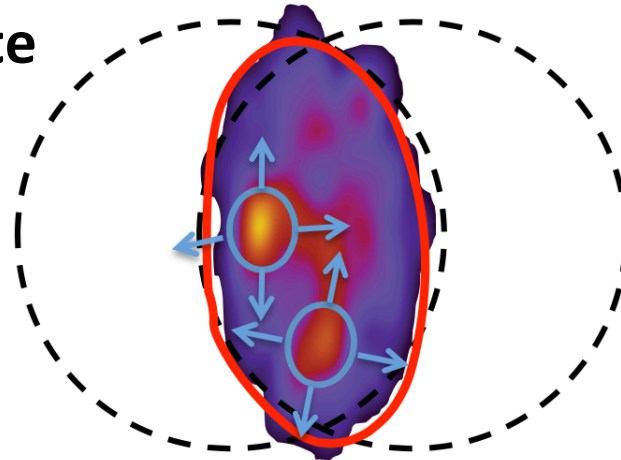
$$f(p_T, \phi, \eta) \sim 1 + 2v_2(p_T, \eta) \cos [2(\phi - \Psi_2(p_T, \eta))] + 2v_3(p_T, \eta) \cos [3(\phi - \Psi_3(p_T, \eta))] + 2v_4(p_T, \eta) \cos [4(\phi - \Psi_4(p_T, \eta))] + 2v_5(p_T, \eta) \cos [5(\phi - \Psi_5(p_T, \eta))] + \dots$$

Elliptic flow
Triangular flow

Initial-state granularity



Closer look at initial-state and its fluctuations:



Ψ_{EP} : Direction of maximum particle density

Final state:

$$f(p_T, \phi, \eta) \sim 1 + 2 \sum_n v_n(p_T, \eta) \cos [2(\phi - \Psi_n(p_T, \eta))]$$

- ❖ Local hot spots: **decorrelation?**
- ❖ $\Psi_n(p_T, \eta)$: details about **3-D initial state fluctuations** (r, ϕ, η)

Most of flow measurement assume factorization:

$$V_{n\Delta}(p_T^a, p_T^b) = v_n(p_T^a) \times v_n(p_T^b)$$

$$r_n(p_T^a, p_T^b) \equiv \frac{V_{n\Delta}(p_T^a, p_T^b)}{\sqrt{V_{n\Delta}(p_T^a, p_T^a)} \sqrt{V_{n\Delta}(p_T^b, p_T^b)}} = 1$$



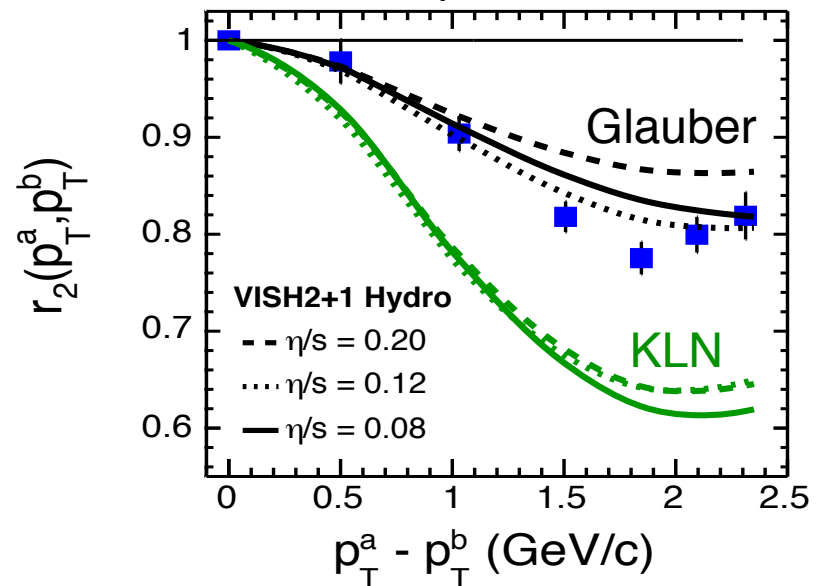
Due to EP $\Psi_n(p_T)$ caused by **lumpy** initial state:

$$V_{n\Delta}(p_T^a, p_T^b) \neq v_n(p_T^a) \times v_n(p_T^b)$$

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0-0.2% ultra-central PbPb

$2.5 < p_T^a < 3.0 \text{ GeV}/c$



arXiv:1503.01692

Flow factorization breaking in p_T

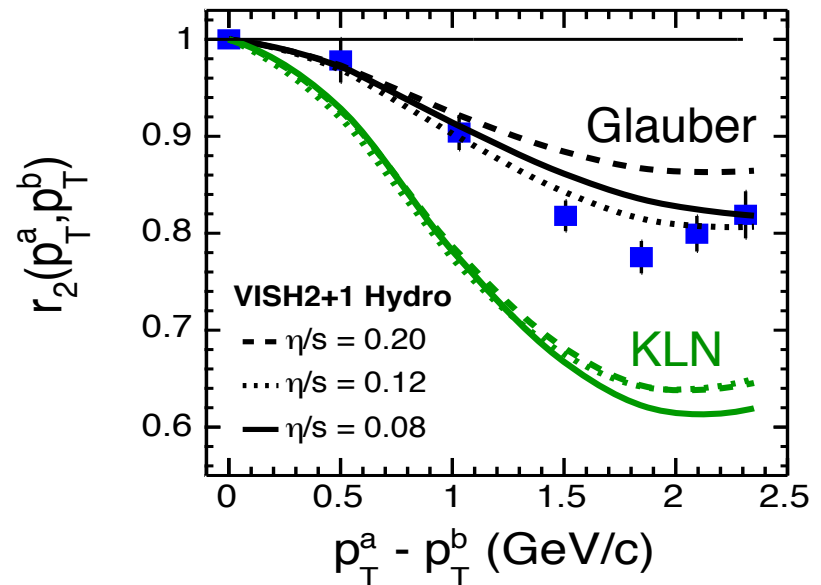


Due to EP $\Psi_n(p_T)$ caused by **lumpy** initial state:

Does not depend much on η/s
access to initial state effect only

0-0.2% ultra-central PbPb

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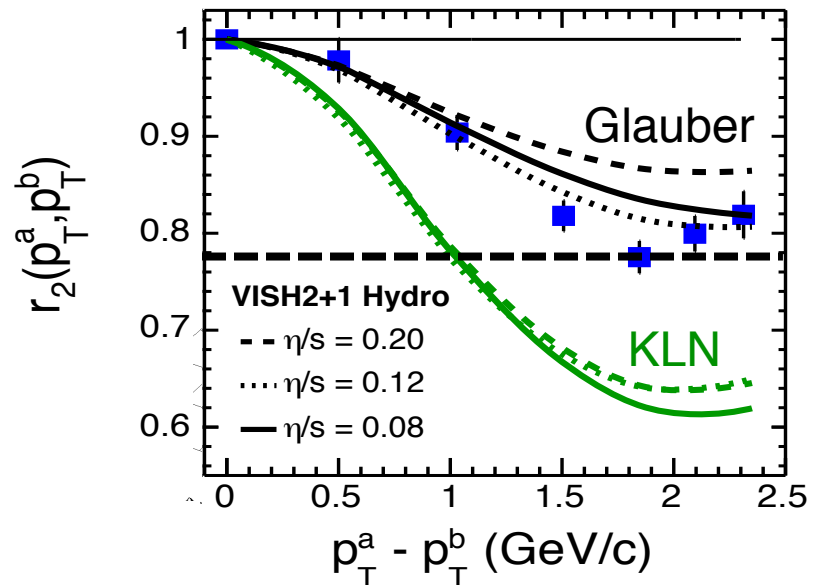
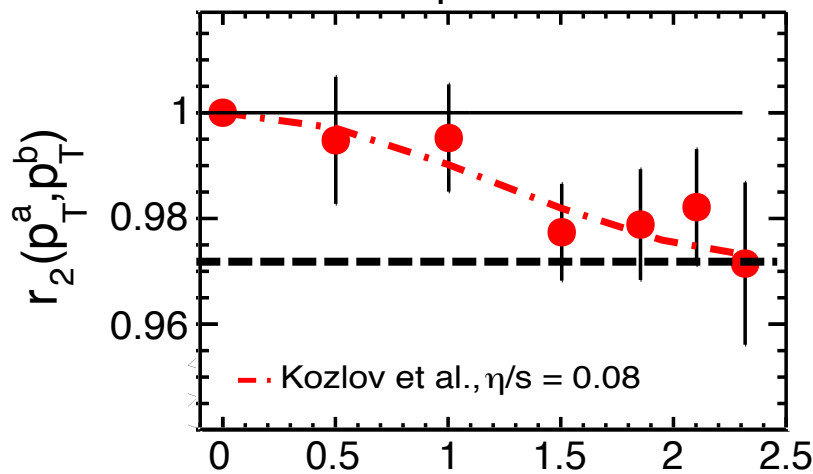
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pPb, $220 < N_{\text{trk}} < 260$

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$2.5 < p_T^a < 3.0 \text{ GeV}/c$

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$p_T^a - p_T^b$ (GeV/c)

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Flow factorization breaking in p_T

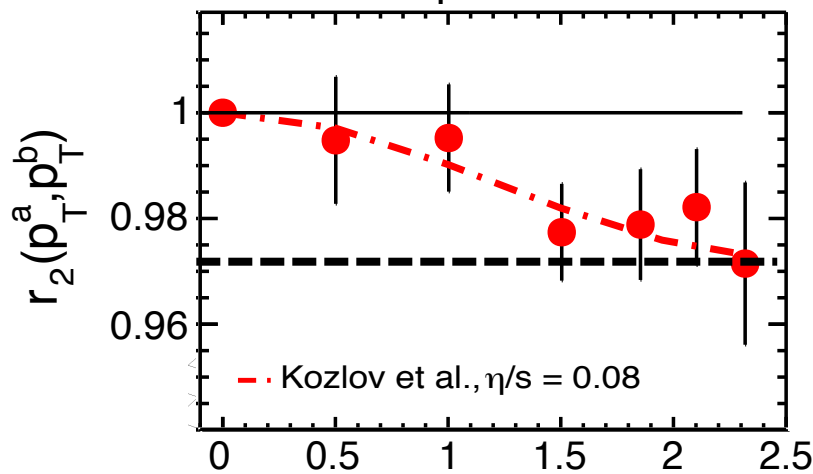


Due to EP $\Psi_n(p_T)$ caused by **lumpy** initial state:

**Bigger effect in Pb-Pb (20% at maximum)
than in p-Pb (3% at maximum)**

pPb, $220 < N_{\text{trk}} < 260$

$2.5 < p_T^a < 3.0 \text{ GeV/c}$

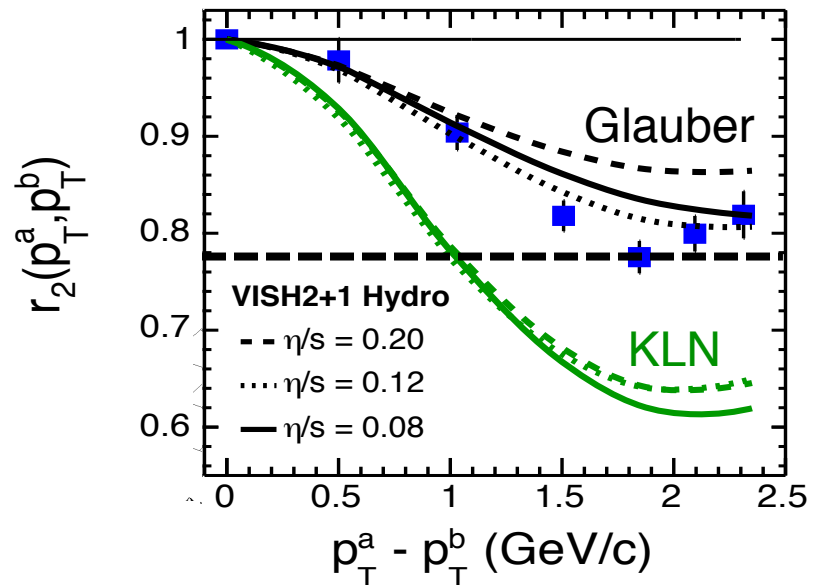


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$p_T^a - p_T^b \text{ (GeV/c)}$

0-0.2% ultra-central PbPb

$2.5 < p_T^a < 3.0 \text{ GeV/c}$



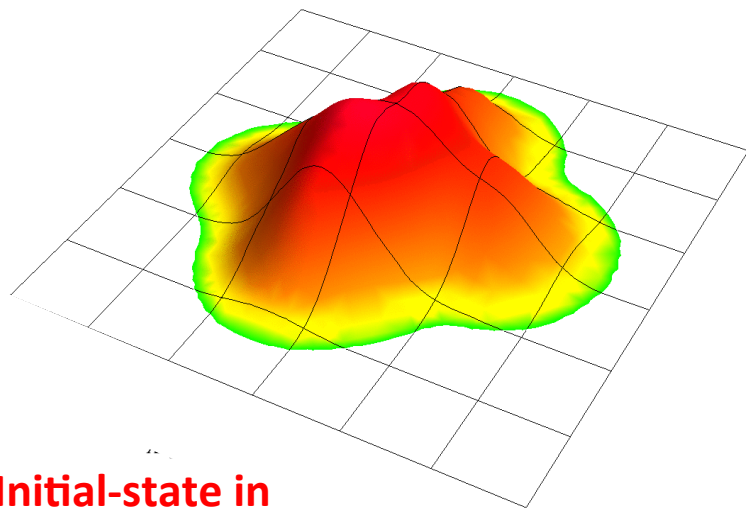
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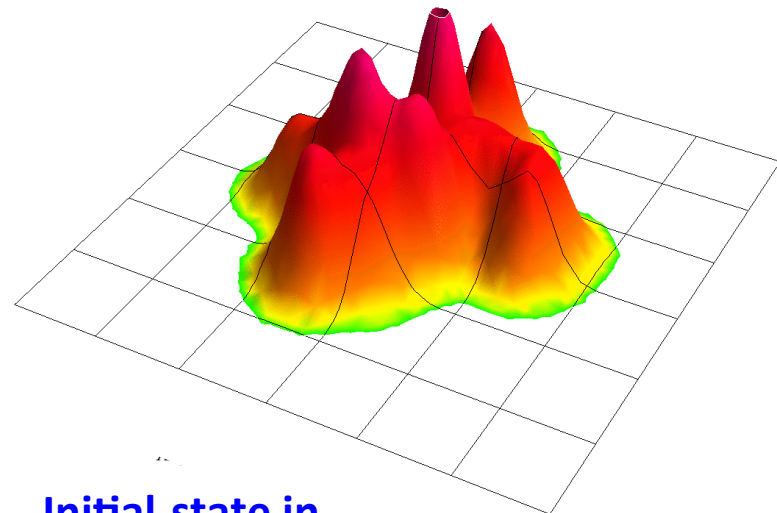
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pPb, $220 < N_{\text{trk}} < 260$



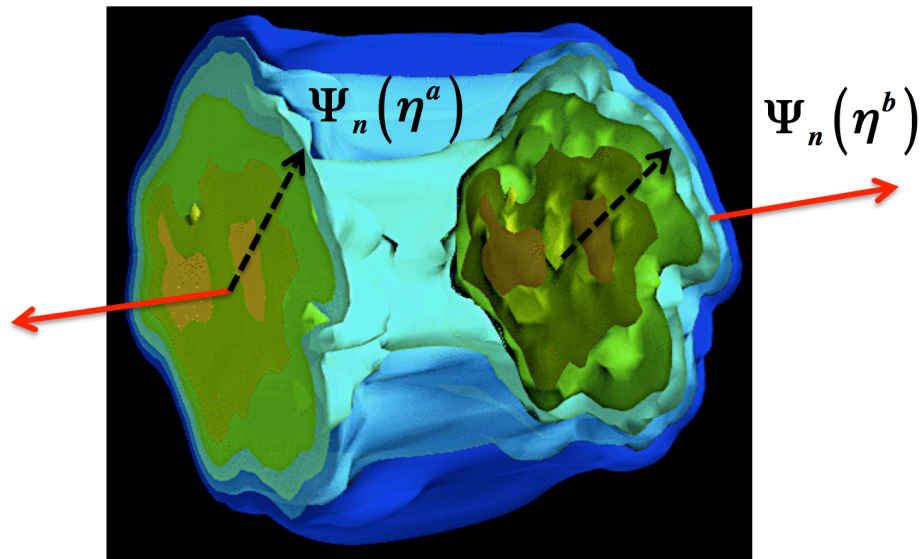
**Initial-state in
p-Pb is smoother**

0-0.2% ultra-central PbPb



**Initial-state in
Pb-Pb is lumpier**

Why looking at longitudinal dynamics?



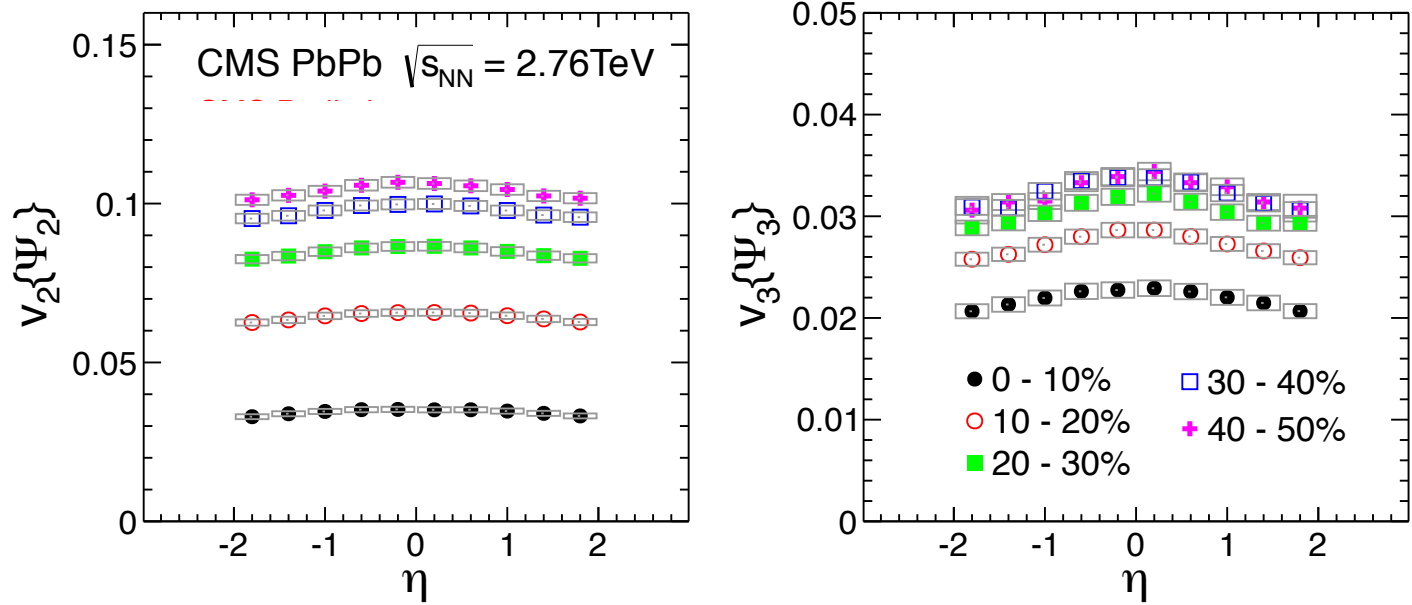
$$f(p_T, \phi, \eta) \sim 1 + 2 \sum_n v_n(p_T, \eta) \cos [2(\phi - \Psi_n(p_T, \eta))]$$

- ❖ Access the **full granularity of initial-state fluctuations** and dynamics of the system: **3D picture**

Longitudinal expansion



Why looking at longitudinal dynamics?

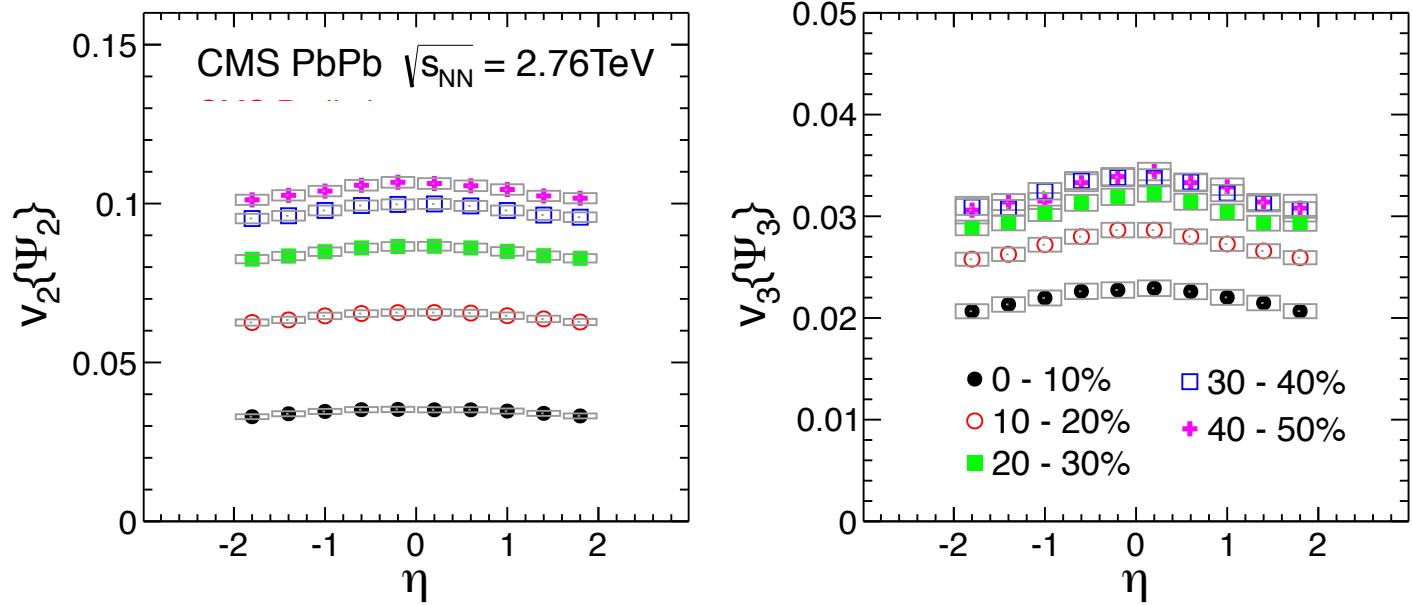


❖ v_n with respect to Ψ_n depends on η

Longitudinal expansion



Why looking at longitudinal dynamics?



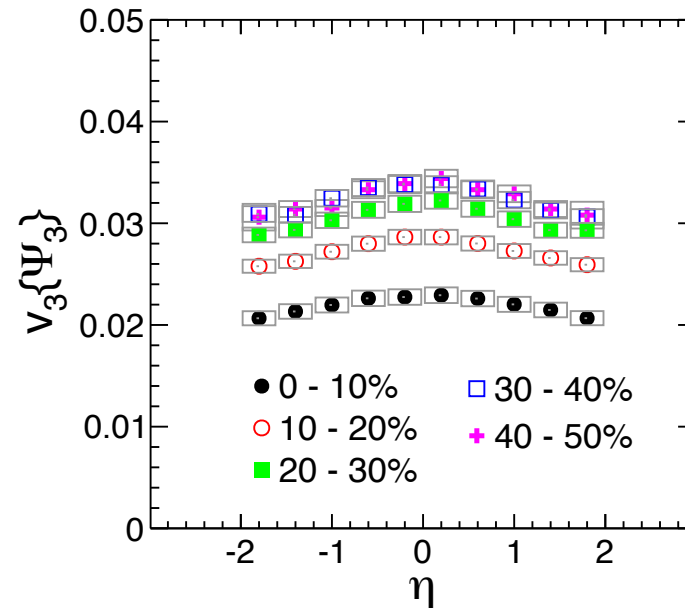
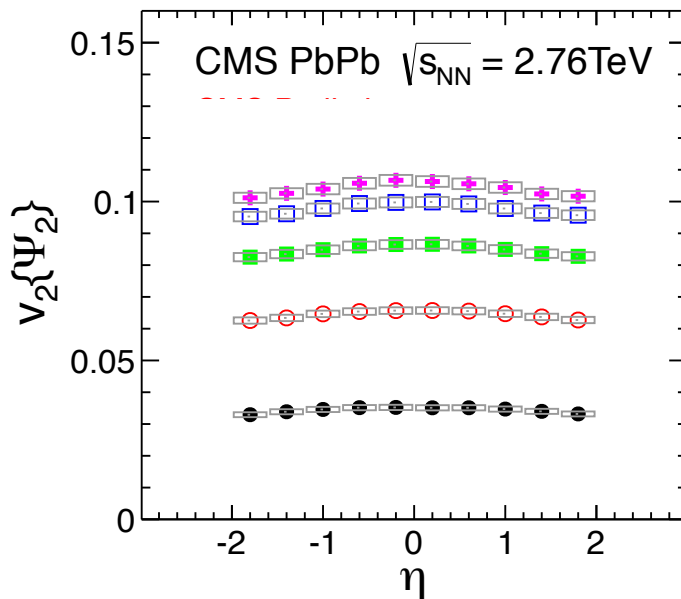
❖ v_n with respect to Ψ_n depends on η

In mid-peripheral events: $\approx 5\%$ effect on v_2 and $\approx 10\%$ on v_3

Longitudinal expansion



Why looking at longitudinal dynamics?

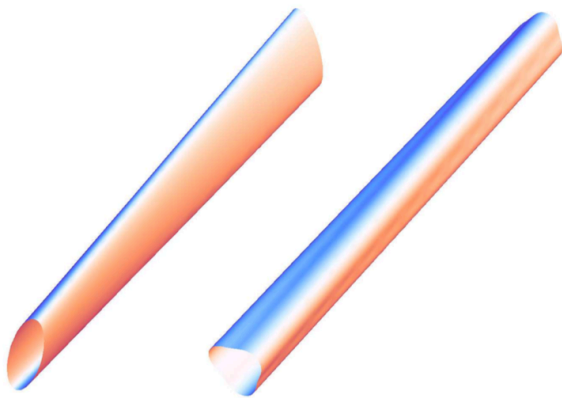


- ❖ v_n with respect to Ψ_n depends on η
- ❖ Where the η dependence come from?
 - **v_n magnitude:** energy density, η/s , ...
 - **Ψ_n orientation:** geometry, initial state, ...

Longitudinal expansion

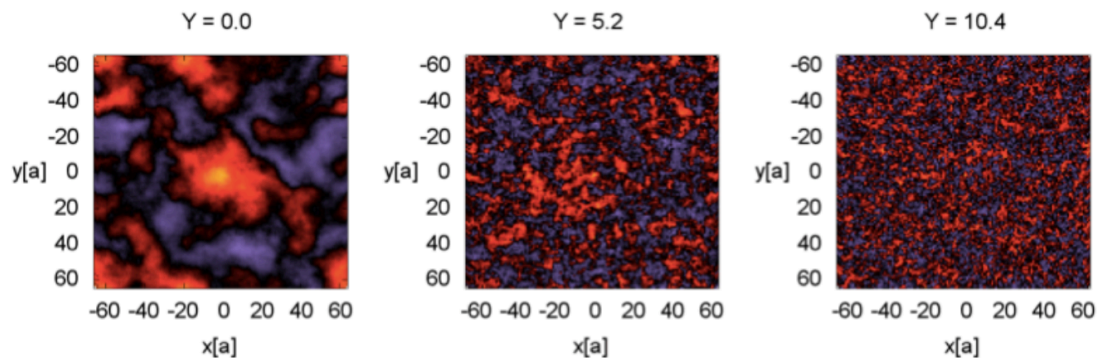
How does $\Psi_n(\eta)$ fluctuate?

Torqued fireball



Bozek et.al., arXiv:1011.3354

Correlation length of gluon field JIMWLK



Dumitru et. al., arXiv:1108.4764

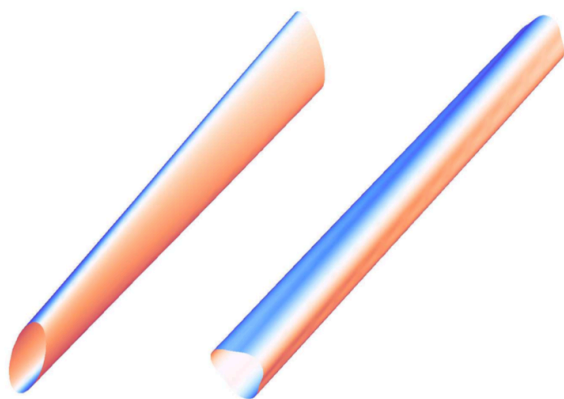
❖ Global twist

❖ Rapidity dependent granularity of gluon field fluctuations

Longitudinal expansion

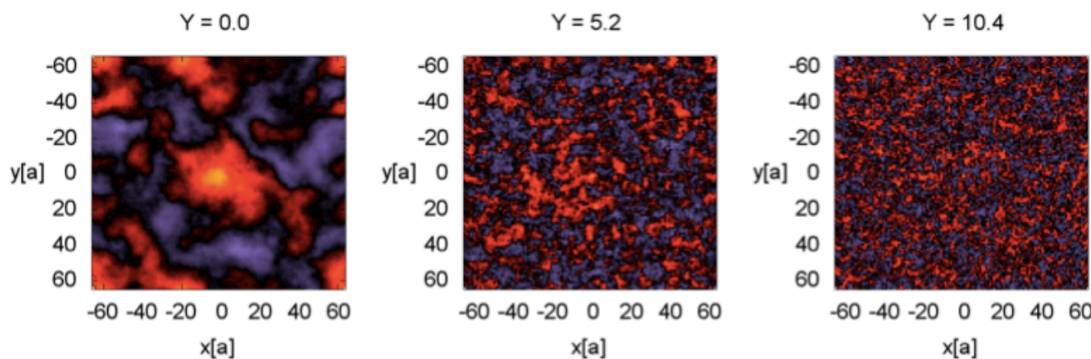
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❖ Global twist

❖ Rapidity dependent granularity of gluon field fluctuations

How to probe $\Psi_n(\eta)$ fluctuations experimentally?

Flow factorization breaking in η



Is it possible to use the same method as for the p_T study?

$$r_n(\eta^a, \eta^b) \equiv \frac{V_{n\Delta}(\eta^a, \eta^b)}{\sqrt{V_{n\Delta}(\eta^a, \eta^a)}\sqrt{V_{n\Delta}(\eta^b, \eta^b)}} \sim \langle \cos [n(\Psi_n(\eta^a) - \Psi_n(\eta^b))] \rangle$$



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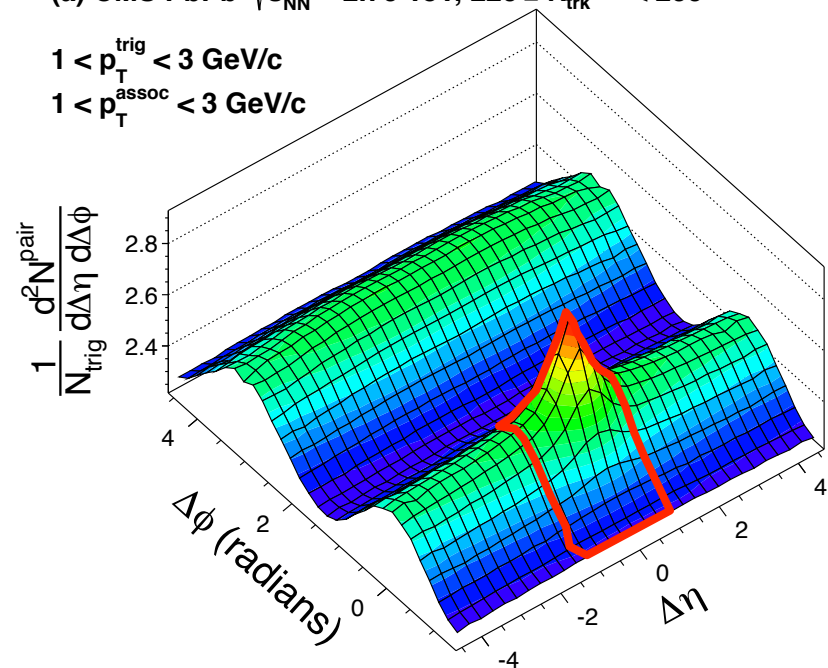
❖ At small $\Delta\eta$ ($\Delta\eta \approx 0$) significant non-flow from **jet contribution**

❖ Need to include a large η gap for all pairs

(a) CMS PbPb $\sqrt{s_{NN}} = 2.76$ TeV, $220 \leq N_{trk}^{offline} < 260$

$1 < p_T^{trig} < 3$ GeV/c

$1 < p_T^{assoc} < 3$ GeV/c

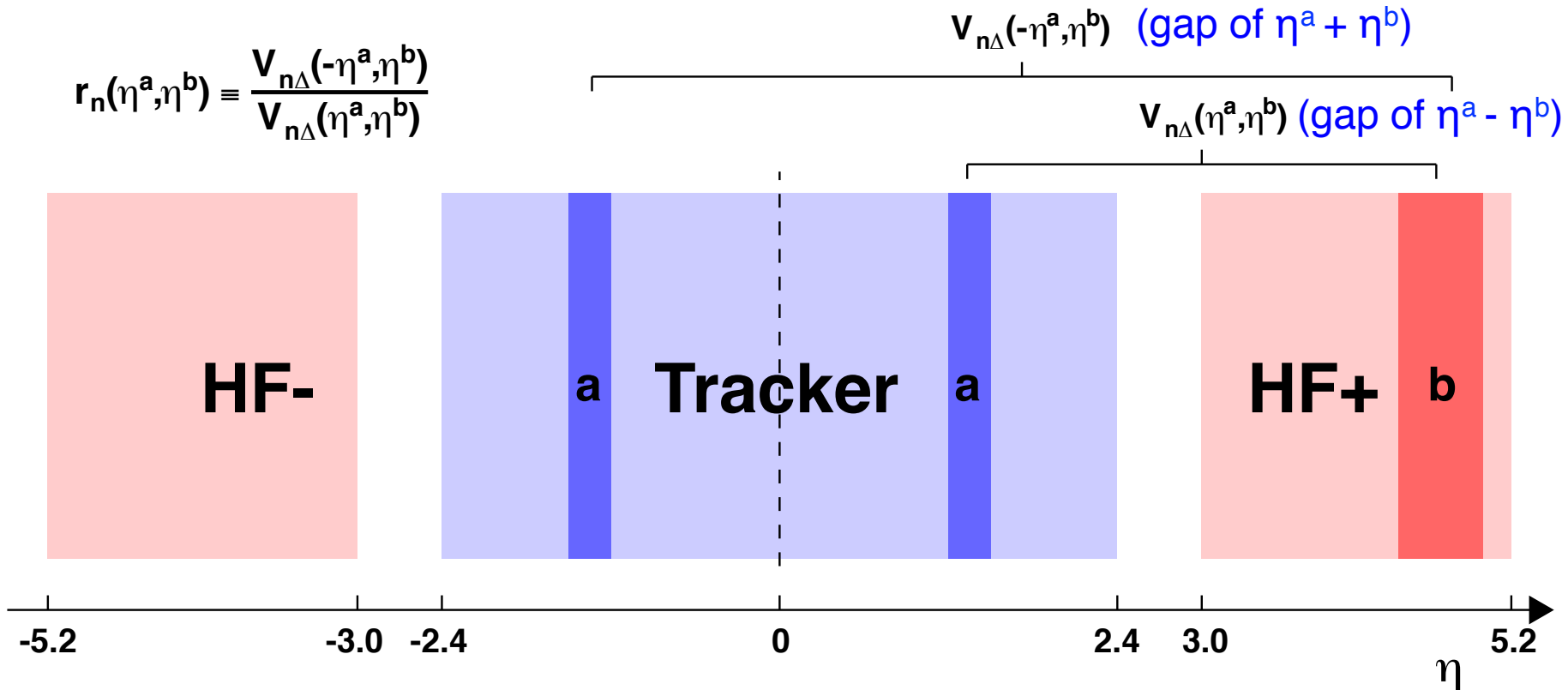


Analysis method



❖ Ensure an η gap of at least 2 units

$$r_n(\eta^a, \eta^b) \equiv \frac{V_{n\Delta}(-\eta^a, \eta^b)}{V_{n\Delta}(\eta^a, \eta^b)}$$

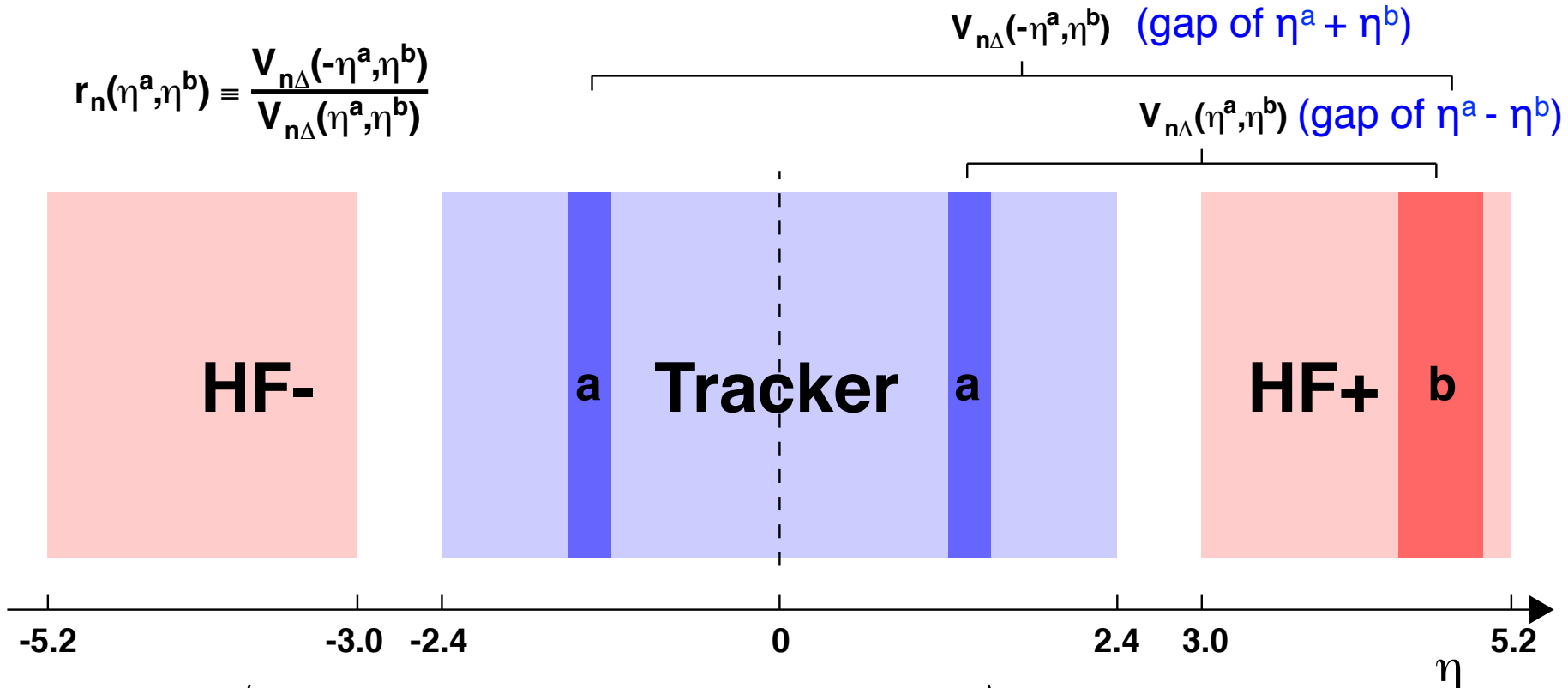


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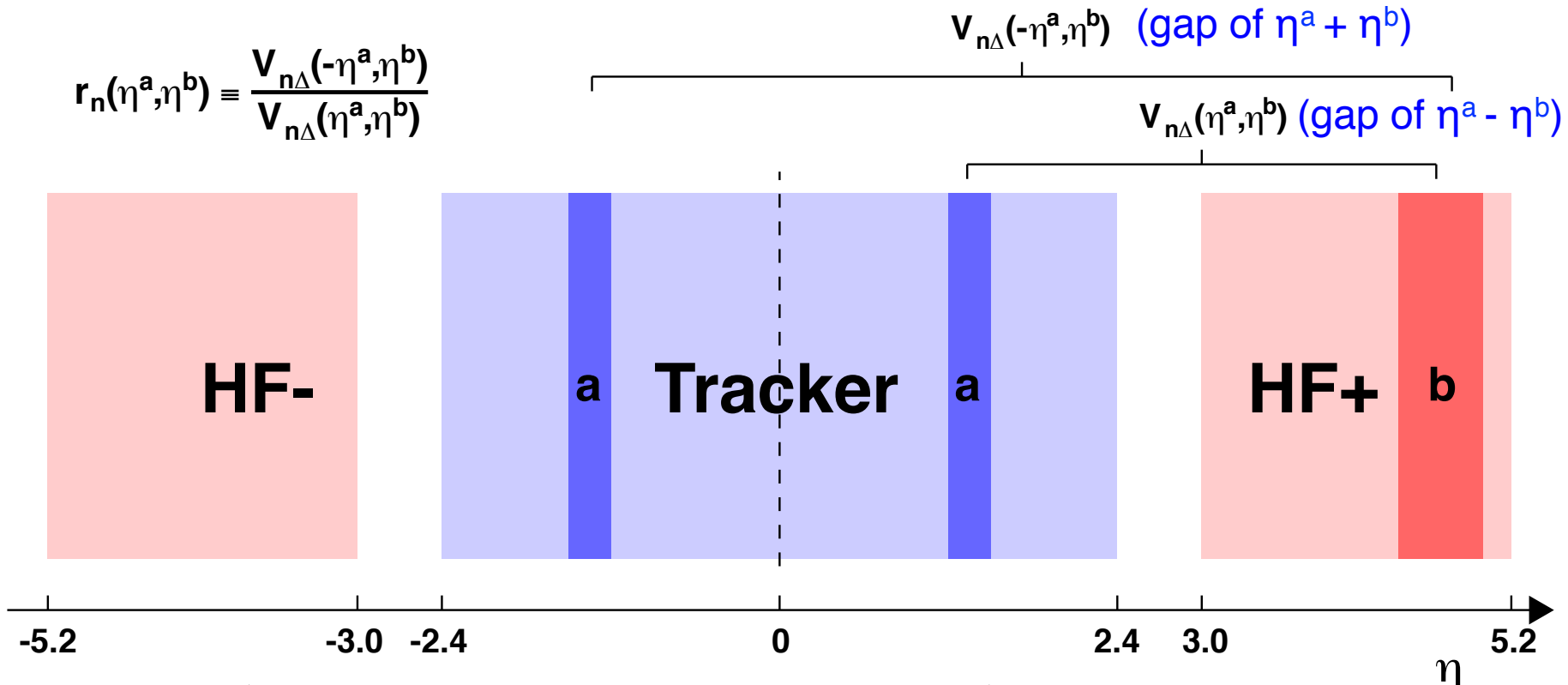


$$r_n(\eta^a, \eta^b) = \frac{\langle v_n(-\eta^a) v_n(\eta^b) \cos[n(\Psi_n(-\eta^a) - \Psi_n(\eta^b))] \rangle}{\langle v_n(\eta^a) v_n(\eta^b) \cos[n(\Psi_n(\eta^a) - \Psi_n(\eta^b))] \rangle} \sim \langle \cos[n(\Psi_n(\eta^a) - \Psi_n(-\eta^a))] \rangle$$

Analysis method

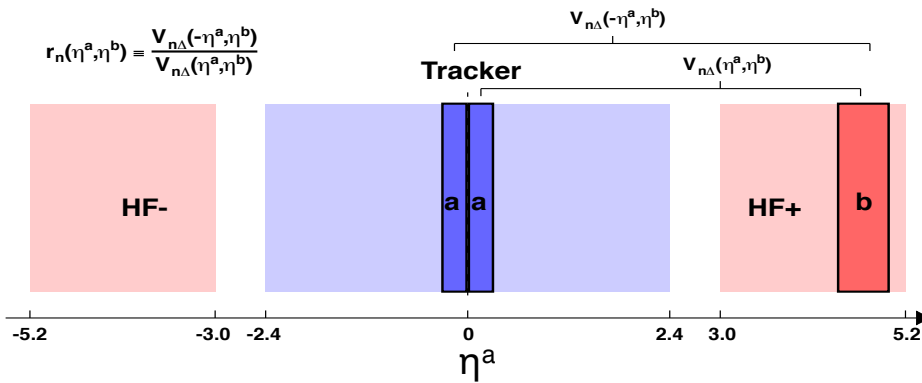
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Pb-Pb results

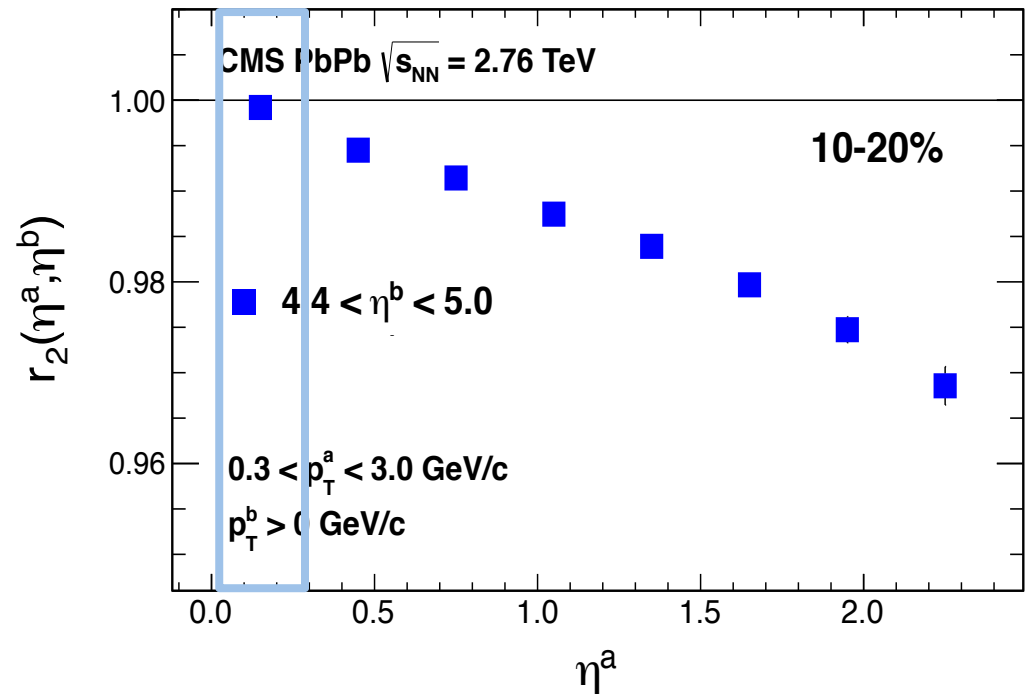


$$r_2(\eta^a, \eta^b) \approx \langle \cos[2(\Psi_2(\eta^a) - \Psi_2(-\eta^a))] \rangle$$

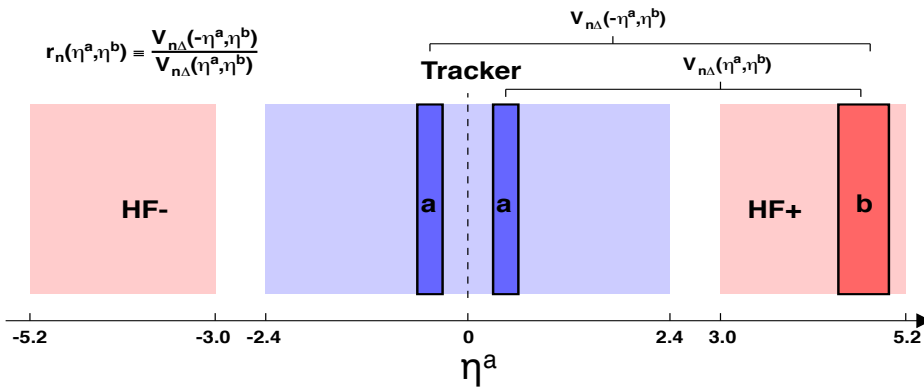
$$\Delta\eta = 2\eta^a$$

η gap ≥ 2 units:

- ❖ De-correlation of Ψ_2 increases as $\Delta\eta$ increases



Pb-Pb results

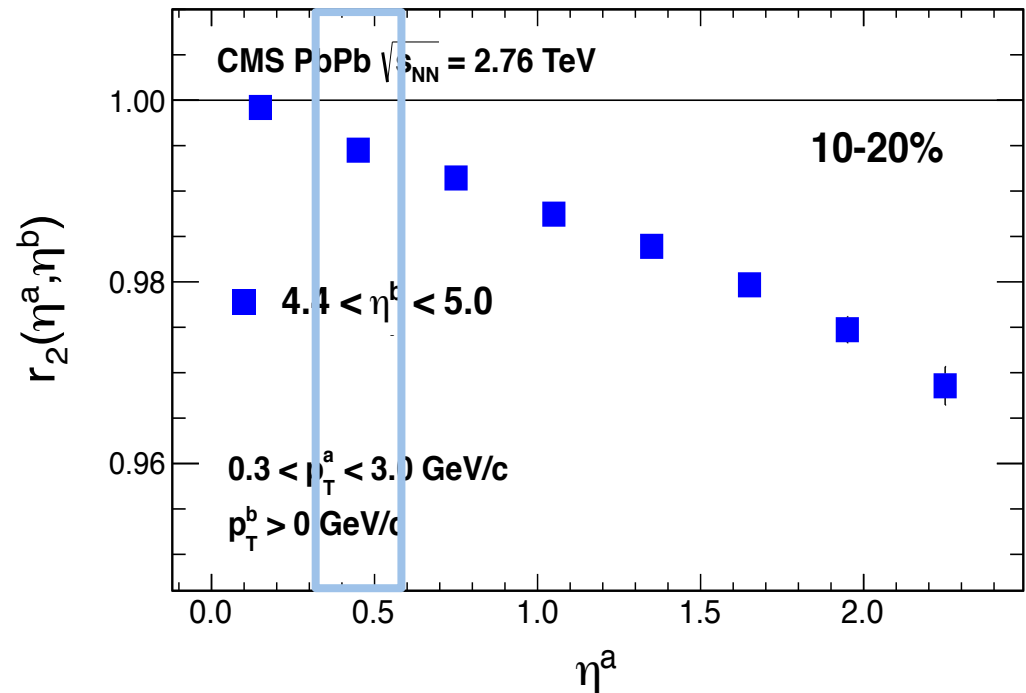


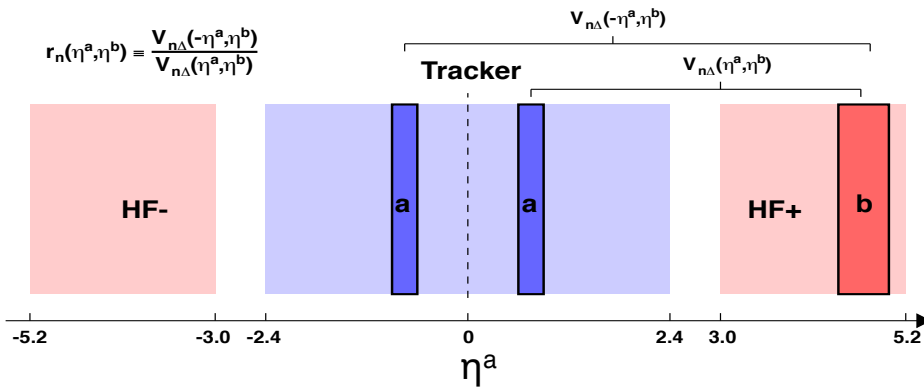
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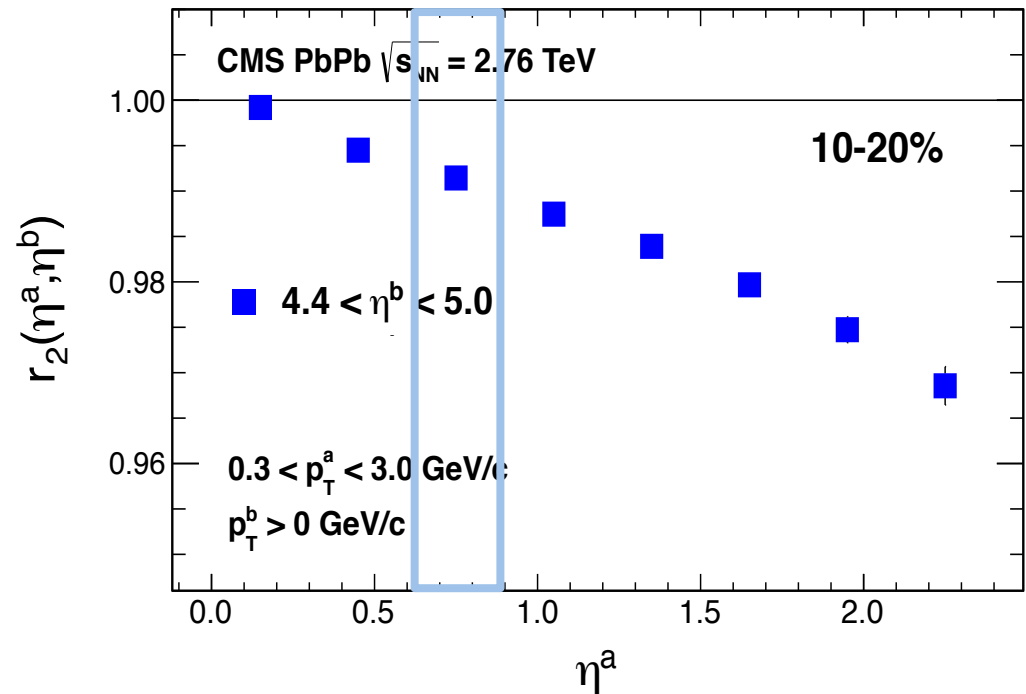


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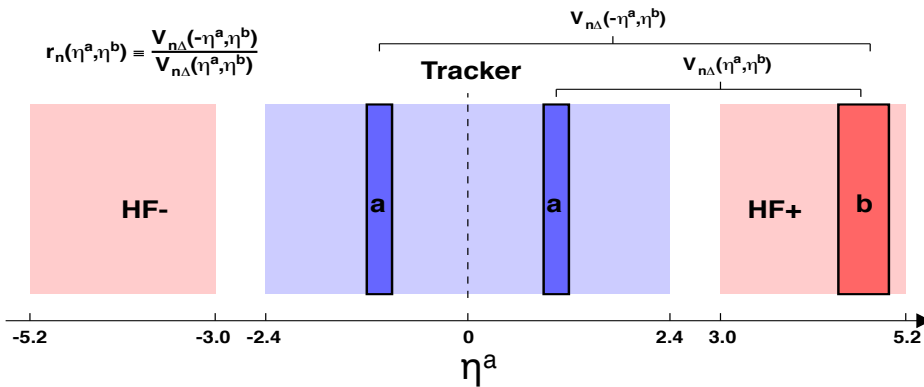
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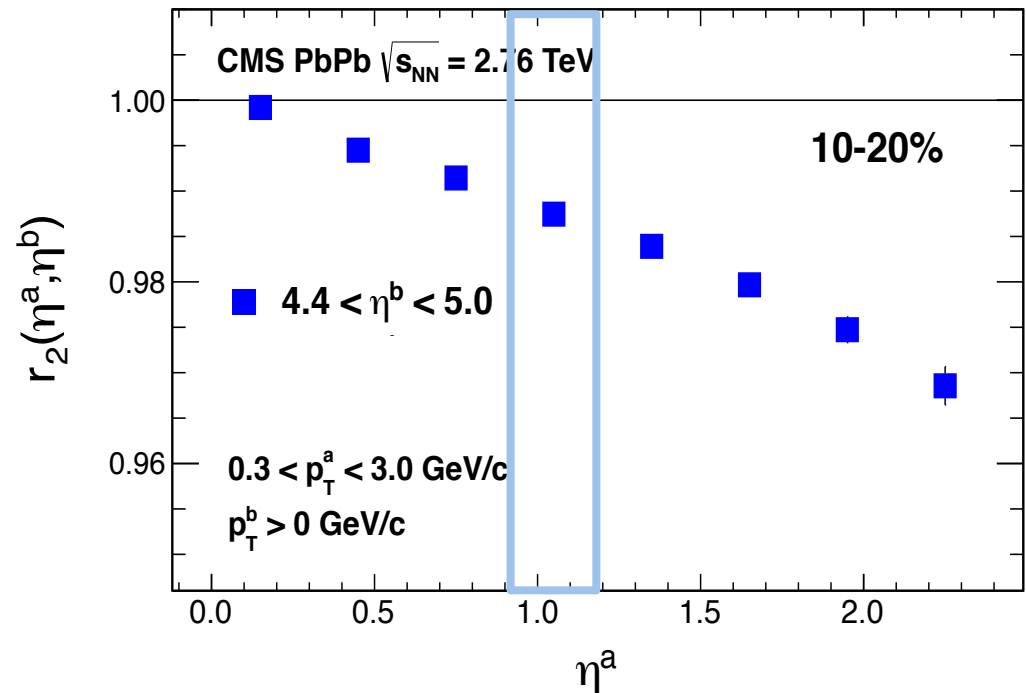


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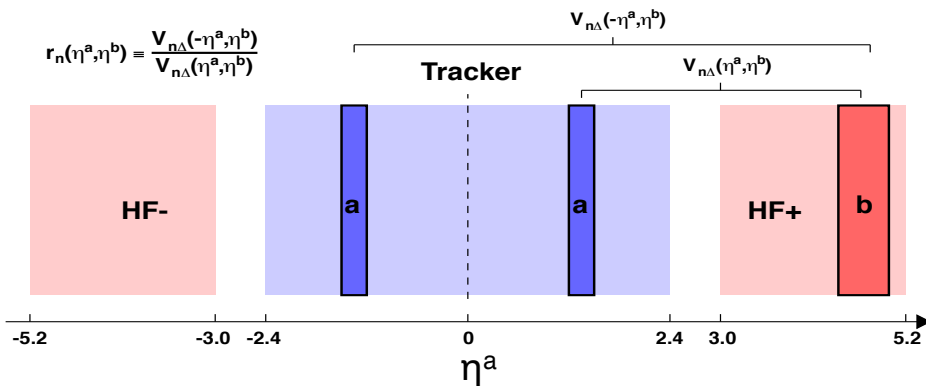
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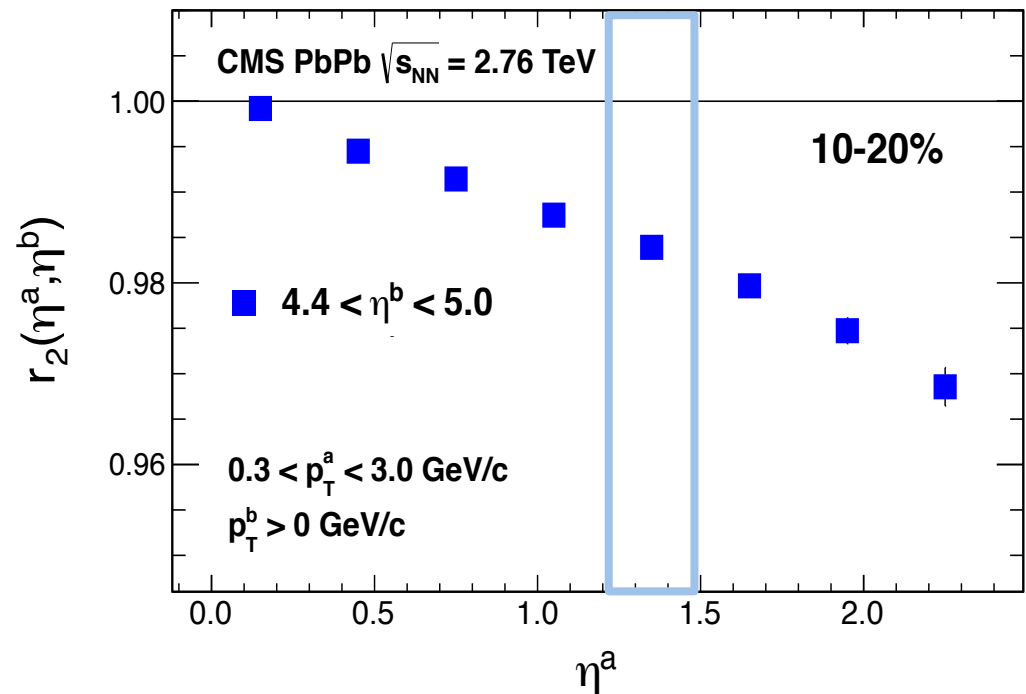


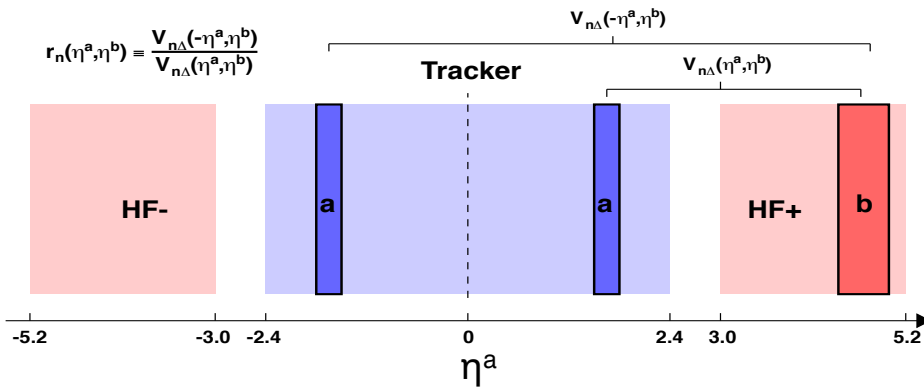
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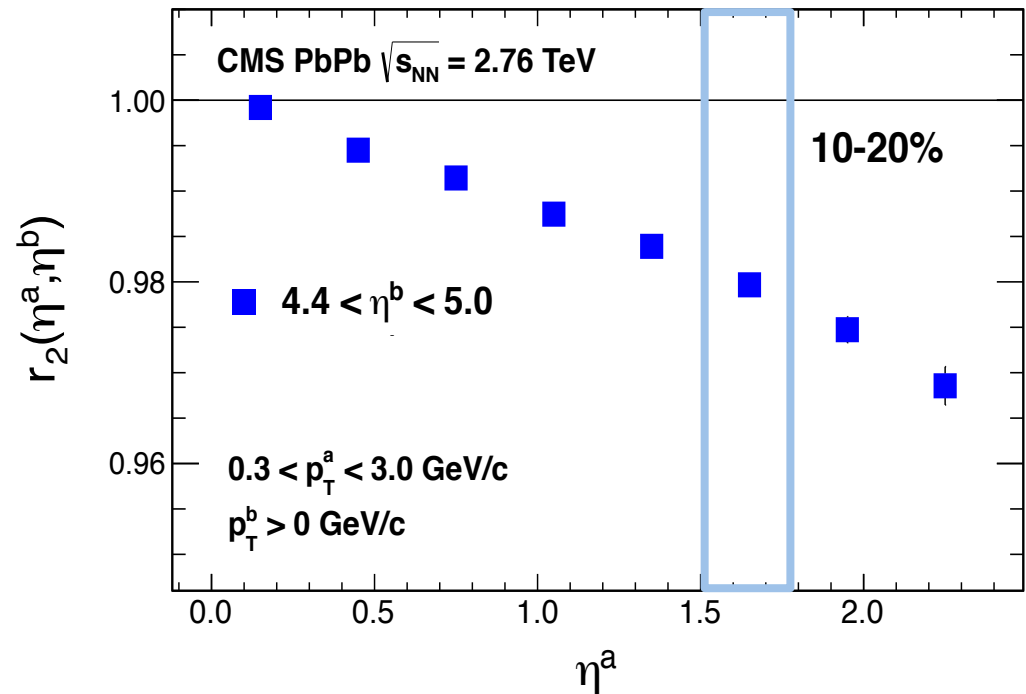


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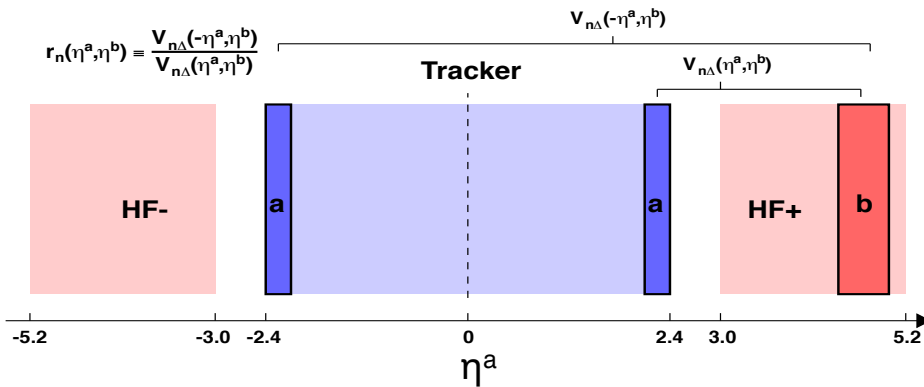
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Pb-Pb results

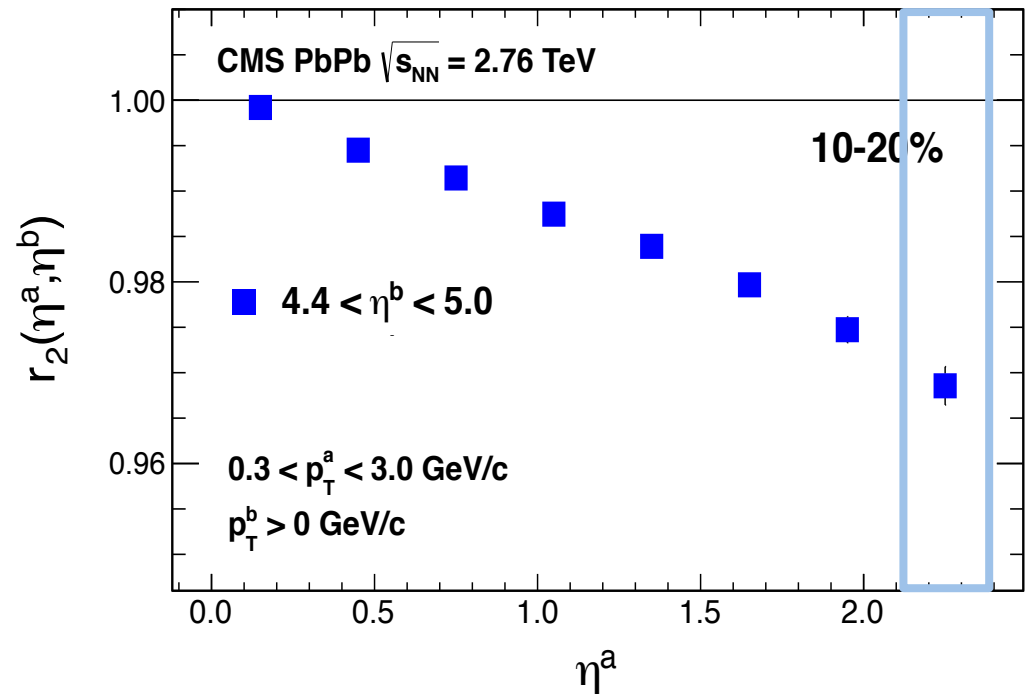


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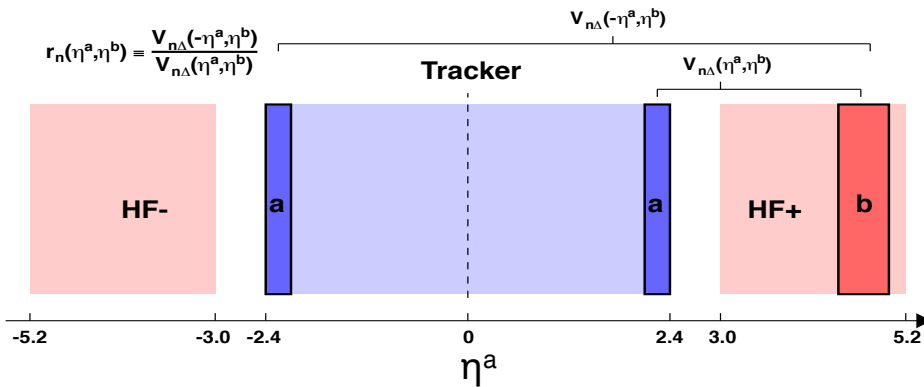
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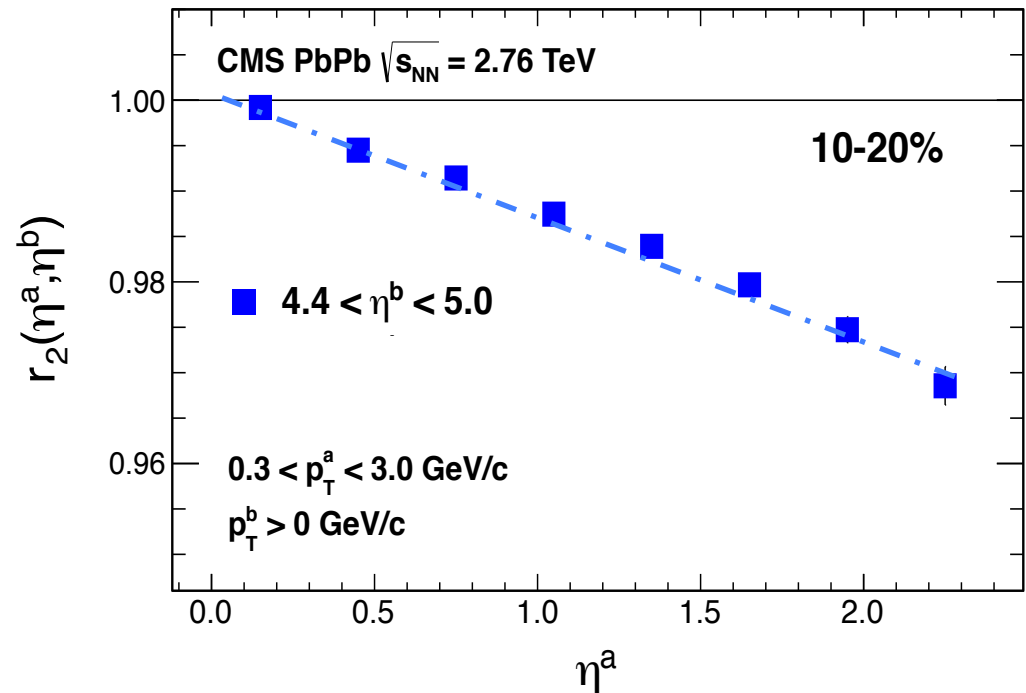


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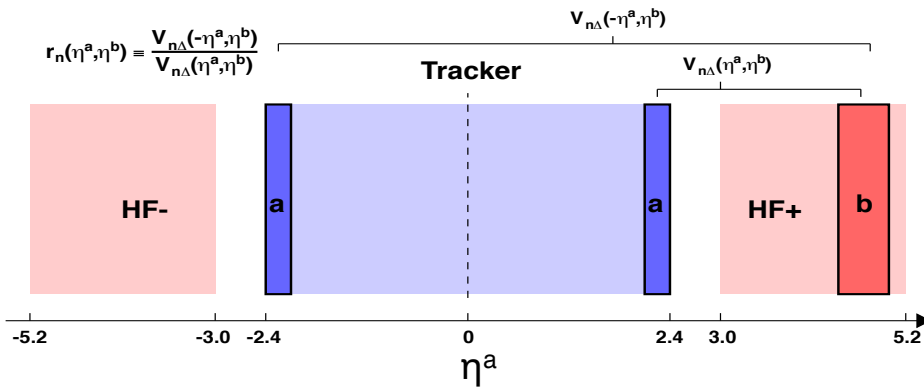
$$\Delta\eta = 2\eta^a$$

η gap ≥ 2 units:

- ❖ De-correlation of Ψ_2 increases as $\Delta\eta$ increases
- ❖ Mostly **linear**



Pb-Pb results

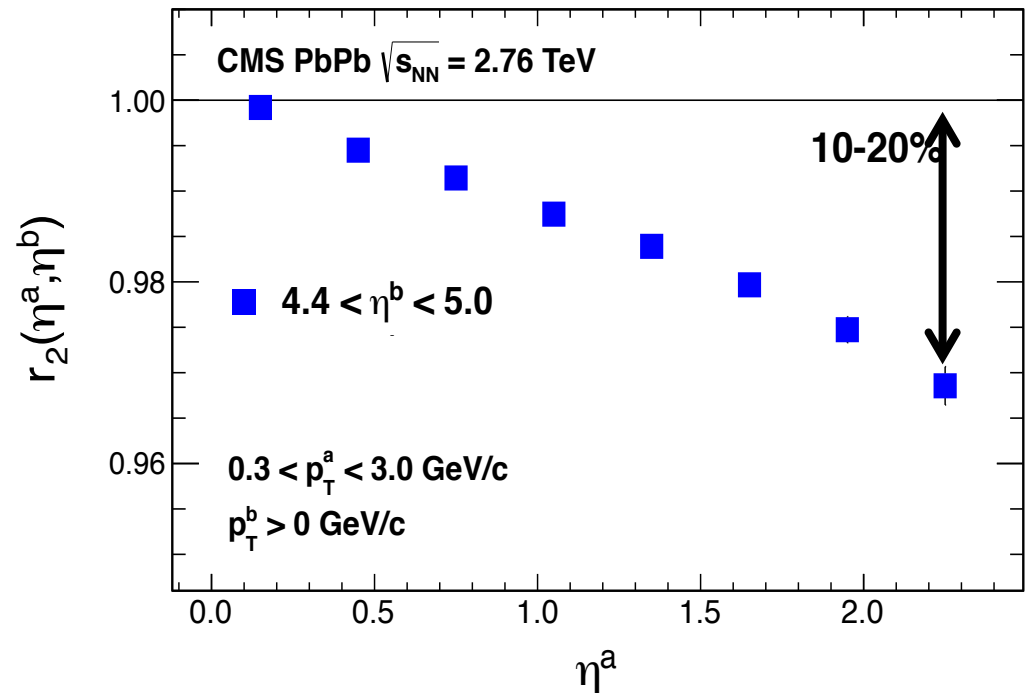


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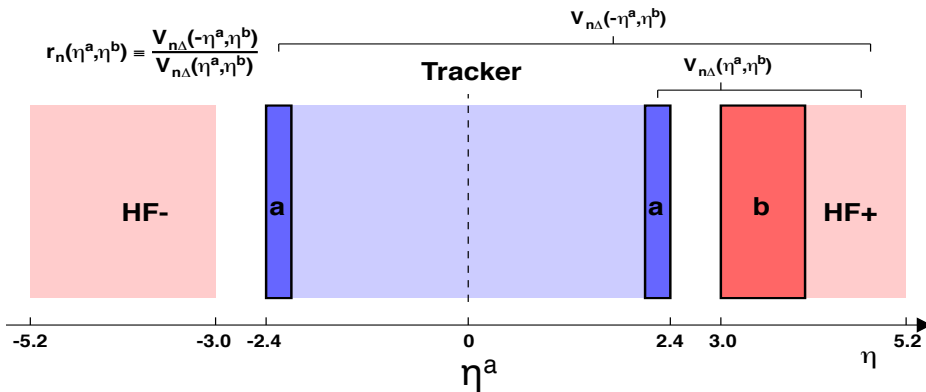
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η gap ≥ 2 units:

- ❖ De-correlation of Ψ_2 increases as $\Delta\eta$ increases
- ❖ Mostly linear
- ❖ 3-4% effect



Pb-Pb results

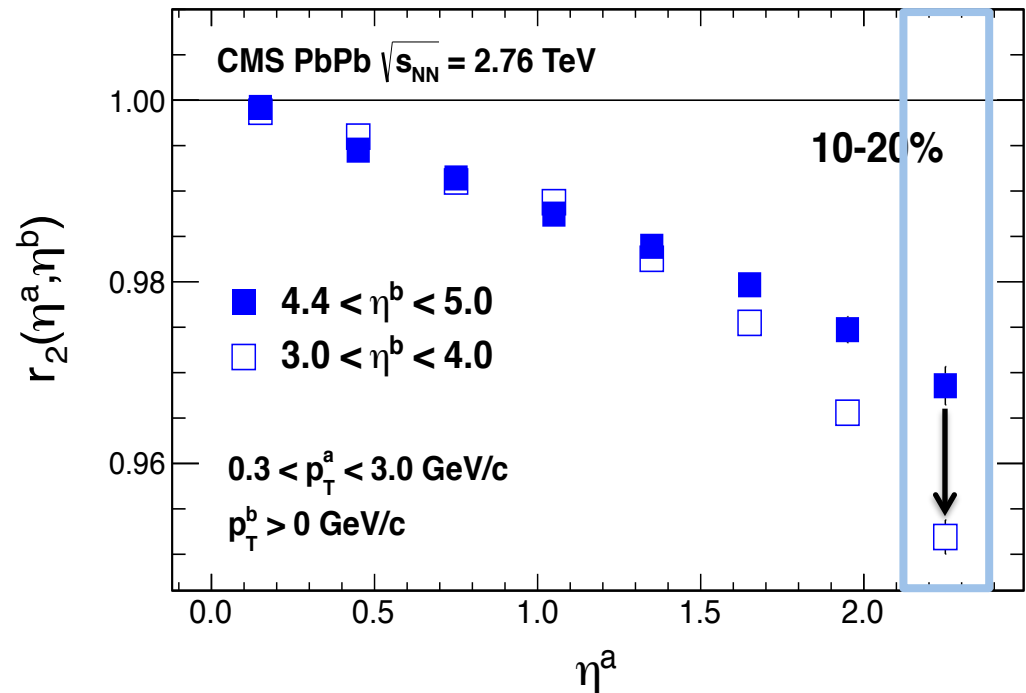


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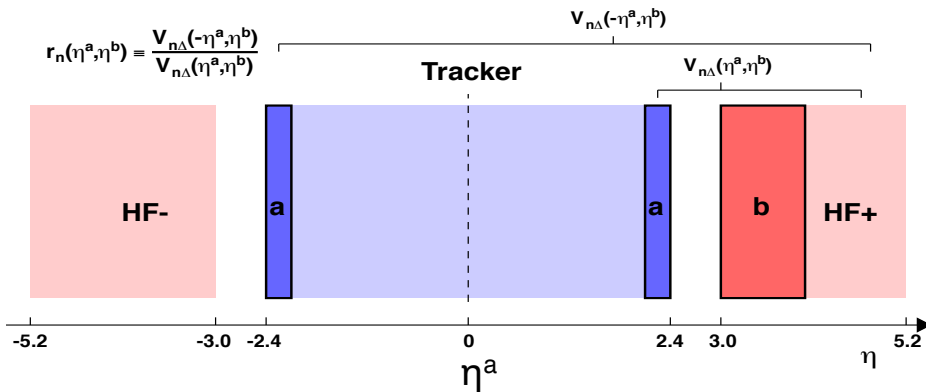
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η gap < 2 units:

❖ r_2 decreases faster for short-range



Pb-Pb results



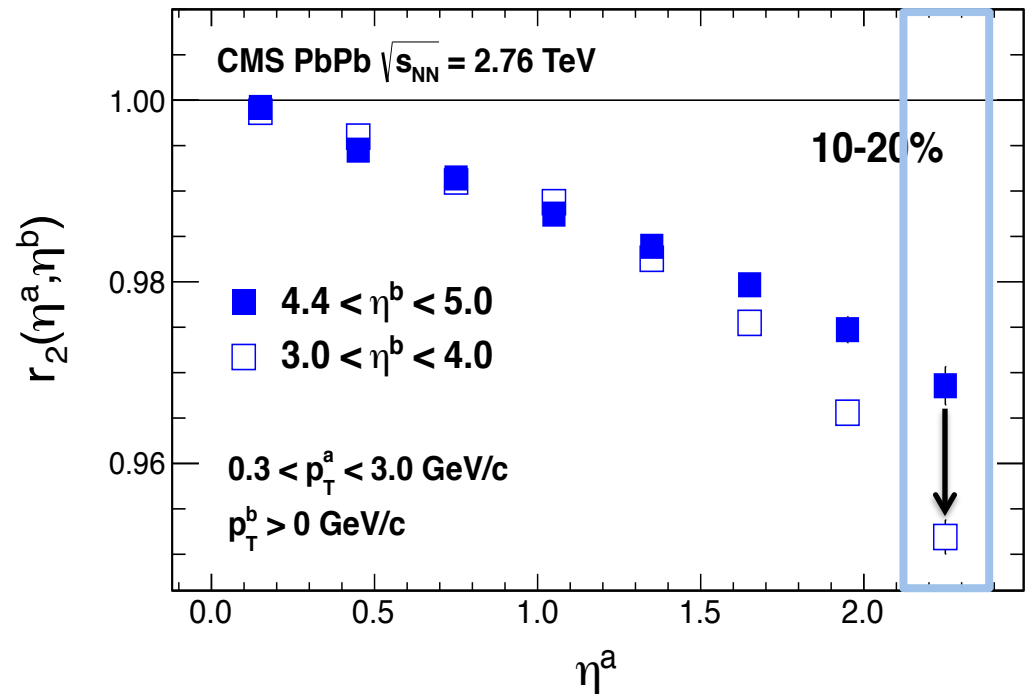
$$r_2(\eta^a, \eta^b) \approx \langle \cos[2(\Psi_2(\eta^a) - \Psi_2(-\eta^a))] \rangle$$

$$\Delta\eta = 2\eta^a$$

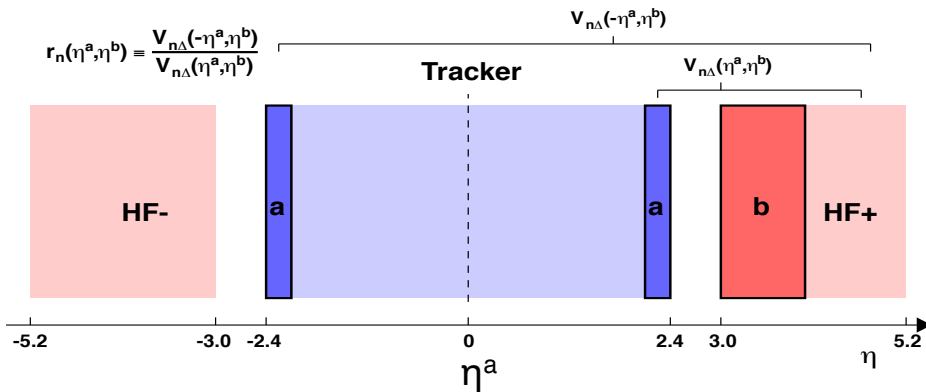
η gap < 2 units:

❖ r_2 decreases faster for short-range

$$r_n(\eta^a, \eta^b) \equiv \frac{V_{n\Delta}(-\eta^a, \eta^b)}{V_{n\Delta}(\eta^a, \eta^b)}$$



Pb-Pb results



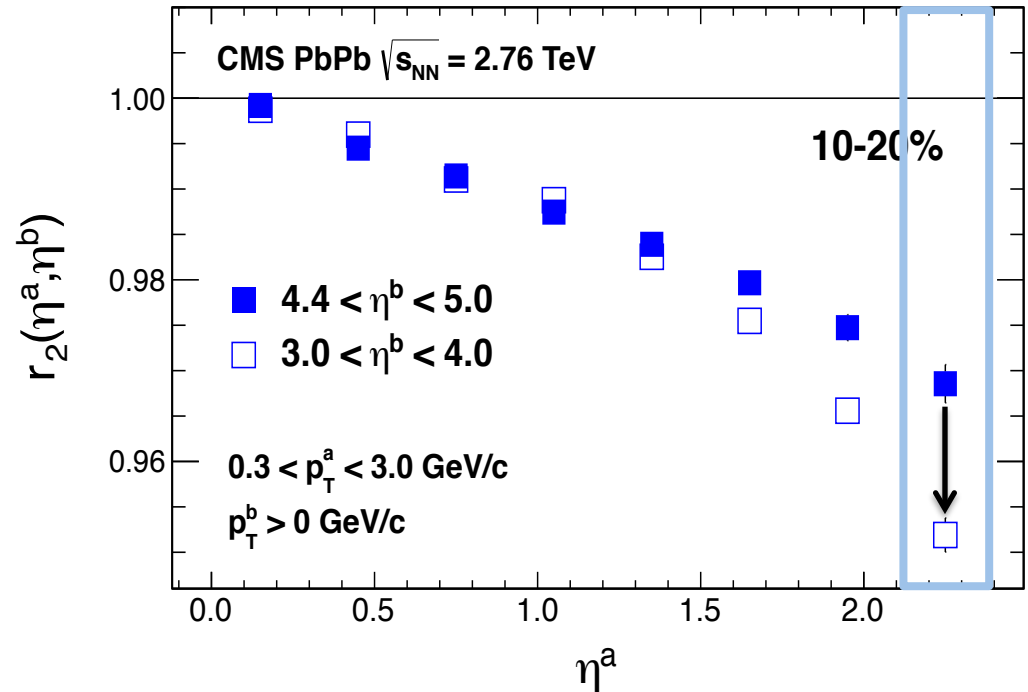
$$r_2(\eta^a, \eta^b) \approx \langle \cos[2(\Psi_2(\eta^a) - \Psi_2(-\eta^a))] \rangle$$

$$\Delta\eta = 2\eta^a$$

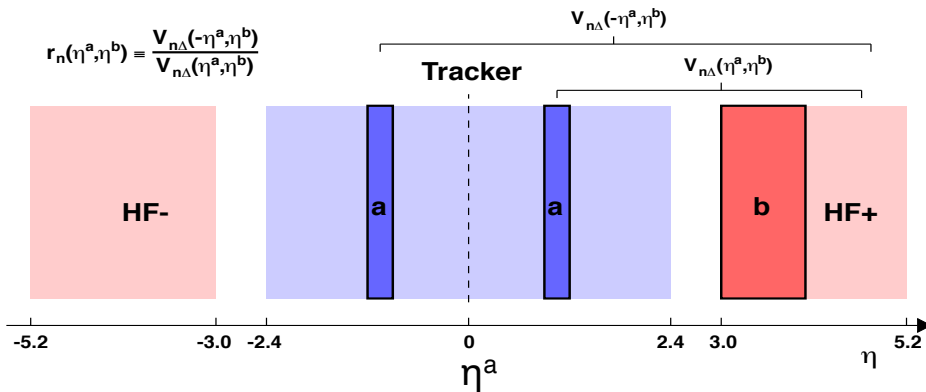
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Pb-Pb results

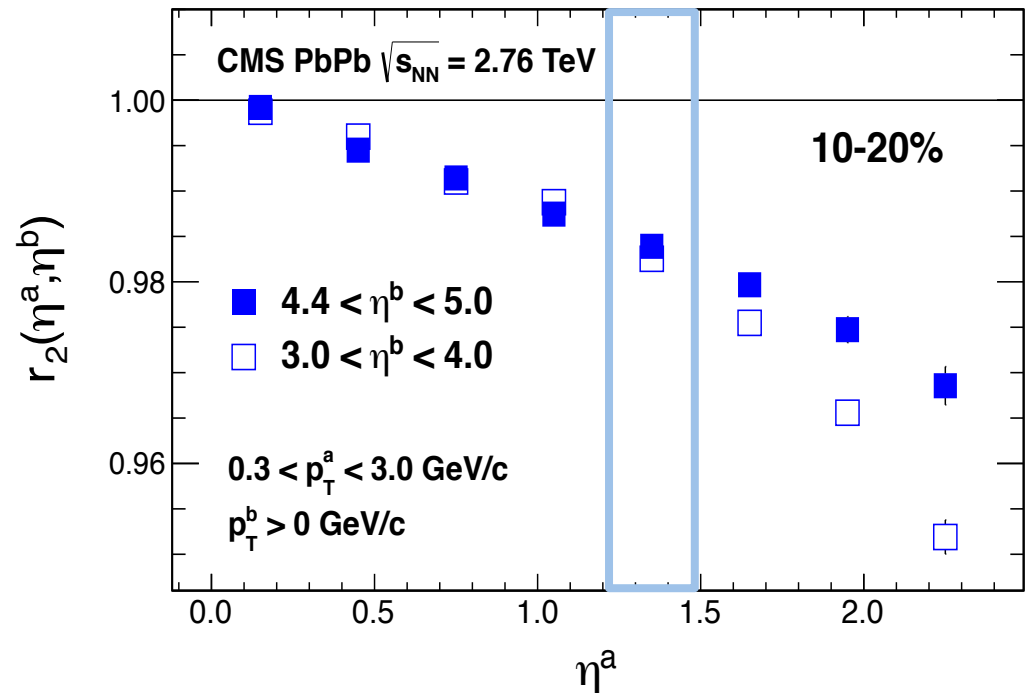


$$r_2(\eta^a, \eta^b) \approx \langle \cos[2(\Psi_2(\eta^a) - \Psi_2(-\eta^a))] \rangle$$

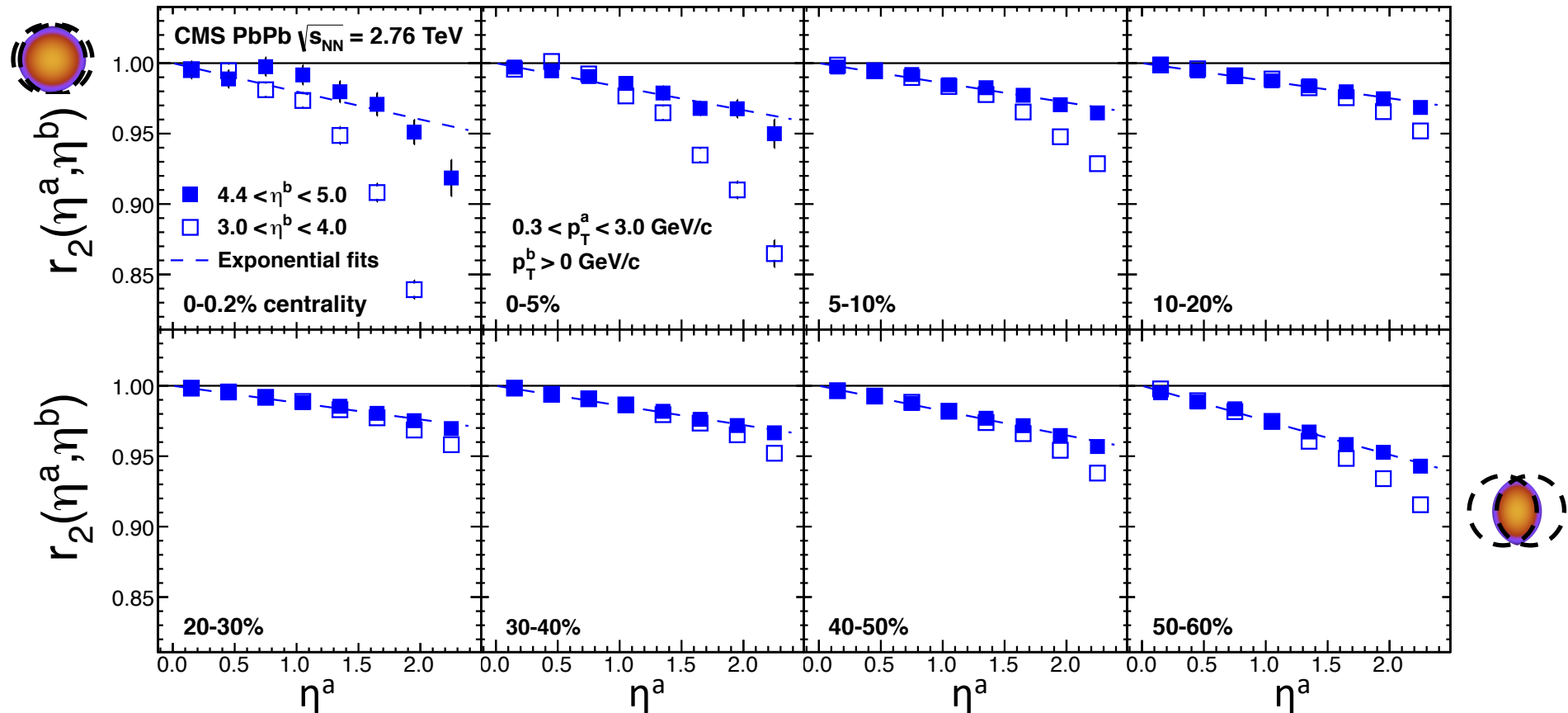
$$\Delta\eta = 2\eta^a$$

η gap < 2 units:

- ❖ r_2 decreases faster for short-range
- ❖ r_2 mostly insensitive for long-range



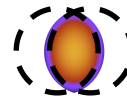
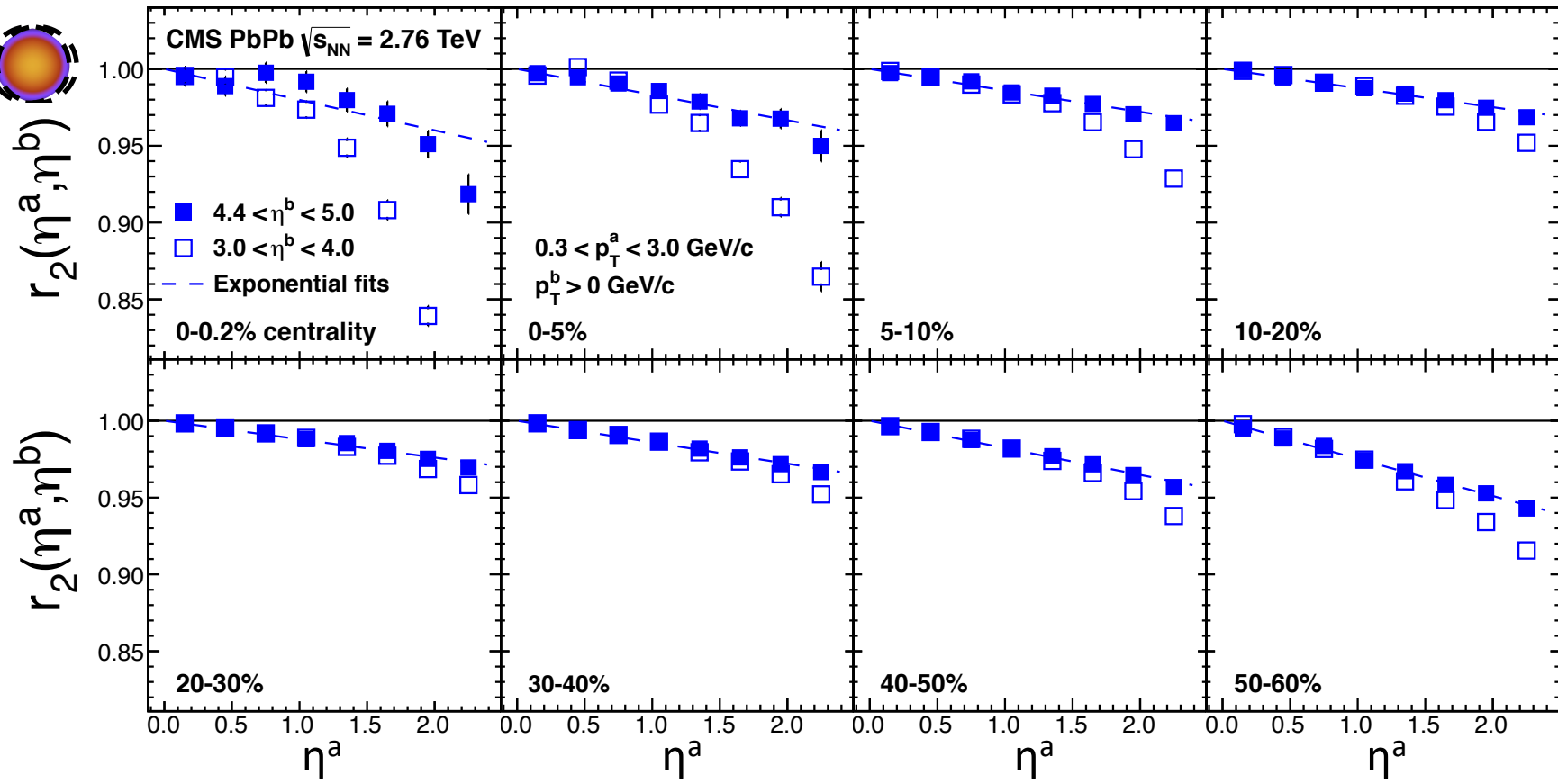
Pb-Pb results



As a function of centrality:

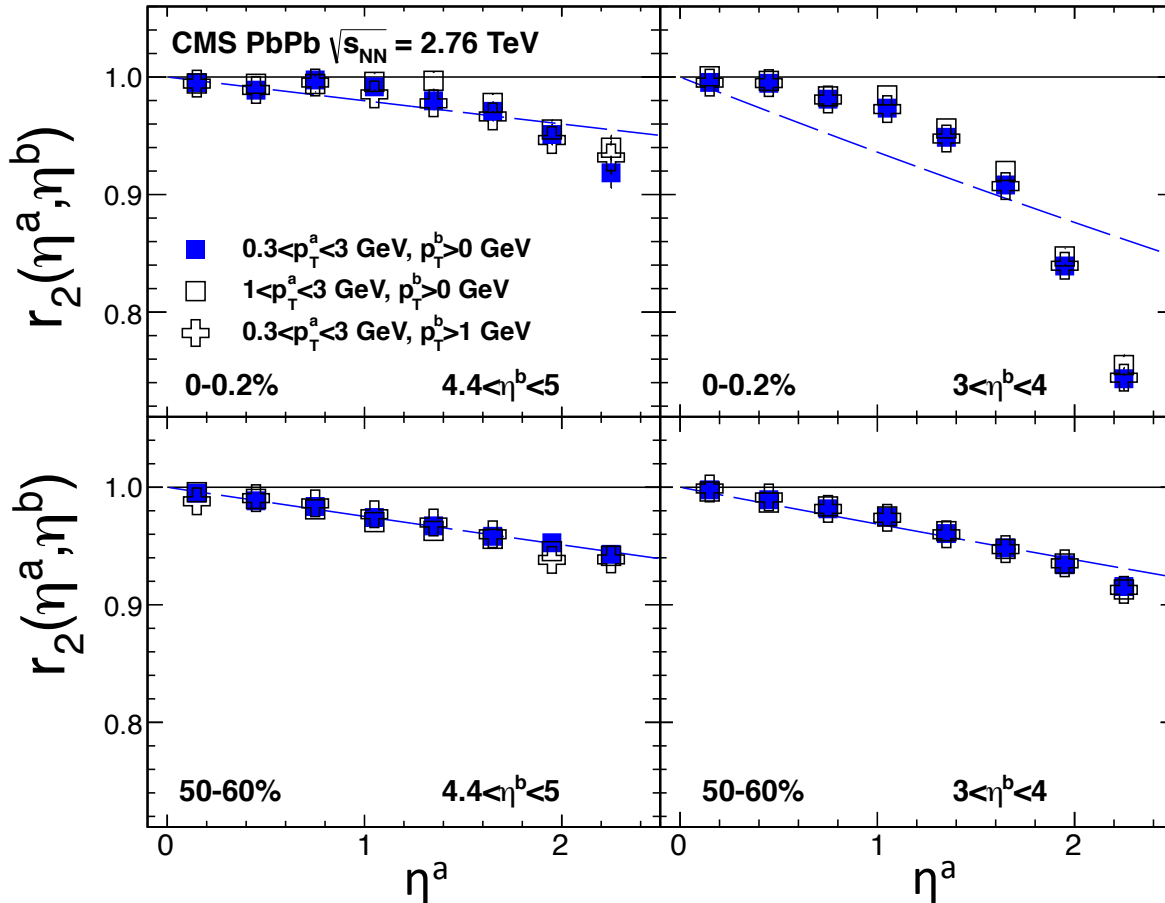
❖ **Same trend in all centrality bins except for ultra-central bin (0-0.2%)**

Pb-Pb results



$\approx 5\%$ effect \rightarrow play a non-negligible role in v_2 η -dependence

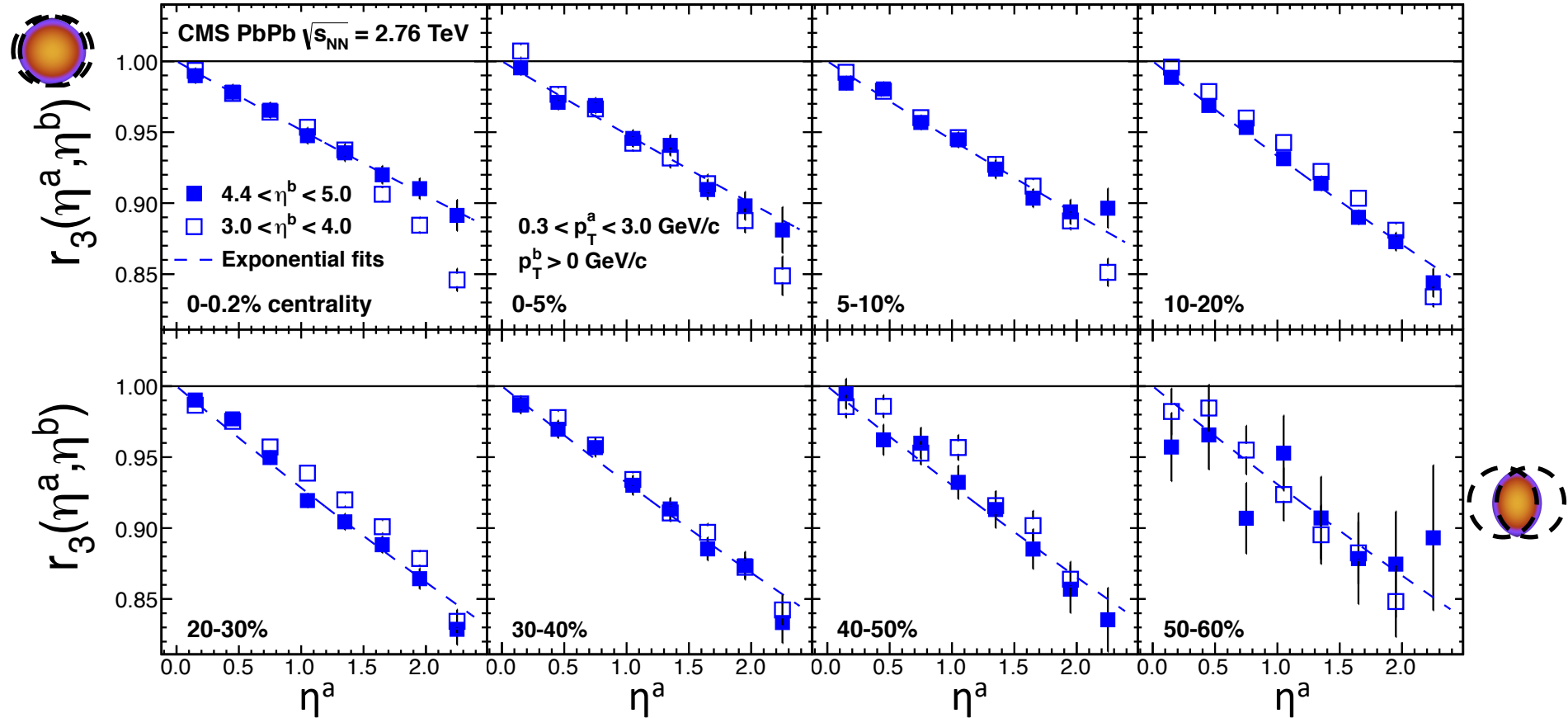
Pb-Pb results



p_T -dependance:

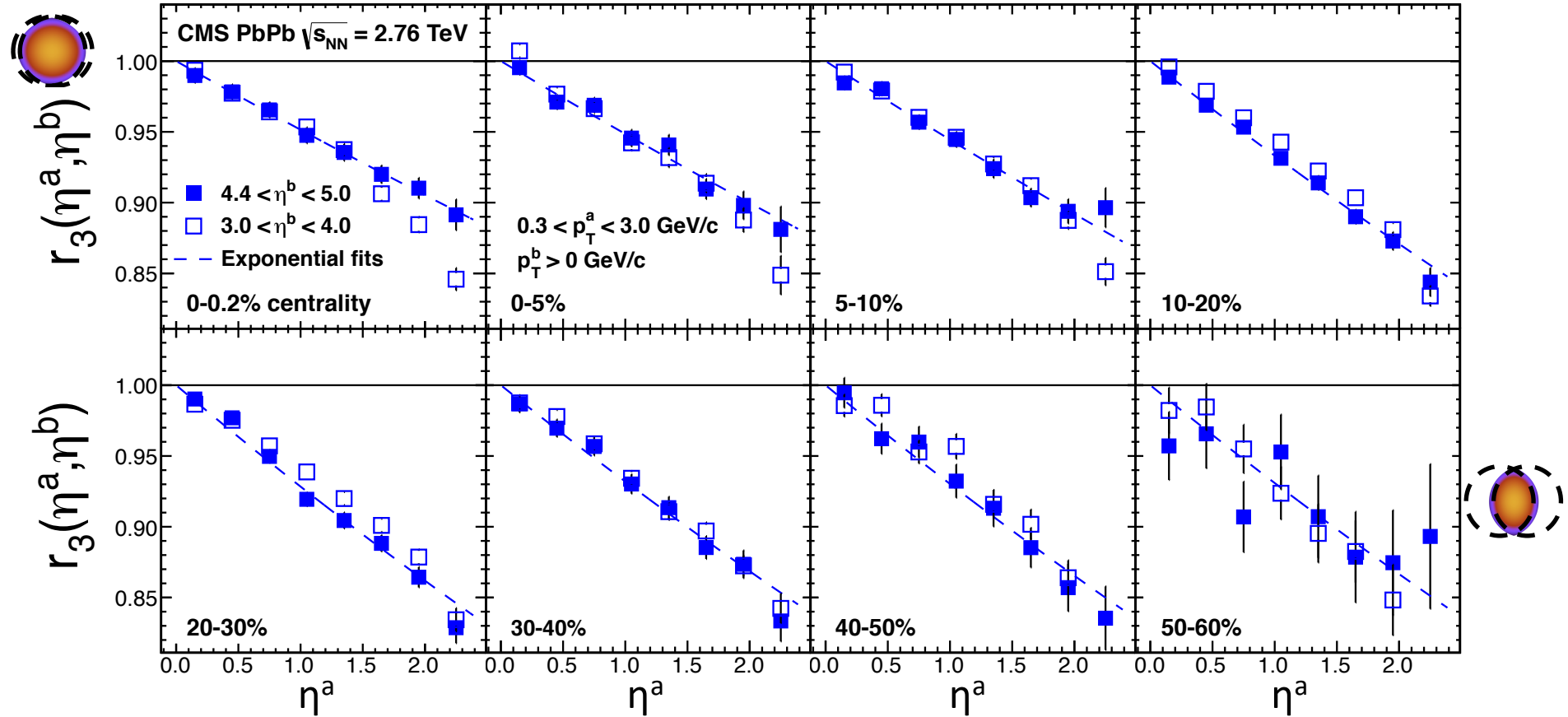
❖ No significant p_T dependence observed: **Initial-state effect?**

Pb-Pb results



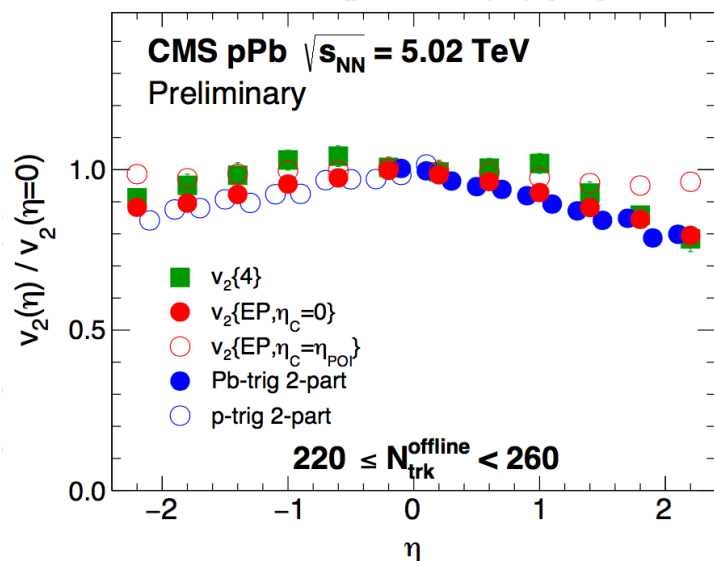
Higher order

❖ Stronger (**15%**) and slightly centrality dependent



Stronger \rightarrow **Fluctuation driven effect**
 Magnitude \rightarrow **Compatible with v_3 η -dependence**

How does it look like in p-Pb?



❖ Asymmetric η -dependence is observed

- See Quan's talk on Monday for more details (QGP in small system I, 18:00)

❖ Does it come from v_n magnitude or Ψ_n de-correlation?

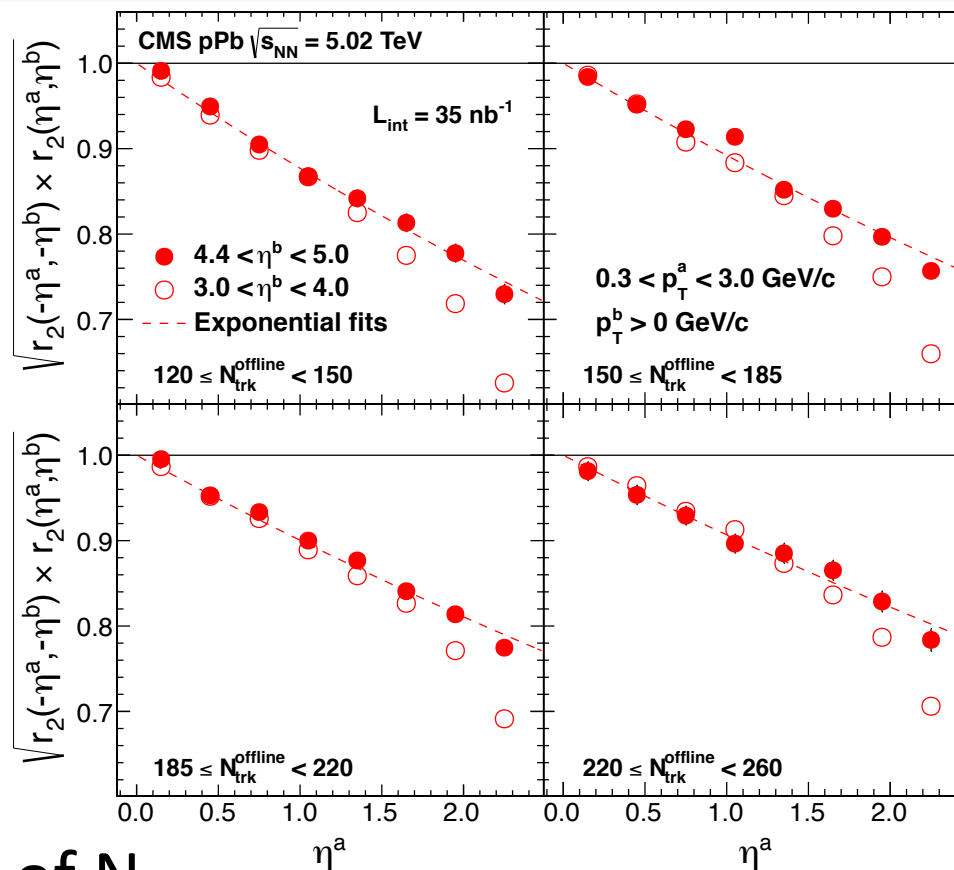
Analysis method:

❖ **Geometric mean** (account for the asymmetry as a function of η):

$$\sqrt{r_n(\eta^a, \eta^b) \times r_n(-\eta^a, -\eta^b)}$$

❖ **p- and Pb-side are averaged and not differentiable** with this method

p-Pb results



As a function of N_{trk}

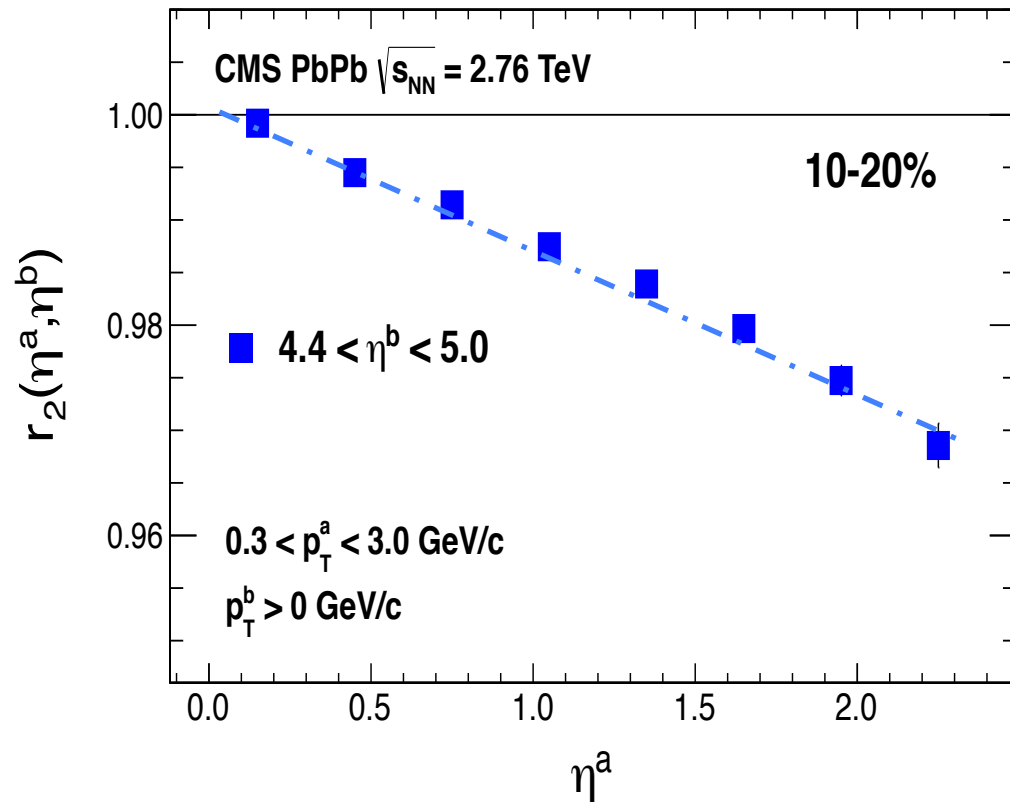
- ❖ Same trend (linear) in all N_{trk} bins
- ❖ Bigger effect (20%): larger fluctuation in p-Pb than in Pb-Pb

p-Pb and Pb-Pb comparison



Empirical parameterization:

$$\diamond r_n(\eta^a, \eta^b) = e^{-2F_n^\eta \eta^a} \sim 1 - 2F_n^\eta \eta^a$$



p-Pb and Pb-Pb comparison

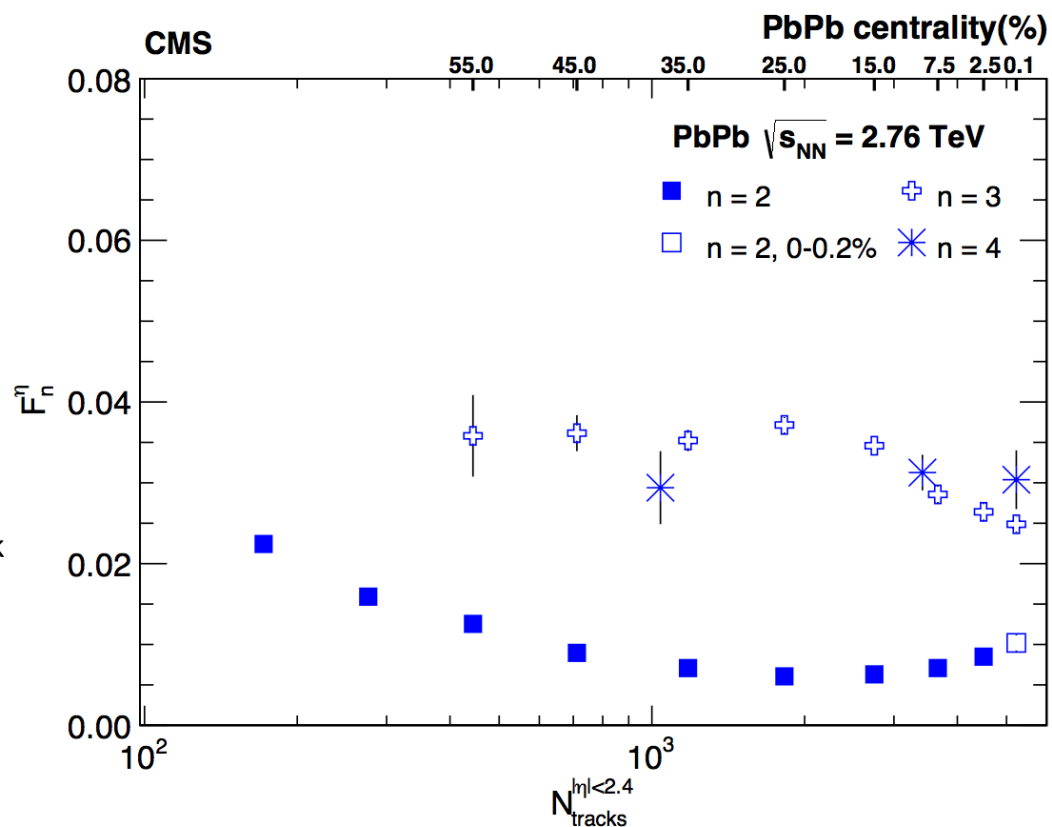


Empirical parameterization:

$$r_n(\eta^a, \eta^b) = e^{-2F_n^\eta \eta^a} \sim 1 - 2F_n^\eta \eta^a$$

Pb-Pb:

- For $n = 2$, F^n increases toward more peripheral events
- Higher factorization breakdown for $n = 3, 4$ than $n = 2$
- No centrality dependence on N_{trk} for $n = 4$



p-Pb and Pb-Pb comparison



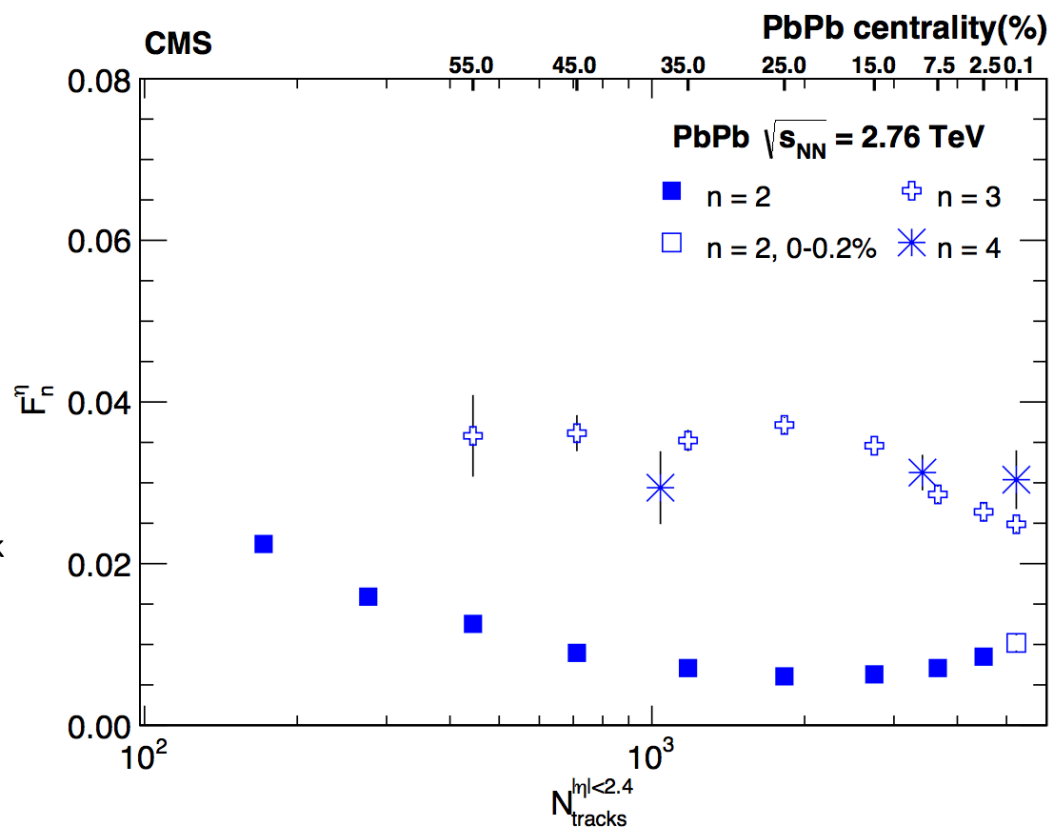
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Pb-Pb:

- ❖ For $n = 2$, F^n increases toward more peripheral events
- ❖ Higher factorization breakdown for $n = 3, 4$ than $n = 2$
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$$\diamond \text{If } \Psi_4 \approx \Psi_2 \rightarrow F_4^\eta \approx 4 \cdot F_2^\eta$$



p-Pb and Pb-Pb comparison



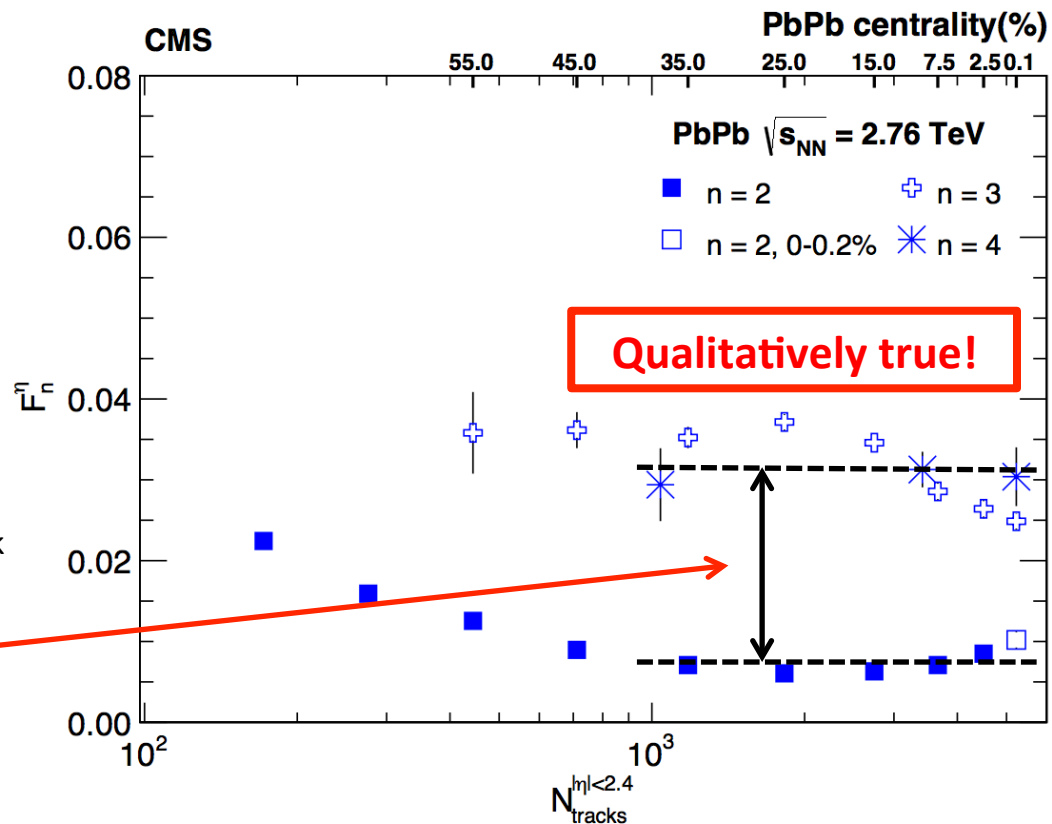
Empirical parameterization:

$$r_n(\eta^a, \eta^b) = e^{-2F_n^\eta \eta^a} \sim 1 - 2F_n^\eta \eta^a$$

Pb-Pb:

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p-Pb and Pb-Pb comparison



Empirical parameterization:

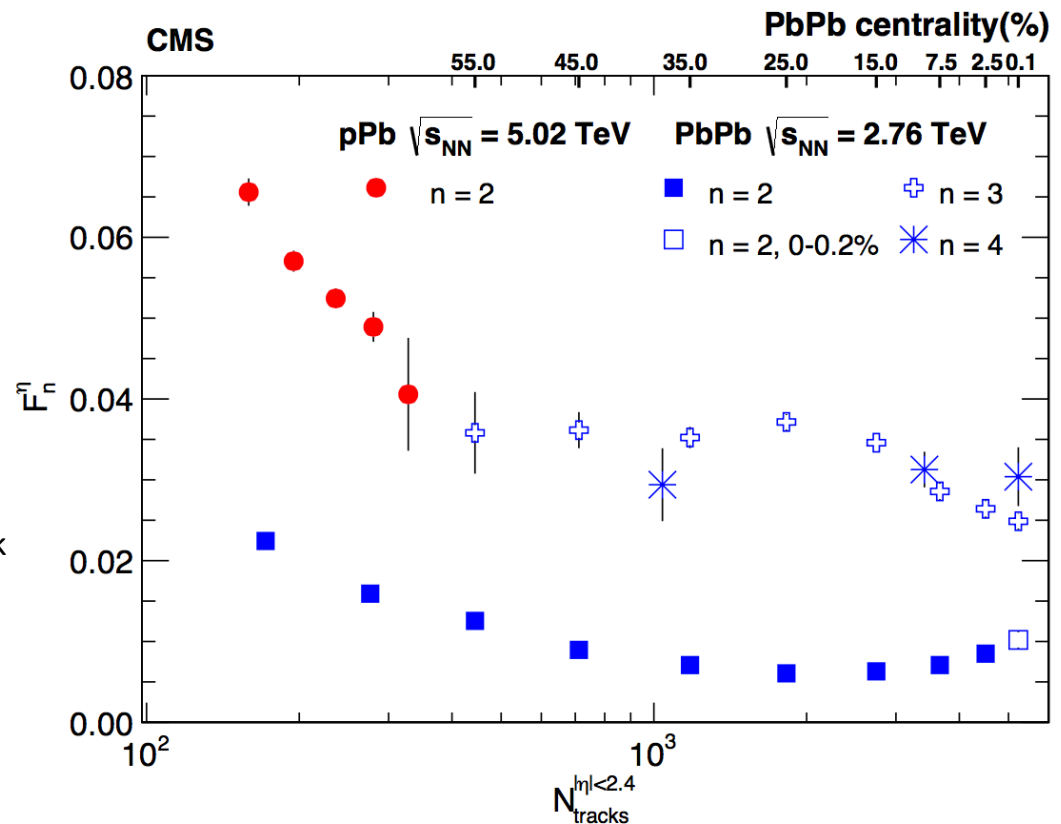
$$r_n(\eta^a, \eta^b) = e^{-2F_n^\eta \eta^a} \sim 1 - 2F_n^\eta \eta^a$$

Pb-Pb:

- For $n = 2$, F^n increases toward more peripheral events
- Higher factorization breakdown for $n = 3, 4$ than $n = 2$
- No centrality dependence on N_{trk} for $n = 4$
- If $\Psi_4 \approx \Psi_2 \rightarrow F_4^\eta \approx 4 \cdot F_2^\eta$

p-Pb:

- Much higher value for $n = 2$
- Decrease faster toward high multiplicity events



- ❖ Fluctuations and Ψ_n de-correlation along the longitudinal direction were observed in Pb-Pb and p-Pb
- ❖ Access to the full granularity (3D) of the initial state
 - **More constraints on longitudinal dynamics**
- ❖ No p_T -dependence observed in Pb-Pb
 - Could point to an **initial state effect**
- ❖ The effect is larger in p-Pb than in Pb-Pb
 - **Larger fluctuation along longitudinal direction** in the initial-state in p-Pb
- ❖ EP de-correlation have **non-negligible effect on v_n η -dependence**

BACKUP

QM2015

CMS detector

CMS DETECTOR

Total weight : 14,000 tonnes
 Overall diameter : 15.0 m
 Overall length : 28.7 m
 Magnetic field : 3.8 T

STEEL RETURN YOKE
 12,500 tonnes

SILICON TRACKERS
 Pixel (100x150 μm) $\sim 16\text{m}^2 \sim 66\text{M}$ channels
 Microstrips (80x180 μm) $\sim 200\text{m}^2 \sim 9.6\text{M}$ channels

SUPERCONDUCTING SOLENOID
 Niobium titanium coil carrying $\sim 18,000\text{A}$

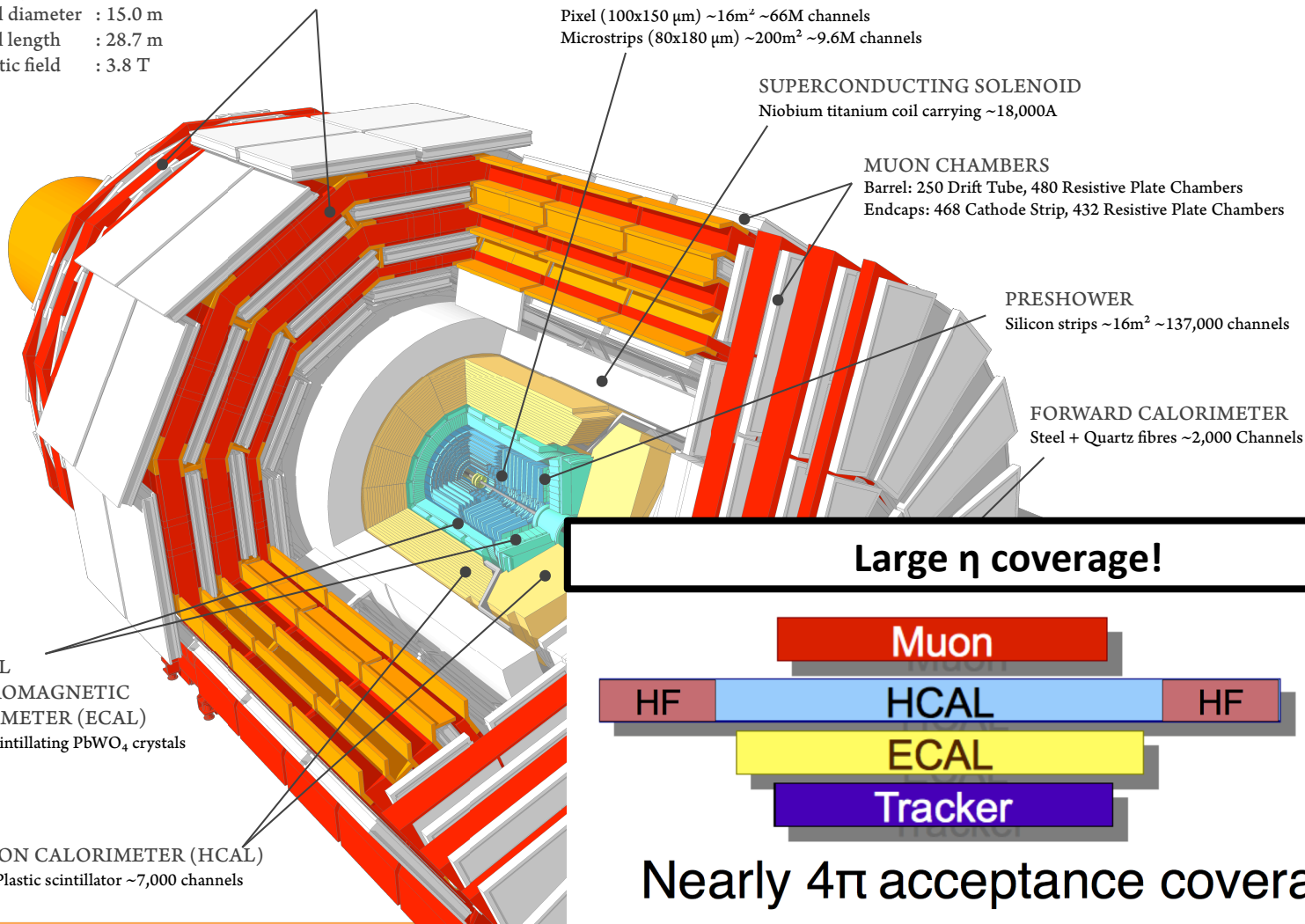
MUON CHAMBERS
 Barrel: 250 Drift Tube, 480 Resistive Plate Chambers
 Endcaps: 468 Cathode Strip, 432 Resistive Plate Chambers

PRESHOWER
 Silicon strips $\sim 16\text{m}^2 \sim 137,000$ channels

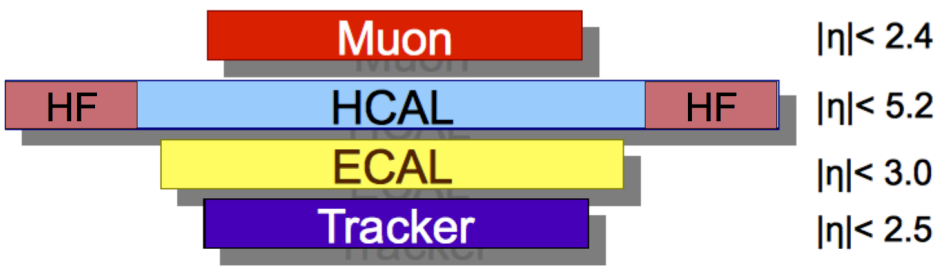
FORWARD CALORIMETER
 Steel + Quartz fibres $\sim 2,000$ Channels

CRYSTAL ELECTROMAGNETIC CALORIMETER (ECAL)
 $\sim 76,000$ scintillating PbWO_4 crystals

HADRON CALORIMETER (HCAL)
 Brass + Plastic scintillator $\sim 7,000$ channels



Large η coverage!



Nearly 4π acceptance coverage

How $r_n(\eta^a, \eta^b)$ is related to factorization and $\Psi_n(\eta)$ fluctuations?

If $V_{n\Delta}$ factorizes or $\Psi_n(\eta)$ indep. of η ,

$$r_n(\eta^a, \eta^b) = \frac{\langle v_n(-\eta^a) v_n(\eta^b) \rangle}{\langle v_n(\eta^a) v_n(\eta^b) \rangle} = 1 \quad (\text{for symmetric system})$$

Otherwise,

$$r_n(\eta^a, \eta^b) = \frac{\langle v_n(-\eta^a) v_n(\eta^b) \cos[n(\Psi_n(-\eta^a) - \Psi_n(\eta^b))] \rangle}{\langle v_n(\eta^a) v_n(\eta^b) \cos[n(\Psi_n(\eta^a) - \Psi_n(\eta^b))] \rangle}$$

$$\sim \frac{\langle \cos[n(\Psi_n(-\eta^a) - \Psi_n(\eta^b))] \rangle}{\langle \cos[n(\Psi_n(\eta^a) - \Psi_n(\eta^b))] \rangle} \quad (\text{for symmetric system})$$

$$\sim \langle \cos[n(\Psi_n(\eta^a) - \Psi_n(-\eta^a))] \rangle$$

(two EPs separated a gap of $2\eta^a$)

p-Pb analysis method

A subtlety in pPb as $v_n(-\eta^a) \neq v_n(\eta^a)$

$$r_n(\eta^a, \eta^b) = \frac{\langle v_n(-\eta^a) \times v_n(\eta^b) \times \cos[n(\Psi_n(-\eta^a) - \Psi_n(\eta^b))] \rangle}{\langle v_n(\eta^a) \times v_n(\eta^b) \times \cos[n(\Psi_n(\eta^a) - \Psi_n(\eta^b))] \rangle}$$

Do not cancel!

Let's take a 'geometric mean'

$$\begin{aligned} \sqrt{r_n(\eta^a, \eta^b) \times r_n(-\eta^a, -\eta^b)} &= \sqrt{\frac{V_{n\Delta}(-\eta^a, \eta^b)}{V_{n\Delta}(\eta^a, \eta^b)} \frac{V_{n\Delta}(\eta^a, -\eta^b)}{V_{n\Delta}(-\eta^a, -\eta^b)}} \\ &= \sqrt{\frac{\langle v_n(-\eta^a) v_n(\eta^b) \cos[n(\Psi_n(-\eta^a) - \Psi_n(\eta^b))] \rangle}{\langle v_n(\eta^a) v_n(\eta^b) \cos[n(\Psi_n(\eta^a) - \Psi_n(\eta^b))] \rangle} \frac{\langle v_n(\eta^a) v_n(-\eta^b) \cos[n(\Psi_n(\eta^a) - \Psi_n(-\eta^b))] \rangle}{\langle v_n(-\eta^a) v_n(-\eta^b) \cos[n(\Psi_n(-\eta^a) - \Psi_n(-\eta^b))] \rangle}} \\ &\sim \sqrt{\frac{\langle \cos[n(\Psi_n(-\eta^a) - \Psi_n(\eta^b))] \rangle}{\langle \cos[n(\Psi_n(\eta^a) - \Psi_n(\eta^b))] \rangle} \frac{\langle \cos[n(\Psi_n(\eta^a) - \Psi_n(-\eta^b))] \rangle}{\langle \cos[n(\Psi_n(-\eta^a) - \Psi_n(-\eta^b))] \rangle}} \boxed{\langle \cos[n(\Psi_n(\eta^a) - \Psi_n(-\eta^a))] \rangle} \end{aligned}$$

Drawback: p- and Pb-side averaged ,not differentiable

A subtlety in pPb as $v_n(-\eta^a) \neq v_n(\eta^a)$

$$r_n(\eta^a, \eta^b) = \frac{\langle v_n(-\eta^a) \times v_n(\eta^b) \times \cos[n(\Psi_n(-\eta^a) - \Psi_n(\eta^b))] \rangle}{\langle v_n(\eta^a) \times v_n(\eta^b) \times \cos[n(\Psi_n(\eta^a) - \Psi_n(\eta^b))] \rangle}$$

Do not cancel!

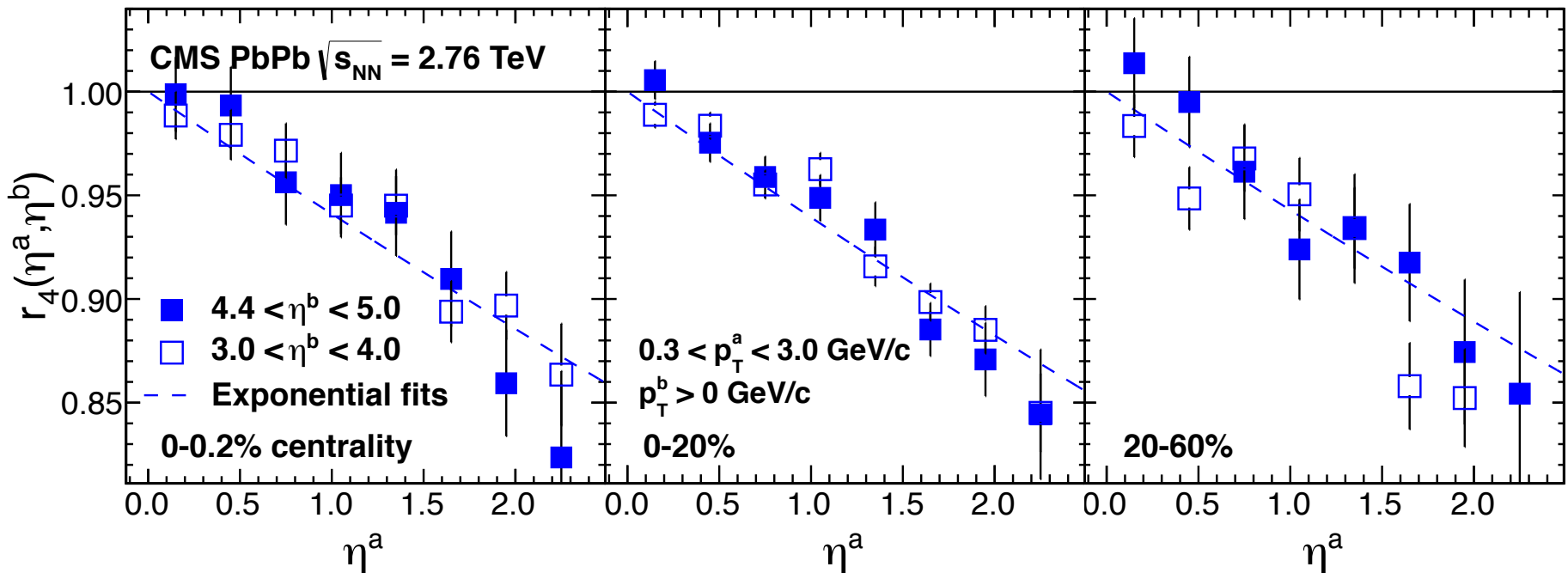
Let's take a 'geometric mean'

$$\begin{aligned} \sqrt{r_n(\eta^a, \eta^b) \times r_n(-\eta^a, -\eta^b)} &= \sqrt{\frac{V_{n\Delta}(-\eta^a, \eta^b)}{V_{n\Delta}(\eta^a, \eta^b)} \frac{V_{n\Delta}(\eta^a, -\eta^b)}{V_{n\Delta}(-\eta^a, -\eta^b)}} \\ &= \sqrt{\frac{\langle v_n(-\eta^a) v_n(\eta^b) \cos[n(\Psi_n(-\eta^a) - \Psi_n(\eta^b))] \rangle}{\langle v_n(\eta^a) v_n(\eta^b) \cos[n(\Psi_n(\eta^a) - \Psi_n(\eta^b))] \rangle} \frac{\langle v_n(\eta^a) v_n(-\eta^b) \cos[n(\Psi_n(\eta^a) - \Psi_n(-\eta^b))] \rangle}{\langle v_n(-\eta^a) v_n(-\eta^b) \cos[n(\Psi_n(-\eta^a) - \Psi_n(-\eta^b))] \rangle}} \\ &\sim \sqrt{\frac{\langle \cos[n(\Psi_n(-\eta^a) - \Psi_n(\eta^b))] \rangle}{\langle \cos[n(\Psi_n(\eta^a) - \Psi_n(\eta^b))] \rangle} \frac{\langle \cos[n(\Psi_n(\eta^a) - \Psi_n(-\eta^b))] \rangle}{\langle \cos[n(\Psi_n(-\eta^a) - \Psi_n(-\eta^b))] \rangle}} \boxed{\langle \cos[n(\Psi_n(\eta^a) - \Psi_n(-\eta^a))] \rangle} \end{aligned}$$

Drawback: p- and Pb-side averaged ,not differentiable

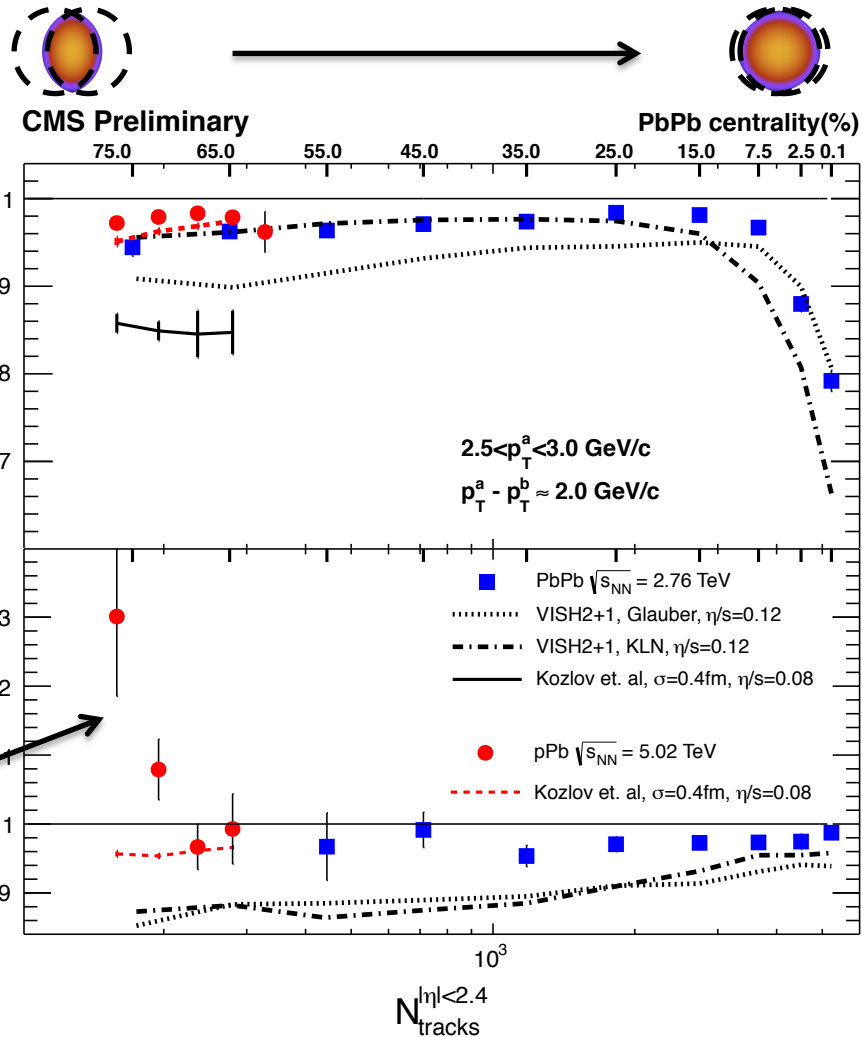
Centrality dependence of r_4

$$r_4(\eta^a, \eta^b) \approx \langle \cos[4(\Psi_4(\eta^a) - \Psi_4(-\eta^a))] \rangle$$



- ❖ Also roughly linear increase with η gap
- ❖ r_4 is related to r_2 , esp. for peripheral events (linear vs non-linear contributions)

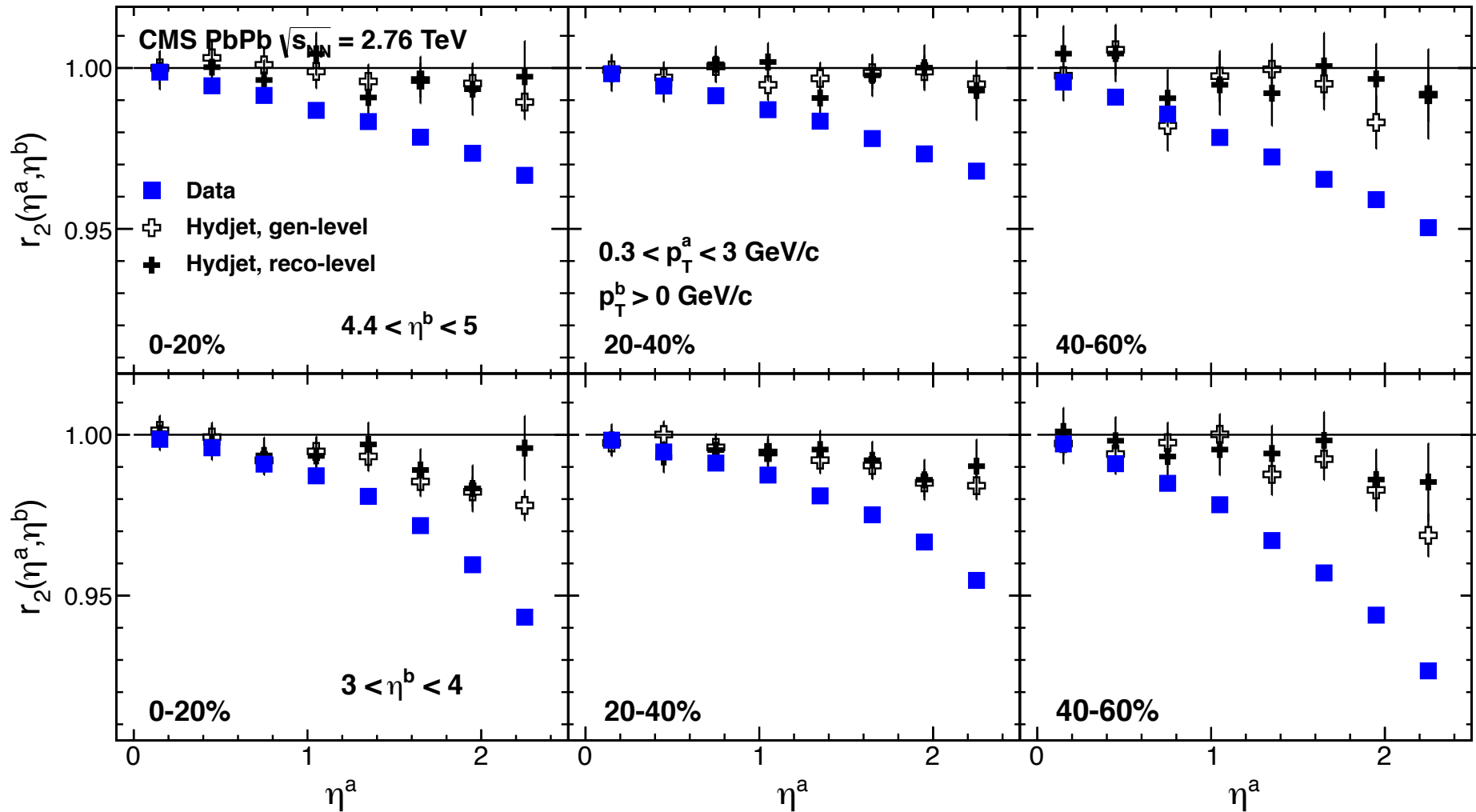
$\Psi_n(p_T)$ fluctuations in p-Pb and Pb-Pb



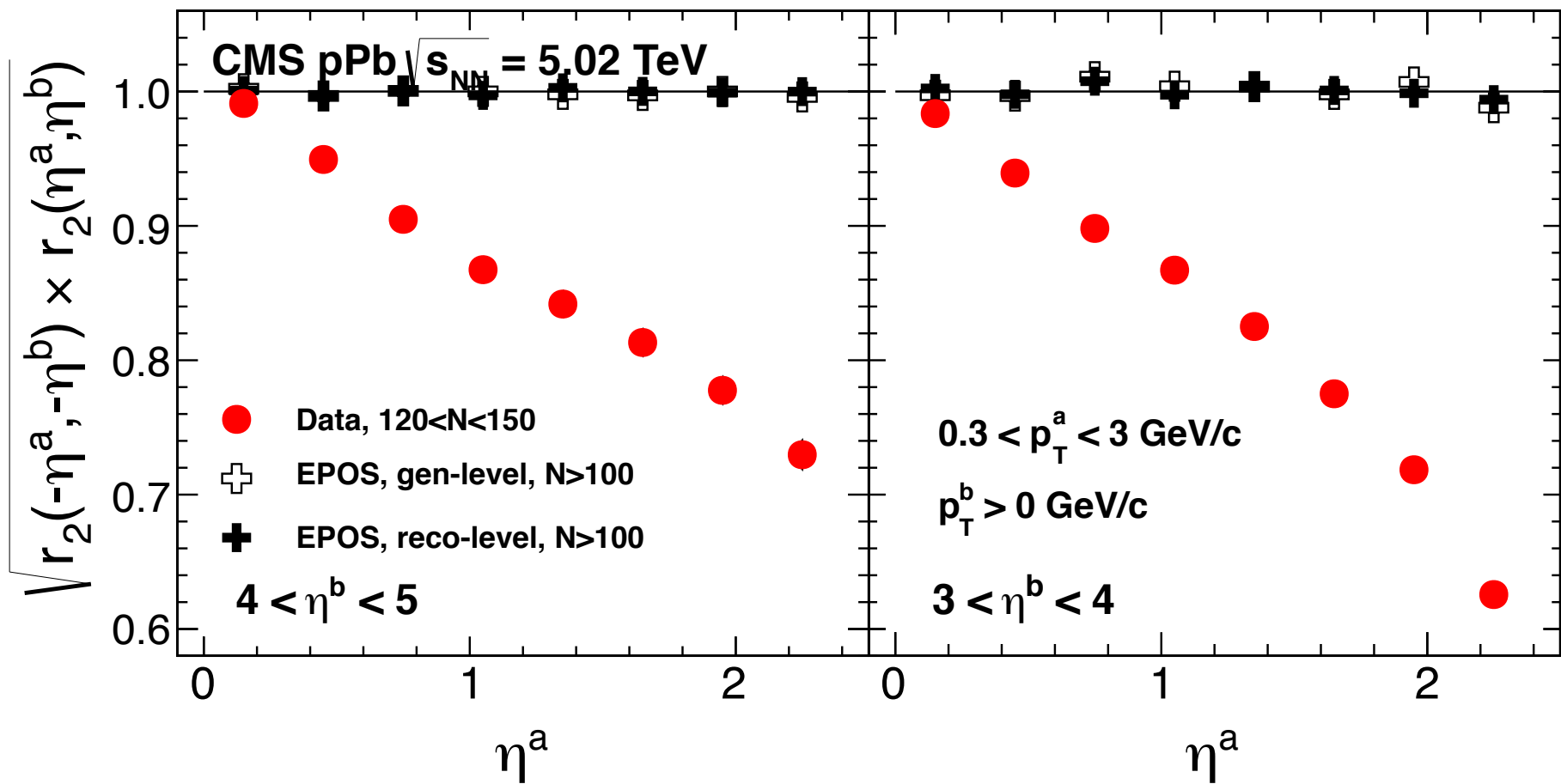
Significant effect toward central PbPb

nonflow

Comparison with HYDJET



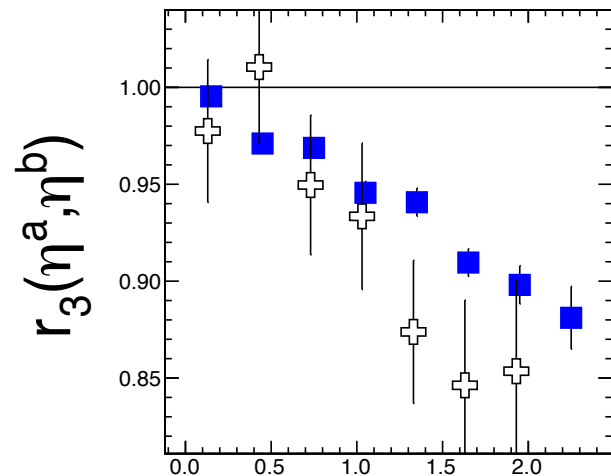
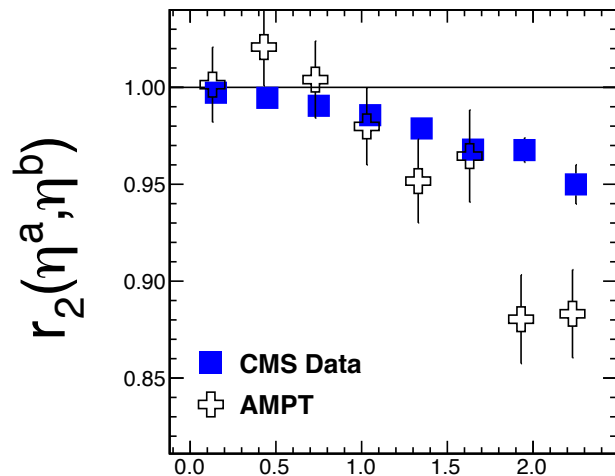
Comparison with EPOS



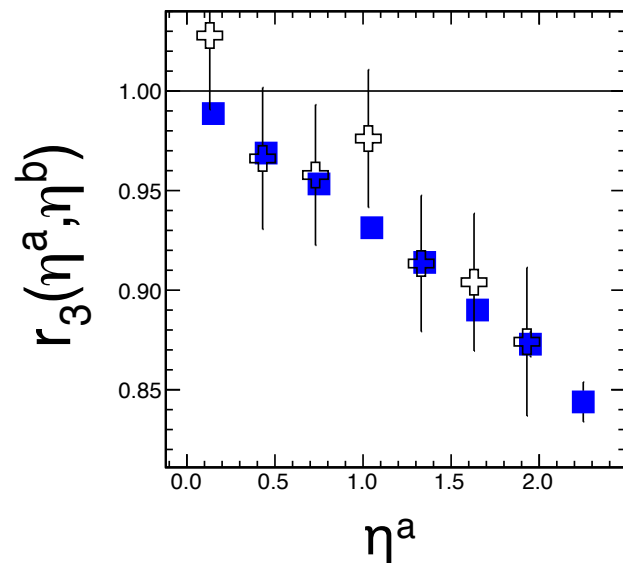
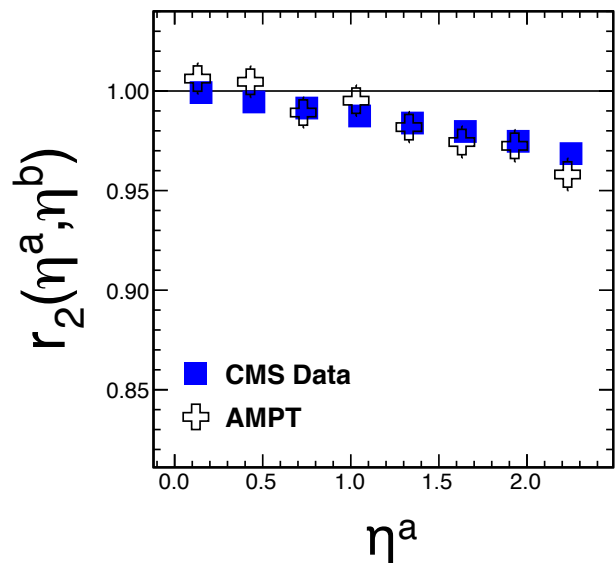
Comparison with AMPT



0-5%

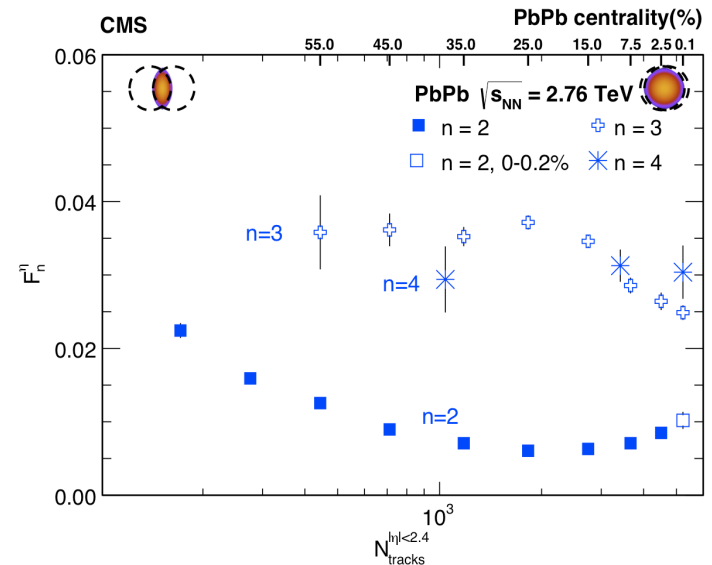
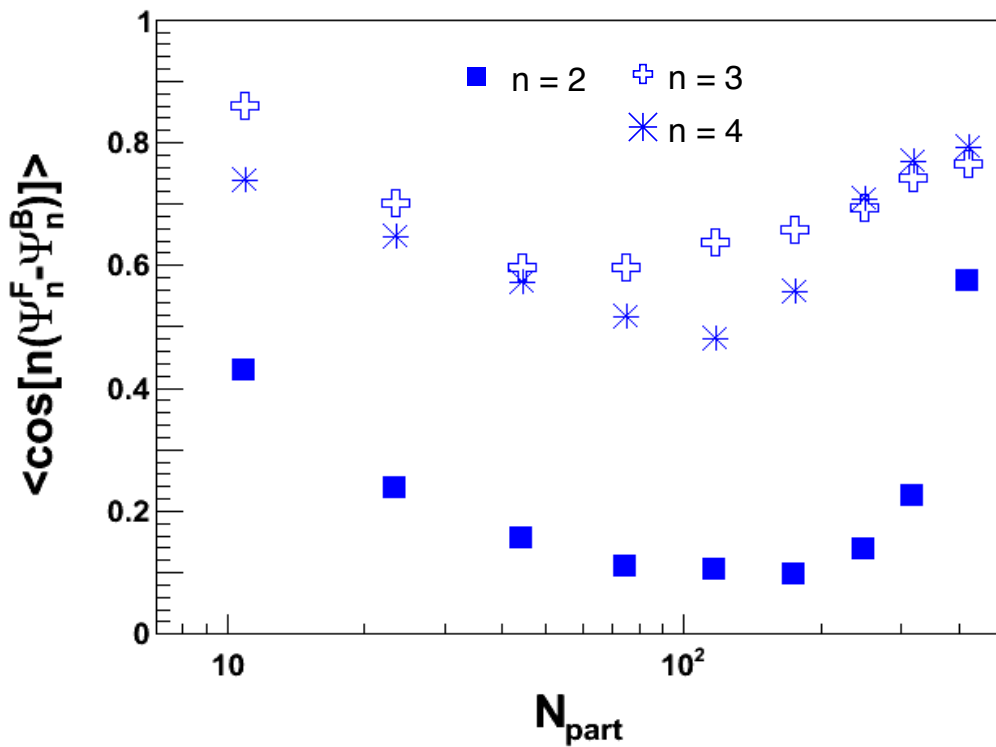


10-20%



Glauber model

- ❖ Trend **qualitatively consistent** with participant fluctuations in Glauber model
- ❖ Details **depend on dynamics**



v_n η -dependence scaled

