Thermal Blurring Effects on Fluctuations of Conserved Charges in Rapidity Space

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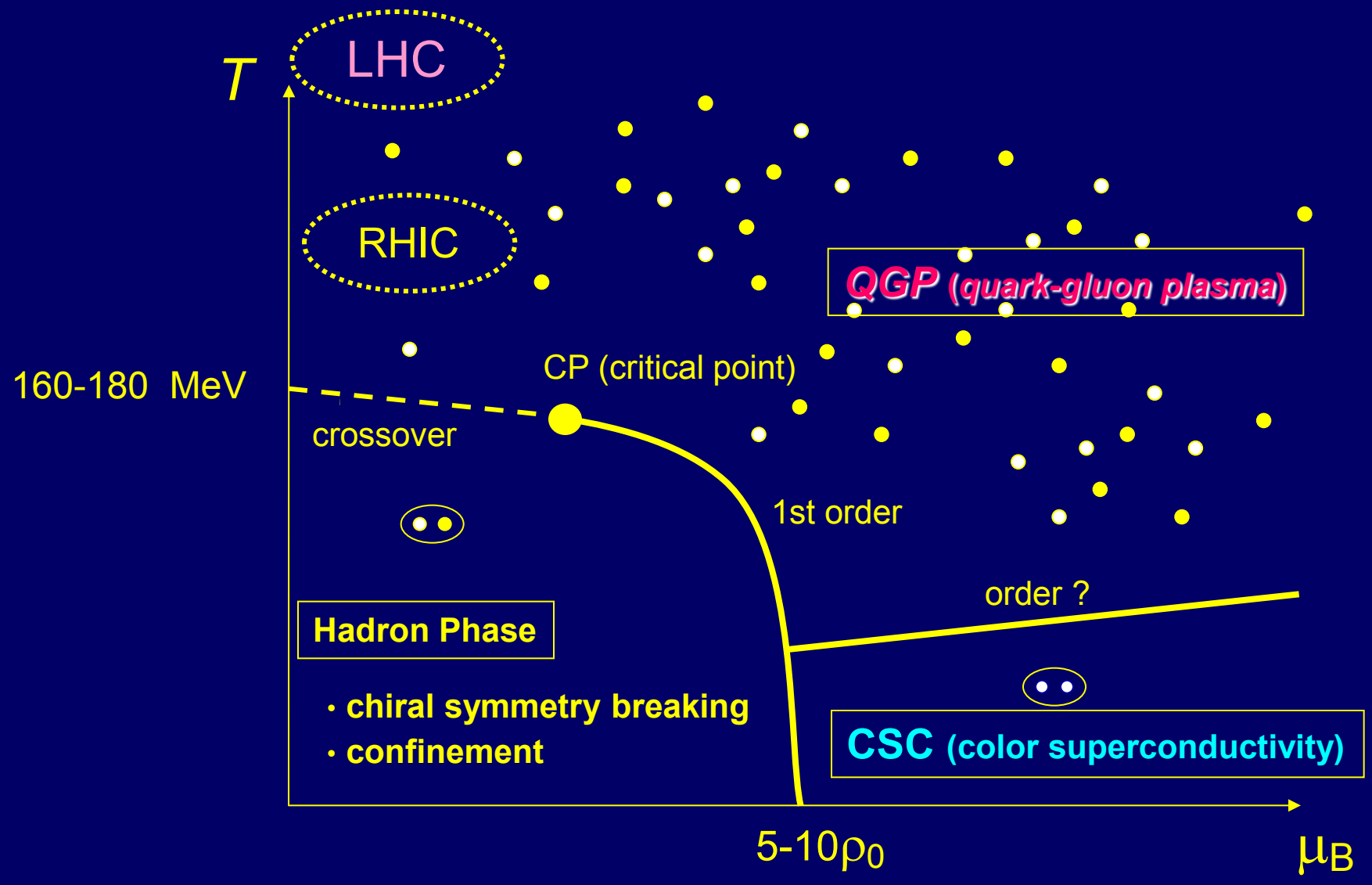
With M. Kitazawa, Y. Onishi, and M. Sakaida

QCD Phase Diagram



Why Conserved Charge Fluctuations ?

- Their values do not change during the phase transition
- Their values in QGP and Hadron Phase are different
- They change in Hadron Phase only by diffusion

D measure for electromagnetic charge fluctuation

Heinz, Müller, M.A., Jeon, Koch, 2000

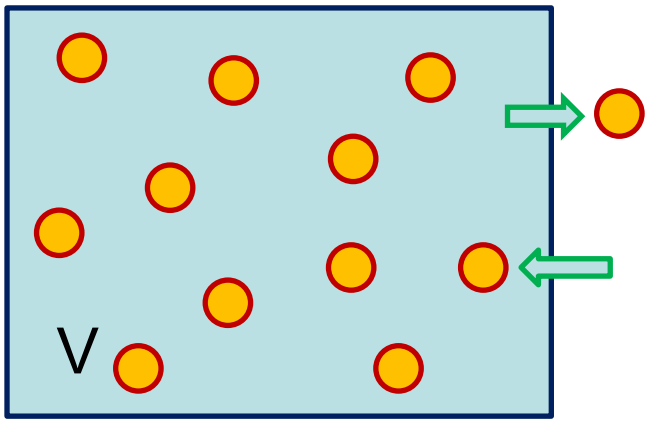
- Charge Fluctuation and Baryon Number Fluctuation are well-defined quantities, and can be measured on the lattice

- Lattice results and Effective Model results (equilibrium thermodynamics) are often compared with experimental results directly

  Does this make sense?

Conserved and Non-Conserved Charge Fluc.

Conserved Charge

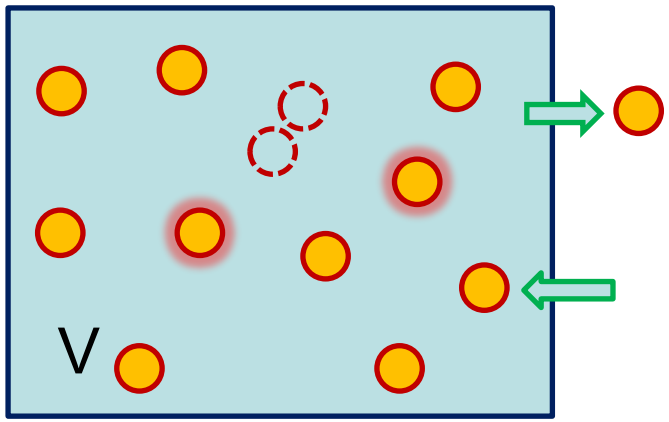


Only diffusion changes
the number of charge

relaxation time $\tau \to \infty$

for $V \to \infty$

Non-Conserved Quantity



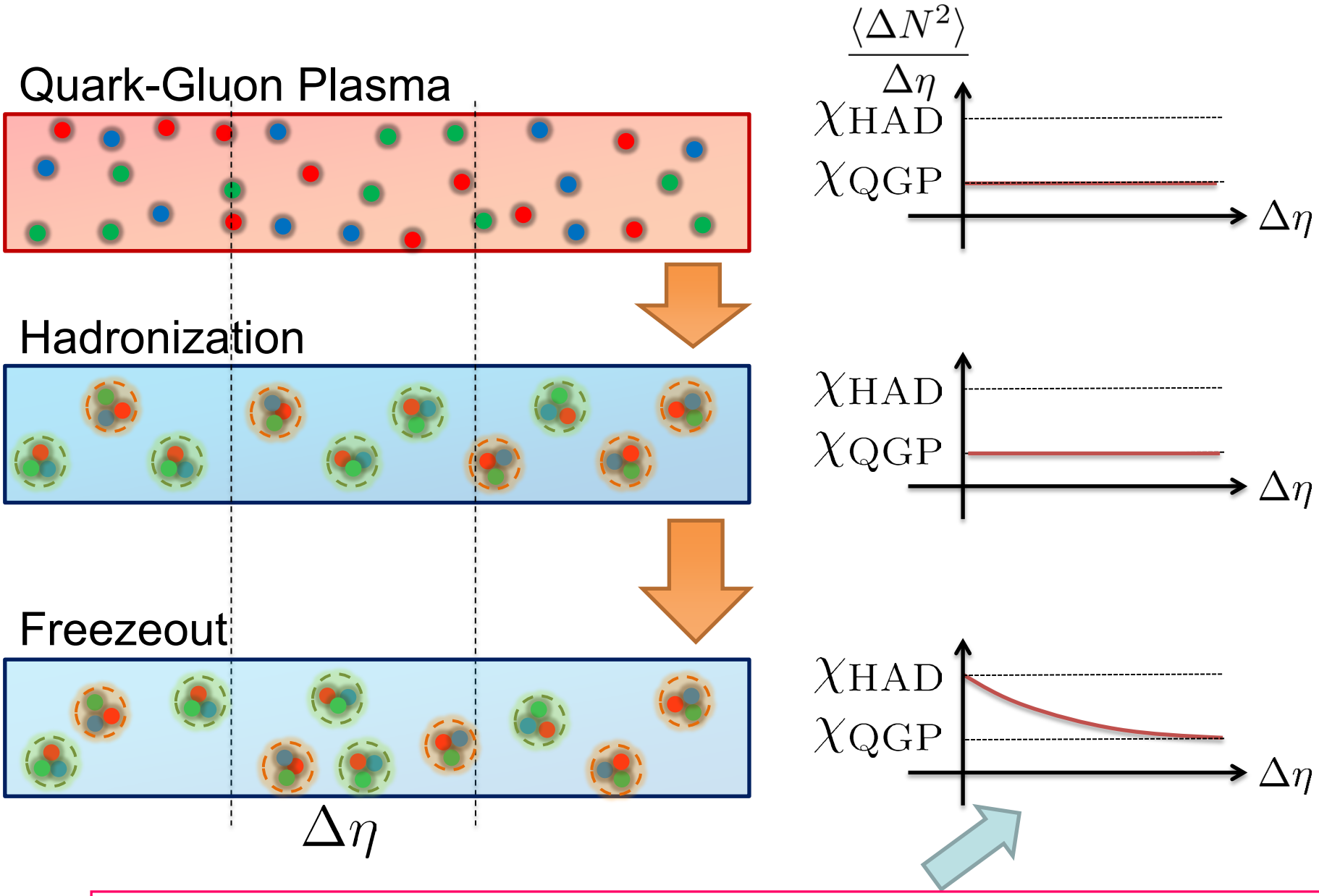
Quantity can change
anywhere in the volume

$\tau \to$ finite

for $V \to \infty$

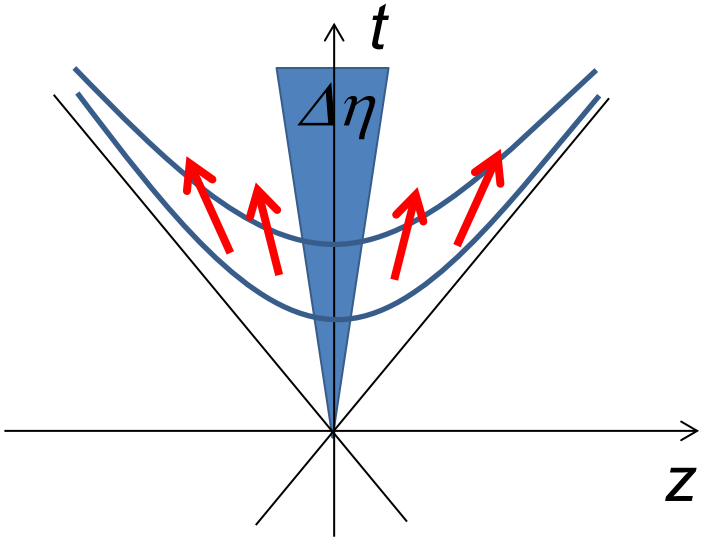
Necessary to consider dynamical evolution of fluctuation!

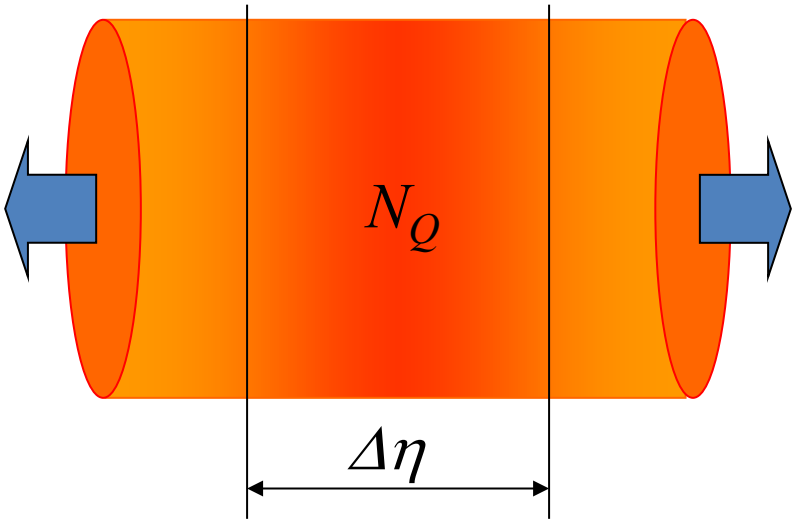
Time Evolution of C.C. fluctuation



In the $\Delta\eta$ dependence of C.C. Fluctuation, history of system is encoded

Theoretical Side



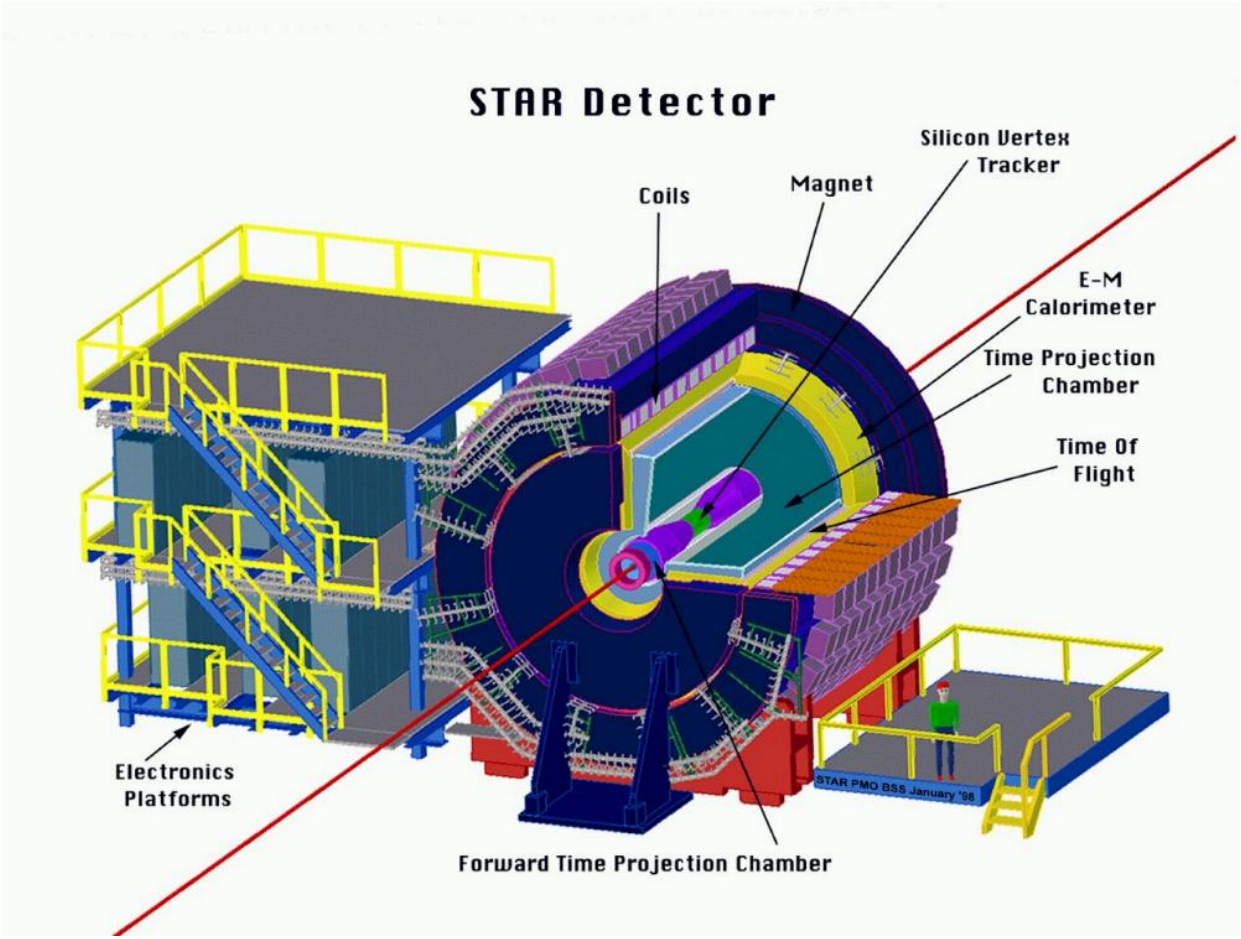


Fluctuation of conserved charges in $\Delta\eta$ (space-time rapidity)

||

What is usually calculated

Experimental Side



Detectors (usually) cover Δy ((pseudo) rapidity)

Relation between y and η ?

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$y = \eta$ in Bjorken picture

Rapidity

$$y = \frac{1}{2}\ln\frac{E+P_z}{E-P_z} = \frac{1}{2}\ln\frac{1+v_z}{1-v_z}$$

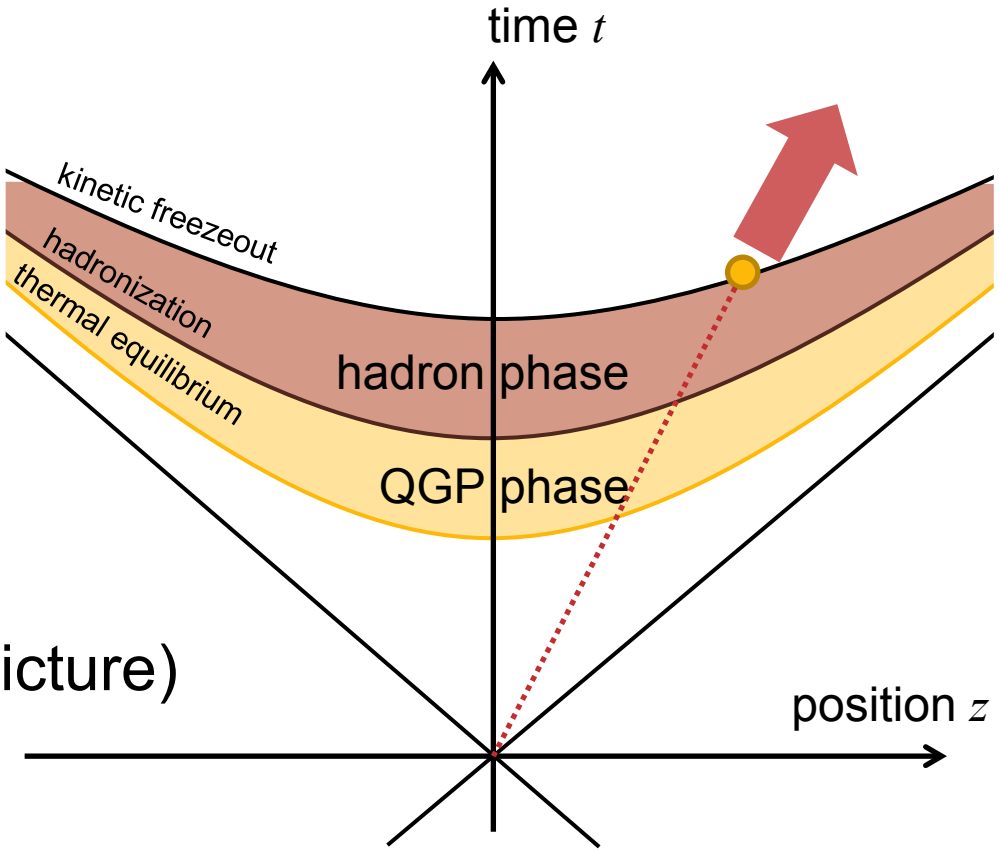
Momentum P_z

identified
(in Bjorken picture)

Spacetime Rapidity

$$\eta = \frac{1}{2}\ln\frac{t+z}{t-z} = \frac{1}{2}\ln\frac{1+z/t}{1-z/t}$$

position z



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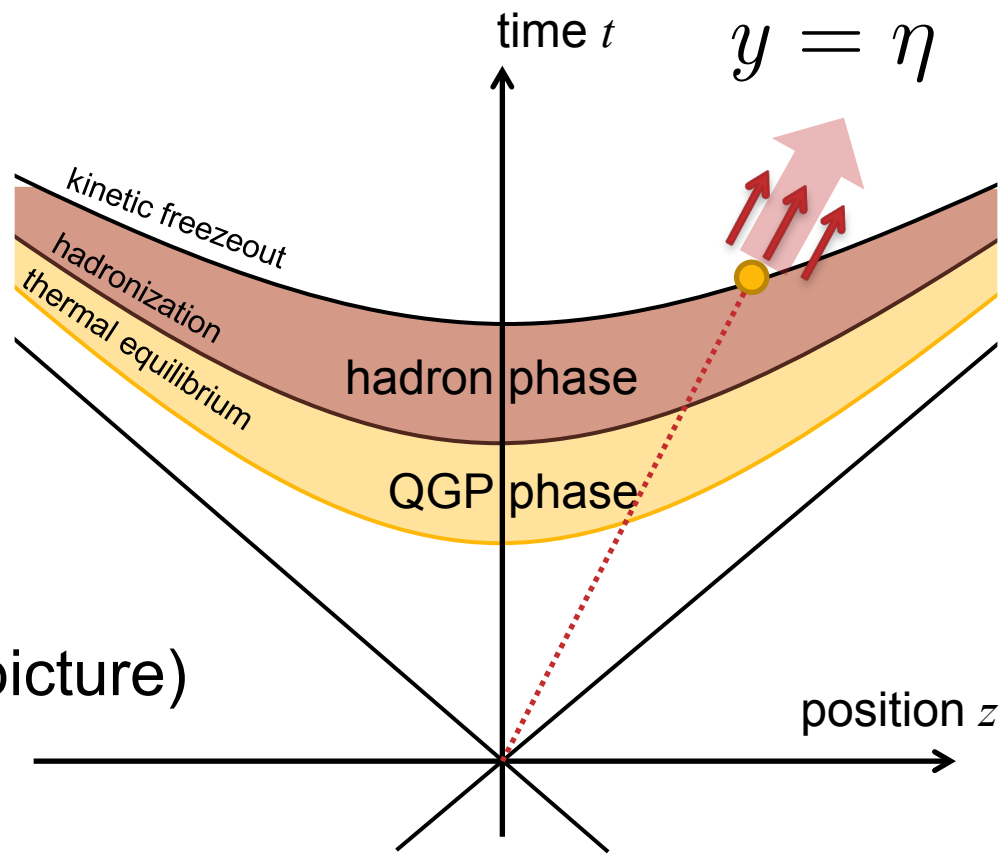

But this is a relation for FLOW rapidity

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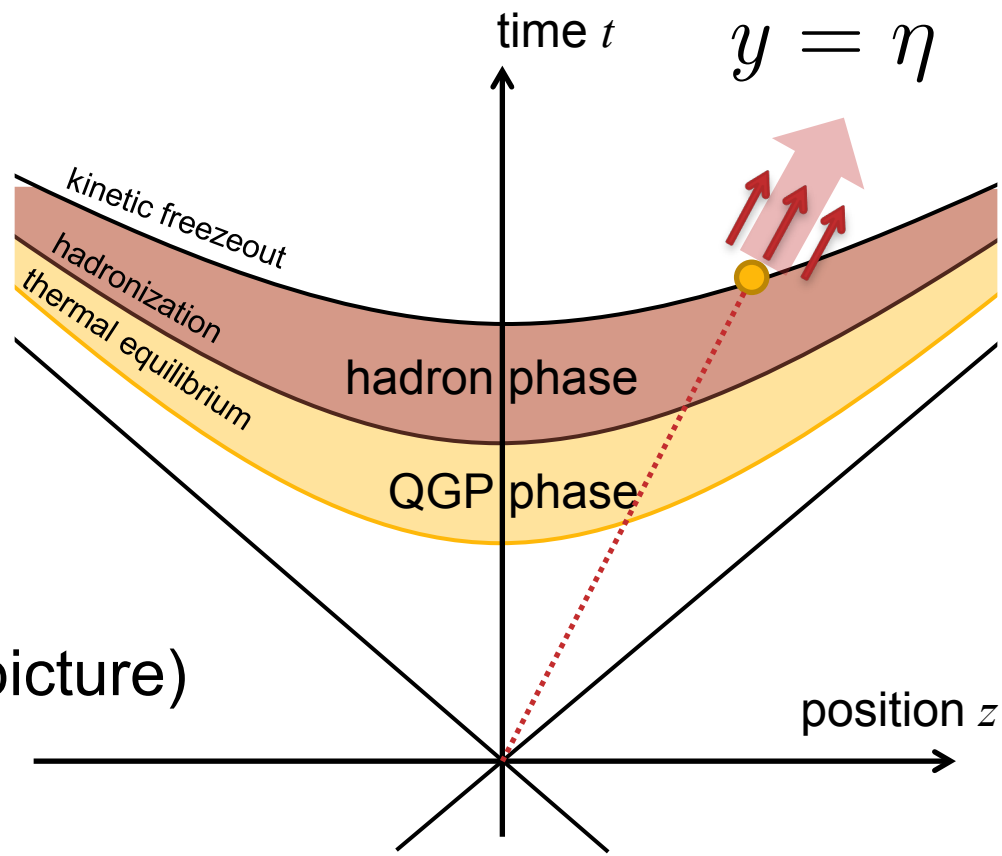

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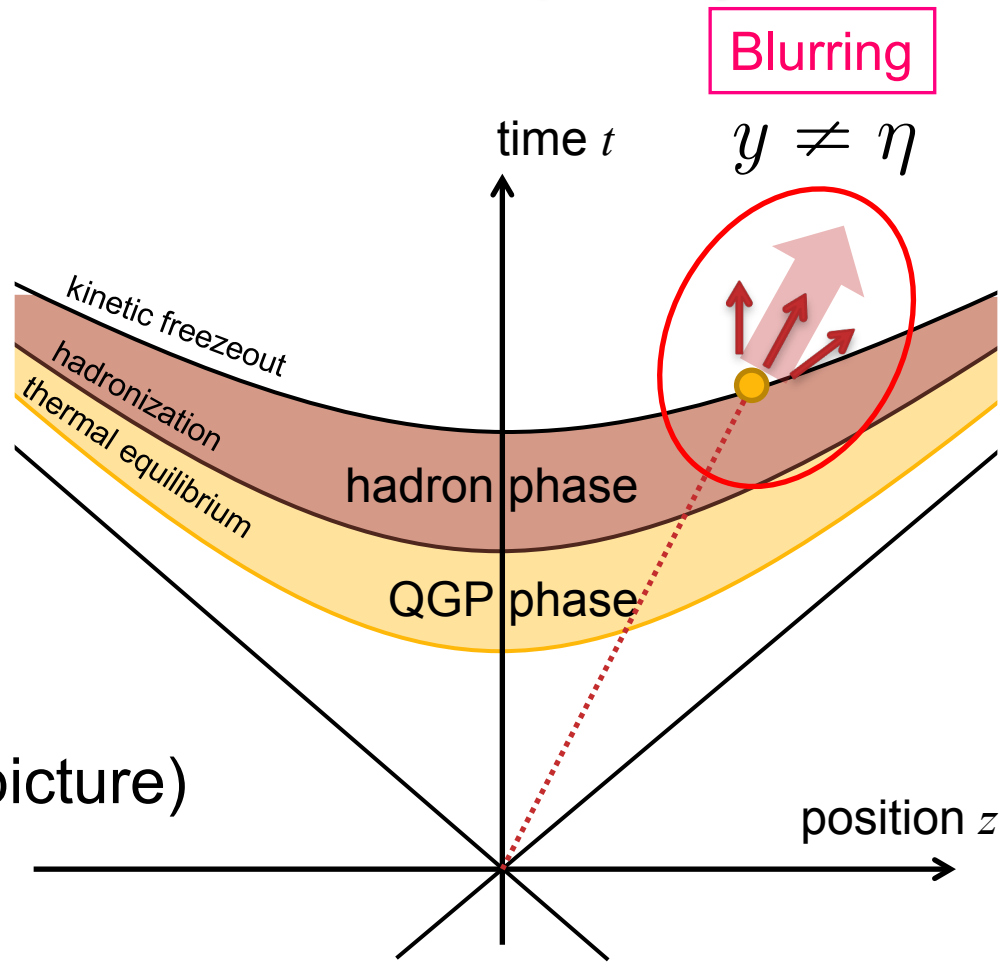

For a fixed (pseudo)rapidity

$y = \eta$ in Bjorken picture

Rapidity

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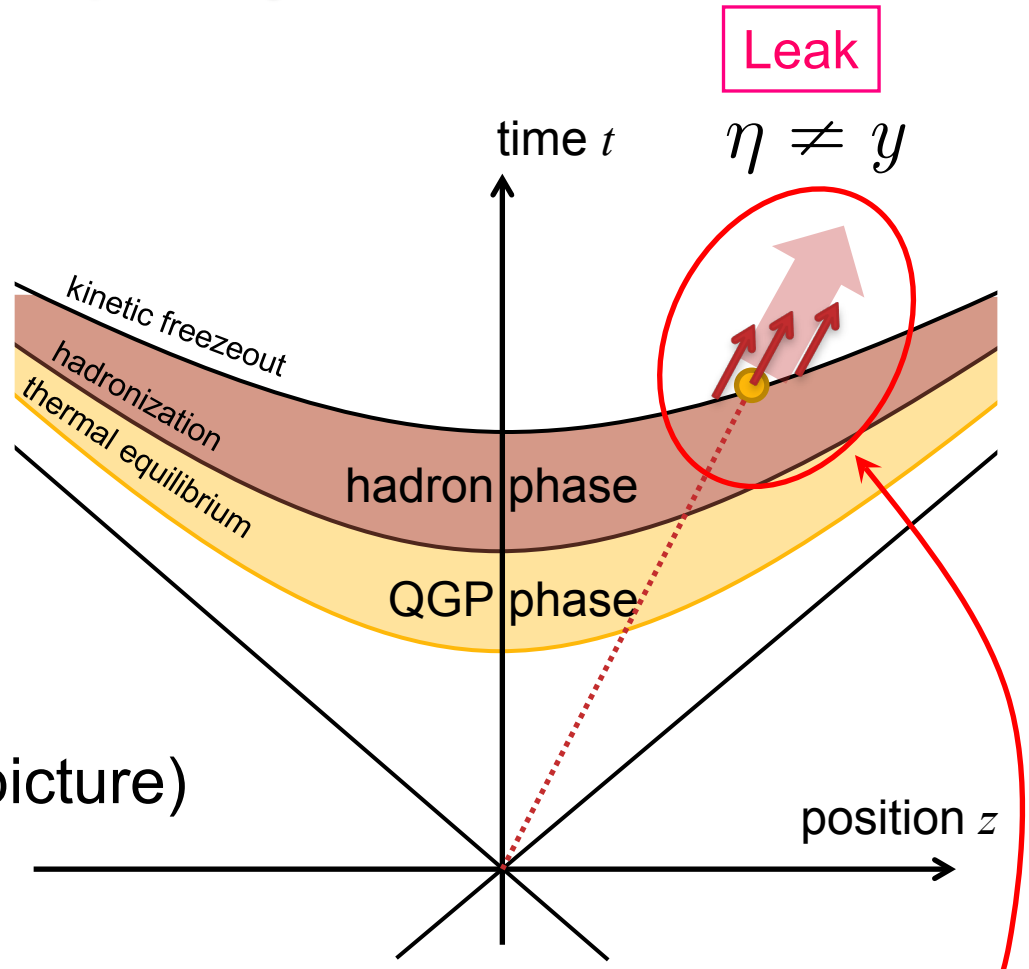


Blurring or Leak in rapidity space takes place !

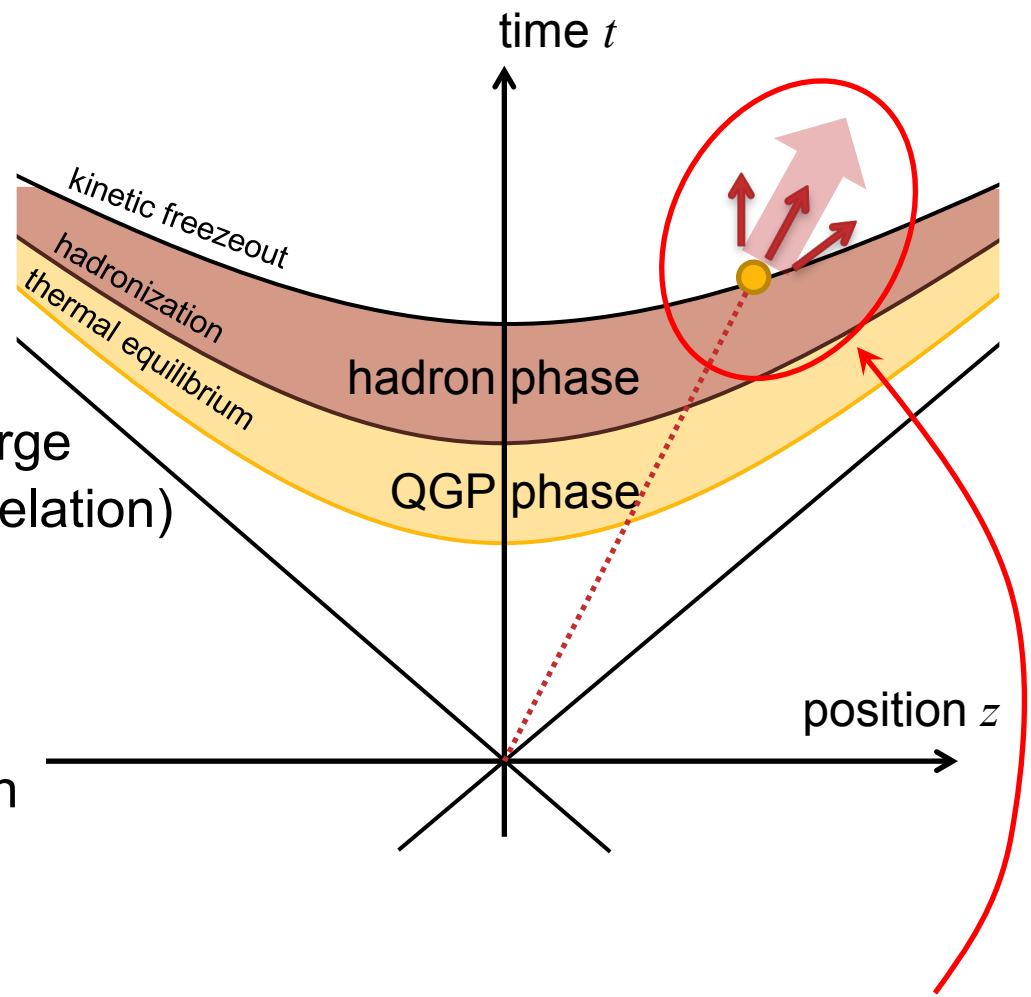
Thermal Blurring or Leak

$y = \eta$ in Bjorken picture

is blurred in one particle distribution owing to thermal motion

Accordingly, conserved charge fluctuation (two particle correlation) is modified

In the following, we use boosted Maxwell-Boltzmann distribution (with radial and longitudinal flow)



Blurring or Leak in rapidity space takes place !

Strategy: Chapman-Kolmogorov Equation

Chemical freeze-out

$P_1(\eta \to \eta')$ *Diffusion*

spacetime rapidity space

Kinetic freeze-out

$P_2(\eta' \to y)$ *Blurring (Leak)*

spacetime rapidity space
→ rapidity space

Observables at (pseudo)rapidity y

$$P(\eta \to y)$$

$$P(\eta \to y) = \int_{-\infty}^{\infty} d\eta' P_1(\eta \to \eta') P_2(\eta' \to y)$$

Chapman-Kolmogorov Equation

Computing Net Charge Cumulants with Initial Fluc.

1. Assume diffusions of positive and negative charges are independent

2. Assume initial system is uniform and that fluctuation is local (fluctuations at different points on the timelike initial hypersurface are not correlated)

$$\boxed{\begin{aligned}&\left\langle M_{i_1}(\eta_1)M_{i_2}(\eta_2)\cdots M_{i_l}(\eta_l)\right\rangle_{\mathrm{c}}\\&\quad=[M_{i_1}M_{i_2}\cdots M_{i_l}]_{\mathrm{c}}\delta(\eta_1-\eta_2)\cdots\delta(\eta_1-\eta_l)\end{aligned}}$$

3. From these assumptions and Chapman-Kolmogorov equation, we obtain

2nd and 4th Cumulants

2nd

$$\frac{\left\langle Q^2_{(\mathrm{net})}\right\rangle_c}{\left\langle Q_{(\mathrm{tot})}\right\rangle_c} = 1 + \left(\frac{\left\langle M^2_{(\mathrm{net})}\right\rangle_c}{\left\langle M_{(\mathrm{tot})}\right\rangle_c} - 1\right)\left(\frac{1}{\Delta}\int_{-\infty}^{\infty} d\eta \left[\int_{-\Delta/2}^{\Delta/2} dy \int_{-\infty}^{\infty} d\eta' P_1(\eta \to \eta') P_2(\eta' \to y)\right]^2\right)$$

4th

$$\frac{\left\langle Q^4_{(\mathrm{net})}\right\rangle_c}{\left\langle Q_{(\mathrm{tot})}\right\rangle_c} = 1 - 7F_2 + 12F_3 - 6F_4 + \frac{3\left\langle M^2_{(\mathrm{net})}\right\rangle_c}{\left\langle M_{(\mathrm{tot})}\right\rangle_c}\left(F_2 - 2F_3 + F_4\right)$$

$$F_n = \frac{1}{\Delta}\int_{-\infty}^{\infty} d\eta \left[\int_{-\Delta/2}^{\Delta/2} dy \int_{-\infty}^{\infty} d\eta' P_1(\eta \to \eta') P_2(\eta' \to y)\right]^n$$

Concrete Forms of P_1 and P_2

Diffusion + Blurring(Leak) Diffusion Blurring(Leak)

$$P(\eta \to y) = \int_{-\infty}^{\infty} d\eta' P_1(\eta \to \eta') P_2(\eta' \to y)$$

Diffusion

$$P_1(\eta \to \eta') = \frac{1}{2\sqrt{\pi D t}} \exp\left(-\frac{(\eta' - \eta)^2}{4Dt} \right)$$

Continuum limit of solution of Diffusion Master Equation

Blurring(Leak)

$$P_2(\eta' \to y, \beta) = \int_0^{\infty} dp_T \frac{z}{2m^3 K_2(z)} \exp\left(-z\gamma \cosh(y - \eta') \sqrt{1 + \left(\frac{p_T}{m}\right)^2} \right) \times \left[-\gamma\beta p_T^2 I_1(\gamma\beta p_T/T) + \gamma p_T \sqrt{m^2 + p_T^2} \cosh(y - \eta') I_0(\gamma\beta p_T/T) \right]$$

Boosted Maxwell-Boltzmann distribution

y: rapidity of a particle, $z = m/T$, β : transverse flow velocity

2nd Cumulant without Diffusion or Blurring

2nd (full)

$$\frac{\left\langle Q^2_{(\mathrm{net})}\right\rangle_c}{\left\langle Q_{(\mathrm{tot})}\right\rangle_c} = 1 + \left(\frac{\left\langle M^2_{(\mathrm{net})}\right\rangle_c}{\left\langle M_{(\mathrm{tot})}\right\rangle_c} - 1\right)\left(\frac{1}{\Delta}\int_{-\infty}^{\infty} d\eta \left[\int_{-\Delta/2}^{\Delta/2} dy \int_{-\infty}^{\infty} d\eta' P_1(\eta \to \eta') P_2(\eta' \to y)\right]^2\right)$$

diffusion only

$$\frac{\left\langle Q^2_{(\mathrm{net})}\right\rangle_c}{\left\langle Q_{(\mathrm{tot})}\right\rangle_c} = 1 + \left(\frac{\left\langle M^2_{(\mathrm{net})}\right\rangle_c}{\left\langle M_{(\mathrm{tot})}\right\rangle_c} - 1\right)\left(\frac{1}{\Delta}\int_{-\infty}^{\infty} d\eta \left[\int_{-\Delta/2}^{\Delta/2} d\eta' P_1(\eta \to \eta')\right]^2\right)$$

blurring only

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none (most papers)

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2nd Cumulant without Diffusion or Blurring

diffusion only

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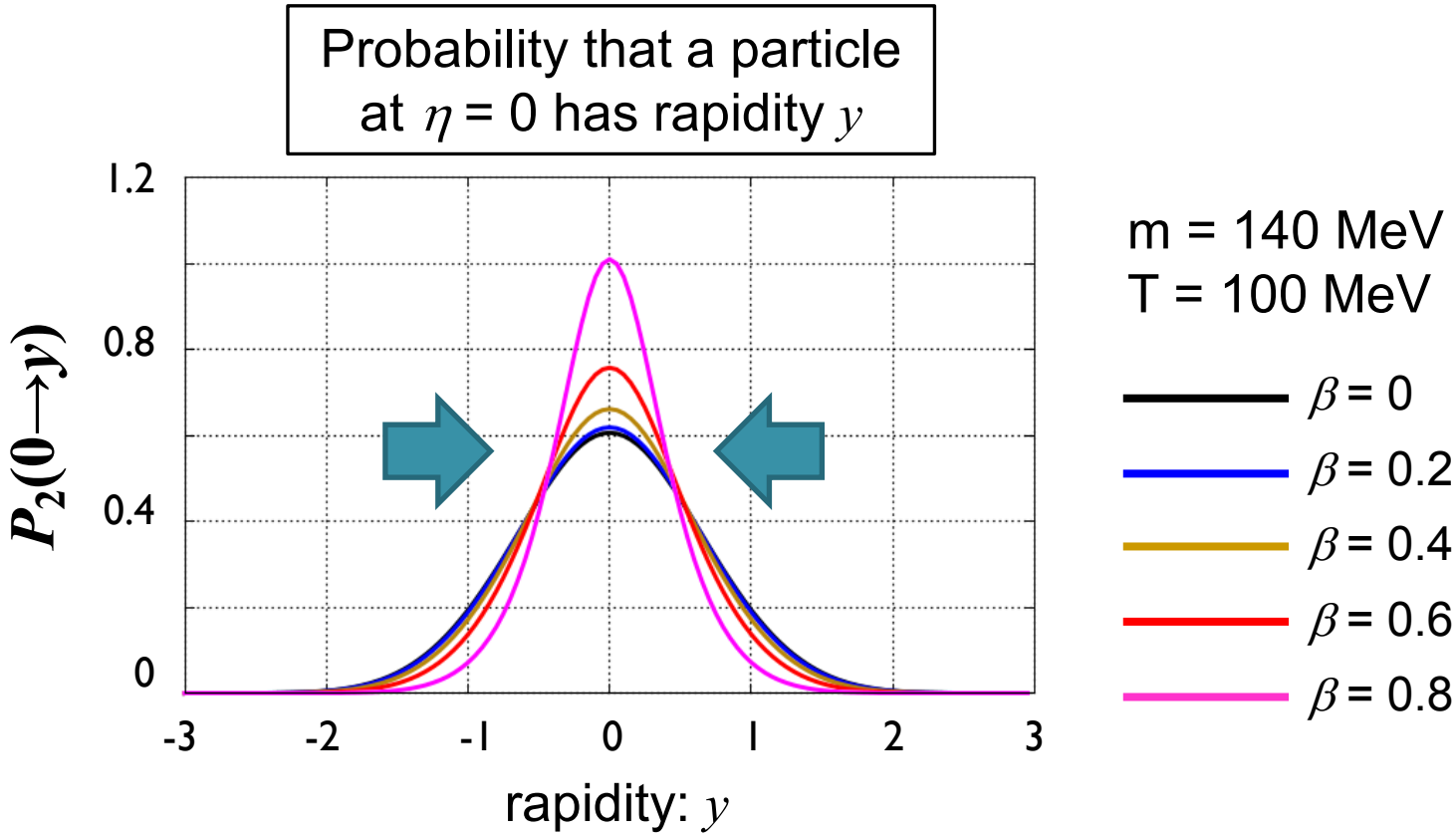
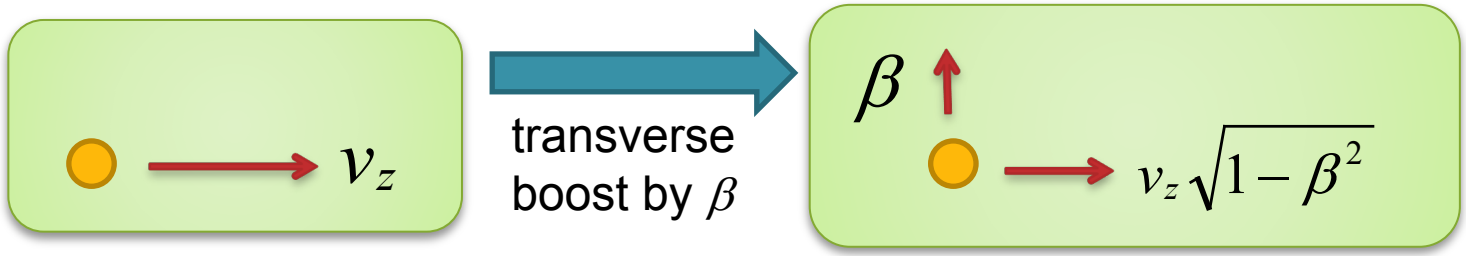
~~Freeze-out parameters: lattice meets experiment~~

Results

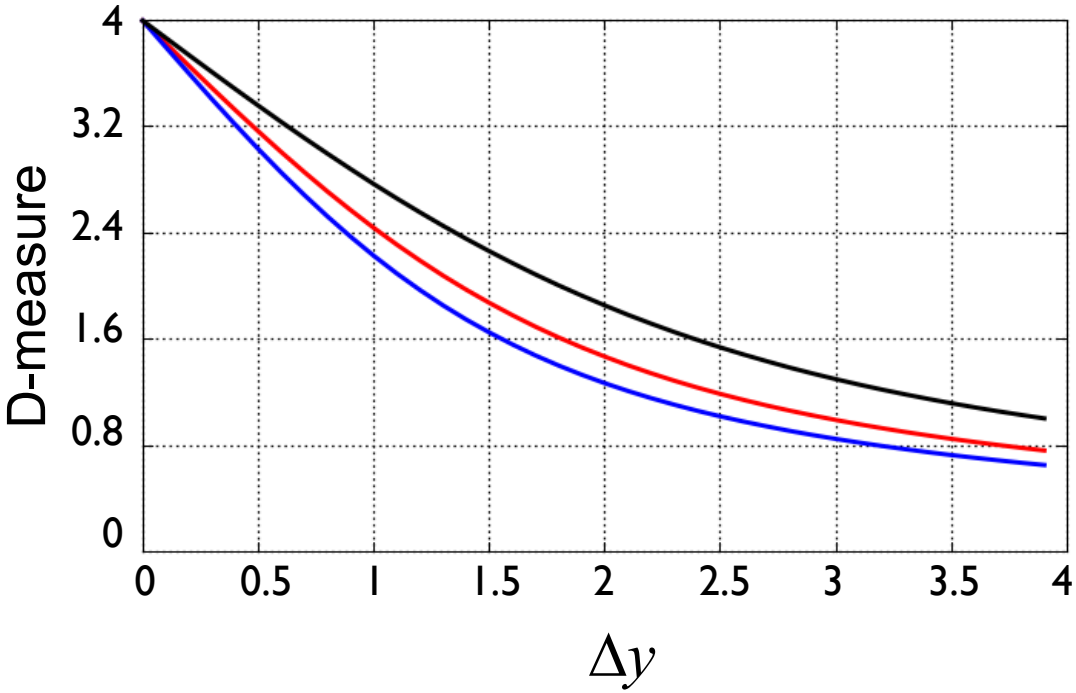
Blurring Effect and Transverse Flow β



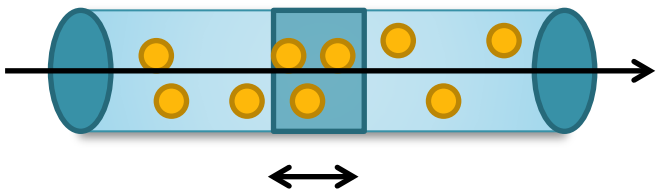
The larger β, the less blurring. This is a relativistic effect.



Result (with β=0)



Detector coverage



Δy (in rapidity space)

Diffusion only

Blurring only

Diffusion + Blurring

Initial fluctuation

$$\frac{\text{Initial fluctuation}}{\text{Thermal fluctuation in hadron phase}} = 0$$

m = 140 MeV

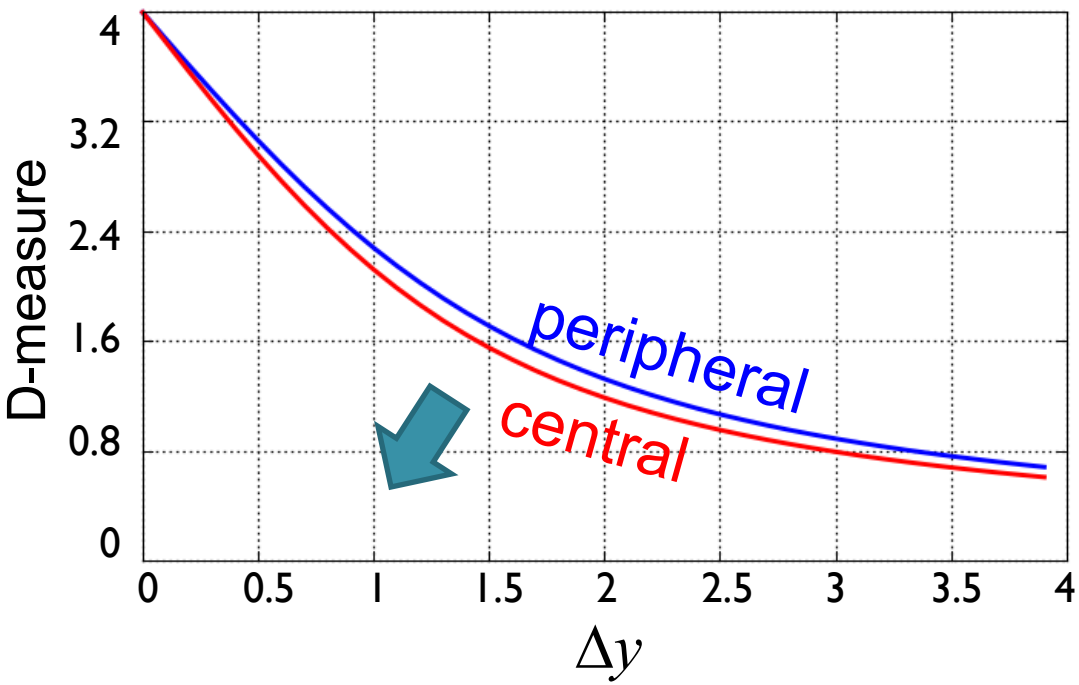
T = 100 MeV

$\beta = 0$

$\sqrt{Dt} = 0.4$ (D : diffusion const.)

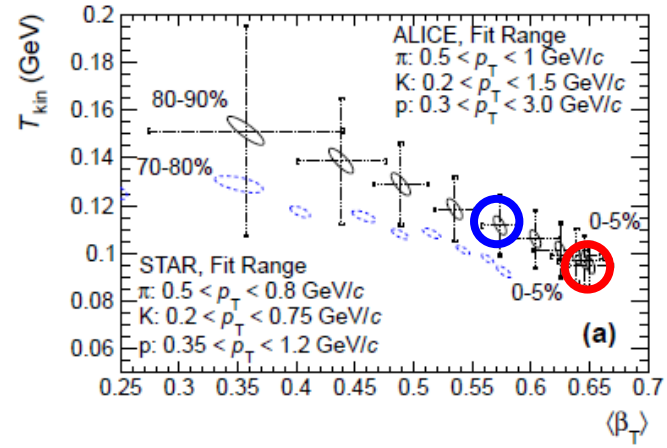
Sakaida, Asakawa, Kitazawa, PRC 2014

Result (with $\beta \neq 0$): Blurring only



β = 0.65, T = 100 MeV (centrality: 0 – 5 %)

β = 0.57, T= 120MeV (centrality: 40 – 50 %)



Initial fluctuation

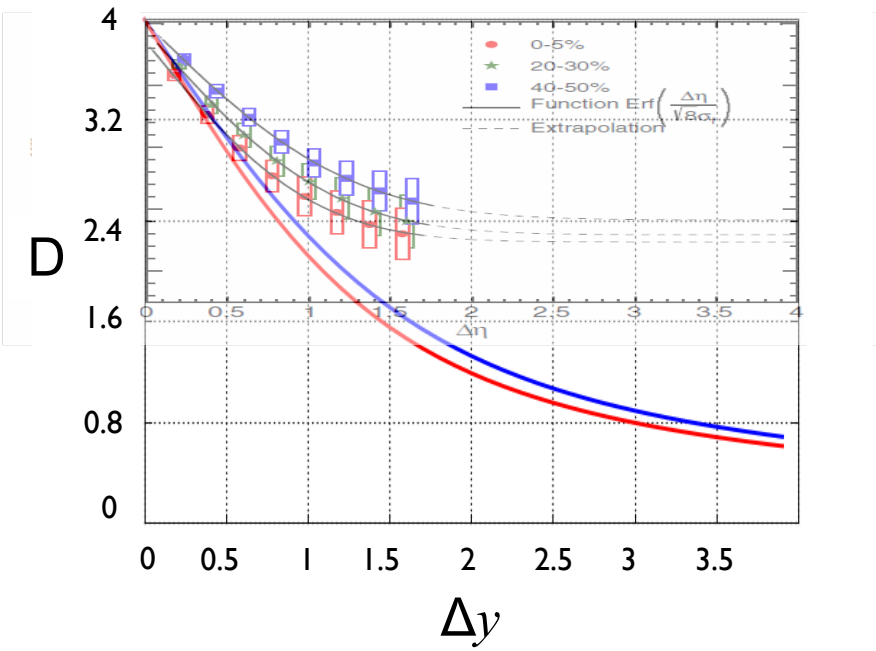
$$\frac{\text{Initial fluctuation}}{\text{Thermal fluctuation in hadron phase}} = 0$$

The more central, the less blurring (because of lower T)

Comparison with ALICE result: Blurring only

Initial fluctuation

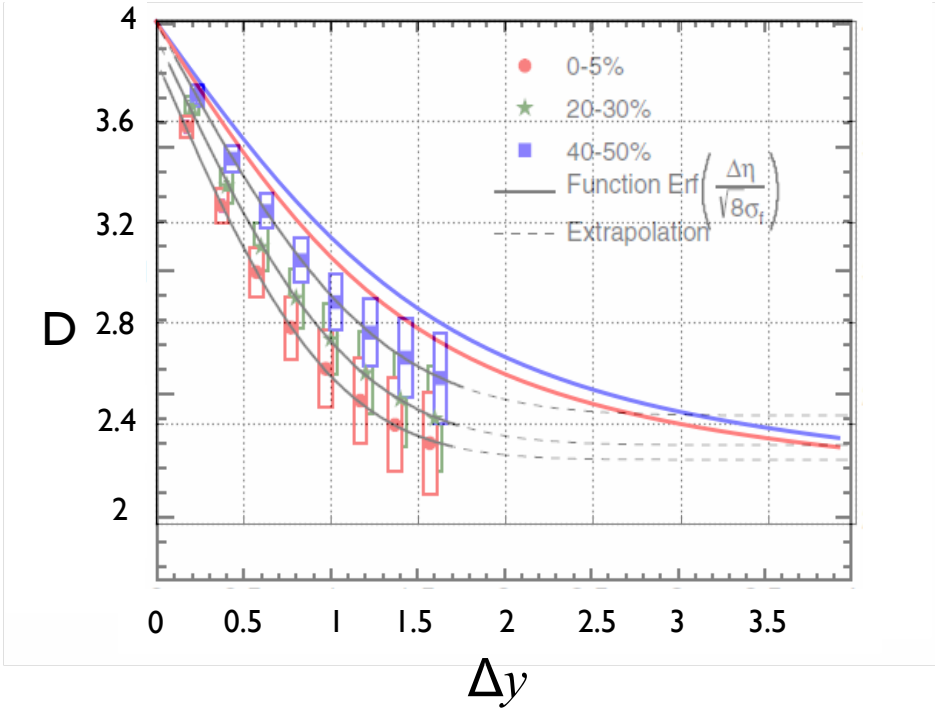
$$\frac{\text{Initial fluctuation}}{\text{Thermal fluctuation in hadron phase}} = 0$$



Room for other effects

Initial fluctuation

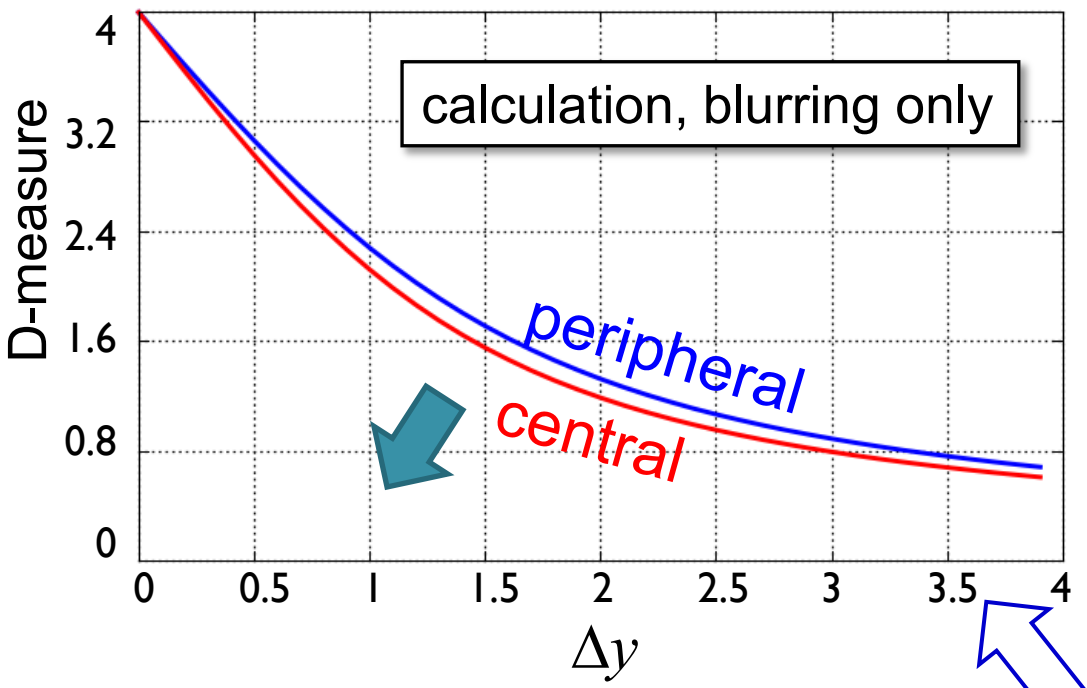
$$\frac{\text{Initial D-measure}}{\text{Thermal D-measure in hadron phase}} = 0.5$$



Only with blurring, the result exceeds the experimental data

This init. cond. is rejected

Comparison with ALICE result: Blurring only



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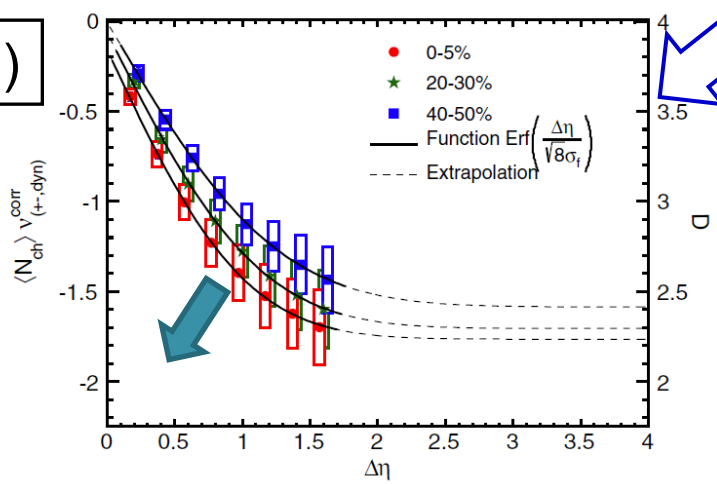
ALICE (exp)



- Qualitatively similar
- But note that diffusion would give opposite tendency (more central = longer diffusion)

Summary and Perspectives

- We have developed a formulation to evaluate the effects of diffusion and thermal blurring (leak) for conserved charge fluctuation

- We have assumed Bjorken scaling

  (not essential in the formulation)

At lower energies (e.g., BES energies)

- Violation of Bjorken scaling

  (lost correspondence between spacetime rapidity and rapidity)

- Larger effect of global charge conservation

  (smaller system size)

- Shorter diffusion duration

  (smaller time scale)

Summary and Perspectives

At lower energies (e.g., BES energies)

- Violation of Bjorken scaling

  (lost correspondence between spacetime rapidity and rapidity, i.e., $y \neq \eta$)

- Non-negligible effect of global charge conservation

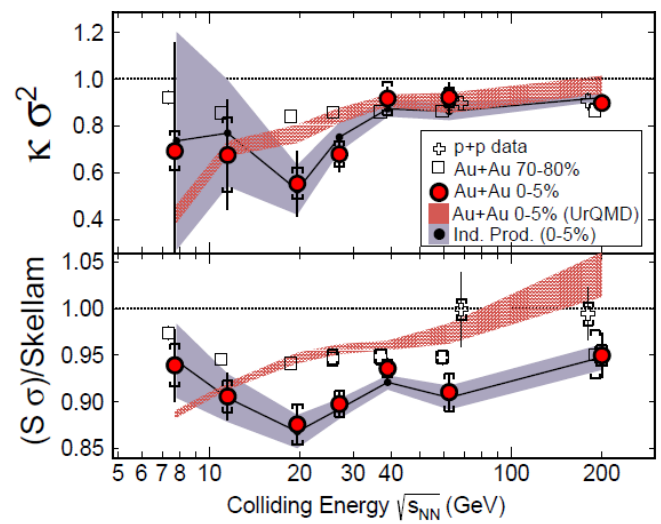
  (smaller system size)

- Shorter diffusion duration

  (smaller time scale)

- Still blurring (leak) exists

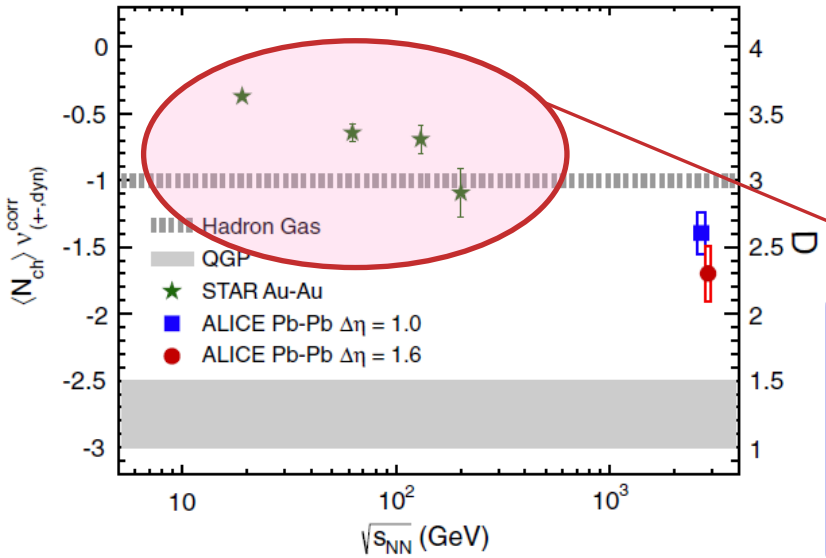
  (similar temperature scale)



Results like this are different from what is calculated theoretically in most cases and are not supposed to be compated with theoretical results without taking these effects into account

Perspective (Backup)

E.M. charge fluctuation

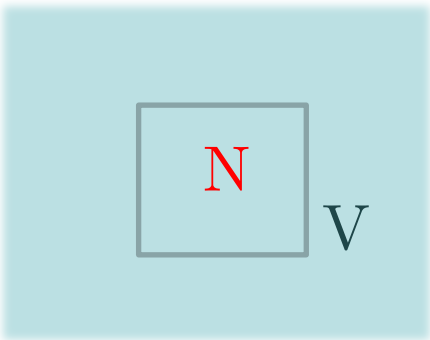


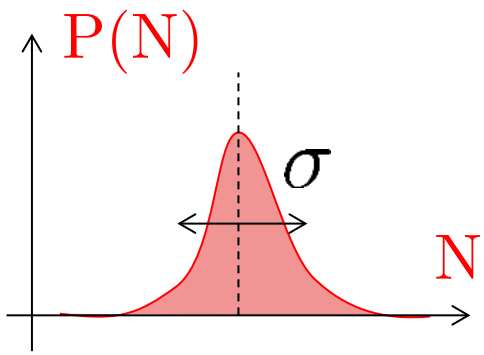
ALICE , PRL 110, 152301 (2013)

To understand SPS & RHIC result, these effects (diffusion, blurring, and Bjorken scaling violation, $y \neq \eta$) would be crucial

Fluctuations, or Cumulants $\langle \delta N^n \rangle_c$

Observables in equilibrium are fluctuating.

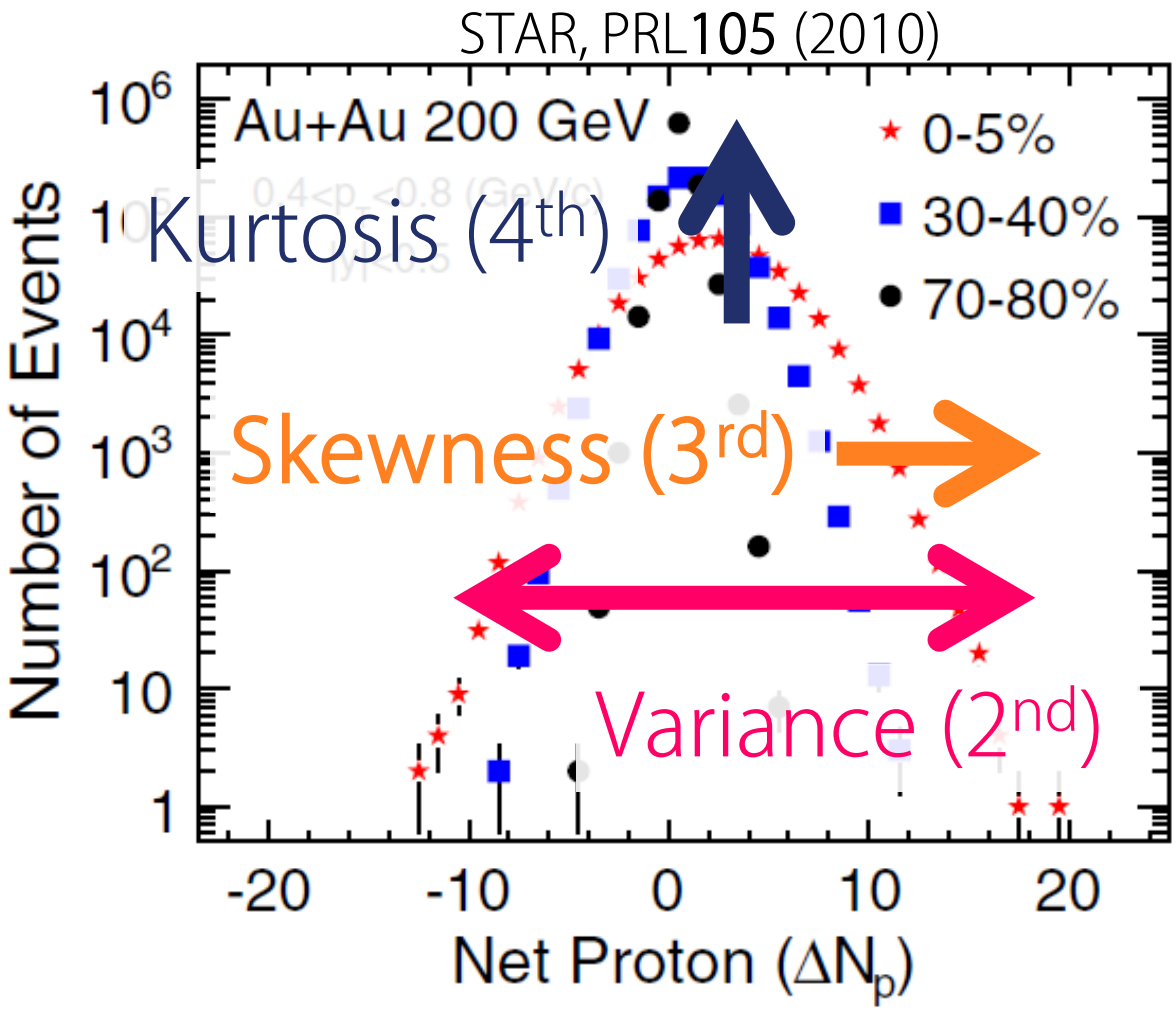




- Variance: $\langle \delta N^2 \rangle = V \chi_2 = \sigma^2$ $\quad \delta N = N - \langle N \rangle$
- Skewness: $S = \dfrac{\langle \delta N^3 \rangle}{\sigma^3}$
- Kurtosis: $\kappa = \dfrac{\langle \delta N^4 \rangle - 3 \langle \delta N^2 \rangle^2}{\chi_2 \sigma^2}$

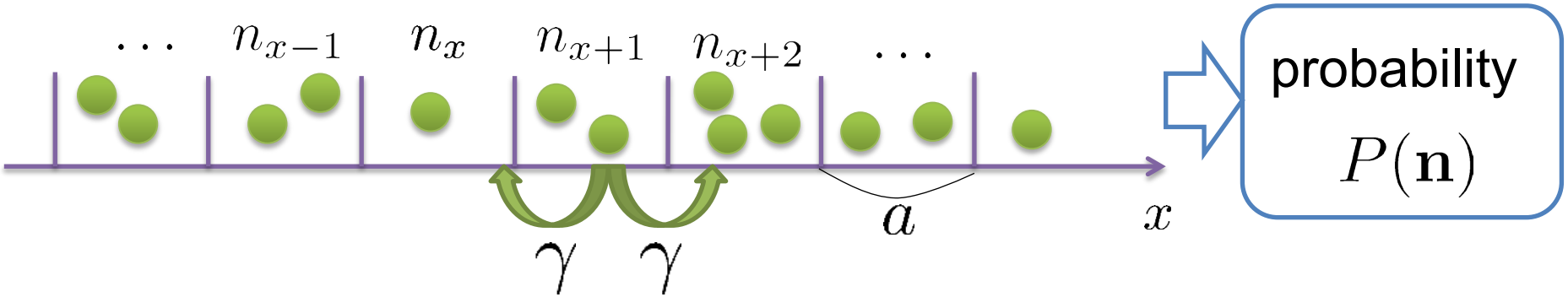
Non-Gaussianity

Variance, Skewness, and Kurtosis



Diffusion Master Equation (DME)

Divide spatial coordinate into discrete cells



Master Equation for *P*(*n*)

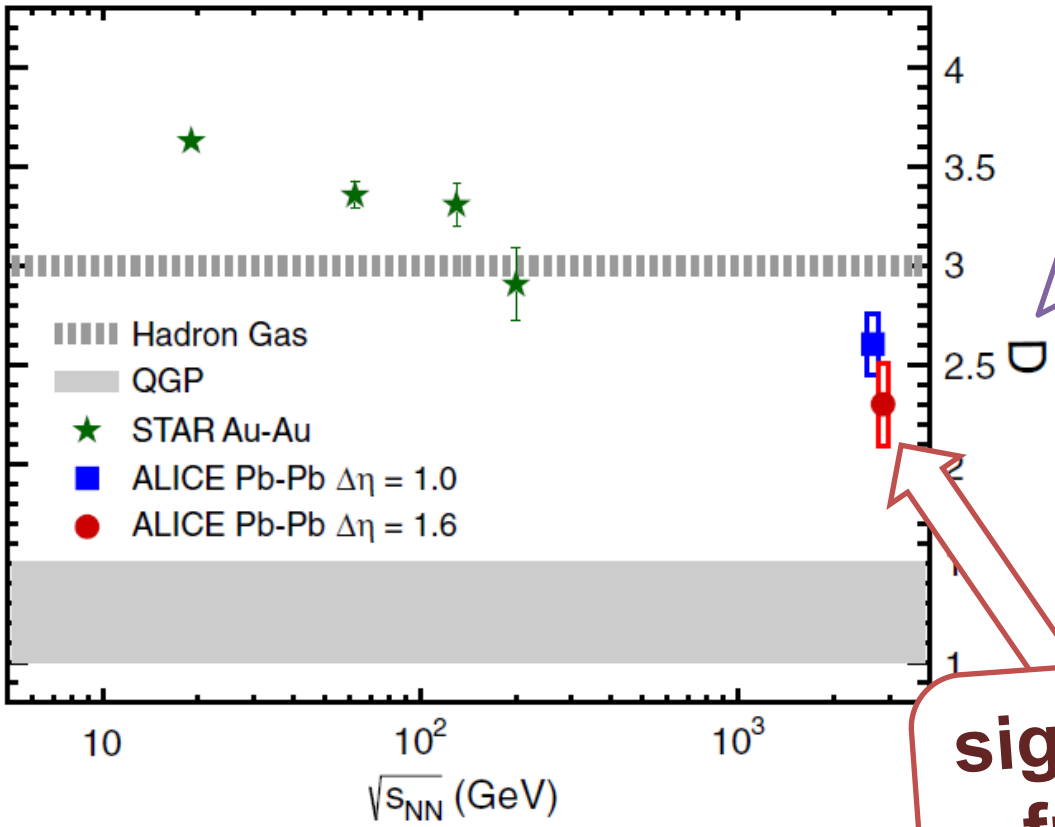
$$\frac{\partial}{\partial t}P(\mathbf{n}) = \gamma \sum_x [(n_x+1)\{P(\mathbf{n}+\mathbf{e}_x-\mathbf{e}_{x+1}) + P(\mathbf{n}+\mathbf{e}_x-\mathbf{e}_{x-1})\} - 2n_x P(\mathbf{n})]$$

Solve the DME **exactly**, and take *a*→0 limit

No approximation is needed

Ono, Kitazawa, M.A., PLB 2014

Charge Fluctuation @LHC



ALICE, PRL110,152301(2013)

D-measure

$$D = 4\frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}$$

- D ~ 3-4 Hadronic
- D ~ 1 Quark

significant suppression from hadronic value at LHC energy!

$\langle \delta N_Q^2 \rangle$ is not equilibrated at freeze-out at LHC energy!