Thermal Blurring Effects on Fluctuations of Conserved Charges in Rapidity Space

Masayuki Asakawa
Department of Physics, Osaka University

With M. Kitazawa, Y. Onishi, and M. Sakaida
QCD Phase Diagram

- **QGP (quark-gluon plasma)**
- **CSC (color superconductivity)**
- Hadron Phase
  - chiral symmetry breaking
  - confinement

- **1st order crossover**
- **CP (critical point)**
- **160-180 MeV**
- **RHIC**
- **LHC**
- **5-10ρ₀**
- **μₐ**

M. Asakawa (Osaka University)
Why Conserved Charge Fluctuations?

- Their values do not change during the phase transition
- Their values in QGP and Hadron Phase are different
- They change in Hadron Phase only by diffusion

D measure for electromagnetic charge fluctuation

Heinz, Müller, M.A., Jeon, Koch, 2000

- Charge Fluctuation and Baryon Number Fluctuation are well-defined quantities, and can be measured on the lattice

- Lattice results and Effective Model results (equilibrium thermodynamics) are often compared with experimental results directly

Does this make sense?
Conserved and Non-Conserved Charge Fluc.

Conserved Charge

Only diffusion changes the number of charge

relaxation time \( \tau \rightarrow \infty \)

for \( V \rightarrow \infty \)

Non-Conserved Quantity

Quantity can change anywhere in the volume

\( \tau \rightarrow \text{finite} \)

for \( V \rightarrow \infty \)

Necessary to consider dynamical evolution of fluctuation!
In the $\Delta \eta$ dependence of C.C. Fluctuation, history of system is encoded.
Theoretical Side

Fluctuation of conserved charges in $\Delta \eta$ (space-time rapidity)

What is usually calculated
Detectors (usually) cover $\Delta y$ ((pseudo) rapidity)
Relation between $y$ and $\eta$?
Relation between $y$ and $\eta$?

$y = \eta$ in Bjorken picture

Rapidity

$y = \frac{1}{2} \ln \frac{E + P_z}{E - P_z} = \frac{1}{2} \ln \frac{1 + v_z}{1 - v_z}$

Momentum $P_z$

identified (in Bjorken picture)

Spacetime Rapidity

$\eta = \frac{1}{2} \ln \frac{t + z}{t - z} = \frac{1}{2} \ln \frac{1 + z/t}{1 - z/t}$

position $z$

time $t$

position $z$

kinetic freezeout
hadronization
thermal equilibrium
hadron phase
QGP phase
Relation between $y$ and $\eta$?

$y = \eta$ in Bjorken picture

Rapidity

$$y = \frac{1}{2} \ln \frac{E + P_z}{E - P_z} = \frac{1}{2} \ln \frac{1 + v_z}{1 - v_z}$$

Momentum $P_z$

Identified (in Bjorken picture)

Spacetime Rapidity

$$\eta = \frac{1}{2} \ln \frac{t + z}{t - z} = \frac{1}{2} \ln \frac{1 + z/t}{1 - z/t}$$

Position $z$
But this is a relation for FLOW rapidity

\[ y = \eta \text{ in Bjorken picture} \]

Rapidity

\[ y = \frac{1}{2} \ln \frac{E + P_z}{E - P_z} = \frac{1}{2} \ln \frac{1 + v_z}{1 - v_z} \]

Momentum \( P_z \)

Spacetime Rapidity

\[ \eta = \frac{1}{2} \ln \frac{t + z}{t - z} = \frac{1}{2} \ln \frac{1 + z/t}{1 - z/t} \]

identified (in Bjorken picture)

\[ y^\eta = \ln \frac{1 + v_z}{1 - v_z} \in [0, \infty) \]

time \( t \)

position \( z \)

kinetic freezeout

hadronization

thermal equilibrium

hadron phase

QGP phase

\( \eta \text{ in Bjorken picture} \)
But this is a relation for FLOW rapidity

\[ y = \eta \quad \text{in Bjorken picture} \]

**Rapidity**

\[ y = \frac{1}{2} \ln \frac{E + P_z}{E - P_z} = \frac{1}{2} \ln \frac{1 + v_z}{1 - v_z} \]

**Momentum** \( P_z \)

**Spacetime Rapidity**

\[ \eta = \frac{1}{2} \ln \frac{t + z}{t - z} = \frac{1}{2} \ln \frac{1 + z/t}{1 - z/t} \]

**Position** \( z \)
For a fixed (pseudo)rapidity

\[ y = \eta \quad \text{in Bjorken picture} \]

### Rapidity

\[ y = \frac{1}{2} \ln \frac{E + P_z}{E - P_z} = \frac{1}{2} \ln \frac{1 + v_z}{1 - v_z} \]

### Momentum \( P_z \)

Identified (in Bjorken picture)

### Spacetime Rapidity

\[ \eta = \frac{1}{2} \ln \frac{t + z}{t - z} = \frac{1}{2} \ln \frac{1 + z/t}{1 - z/t} \]

Position \( z \)

Blurring or Leak in rapidity space takes place!
Thermal Blurring or Leak

\[ y = \eta \] in Bjorken picture

is blurred in one particle distribution owing to thermal motion

Accordingly, conserved charge fluctuation (two particle correlation) is modified

In the following, we use boosted Maxwell-Boltzmann distribution (with radial and longitudinal flow)

Blurring or Leak in rapidity space takes place!
Strategy: Chapman-Kolmogorov Equation

Chemical freeze-out

\( P_1(\eta \rightarrow \eta') \) Diffusion

spacetime rapidity space

Kinetic freeze-out

\( P_2(\eta' \rightarrow y) \) Blurring (Leak)

spacetime rapidity space

\rightarrow \) rapidity space

Observables at (pseudo)rapidity \( y \)

\[
P(\eta \rightarrow y) = \int_{-\infty}^{\infty} d\eta' P_1(\eta \rightarrow \eta')P_2(\eta' \rightarrow y)
\]

Chapman-Kolmogorov Equation
Computing Net Charge Cumulants with Initial Fluc.

1. Assume diffusions of positive and negative charges are independent.

2. Assume initial system is uniform and that fluctuation is local (fluctuations at different points on the timelike initial hypersurface are not correlated).

\[
\left\langle M_{i_1}(\eta_1)M_{i_2}(\eta_2)\ldots M_{i_l}(\eta_l) \right\rangle_c = [M_{i_1}M_{i_2}\ldots M_{i_l}]_c \delta(\eta_1 - \eta_2)\ldots \delta(\eta_1 - \eta_l)
\]

3. From these assumptions and Chapman-Kolmogorov equation, we obtain
2nd and 4th Cumulants

**2nd**

\[
\frac{\langle Q^2_{(\text{net})} \rangle_c}{\langle Q_{(\text{tot})} \rangle_c} = 1 + \left( \frac{\langle M^2_{(\text{net})} \rangle_c}{\langle M_{(\text{tot})} \rangle_c} - 1 \right) \left( \frac{1}{\Delta} \int_{-\infty}^{\infty} d\eta \left[ \int_{-\Delta/2}^{\Delta/2} dy \int_{-\infty}^{\infty} d\eta' P_1(\eta \to \eta')P_2(\eta' \to y) \right]^2 \right)
\]

**4th**

\[
\frac{\langle Q^4_{(\text{net})} \rangle_c}{\langle Q_{(\text{tot})} \rangle_c} = 1 - 7F_2 + 12F_3 - 6F_4 + \frac{3 \langle M^2_{(\text{net})} \rangle_c}{\langle M_{(\text{tot})} \rangle_c} (F_2 - 2F_3 + F_4)
\]

\[
F_n = \frac{1}{\Delta} \int_{-\infty}^{\infty} d\eta \left[ \int_{-\Delta/2}^{\Delta/2} dy \int_{-\infty}^{\infty} d\eta' P_1(\eta \to \eta')P_2(\eta' \to y) \right]^n
\]
Concrete Forms of $P_1$ and $P_2$

$$P(\eta \to y) = \int_{-\infty}^{\infty} d\eta' P_1(\eta \to \eta') P_2(\eta' \to y)$$

**Diffusion**

$$P_1(\eta \to \eta') = \frac{1}{2\sqrt{\pi Dt}} \exp\left(-\frac{(\eta' - \eta)^2}{4Dt}\right)$$

**Continuum limit of solution of Diffusion Master Equation**

**Blurring(Leak)**

$$P_2(\eta' \to y, \beta) = \int_0^\infty dp_T \frac{z}{2m^3 K_2(z)} \exp\left(-z\gamma \cosh(y - \eta') \sqrt{1 + \left(\frac{p_T}{m}\right)^2}\right)$$

$$\times \left[-\gamma\beta p_T^2 I_1(\gamma\beta p_T/T)\right.$$

$$+ \left.\gamma p_T \sqrt{m^2 + p_T^2} \cosh(y - \eta') I_0(\gamma\beta p_T/T)\right]$$

**Boosted Maxwell-Boltzmann distribution**

$y$: rapidity of a particle, $z = m/T$, $\beta$: transverse flow velocity
2nd Cumulant without Diffusion or Blurring

**2nd (full)**

\[
\frac{\langle Q_{(net)}^2 \rangle_c}{\langle Q_{(tot)} \rangle_c} = 1 + \left( \frac{\langle M_{(net)}^2 \rangle_c}{\langle M_{(tot)} \rangle_c} - 1 \right) \left( \frac{1}{\Delta} \int_{-\infty}^{\infty} d\eta \left[ \int_{-\Delta/2}^{\Delta/2} dy \int_{-\infty}^{\infty} d\eta' P_1(\eta \to \eta') P_2(\eta' \to y) \right]^2 \right)
\]

**diffusion only**

\[
\frac{\langle Q_{(net)}^2 \rangle_c}{\langle Q_{(tot)} \rangle_c} = 1 + \left( \frac{\langle M_{(net)}^2 \rangle_c}{\langle M_{(tot)} \rangle_c} - 1 \right) \left( \frac{1}{\Delta} \int_{-\infty}^{\infty} d\eta \left[ \int_{-\Delta/2}^{\Delta/2} d\eta' P_1(\eta \to \eta') \right]^2 \right)
\]

**blurring only**

\[
\frac{\langle Q_{(net)}^2 \rangle_c}{\langle Q_{(tot)} \rangle_c} = 1 + \left( \frac{\langle M_{(net)}^2 \rangle_c}{\langle M_{(tot)} \rangle_c} - 1 \right) \left( \frac{1}{\Delta} \int_{-\infty}^{\infty} d\eta' \left[ \int_{-\Delta/2}^{\Delta/2} dy P_2(\eta' \to y) \right]^2 \right)
\]
2nd Cumulant without Diffusion or Blurring

**Diffusion only**

\[
\frac{\langle Q^2_{\text{(net)}} \rangle_c}{\langle Q_{\text{(tot)}} \rangle_c} = 1 + \left( \frac{\langle M^2_{\text{(net)}} \rangle_c}{\langle M_{\text{(tot)}} \rangle_c} - 1 \right) \left( \frac{1}{\Delta} \int_{-\infty}^{\infty} d\eta \left[ \int_{-\Delta/2}^{\Delta/2} d\eta' P_1(\eta \rightarrow \eta') \right]^2 \right)
\]

**Blurring only**

\[
\frac{\langle Q^2_{\text{(net)}} \rangle_c}{\langle Q_{\text{(tot)}} \rangle_c} = 1 + \left( \frac{\langle M^2_{\text{(net)}} \rangle_c}{\langle M_{\text{(tot)}} \rangle_c} - 1 \right) \left( \frac{1}{\Delta} \int_{-\infty}^{\infty} d\eta' \left[ \int_{-\Delta/2}^{\Delta/2} dy P_2(\eta' \rightarrow y) \right]^2 \right)
\]

**None (most papers)**

\[
\frac{\langle Q^2_{\text{(net)}} \rangle_c}{\langle Q_{\text{(tot)}} \rangle_c} = 1 + \left( \frac{\langle M^2_{\text{(net)}} \rangle_c}{\langle M_{\text{(tot)}} \rangle_c} - 1 \right) = \frac{\langle M^2_{\text{(net)}} \rangle_c}{\langle M_{\text{(tot)}} \rangle_c}
\]
2nd Cumulant without Diffusion or Blurring

**diffusion only**

\[
\frac{\langle Q^2_{(\text{net})} \rangle_c}{\langle Q_{(\text{tot})} \rangle_c} = 1 + \left( \frac{\langle M^2_{(\text{net})} \rangle_c}{\langle M_{(\text{tot})} \rangle_c} - 1 \right) \left( \frac{1}{\Delta} \int_{-\infty}^{\infty} d\eta \left[ \int_{-\Delta/2}^{\Delta/2} d\eta' P_1(\eta \rightarrow \eta') \right]^2 \right)
\]

**blurring only**

\[
\frac{\langle Q^2_{(\text{net})} \rangle_c}{\langle Q_{(\text{tot})} \rangle_c} = 1 + \left( \frac{\langle M^2_{(\text{net})} \rangle_c}{\langle M_{(\text{tot})} \rangle_c} - 1 \right) \left( \frac{1}{\Delta} \int_{-\infty}^{\infty} d\eta' \left[ \int_{-\Delta/2}^{\Delta/2} dy P_2(\eta' \rightarrow y) \right]^2 \right)
\]

**none (most papers)**

\[
\frac{\langle Q^2_{(\text{net})} \rangle_c}{\langle Q_{(\text{tot})} \rangle_c} = 1 + \left( \frac{\langle M^2_{(\text{net})} \rangle_c}{\langle M_{(\text{tot})} \rangle_c} - 1 \right) = \frac{\langle M^2_{(\text{net})} \rangle_c}{\langle M_{(\text{tot})} \rangle_c}
\]

Freeze-out parameters: lattice meets experiment
Results
**Blurring Effect and Transverse Flow $\beta$**

The larger $\beta$, the less blurring. This is a relativistic effect.

- $P_2(0 \rightarrow y)$
- $m = 140$ MeV
- $T = 100$ MeV
- $\beta = 0$
- $\beta = 0.2$
- $\beta = 0.4$
- $\beta = 0.6$
- $\beta = 0.8$
Result (with $\beta=0$)

- $m = 140$ MeV
- $T = 100$ MeV
- $\beta = 0$
- $\sqrt{Dt} = 0.4$ ( $D$ : diffusion const.)

Sakaida, Asakawa, Kitazawa, PRC 2014

Graph showing:
- $\Delta y$ (in rapidity space)
- D-measure vs $\Delta y$
- Three curves: Diffusion only, Blurring only, Diffusion + Blurring

Initial fluctuation

- Initial fluctuation = 0
- Thermal fluctuation in hadron phase

Detector coverage
**Result (with $\beta \neq 0$): Blurring only**

Initial fluctuation

\[
\text{Initial fluctuation} = 0
\]

Thermal fluctuation in hadron phase

The more central, the less blurring (because of lower T)
Comparison with ALICE result: Blurring only

Initial fluctuation

\[
\text{Initial fluctuation} = 0
\]

Thermal fluctuation in hadron phase

Initial D-measure

\[
\text{Thermal D-measure in hadron phase} = 0.5
\]

Initial fluctuation

\[
\Delta y
\]

Room for other effects

Only with blurring, the result exceeds the experimental data

This init. cond. is rejected
Comparison with ALICE result: Blurring only

- Qualitatively similar
- But note that diffusion would give opposite tendency (more central = longer diffusion)

$\beta = 0.57, T = 120\text{ MeV}$ (centrality: 40 – 50 %)

$\beta = 0.65, T = 100\text{ MeV}$ (centrality: 0 – 5 %)
Summary and Perspectives

- We have developed a formulation to evaluate the effects of diffusion and thermal blurring (leak) for conserved charge fluctuation.
- We have assumed Bjorken scaling (not essential in the formulation).

At lower energies (e.g., BES energies):

- Violation of Bjorken scaling (lost correspondence between spacetime rapidity and rapidity).
- Larger effect of global charge conservation (smaller system size).
- Shorter diffusion duration (smaller time scale).
Summary and Perspectives

At lower energies (e.g., BES energies)

- Violation of Bjorken scaling
  (lost correspondence between spacetime rapidity and rapidity, i.e., $y \neq \eta$)
- Non-negligible effect of global charge conservation
  (smaller system size)
- Shorter diffusion duration
  (smaller time scale)
- Still blurring (leak) exists
  (similar temperature scale)

Results like this are different from what is calculated theoretically in most cases and are not supposed to be compared with theoretical results without taking these effects into account.
To understand SPS & RHIC result, these effects (diffusion, blurring, and Bjorken scaling violation, $y \neq \eta$) would be crucial.
Fluctuations, or Cumulants $\langle \delta N^n \rangle_c$

Observables in equilibrium are fluctuating.

- **Variance:** $\langle \delta N^2 \rangle = V \chi_2 = \sigma^2$
  $$\delta N = N - \langle N \rangle$$

- **Skewness:** $S = \frac{\langle \delta N^3 \rangle}{\sigma^3}$

- **Kurtosis:** $\kappa = \frac{\langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2}{\chi_2 \sigma^4}$

Non-Gaussianity
Variance, Skewness, and Kurtosis

STAR, PRL 105 (2010)

Kurtosis (4th)

Skewness (3rd)

Variance (2nd)
Diffusion Master Equation (DME)

Divide spatial coordinate into discrete cells

\[ P(n) \]

Master Equation for \( P(n) \)

\[
\frac{\partial}{\partial t} P(n) = \gamma \sum_x \left[ (n_x + 1) \left\{ P(n + e_x - e_{x+1}) + P(n + e_x - e_{x-1}) \right\} - 2n_x P(n) \right]
\]

Solve the DME exactly, and take \( a \to 0 \) limit

No approximation is needed

Ono, Kitazawa, M.A., PLB 2014
Charge Fluctuation @LHC

ALICE, PRL110, 152301 (2013)

D-measure

\[
D = 4 \frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}
\]

- \( D \sim 3-4 \) Hadronic
- \( D \sim 1 \) Quark

significant suppression from hadronic value at LHC energy!

\( \langle \delta N_Q^2 \rangle \) is not equilibrated at freeze-out at LHC energy!