Nuclear Symmetry Energy in QCD degree of freedom

Kie Sang Jeong and Su Houng Lee

Department of Physics and Institute of Physics and Applied Physics, Yonsei University, Seoul 120-749, Korea

[1] Kie Sang Jeong, Su Houng Lee, Phys. Rev. C87 (2013) 015204, [2] Kie Sang Jeong, Su Houng Lee, arXiv:1506:01447 (will be published in NPA)

1. Nuclear Symmetry Energy from QCD sum rules

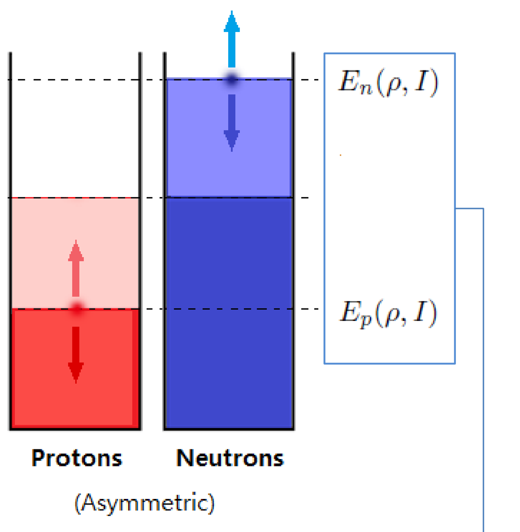
• Phenomenology

Bethe-Weisaker formula

$$m_{tot} = Nm_n + Zm_p - E_B/c^2$$
$$E_B = a_V A - a_S A^{\frac{2}{3}} - a_C(Z(Z-1))A^{-\frac{1}{3}} - a_A I^2 A + \delta(A,Z)$$
$$I = (N-Z)/A$$

In continuous matter

$$\bar{E}(\rho_N, I) = \bar{E}(\rho_N) + \bar{E}_{sym}(\rho_N) I^2 + \cdots$$
$$I = (\rho_n - \rho_p)/\rho$$
$$\bar{E} = \frac{1}{\int d^3k_n d^3k_p} \int d^3k_n d^3k_p E(\rho_n, \rho_p)$$
$$\Rightarrow E_{sym} = \frac{1}{2I} \cdot (\bar{E}_n - \bar{E}_p) \quad \text{(Up to linear density order)}$$



RMFT propagator

$$G(q) = -i \int d^4x e^{iqx} \langle \Psi_0 | \mathrm{T}[\psi(x)\bar{\psi}(0)] | \Psi_0 \rangle = \frac{1}{\not{q} - M_n - \Sigma(q)} \to \lambda^2 \frac{\not{q} + M^* - \not{u}\Sigma_v}{(q_0 - E_q)(q_0 - \bar{E}_0)}$$

- Self energies can be found as residues of quasi-nucleon pole
- In QCD sum rule, only assume this phenomenology

• QCD sum rules in Iso-spin asymmetric nuclear matter

Correlation function

$$\Pi(q) \equiv i \int d^4x\, e^{iqx} \langle \Psi_0 | \mathrm{T}[\eta(x)\bar{\eta}(0)] | \Psi_0 \rangle = \Pi_s(q^2, q\cdot u) + \Pi_q(q^2, q\cdot u)\not{q} + \Pi_u(q^2, q\cdot u)\not{u}$$

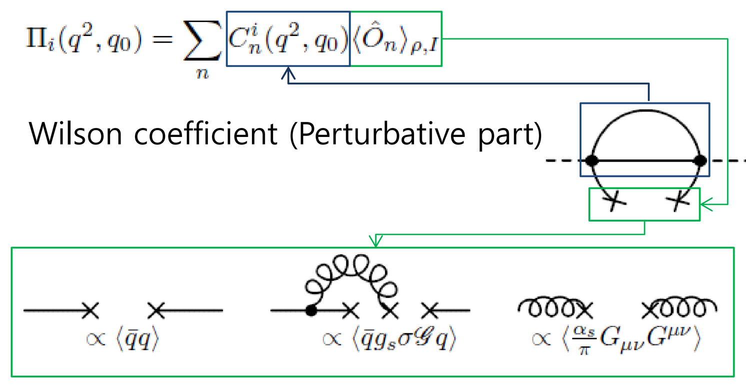
Ioffe's interpolating field for proton

$$\eta(x) = \epsilon_{abc}[u_a^T(x) C\gamma_\mu u_b(x)]\gamma_5\gamma^\mu d_c(x)$$

OPE at short distance

$$\Pi_i(q^2, q_0) = \sum_n C_n^i(q^2, q_0) \langle \hat{O}_n \rangle_{\rho, I}$$

Wilson coefficient (Perturbative part)



Condensate (Non perturbative part)

(These figures are quoted from Ph.D. thesis of Thomas Hilger)

Energy dispersion relation

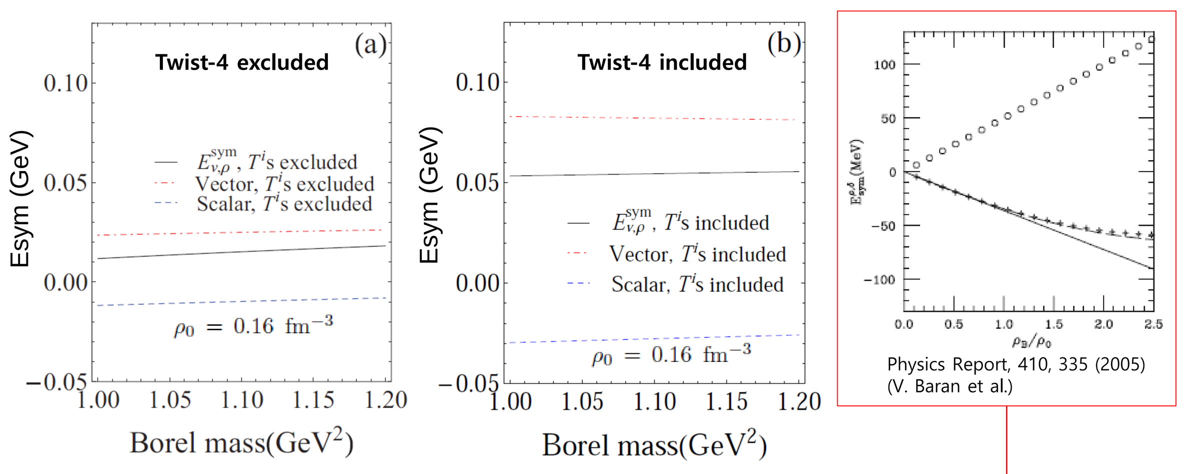
$$\Pi_i(q_0, |\vec{q}|) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} d\omega \frac{2\mathrm{Im}\Pi_i(\omega, |\vec{q}|)}{\omega - q_0} + \text{polynomials}$$

Contains **all possible hadronic resonance states** in **QCD degree of freedom**

$$\Pi(q_0, |\vec{q}|) \sim \frac{1}{(q^\mu - \bar{\Sigma}_v^\mu)\gamma_\mu - M_N^*}$$

Equating both sides, nucleon self energies can be expressed in QCD degree of freedom

Iso-vector scalar / vector decomposition of symmetry energy

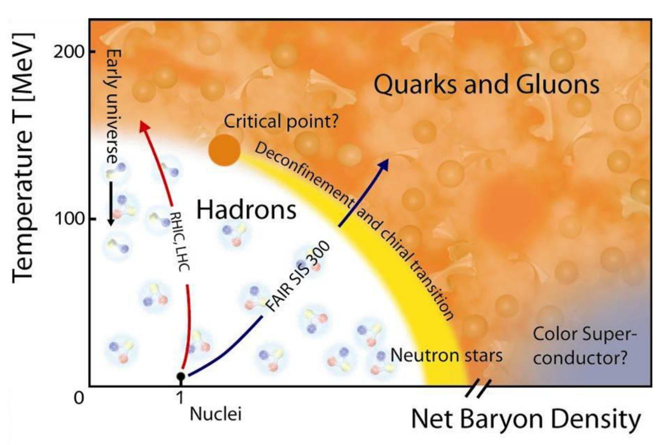


$$E_V^{sym} = \frac{1}{2}\left[f_\rho - f_\delta\left(\frac{m^*}{E_F^*}\right)\right]\rho_B$$

Iso-vector meson exchange
High density behavior -> high density dependence of condensates

2. Symmetry Energy with HDL resummation

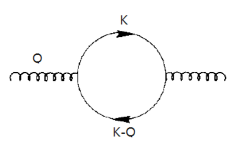
• QCD phase transition



- At high temperature or high density, matter becomes QCD phase
- The core of neutron star can be regarded as cold-dense nuclear matter.

• Hard Dense Loop resummation

Gluon self energies with soft external momenta

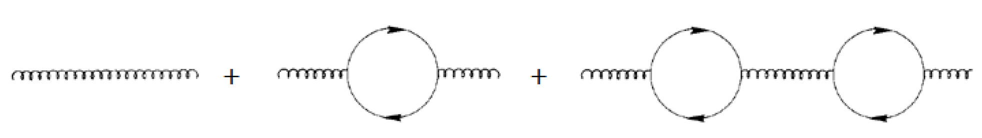


$$\Pi_{\mu\nu}^{ab}(Q) = g^2\delta^{ab} \int \frac{d^4K}{(2\pi)^4} \mathrm{Tr}[\gamma_\mu S_F(K)\gamma_\nu S_F(K-Q)]$$
$$= m^2\delta^{ab} \int \frac{d\Omega}{4\pi}\left(\delta_{\mu 4}\delta_{\nu 4} + \hat{K}_\mu \hat{K}_\nu \frac{i\omega}{Q\cdot \hat{K}}\right),$$

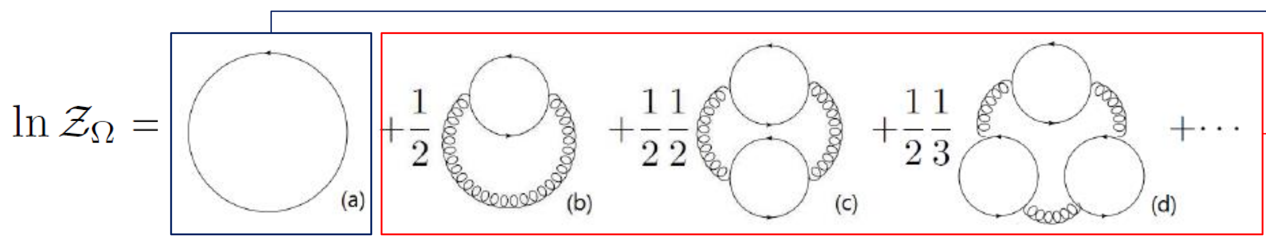
$K_\mu = (-i, k)$, $Q_\mu = (-\omega, q)$

$$m^2 = \frac{1}{3}g^2T^2\left(C_A + \frac{1}{2}n_f\right) + \frac{1}{2}g^2 \sum_f \frac{\mu_f^2}{\pi^2} \quad g^2T^2(g^2\mu^2) \text{ order}$$

All order of diagrams are in same order -> should be resumed



• Grand potential with dense loop



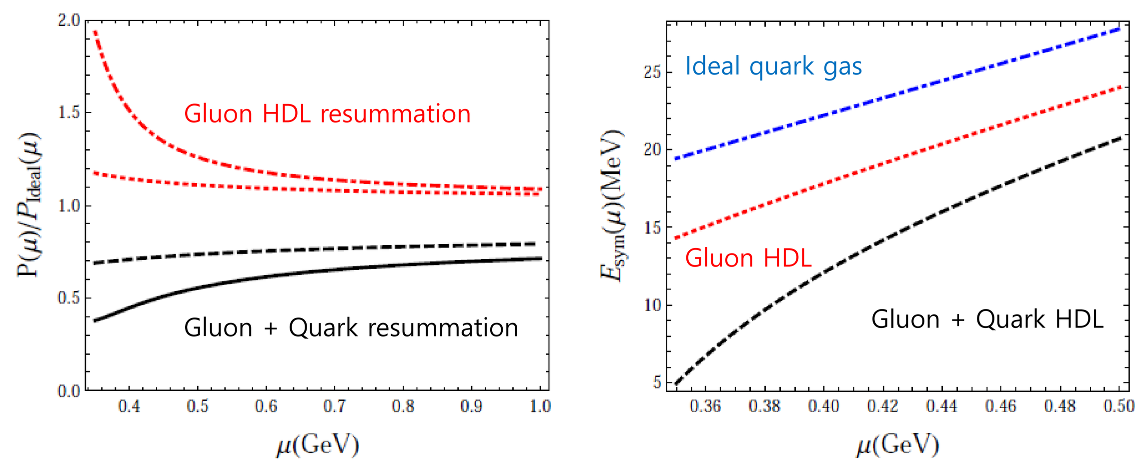
Ideal quark gas contribution (N_f=2)

HDL resummed gluon loop contribution

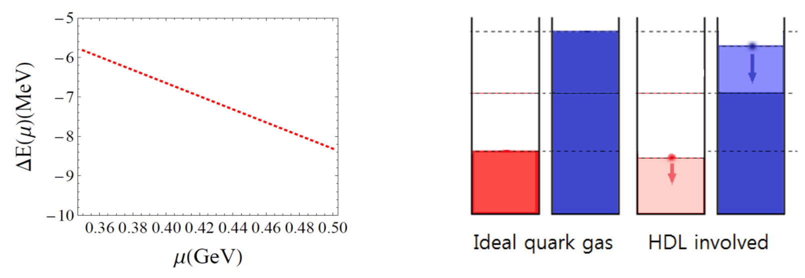
Where [diagram] in m_f, $T \sim g\mu$ and $\mathcal{Z}_\Omega = \mathrm{Tr}\exp\left[-\beta[\hat{H} - \vec{\mu}\cdot\vec{N}]\right]$

• Symmetry Energy from HDL involved grand potential

$$\Omega(\mu) = \langle \hat{H} \rangle - \vec{\mu}\cdot\langle \vec{N} \rangle = -\frac{1}{\beta V}\ln \mathcal{Z}_\Omega$$
$$\epsilon(\mu) = \frac{\langle \hat{H} \rangle}{V} = -\frac{1}{V}\left(\frac{\partial}{\partial\beta} - \frac{1}{\beta}\vec{\mu}\cdot\frac{\partial}{\partial\vec{\mu}}\right)\ln \mathcal{Z}_\Omega$$
$$\frac{\epsilon(\mu, I_B)}{\rho(\mu, I_B)} = \frac{E(\mu, I_B)}{N_B} = \bar{E}(\mu, I_B) = \bar{E}(\mu) + \bar{E}_{sym}(\mu) I_B^2 + \cdots$$
$$\bar{E}_{sym}(\mu) = \frac{1}{2}\frac{\partial^2}{\partial I_B^2}\bar{E}(\mu, I_B) \qquad I_B = 3\frac{\rho_d - \rho_u}{\rho_d + \rho_u}$$



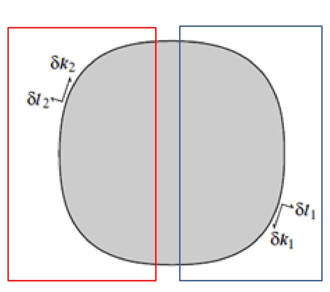
Suppression of quasi-Fermi sea



As density becomes higher, **suppression becomes stronger**
The difference between quasi-Fermi seas becomes smaller
- Costs less energy than ideal gas
- **Reduced symmetry energy**

3. Symmetry Energy in cold superconducting matter

• BCS pairing near Fermi-sea



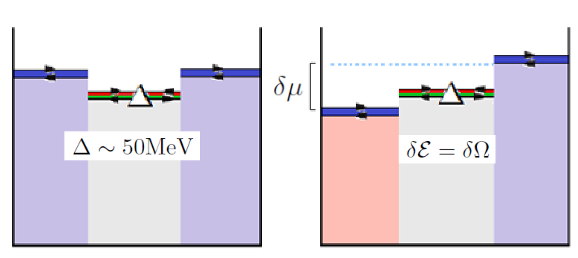
- 4-Fermion interaction with opposite momenta $S_{4q} \sim (\psi^\dagger(-p_f)\psi(-p_i))(\psi^\dagger(p_f)\psi(p_i))$ is marginal along to Fermi velocity
- Fermion – conjugated fermion interaction
- When $V<0$ two states form **a condensate (gap)**

- BCS action as Fermion – conjugated Fermion
$$S_{BCS} \sim \frac{1}{2}\left[\psi(-p)^T C \Delta(p)\psi(p) + \bar{\psi}(p)\bar{\Delta}(p) C\bar{\psi}^T(-p)\right]$$
- Effective Lagrangian – relevant degree of freedom
$$\mathcal{L}_D = \sum_{\vec{v}}\left[\psi^\dagger iV\cdot D\psi - \psi^\dagger \frac{1}{2\mu + i\tilde{V}\cdot D}D_\perp^2\psi\right]$$

Diagrammatically described gapped quasi-state

$$S_\Delta(l) = \frac{l_0 + l_v}{l_0^2 - l_v^2 - \Delta^2}$$

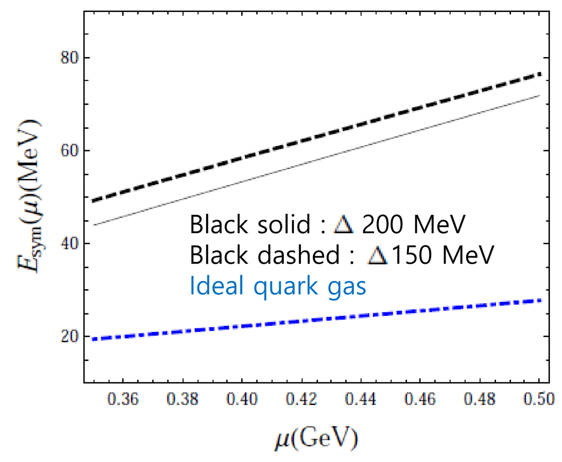
- **Only blue state (1/3)** can be iso-spin asymmetrized



- BCS phase remains in $\delta\mu < (1/\sqrt{2})\Delta \sim \Lambda$ (Phys. Rev. Lett. 9, 266 (1962) A. M. Clogston)
- Only u-d blue states can be asymmetrized
- The other 4 gapped quasi-states are **locked**
- In quasi-state approximation $\delta\Omega = -\frac{\mu^2}{\pi^2}\Delta^2$

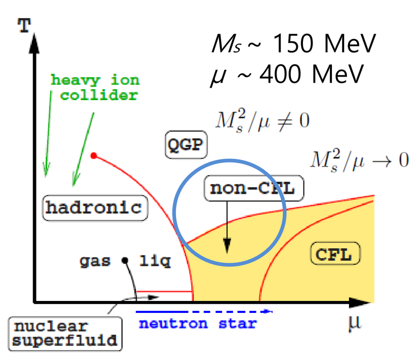
- **Symmetry Energy**

$$\frac{\epsilon(\mu, \Delta, I_{B_\Delta})}{\rho(\mu, \Delta, I_{B_\Delta})} = \frac{E(\mu, \Delta, I_{B_\Delta})}{N_{B_\Delta}} = \bar{E}(\mu, \Delta) + \bar{E}_{sym}(\mu, \Delta) I_{B_\Delta}^2 + \cdots$$
$$\bar{E}_{sym}(\mu, \Delta) = \frac{1}{2}\frac{\partial^2}{\partial I_{B_\Delta}^2}\bar{E}_\Delta(\mu, I_{B_\Delta})$$
$$I_{B_\Delta} = 3\frac{\rho_d - \rho_u}{\rho_d + \rho_u} \times \frac{1}{3}$$

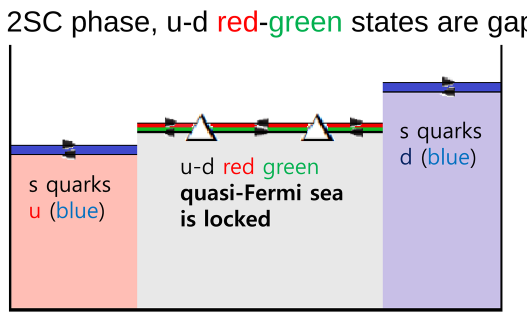


• 2 color superconductivity

- In QCD, color anti-triplet gluon exchange interaction is attractive ($V<0$)
$$\langle \psi_a^\alpha C\gamma_5\psi_b^\beta \rangle \sim \Delta_1\epsilon^{\alpha\beta 1}\epsilon_{ab1} + \Delta_2\epsilon^{\alpha\beta 2}\epsilon_{ab2} + \Delta_3\epsilon^{\alpha\beta 3}\epsilon_{ab3}$$
- In non negligible M_s^2/μ, **2SC** state is favored



In 2SC phase, u-d red-green states are gapped

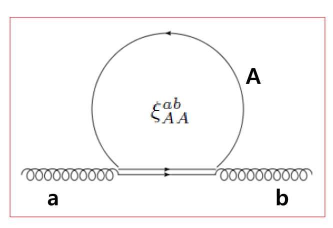


Only s quarks and u-d blue quarks are liberal

- High density effective Lagrangian
  - **2SC description** as linear combination of **Gellman matrices**
$$\psi_{+,ai} = \sum_{A=0}^{5}\frac{(\hat{\lambda}_A)_{ai}}{\sqrt{2}}\psi_+^A \qquad \chi = \begin{pmatrix}\psi_+ \\ C\psi_-^*\end{pmatrix} \quad \text{+ and − represents Fermi velocity}$$
$$\hat{\lambda}_0 = \frac{1}{\sqrt{3}}\lambda_8 + \frac{2}{3}I;\ \hat{\lambda}_A = \lambda_A\ (A=1,2,3);\ \hat{\lambda}_4 = \frac{1}{\sqrt{2}}(\lambda_4 - i\lambda_5);\ \hat{\lambda}_5 = \frac{1}{\sqrt{2}}(\lambda_6 - i\lambda_7),$$
  - **High density effective Lagrangian**
$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \sum_{\vec{v}_F}\sum_{A,B=0}^{\text{half}}\chi^{A\dagger}\begin{pmatrix} iV\cdot\delta_{AB} & \Delta_{AB} \\ \Delta_{AB} & i\tilde{V}\cdot\delta_{AB}\end{pmatrix}\chi^B + ig A_\mu^a \chi^{A\dagger}\begin{pmatrix} iV^\mu\kappa_{AaB} & 0 \\ 0 & -i\tilde{V}^\mu\kappa_{AaB}^*\end{pmatrix}\chi^B$$
$$+ g^2 A_\mu^a A_\nu^b \chi^{A\dagger}\begin{pmatrix}\frac{1}{2\mu + i\tilde{V}\cdot D}\xi_{AB}^{ab} & 0 \\ 0 & \frac{1}{2\mu + iV\cdot D^*}\xi_{AB}^{*ab}\end{pmatrix}P^{\mu\nu}\chi^B + (L \to R), \qquad P^{\mu\nu} = g^{\mu\nu} - \frac{1}{2}(V^\mu\tilde{V}^\nu + V^\nu\tilde{V}^\mu)$$

- Does **HDL reduction effect** survive in **color superconductor**?

- Quasi-fermionic loops in color superconductor

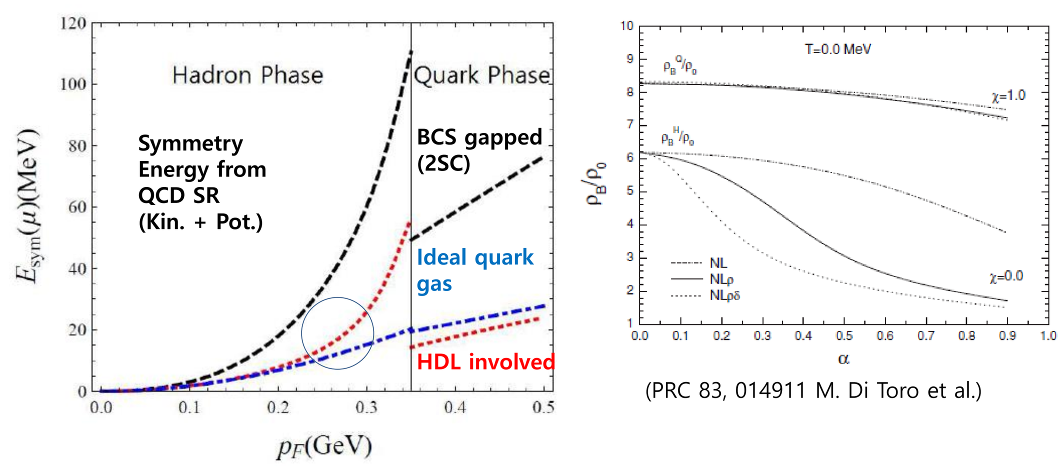


$m^2 = (g^2\mu^2/\pi^2)$

	$a,b=1,2,3$	$a,b=4,5,6,7$	$a,b=8$
$-\Pi_{00}^{ab}(0)$	0	$\frac{1}{2}m^2\delta^{ab}$	$\frac{1}{3}m^2\delta^{ab}$
$-\Pi_{ii}^{ab}(0)$	0	$\frac{1}{12}g^2\sum_f^{1,2}(\mu_f^2/\pi^2)\delta_{ij}\delta^{ab}$	$\frac{1}{9}g^2\sum_f^{1,2}(\mu_f^2/\pi^2)\delta_{ij}\delta^{ab}$

- Only Meissner mass has iso-spin dependence -> portion is very small : minimal effect for static quantities
- Gluonic correction in 2SC phase is subleading

• Nuclear Symmetry Energy



- Large symmetry energy can cause iso-spin evaporation and early appearance of quark phase
- As nuclear symmetry energy is **larger** than quark matter symmetry energy, iso-spin distillation can occur at mixed phase
- At 2SC, the distillation will be **reduced**
- Eventually, π^-/π^+ ratio will be affected by appearance of 2SC phase

Future goal :
Iso-spin asymmetric mixed phase with strangeness