Functional Renormalization Group Approach to Nuclear Matter in Compact Stars

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INTRODUCTION TO THE FUNCTIONAL RENORMALIZATION GROUP METHOD (FRG)

The basic principle: computation of *quantum n-point correlation functions* by gradual path integration. To achieve this, the generator functional is modified with a kinetic term, called the Regulator. The Regulator is constructed to suppress modes below a certain scale: *k*. The **scale dependent effective action,** witch is the generator of the connected Feynmann-diagrams, is calculated by the Legendre-transformaiton of the Schwinger-functional.

$$Z_k[J] = \int \left(\prod_a d\Psi_a\right) e^{-S[\Psi] - \frac{1}{2}R_{k,ab}\Psi_a\Psi_b + \Psi_a J_a}$$

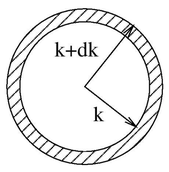
Legendre-transformation

$$\Gamma_k[\psi] = \sup_J \left(\psi_a J_a - W[J]\right) - \frac{1}{2}R_{k,ab}\psi_a\psi_b$$

The Scale dependence of the effective action is determined by the **Wetterich-equation**, witch is an exact equation if the form of effective potential is given. The theory is defined at some UV scale, by the UV-scale effective action. Using this as the initial condition we integrate the Wetterich equation to determine the effective action at *k=0.* This way the effective action at *k=0,* contains all quantum mechanical effects.

$$\partial_k \Gamma_k = \frac{1}{2} Str\left[\left(\partial_k R_k\right)\left(\Gamma_k^{(1,1)} + R_k\right)^{-1}\right]$$

k starts at UV scale: classical → **k=0 : included quantum effects**



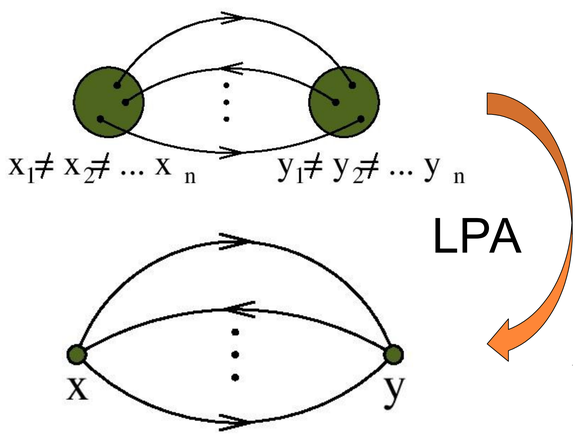
LOCAL POTENTIAL APPROXIMATION (LPA)

In the theory we should include all operators, in the UV effective action, even the most exotic ones, because they might become relevant at lower scale values. Since this task is impossible to do, in practice an ansatz is neded for the effective action.

The local potential approximation, is based on the assumption that propagators vary in spacetime much slower than vertices. This implies that the UV-scale effective action has the following form:

$$\Gamma_k[\psi] = \int d^4x \left[\frac{1}{2}\psi_i K_{k,ij}\psi_j + U_k(\psi)\right]$$

The effective action consists of a kinetic term with some kernel *K*, and a potential, *U* witch depends on the invariant combinations of the fields. This way Wetterich-equation becomes a differntial equation for the potential.



Any invariant combination of the fields can appear in the potential, on arbitrary high power, even for fermionic combinations!

Local potential approximation is based on the assumption that the value of the two diagrams are close. Vertex of the upper diagram is:

$$\Gamma_k^{(n)}(x_1, \ldots, x_n)\Psi(x_1)\ldots\Psi(x_{2n})$$

The vertex of the lower diagram:

$$U_k^{(n)} \lim_{\Delta V \to 0}\left(\frac{1}{\Delta V}\int_{\Delta V}\bar{\psi}(x)\psi(x)\right)^n \overset{\text{notation}}{\longrightarrow} U_k^{(n)}(\bar{\psi}(x)\psi(x))^n$$

This means that, the fields have different cooridante variables, but it is a good approximation to take their value at a common cooridante, witch charaterizes the vertex on the upper diagram. This can be interpreted as some compositness scale: above this scale the vertex appears as a point, below this scale we can think of it as a patch of very close points.

FRG at finite temperature

At finite temperature the path integral needs to extend for imaginary time. Since the regulator term is **time-independent**, the Wetterich-equation takes the following form in LPA:

$$\partial_k U = -\frac{i}{2}\int \frac{d^4p}{(2\pi)^4}\left(\partial_k R_{ij}\right)G_{ij}(p) \quad \Rightarrow \quad \partial_k U = -\frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\,\partial_k R_{ij}(\mathbf{p})\left(\frac{1}{2} + n_{\alpha_i}(p_0)\right)\varrho_{ij}(p)$$

where the Fermi-Dirac/Bose-Einstein distribution is denoted by:

$$n_\alpha(\omega) = \frac{\alpha}{e^{\beta\omega} - \alpha} \qquad \alpha = \pm 1$$

and $\varrho_{ij}(p)$ is the spectral function of the system.

Solving FRG-equations numerically

In the LPA approximation the aim is to determine the scale-dependence of the effective potential *U*. The initial condintion: *U* function is given at some *kUV* scale. For one scalar field at *T=0*, the Wetterich-equation for the effective potential is:

$$\frac{\partial}{\partial k}U_k(\phi) = \frac{k^4}{12\pi^2}\frac{1}{\sqrt{k^2 + \frac{\partial^2 U_k(\phi)}{\partial\phi^2}}}$$

Which is a two variable second order non-linear type of differential equation. Possible methods to solve this equation:

1) Newton-Raphson (more widely used)
2) Runge-Kutta type methods (problems with instability)
3) Taylor expansion of the equation and comparison of the coefficients

For fermionic fields at finite temperature the Newton-Raphson method is non-convergent, because derivatives of the Fermi-Dirac distribution at low temperature does not behave well. One has to use an explicite method to avoid the derivation of the Fermi-Dirac distribution function. A modified version of the Dormand-Price method (adaptive Runge-Kutta type) is used to overcome this difficulty, but we have to deal with the instabilities in these explicite methods.

Walecka-type model for nuclear matter

The scale dependent effective action of this model is given by:

$$\Gamma_k = \bar{\psi}\left(\not{\partial} - g_\sigma(\sigma + i\gamma_5\tau_j\pi^j) - g_\omega\not{\omega}\right)\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\pi\partial^\mu\pi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + U(\sigma,\pi)$$

The model contains two fermionic fields (to describe the nucleons), a scalar and a vector meson coupled to the fermionic fields to describe the nuclear interaction. The last term is the scale dependent effective potential which describes the self interactions of the mesons. Using LPA and Mean Field Approximation for the ω-meson we can derive the Wetterich-equation of the system at finite temperature:

$$\partial_k U_k = \frac{k^4}{12\pi^2}\left(\frac{2n_b(\omega_\sigma)+1}{\omega_\sigma} + 3\frac{2n_b(\omega_\pi)+1}{\omega_\pi} - 8\frac{1 - n_f(\omega-\mu) - n_f(\omega+\mu)}{\omega}\right)$$

$$\partial_k\Gamma_k = \frac{k^4}{12\pi^2}\left(bosonic - fermionic\right)$$

$$\omega_\sigma = \sqrt{k^2 + \frac{\partial^2 U_k}{\partial\sigma^2}}$$

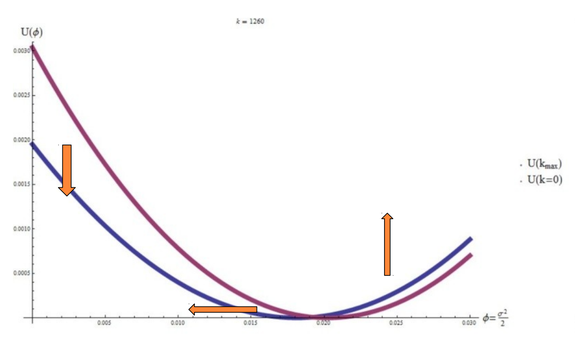
$$\omega_\pi = \sqrt{k^2 + \frac{\partial U_k}{\partial\sigma}}$$

$$\omega = \sqrt{k^2 + (g_\sigma\sigma)^2}$$

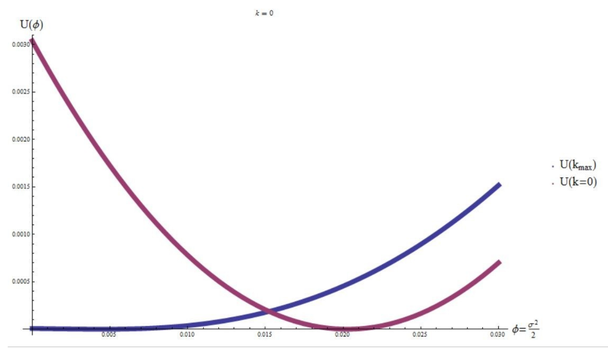
The parametrization of the model which gives back the vacuum expectation value of the scalar-meson is:

- k=1.3 GeV
- m=1.2 GeV2
- λ=7.4

$$U(\phi) = -m\phi + \lambda\phi^2 \qquad \phi = \frac{\sigma^2}{2}$$



UV scale: classical → ***k=0* : included quantum effects**



Scale dependence of the effective potential

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APPLICATIONS

- Fundamental matter is fermionic, Bose-condensation with fermionic bound states
- Higgs-mechanism, composite Higgs-models
- Scale dependent equation of state for nuclear matter, phase stransition points
- Neutron star stability tests

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