v_n from instabilities of the GLR equation

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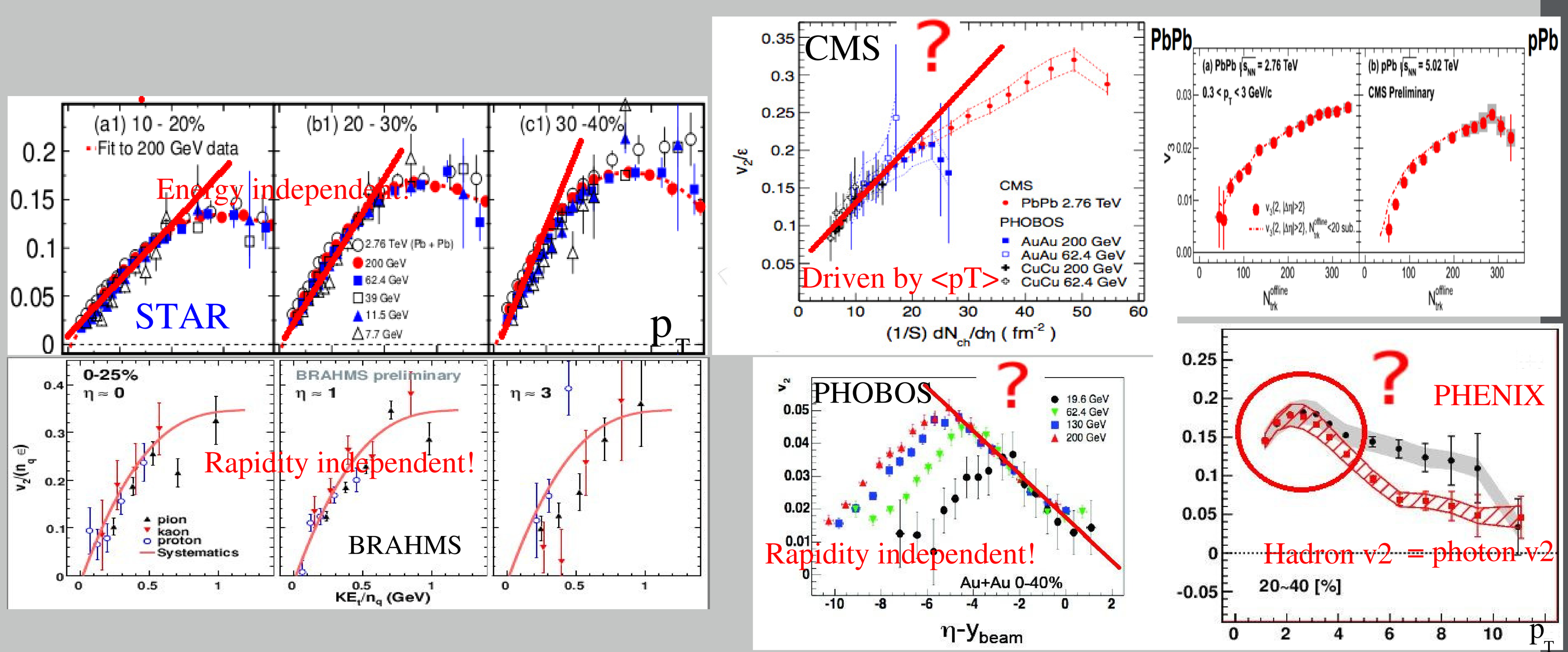
Phenomenological Introduction

Anisotropic coefficients in heavy ion collisions v_n

$$\frac{dN}{d^3p} = \frac{dN}{dp_T dy}\left(1 + 2\sum_n v_n(p_T, y)\cos\left(n\left(\phi - \psi_n\right)\right)\right)$$

have been a very well-studied observable. Its usual phenomenological interpretation is that of hydrodynamics, suggesting matter formed in heavy ion collisions is a "perfect fluid" but some deep puzzles have emerged...



- v_n Similar in photons and hadrons This is strange as photon and hadron evolution very different
- v_n is present in pA,perhaps pp Initial state "Glasma" models exist for small systems, but cannot describe larger AA systems ("many random antennae"). Scaling suggests v_n of same origin in small and large systems
- It seems $v_n(p_T) \sim \epsilon_n F(p_T)$ where ϵ_n is geometrical $F(p_T)$ is universal across $\sqrt{s}$,rapidity, system size . Integrated v_n variations driven by $\langle p_T\rangle \sim \frac{1}{S}\frac{dN}{dy}$ variations. Hydrodynamics doesn't factorize this way!
- This scaling is reminiscent of Bjorken scaling in parton distributions (PDFs , GPDs), where $G(x, Q^2)$ vary weakly with $Q^2 \sim p_T$, strongly with x
- All these features would be natural if somehow $G(x, Q^2) \to G(x, Q^2, \phi)$ while maintaining its QCD RG structure . Can v_n be an initial condition which obeys QCD scaling rather than a manifestation of hydro ? But we need to justify "somehow": $G(x, Q^2)$ is a property of the hadron wavefunction, and its azimuthal symmetry is protected... but...

Calculations

The azimuthally symmetric GLR-MQ evolution equation is given by

$$\frac{Q}{2}\frac{\partial}{\partial Q}\frac{\partial xG(x, Q^2)}{\partial \ln(1/x)} = \frac{\alpha_s N_c}{\pi}xG(x, Q^2) - \frac{\alpha_s^2 N_c \pi}{2C_F S_\perp}\frac{1}{Q^2}[xG(x, Q^2)]^2$$

Its solution $(G_0(x, Q^2))$ is known partially (for two limiting cases: $Q_s(x)/Q \ll 1$ and $Q_s(x)/Q \gg 1$). Therefore, we propose a model for the complete solution as

$$G_0(x, Q^2) = \frac{x^{2\lambda}}{2}\left[1 - \tanh\frac{Q - Q_s(x)}{\zeta} + \frac{(Q_s(x))^2}{Q^2}\left(1 + \tanh\frac{Q - Q_s(x)}{\zeta}\right)\right]$$

Calculations show $\zeta \propto \frac{\alpha_s}{N_c}\Lambda_{QCD}$. The azimuthally asymmetric QLR-MQ equation reads

$$\frac{xQ}{2}\left(\frac{\partial}{\partial Q} + \frac{1}{Q}\frac{\partial}{\partial \phi}\right)\frac{\partial}{\partial x}[xG(x, Q^2, \phi)] = \frac{\alpha_s N_c}{\pi}xG(x, Q^2, \phi) - \frac{\alpha_s^2 N_c \pi}{2C_F S_\perp}\frac{1}{Q^2}[xG(x, Q^2, \phi)]^2$$

We propose a solution based on $G_0(x, Q^2)$:

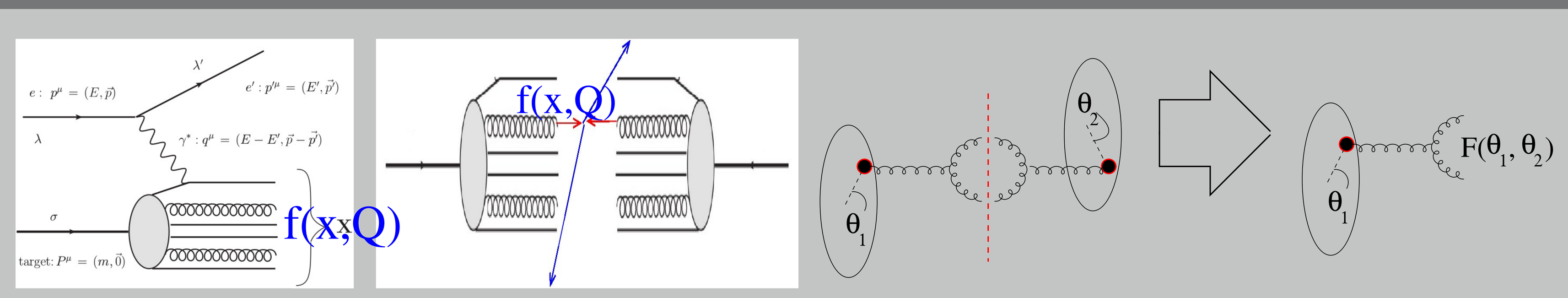
$$G(x, Q^2, \phi) = G_0(x, Q^2)\left(1 + \sum_{n=1}^{\infty} u_n(x, Q^2)\cos(n\phi + \beta_n)\right)$$

As a result of plugging this ansatz intothe asymmetric GLR equation we get a constrain for the β_n $\cos(2\beta_n) = 0$ and for the limiting case $Q_s(x)/Q \ll 1$ we get a set of equations:

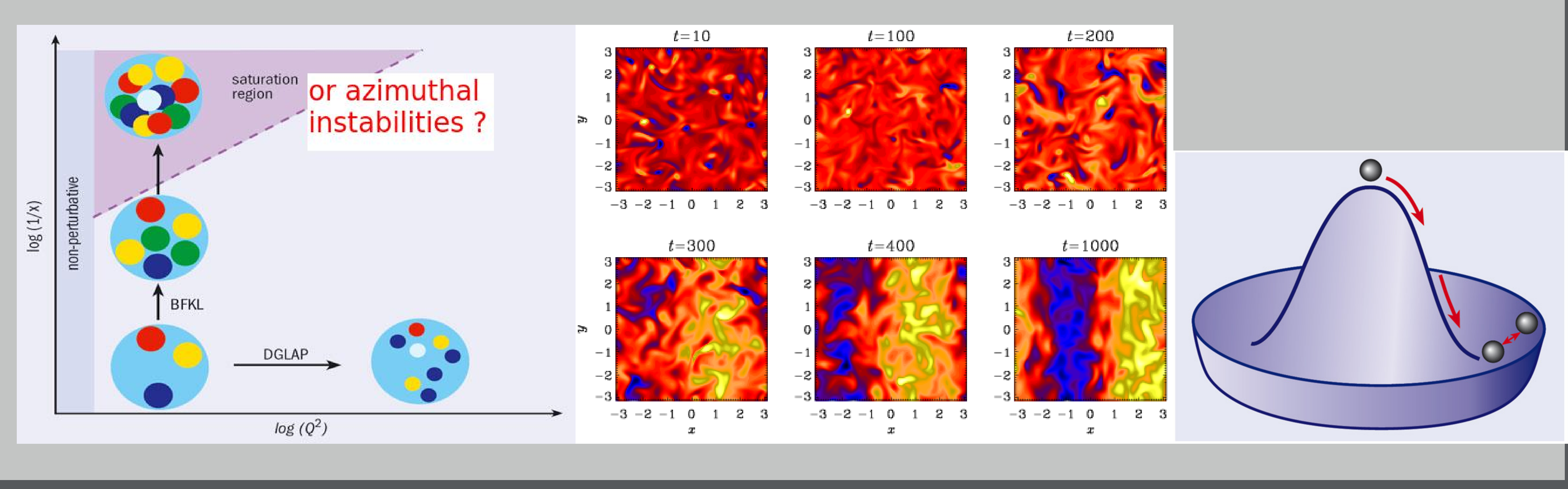
$$x\frac{\partial u_n(x, Q^2)}{\partial x} = -(2\lambda + 1)u_n(x, Q^2)$$

$$+\frac{\alpha_s^2 N_c \pi}{2C_F S_\perp}\frac{1}{Q^2}x^{2\lambda+1}\frac{1}{n}\left[\sum_k^{n-1} u_k(x, Q^2)u_{n-k}(x, Q^2)\sin(\beta_n - \beta_k - \beta_{n-k}) + 2\sum_k u_k(x, Q^2)u_{n+k}(x, Q^2)\sin(\beta_n + \beta_k - \beta_{n+k})\right]$$

The GLR equation and possible instabilities



- Parton distribution functions $G(x, Q^2)$ are the probability of finding a quark of longitudinal momentum xE in a nucleon striking a probe at energy E, where the characteristic momentum of the probe is Q. To leading order $G(x, Q^2) \to G(x)$ but loop corrections bring a Q^2 dependence. Generalized to GPDs,local in $x_\perp$. Azimuthal dependence forbidden by symmetry at leading orders, could arise $\alpha_s^2\epsilon$ ($\ll v_2$) for "extended" probes.
- In BFKL evolution it is well known that, as we decrease x, the parton density grows and diverges at a rate where the total proton-proton cross-section would eventually break the Froissart bound required by unitarity, $\sigma_{tot} \sim \ln^2 s$. A natural way to resolve this is to assume that, at some critical value $x, Q \to Q_s(x)$ the density of partons becomes so large that their wave functions start to overlap , i.e. they interact.
- **Gribov, Levin and Ryskin** considered a a quadratic correction to the BFKL equation, which slows down the growth of the gluon distribution until saturation of parton distributions occurs The full GLR equation is a 2+1 diffusion equation (x, Q, ϕ) , azimuthal symmetry makes it 1+1 (x, Q)
- **Saturation** and the Color-Glass Condensate formalism are thought to arise when non-linear parton evolution is implemented within a renormalization group approach, with the separation in $|\ln(1/x)| \to 0$ between target/projectile and "wee" gluons $x \to 0$ providing a scale variable. But RG evolution could contain IR operators breaking the fundamental symmetries of the Lagrangian!
- **2+1 diffusion equations** symmetric solutions can be unstable against perturbations. 2 dimensional turbulence is characterized by inverse cascades, with high frequency low amplitude perturbations growing in low-frequency high amplitude ones (avoiding the "many antennae" limit of Glasma based models). Thus, for GPDs small transversely local azimuthal perturbations at the target-projectile could seed global breaking of azimuthal symmetry for low x $\to G(x, Q_x, Q_y)$ In a RG approach the EFT lagrangian, not just boundary conditions, asymmetric. tiny anisotropies at $x \to \pm 1$ grow with Q . v_n, with the right scaling,as initial state

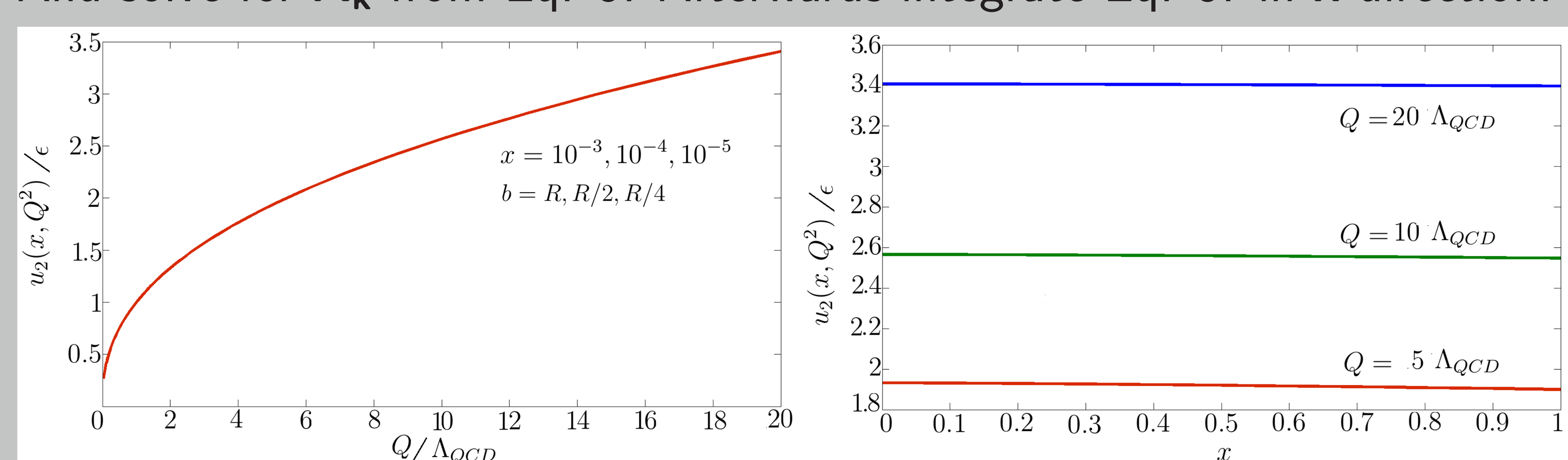


Preliminary results

As an ansatz we propose

$$u_n(x \to 0, Q^2) = \delta_{n,2}\sum_{k=0}^{\infty} A_k\frac{(Bx^C)^k}{k!}Q^{D-2k}$$

And solve for A_k from Eq. 5. Afterwards integrate Eq. 5. in x direction.



Qualitatively reasonable features emerge: Perturbations dependence on Q^2 practically decoupled from x, linear dependence on ϵ, near-decoupling of Fourier modes. A detailed phenomenological study, including factorization, dynamics and fragmentation is forthcoming, but this approach could reproduce scaling seen in the data. v_n/ridges at the EIC eA collisions?