Non-Gaussian eccentricity fluctuations

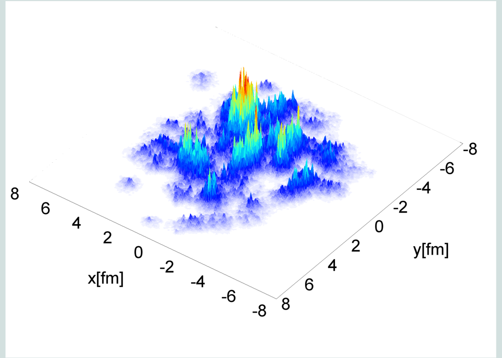
Hanna Grönqvist, Jean-Paul Blaizot & Jean-Yves Ollitrault

Institut de physique théorique, Université Paris-Saclay, CNRS, CEA, F-91191 Gif-sur-Yvette, France

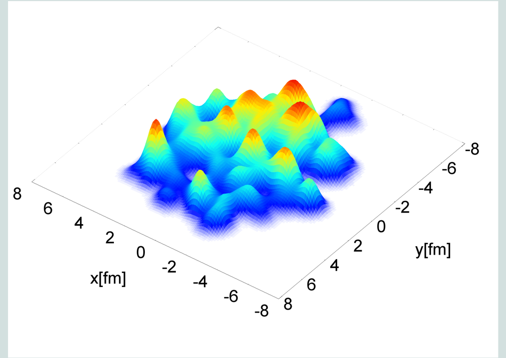
1. Motivation

- Fluctuations in the initial density profile result in flow fluctuations [1].

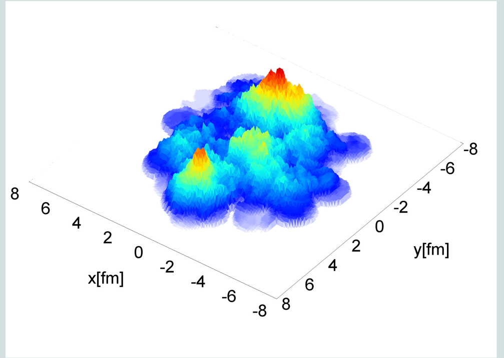
IP Glasma



MC Glauber

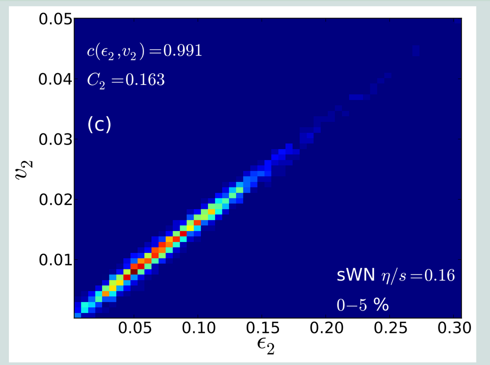


MC KLN

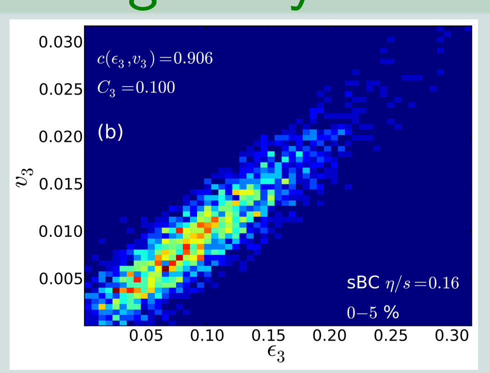


- Anisotropic flow is approximately proportional to the initial anisotropy [2].

Ellipticity



Triangularity



- Therefore the statistics of v_n fluctuations observed in experiments tell us about ε_n fluctuations. in particular, if ε_n fluctuations are Gaussian, one expects Gaussian v_n fluctuations [3].
  - The second and fourth cumulants are

$$\varepsilon_n\{2\}^2 \equiv \langle|\varepsilon_n|^2\rangle$$
$$\varepsilon_n\{4\}^4 \equiv 2\left\langle|\varepsilon_n|^2\right\rangle^2 - \langle|\varepsilon_n|^4\rangle \ ,$$

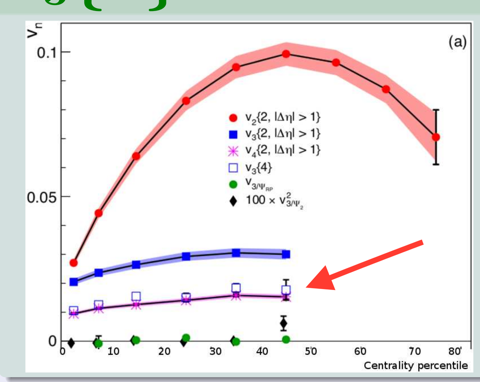
- We consider only fluctuation-driven anisotropies for simplicity (no reaction plane eccentricity). We note that $\varepsilon_n\{4\} = 0$ for Gaussian fluctuations.

2. Measurements

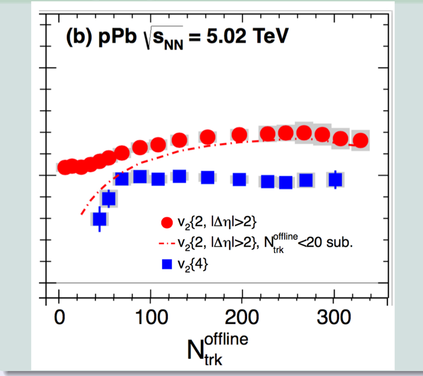
We know that v_n fluctuations are **not** Gaussian:

- non-zero $v_3\{4\}$ in Pb+Pb [4] :
- non-zero $v_2\{4\}$ in p+Pb [5] :

$v_3\{4\}$ @ ALICE



$v_2\{4\}$ @ CMS



Therefore ε_n fluctuations are not Gaussian. I study these non-Gaussianities systematically in order to figure out what we can learn from them.

3. State of the art

a) Eccentricity fluctuations have been studied in the "independent source model" where the initial energy density profile is a superposition of N identical, pointlike sources. Analytic results have been obtained for $N \gg 1$ [6].

$$\left.\begin{aligned}\varepsilon_n\{2\} &\propto N^{-1/2}\\ \varepsilon_n\{4\} &\propto N^{-3/4}\end{aligned}\right\} \Rightarrow \quad \varepsilon_n\{4\} \propto \varepsilon_n\{2\}^{3/2} \ ,$$

where the proportionality coefficient depends on the energy density profile.

b) It has been argued that the non-Gaussianity of eccentricity fluctuations is mostly driven by the condition $|\varepsilon_n| < 1$ [7]. A parameterization of the distribution of $|\varepsilon_n|$ has been proposed which satisfies this constraint and which predicts

$$\varepsilon_n\{4\} \simeq 2^{1/4}\varepsilon_n\{2\}^{3/2} \ .$$

c) There is a contradiction between a) (not universal) and b) (universal).

4. My work

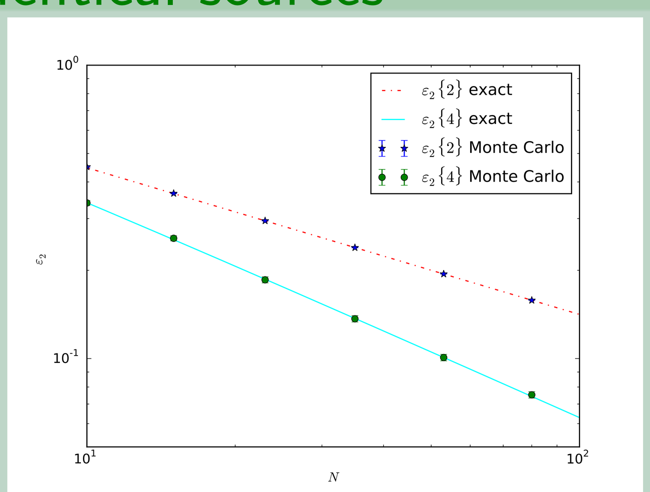
Monte Carlo simulations

N independent, pointlike sources. We test the effect of changing

(1) the density profile in the transverse plane (2d Gaussian versus uniform in a disk)

(2) the energy of each source (identical sources versus negative binomial fluctuations fitted to LHC data)

Gaussian distribution, identical sources

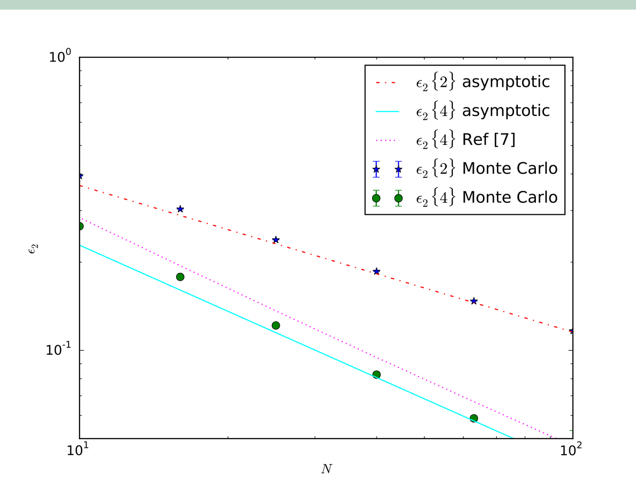


1 exactly solvable

$$\varepsilon_2\{2\} = \sqrt{2/N}$$
$$\varepsilon_2\{4\} = \left(\frac{16}{N^2(N+2)}\right)^{1/4}$$

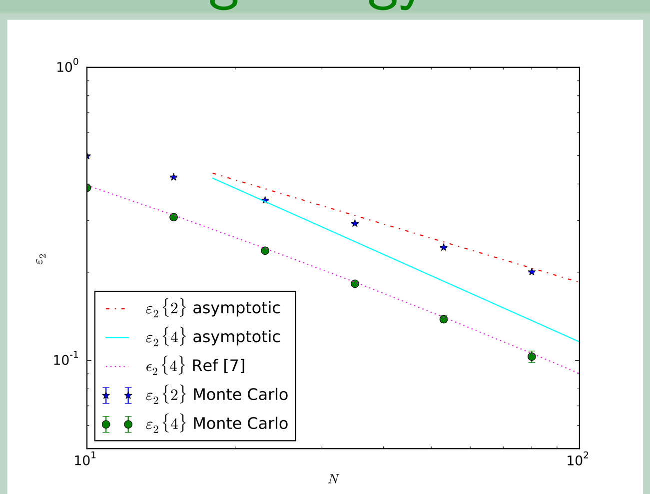
MC simulations fit the exact result perfectly.

Uniform distribution, identical sources



2 The universal statistics from Ref. [7] overestimates the non-Gaussianity $\varepsilon_2\{4\}$. The asymptotic result from Ref. [6] is better for $N > 15$.

Uniform distribution, fluctuating energy

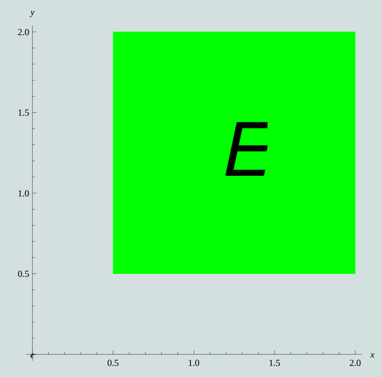


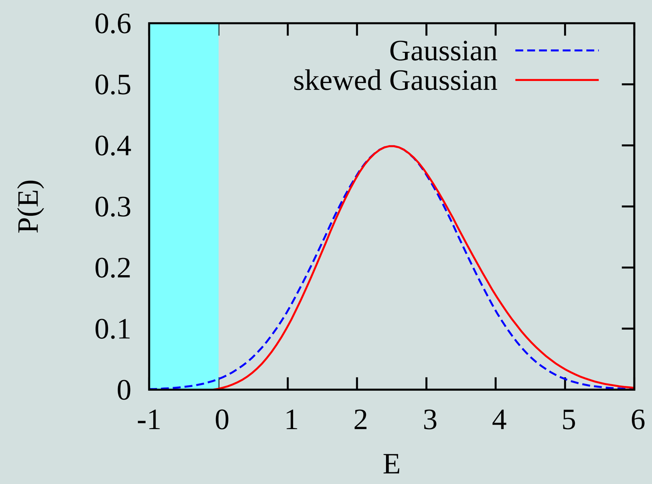
3 Fluctuations in the energy of each source increase both anisotropies ($\varepsilon_2\{2\}$) and non-Gaussianities (smaller splitting between $\varepsilon_2\{2\}$ and $\varepsilon_2\{4\}$). These trends are also present in asymptotic results (generalization of Ref. [6]), yet convergence to the asymptotic regime is very slow. The universal statistics from [7] works significantly better than asymptotic results.

Analytical results

Results from the independent-source model generalized to a continuous fluctuating energy density profile [9].

The energy E contained in a given area of the transverse plane has non-Gaussian event-to-event fluctuations because of the constraint $E \geq 0$.





The distribution is typically right-skewed $\langle(E - \langle E\rangle)^3\rangle > 0$ We treat non-gaussianities as small perturbations and obtain the general expression of $\varepsilon_n\{4\}$ to leading order (not shown). For identical pointlike sources we recover the result of [6]; each term has a different physical interpretation.

$$\varepsilon_n\{4\}^4 = \frac{1}{N^3}\left(-\underbrace{\frac{\langle r^{2n}\rangle^3}{\langle r^n\rangle^6}}_{\text{Gaussian}} + \underbrace{\frac{8\langle r^{3n}\rangle\langle r^{2n}\rangle}{\langle r^n\rangle^5}}_{\text{Skewness}} - \underbrace{\frac{\langle r^{4n}\rangle}{\langle r^n\rangle^4}}_{\text{Kurtosis}} + \underbrace{\frac{2\langle r^{2n}\rangle^2}{\langle r^n\rangle^4}}_{\text{E conserv.}}\right)$$

- Gaussian density fluctuations yield slight non-Gaussianities in ε_n fluctuations but they have the wrong sign [10].
- Non gaussian density fluctuations restore positivity.

Conclusion

Fluctuations in the energy of each source yield strong non-Gaussianities which are well described by the universal statistics of Ref. [7]. This may explain why CMS p+Pb results [8] on $v_2\{6\}$ and $v_2\{8\}$ are also well described by this universal statistics.

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