Inhomogeneous chiral condensed phases with an algebraic long-range order

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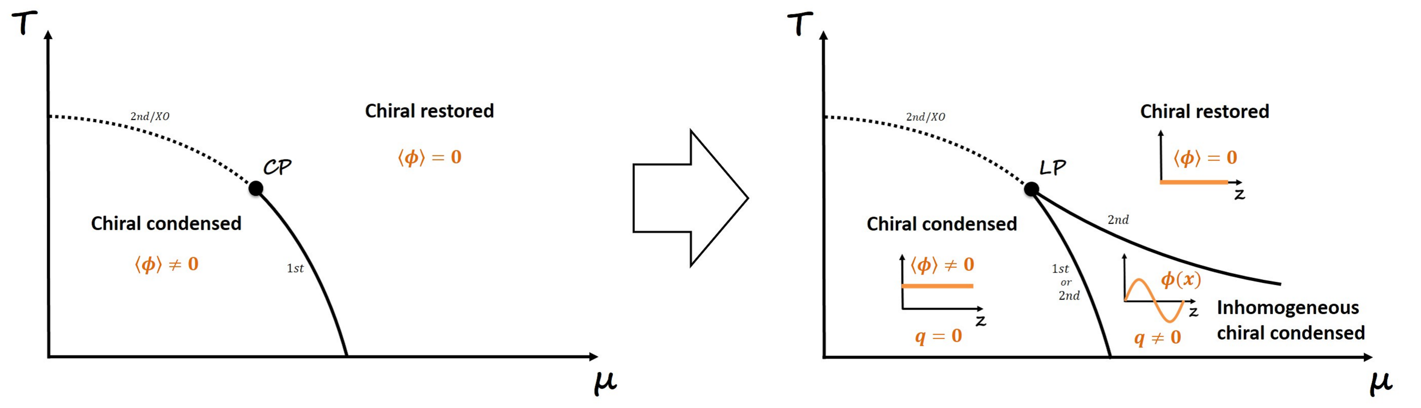
Abstract

We investigate the stability of an inhomogeneous chiral condensed phase against low energy fluctuations about a spatially modulated order parameter. This phase corresponds to the so-called dual chiral density wave of dense quark matter, where the chiral condensate is spatially modulated with a finite wavevector in a single direction. From a symmetry point of view the phase realizes a locking of flavor and translational symmetries. Starting with a Landau-Ginzburg-Wilson effective Lagrangian, we find that the associated Nambu-Goldstone modes, whose dispersion relations are spatially anisotropic and soft in the direction normal to the modulation wavevector, wash out the long-range order at finite temperatures, but support algebraically decaying long-range correlations. This implies that the phase can exhibit a quasi-one-dimensional order as in smectic liquid crystals.

1. Introduction [1,2]



Recent theoretical studies of QCD at finite temperature and density predict the presence of inhomogeneous chiral condensates. If the inhomogeneous phase actually exists, an elementary excitation on the ground state could be experimentally observed. If so, what is the physical d.o.f. in the inhomogeneous chiral condensed phase? In addition, what fate awaits inhomogeneous phases beyond the mean-field approximation when order parameter fluctuations are taken into account?

Main Objectives

To answer the above questions, we discuss the following topics:

1. Nambu-Goldstone (NG) excitations; we first clarify the symmetry breaking pattern in an inhomogeneous chiral condensed phase and explore dispersion relations for the corresponding low-energy collective excitation modes.
2. Landau-Peierls instability; Next, we investigate how much the inhomogeneous chiral condensed phase is stable against low energy fluctuations about a spatially modulated order parameter.

2. Nambu-Goldstone (NG) excitations

Landau-Ginzburg-Wilson effective Lagrangian

$$\mathcal{L}(\phi) = c_2\partial_0\phi\cdot\partial_0\phi - \mathcal{V}(\phi)$$
$$\mathcal{V}(\phi) = a_2(\phi\cdot\phi) + a_{4,1}(\phi\cdot\phi)^2 + a_{4,2}(\nabla\phi\cdot\nabla\phi) + a_{6,1}(\nabla^2\phi\cdot\nabla^2\phi) + a_{6,2}(\nabla\phi\cdot\nabla\phi)(\phi\cdot\phi) + a_{6,3}(\phi\cdot\phi)^3 + a_{6,4}(\phi\cdot\nabla\phi)^2$$

▷ low energy effective Lagrangian density for a general chiral order parameter ϕ
▷ obtained from a microscopic theory by integrating out all higher energy modes
▷ coefficients $a_{i,j}$ can be estimated within NJL/QM model
▷ terms up to 6^{th}-order one with $a_{6,j}$ leads to a stable inhomogeneous phase

Inhomogeneous chiral condensed phase

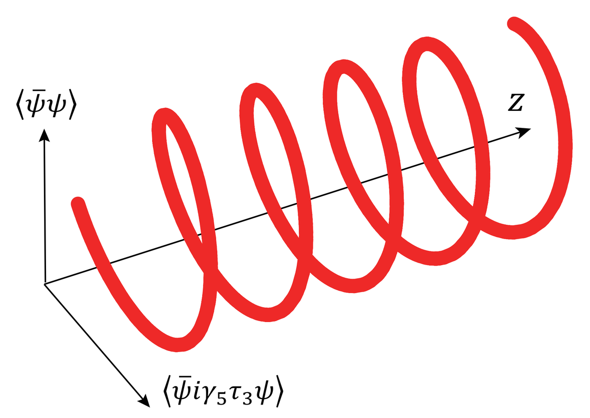


Figure 1: The DCDW condensate given by a plane wave $\langle\bar\psi\psi\rangle + i\langle\bar\psi i\gamma_5\tau_3\psi\rangle = \Delta e^{iqz}$.

We consider a Fulde-Ferrell-type inhomogeneous chiral condensate with a one-dimensional modulation in 3+1 dimensions. In the context of quark matter, this corresponds to the dual chiral density wave (DCDW) characterized by modulated scalar and pseudoscalar condensates (Fig.1):

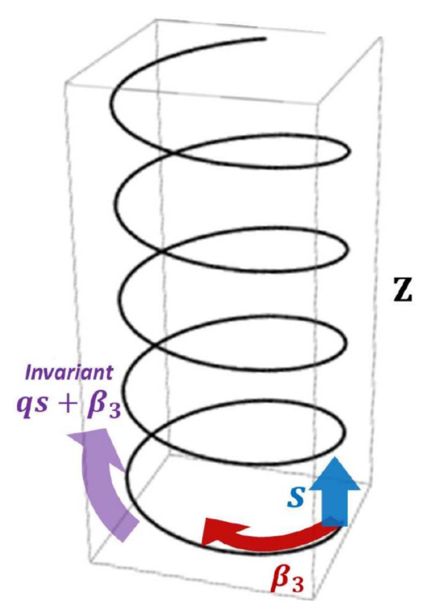
$$\langle\bar\psi\psi\rangle = \Delta\cos(qz),\quad \langle\bar\psi i\gamma_5\tau_3\psi\rangle = \Delta\sin(qz).$$

where a wavenumber q and a constant amplitude Δ corresponding to $\langle\bar\psi e^{i\gamma_5\tau_3 qz}\psi\rangle$.

Spontaneous symmetry breaking and NG modes

▷ Ground state of the DCDW type ($O(4)$ real field for ϕ):

$$\phi_{DCDW} = \Delta\begin{pmatrix}\cos(qz)\\0\\0\\\sin(qz)\end{pmatrix}$$



▷ Symmetry breaking pattern :

- $SU(2)_L\times SU(2)_R$ chiral symmetry + translation in z direction
  - $qs+\beta_3 = 0 \;\to\; H$ (Unbroken generator)
    axial isospin-translation locking symmetry
  - $qs+\beta_3 \neq 0 \;\to\; G/H$ (Broken generator)
    NG mode: $\beta_3(t,\vec x)\ (:= qs+\beta_3)$
    described by out-of-phase operation: $qs(t,\vec x)+\beta_3(t,\vec x)\neq 0$.
- isospin and axial isospin rotations
  ⇒ One of which will be redundant [cf. Low-Manohar(2002)]
  ⇒ Corresponding NG modes: $\beta_{1,2} = \beta_{1,2}(t,\vec x)$
- spatial rotations about x and y axes
  ⇒ local modes θ are completely described by phonons — e.g.) x-axis rotation: $z \to z\cos\theta + y\sin\theta$ (θ: angle parameter)
  ⇒ $\theta(t,\vec x)$: Redundant [cf. Low-Manohar(2002); Watanabe-Murayama(2013); Hayata-Hidaka(2014)]

Infinitesimal translations:

$$\phi_{DCDW} \to \phi_{DCDW} + \Delta\begin{pmatrix}-(qs+\beta_3)\sin qz\\ \beta_1\cos qz - \alpha_2\sin qz\\ \beta_2\cos qz + \alpha_1\sin qz\\ (qs+\beta_3)\cos qz\end{pmatrix}$$

$(\sigma\to\sigma-\vec\beta\cdot\vec\pi,\ \vec\pi\to\vec\pi-\vec\alpha\times\vec\pi+\vec\beta\sigma,\ z\to z+s)$
($\vec\alpha,\vec\beta$: su(2)×su(2) rotation parameters, s: displacement parameter in z direction)

NG modes in DCDW: $dim(G/H) = 3$ (8 → 3 ; independent NG modes remain for inertial and spacetime symmetries)
$\vec\beta = \vec\beta(t,\vec x)$ (NG modes can be chosen as axial isospin rotations)

Low energy collective excitations

■ Fluctuations in DCDW :

$$\phi = (\Delta+\delta)\begin{pmatrix}\cos(qz+\beta_3)\cos\beta_2\cos\beta_1\\ \cos(qz+\beta_3)\cos\beta_2\sin\beta_1\\ \cos(qz+\beta_3)\sin\beta_2\\ \sin(qz+\beta_3)\end{pmatrix} = (\Delta+\delta)U(\beta_i)\begin{pmatrix}\cos(qz)\\0\\0\\\sin(qz)\end{pmatrix}$$

with **amplitude fluctuation** δ and **NG modes** β_i ($U(\vec\beta) := e^{\vec\beta\cdot\vec L}$ with axial isospin generators $\vec L$)

■ Low energy effective theory for fluctuation fields (δ(x) and β_i(x)) : $\tilde\beta_U = \beta_T\cos qz$

$$\mathcal{L} = (\partial_0\delta)^2 + \Delta^2(\partial_0\vec\beta_U)^2 + \Delta^2(\partial_0\beta_3)^2 - (\mathcal{V}_\delta + \mathcal{V}_{\delta\beta} + \mathcal{V}_\beta)$$

(Euclidean action in Fourier space)

$$-S_E = \int dk \begin{pmatrix}\delta^*(k)\\ \Delta\beta_3^*(k)\end{pmatrix}^T \begin{pmatrix}S_{\delta 0}^{-1}(k) & -g(k)\\ g(k) & S_0^{-1}(k)\end{pmatrix}\begin{pmatrix}\delta(k)\\ \Delta\beta_3(k)\end{pmatrix} + \frac14\int dk \begin{pmatrix}\Delta\vec\beta_T^*(k)\\ \Delta\vec\beta_T^*(k+2q\hat z)\end{pmatrix}^T \begin{pmatrix}S_0^{-1}(k) & G(k)\\ G(k) & S_0^{-1}(k+2q\hat z)\end{pmatrix}\begin{pmatrix}\Delta\vec\beta_T(k)\\ \Delta\vec\beta_T(k+2q\hat z)\end{pmatrix}$$

▷ Fluctuations of δ and β_3 mix for nonvanishing wavenumber q
▷ Transverse fluctuations $\vec\beta_T$ with different momenta mix

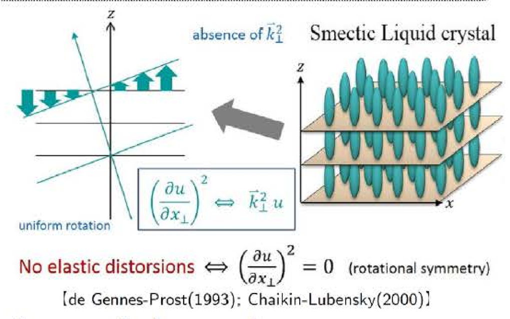
■ Dispersion relations:

► for mixing δ-β_3 (normal modes)

$$\omega_+^2 \simeq M^2 + a_{6,1}\left[u_{z+}^2k_z^2 + \left(\vec k^2\right)^2\right] + a_{6,4}\Delta^2\vec k^2 + A\vec k^2k_z^2 + Bk_z^4$$
$$\omega_-^2 \simeq a_{6,1}\left[u_{z-}^2k_z^2 + \left(\vec k^2\right)^2\right] - A\vec k^2k_z^2 - Bk_z^4$$

► for $\vec\beta_T(=\vec\beta_{1,2})$ (transverse modes)

$$\omega_k^2 = a_{6,1}\left[4q^2k_z^2 + \left(\vec k^2\right)^2\right] + \mathcal{O}(k^6)$$



▷ These dispersion relations are spatially anisotropic
⇒ The lack of $\vec k_\perp^2$ is a consequence of the rotational symmetry about any axis in *x-y* plane (as in SmLC)
▷ Transverse fluctuations are softer than longitudinal ones
▷ $\mathcal{O}(k^6)$: higher-order correction from interactions with background modulation

3. Landau-Peierls instability [3]

Impacts on low energy fluctuation

$$\langle(\Delta+\delta)U(\beta_i)\phi_0\rangle = \Delta\langle U(\beta_i)\phi_0\rangle + \langle\delta U(\beta_i)\phi_0\rangle$$

$$\langle U(\beta_i)\phi_0\rangle = \begin{pmatrix}\cos(qz)\exp(-\sum_i\langle\beta_i^2\rangle/2)\\0\\0\\\sin(qz)\exp(-\langle\beta_3^2\rangle/2)\end{pmatrix}$$

$$\langle\delta U(\beta_i)\phi_0\rangle = \begin{pmatrix}-\sin(qz)\langle\delta\beta_3\rangle\exp(-\sum_i\langle\beta_i^2\rangle/2)\\0\\0\\\cos(qz)\langle\delta\beta_3\rangle\exp(-\langle\beta_3^2\rangle/2)\end{pmatrix}$$

Thermal fluctuation (T > 0)

$$\langle\delta\beta_3\rangle = 0,\quad \langle\beta_{1,2}^2\rangle \simeq \frac{1}{2\Delta^2}\int\frac{d^3k}{(2\pi)^3}\frac{T}{\omega_k^2},\quad \langle\beta_3^2\rangle \simeq \frac{1}{2\Delta^2}\int\frac{d^3k}{(2\pi)^3}\frac{T}{\omega_-^2}$$

⇒ which of all, $\langle\beta_{1,2,3}^2\rangle_{T>0}$, logarithmically diverge due to soft modes in x-y direction.
⇒ low-energy fluctuations wash out the order parameter at $T>0$.
⇒ which, however, does not immediately lead to no existence of DCDW phase.
⇒ which possibly exists as a quasi-1D phase. [Landau-Lifshitz(1969); Baym-Friman-Grinstein (1982)]

Quantum fluctuation (T = 0)

$$\langle\delta\beta_3\rangle = 0,\quad \langle\beta_{1,2}^2\rangle \simeq \frac{1}{4\Delta^2}\int\frac{d^3k}{(2\pi)^3}\frac{T}{\omega_k},\quad \langle\beta_3^2\rangle \simeq \frac{1}{4\Delta^2}\int\frac{d^3k}{(2\pi)^3}\frac{T}{\omega_-}$$

⇒ which of all, $\langle\beta_{1,2,3}^2\rangle_{T=0}$, do not diverge; $\langle(\Delta+\delta)U(\beta_i)\phi_0\rangle\neq 0$
⇒ low-energy fluctuations are not so strong to wash out the order parameter

The DCDW phase is realized as a LRO if T is sufficiently low ($0<T<T_0$)

Long-range correlations

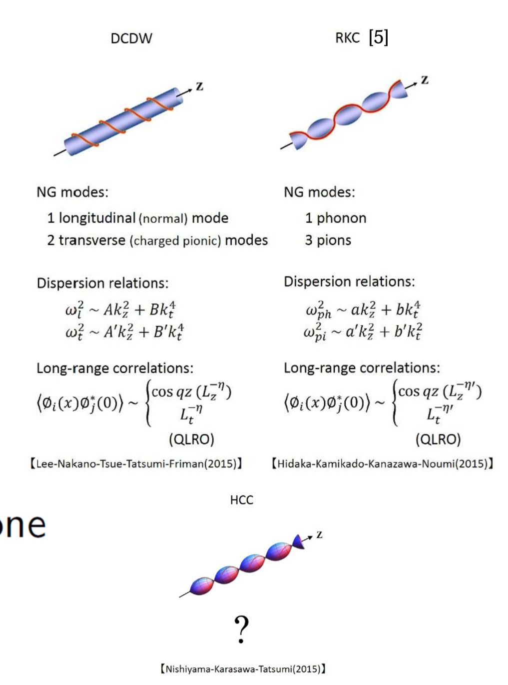
$$\langle\phi(z\hat z)\cdot\phi^*(0)\rangle \sim \frac12\Delta^2\cos qz\left(\frac{z}{z_0}\right)^{-T/T_0}$$
$$\langle\phi(x_t\hat x_t)\cdot\phi^*(0)\rangle \sim \frac12\Delta^2\left(\frac{x_t}{x_0}\right)^{-2T/T_0}$$

algebraic long-range order
(quasi-long-range order) [4]

The DCDW phase can be practically realized as a quasi-one-dimensionally ordered one akin to smectic-A liquid crystals [Als-Nielsen et al.(1977,1980)]

4. Conclusions

► low energy collective excitations in the DCDW phase
► a flavor-translation locking symmetry
► spatially anisotropic dispersion relations
(lack of quadratic terms / soft in transverse direction)
► Landau-Peierls instability
(low energy fluctuations wash out the order parameter at $T>0$)
(but the DCDW phase exhibits an algebraic long-range order)
► The DCDW phase is stable as a quasi-one-dimensional ordered one
(practically realized like smectic liquid crystals)



5. Outlook

► Experimental implications:
- to explore how collective modes in DCDW interact with external probes (quarks/photons) via quasi-Bragg peaks

► Phenomenological implications:
- astronomical implications (novel cooling and EOS/MR relation with DCDW) [cf. Tatsumi-Muto('14), Carignano-Ferrer-Incera-Paulucci('15)]
- to see how NG modes in DCDW affect transport properties in the compact star inner core

References [1] E.Nakano and T.Tatsumi, PRD71,114006(2005); D.Nickel, PRL103,072301(2009); PRD80,074025(2009); D.Müller, M.Buballa, and J.Wambach, PLB727,240(2013). [2] M.Buballa and S.Carignano, PPNP81,39(2015). [3] L.D.Landau and E. M. Lifshitz, *Statistical Physics*, (Pergamon Press,Oxford,1969); G.Baym, B.Friman, and G.Grinstein, NPB210,193(1982). [4] P.M.Chaikin and T.C.Lubensky, *Principles of Condensed Matter Physics*, (Cambridge University Press,2000). [5] Y.Hidaka, K.Kamikado, T.Kanazawa, and T.Noumi, PRD92,034003(2015)