Real Time Evolution of Non-Gaussian Cumulants in the QCD Critical Regime

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with: Raju Venugopalan and Yi Yin

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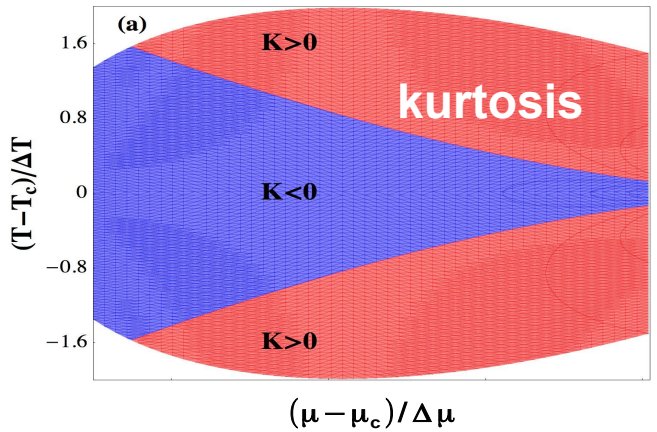
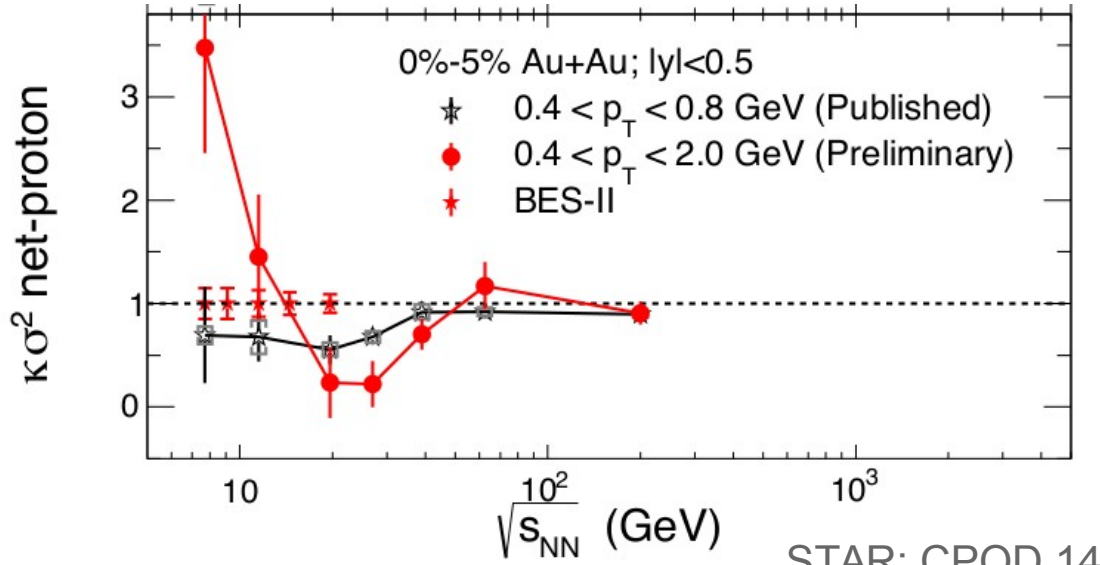
Why non-equilibrium ?



Stephanov: arXiv:1104.1627



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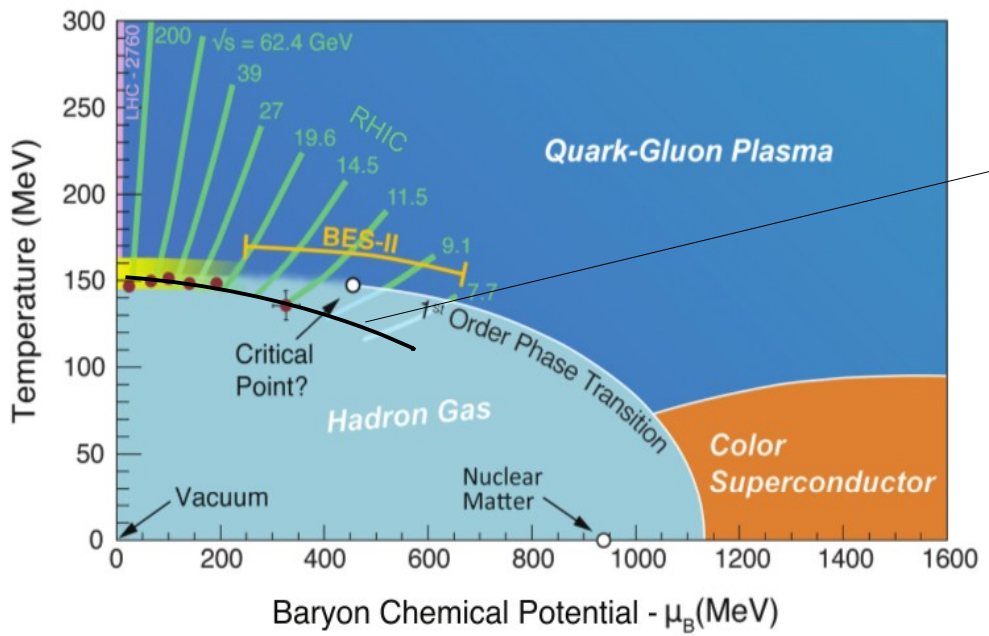


freeze-out ?

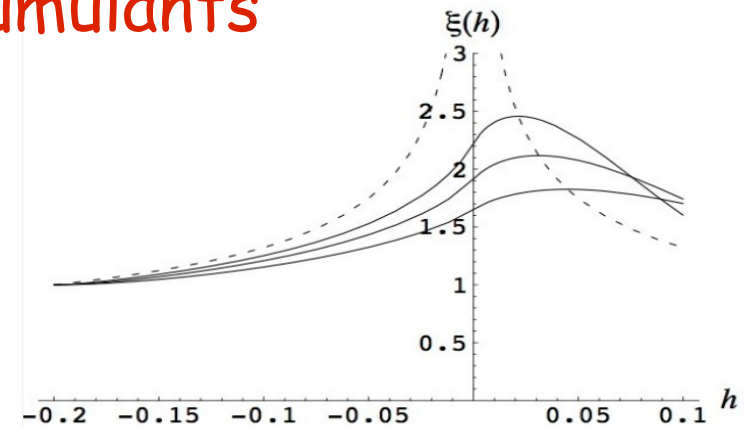
non-equilibrium memory effects are required to preserve remnant of critical signatures

unless accidental freeze-out near critical point

Simple non-equilibrium evolution of cumulants

Ansatz for evolution of correlation length:

$$\partial_\tau \xi^{-1} = -\tau_{\text{eff}}^{-1}\left[\xi^{-1} - \xi_{\text{eq}}^{-1}\right]$$



with dynamical universality: $\tau_{\text{eff}} \sim \xi^z$

Berdnikov, Rajagopal: arXiv:hep-ph/9912274

rely on equilibrium scaling for non-Gaussian cumulants: $\kappa_3^{\text{eq}} \sim \xi_{\text{eq}}^{9/2}$ $\kappa_4^{\text{eq}} \sim \xi_{\text{eq}}^{7}$

- do such scalings really hold out of equilibrium ?
- signs of non-Gaussian cumulants out of equilibrium ?

Effective action & scope

free energy density: $\Omega_0(\sigma) = \frac{1}{2} m_\sigma^2 (\sigma - \sigma_0)^2 + \frac{\lambda_3}{3}(\sigma - \sigma_0)^3 + \frac{\lambda_4}{4}(\sigma - \sigma_0)^4$

equilibrium distribution: $P_0(\sigma) \sim \exp\left(-V\,\Omega_0(\sigma)/T\right)$

$m_\sigma^{-1} \equiv \xi_{eq}$

σ : critical mode (zero-momentum)

→ linear combination of chiral condensate and baryon current

limit to: scaling regime, but not at the critical point $L_{micr} < \xi < L$

$$\epsilon = \sqrt{\xi^3/V} \ll 1$$

mass term ~ σ^2/ξ_{eq}^2

kinetic term (momentum dependence) ~ σ^2/L^2 ← neglected

Cumulants

$$M(\tau)=\langle\sigma\rangle \qquad \kappa_2(\tau)=\langle(\delta\sigma)^2\rangle \qquad \kappa_3(\tau)=\langle(\delta\sigma)^3\rangle \qquad \kappa_4(\tau)=\langle(\delta\sigma)^4\rangle-3\kappa_2^2(\tau)$$

$$\delta\sigma=\sigma-M(\tau) \qquad \langle\cdots\rangle=\int d\sigma(\cdots)P(\sigma;\tau)\ /\ \int d\sigma P(\sigma;\tau)$$

$$M^{eq}=\sigma_0 \qquad \kappa_2^{eq}=\frac{\xi_{eq}^2}{V_4} \qquad \kappa_3^{eq}=-\frac{2\lambda_3}{V_4^2}\xi_{eq}^6 \qquad \kappa_4^{eq}=-\frac{6}{V_4^3}\left[2(\lambda_3\xi_{eq})^2-\lambda_4\right]$$

$$m_\sigma=\xi_{eq}^{-1} \qquad \sigma=\tilde{\sigma}_0 T(T\xi_{eq})^{-1/2} \qquad \lambda_3=\tilde{\lambda}_3 T(T\xi_{eq})^{-3/2} \qquad \lambda_4=\tilde{\lambda}_4(T\xi_{eq})^{-1}$$

$$M^{eq}\sim\xi_{eq}^{-1/2} \qquad \kappa_2^{eq}\sim\xi_{eq}^2 \qquad \kappa_3^{eq}\sim\xi_{eq}^{9/2} \qquad \kappa_4^{eq}\sim\xi_{eq}^7$$

power counting in ϵ: $\dfrac{\epsilon M^{eq}}{b},\ \dfrac{\kappa_2^{eq}}{b},\ \dfrac{\kappa_3^{eq}}{\epsilon b^3},\ \dfrac{\kappa_4^{eq}}{\epsilon^2 b^4}\sim O(1)$

$$\epsilon=\sqrt{\xi_{eq}^3/V}$$

$$b=\sqrt{\kappa_2^{eq}}$$

$$V_4=V/T$$

assume this power counting holds for non-equilibrium evolution of cumulants in the critical regime

and make systematic expansions in ϵ

Real time evolution of cumulants

Langevin dynamics: soft critical mode receives small, random kicks from a bath of hard modes

Fokker-Planck equation: $\partial_\tau P(\sigma;\tau) = \frac{1}{m_\sigma^2 \tau_{eff}}\left[\partial_\sigma\left[\partial_\sigma \Omega_0(\sigma) + V_4^{-1}\partial_\sigma\right]P(\sigma;\tau)\right]$

τ_{eff} : effective relaxation time of the critical modes

$\tau_{eff} \sim \xi^z$ z : dynamical critical exponent

$$\partial_\tau \langle f(\sigma)\rangle = -\frac{1}{m_\sigma^2 \tau_{eff}}\left[\langle f'(\sigma)\Omega_0'(\sigma)\rangle - V_4^{-1}\langle f''(\sigma)\rangle\right]$$

time evolution of the cumulatns and systematic expansion in $\epsilon = \sqrt{\xi^3/V}$

Real time evolution of cumulants: coupled equations

closed set of coupled time evolution equations:

$$\epsilon = \sqrt{\xi^3/V} \ll 1$$

$$\partial_\tau M = -\tau_{\rm eff}^{-1}\, p_1(M)\left[1+O(\epsilon)\right]$$

$$\partial_\tau \kappa_n = -n\,\tau_{\rm eff}^{-1}\, p_n(M,\kappa_2,\cdots,\kappa_n)\left[1+O(\epsilon)\right]$$

evolution of the higher cumulants couples only to lower ones

$p_n \longrightarrow$ polynomials

Real time evolution of cumulants: two limits

Gaussian limit:

$$\Omega_0(\sigma) = \frac{1}{2} m_\sigma^2 (\sigma - \sigma_0)^2 \qquad \kappa_3^{eq} = \kappa_4^{eq} = 0$$

$$\partial_\tau \kappa_n = -n \tau_{eff}^{-1} \left[\kappa_n - \kappa_n^{eq} \right]$$

evolutions decouple,
non-Gaussian cumulants are damped,
higher cumulants damps faster

for n=2 reduces to the old
Berdnikov-Rajagopal Ansatz:

$$\partial_\tau \xi^{-1} = -\tau_{eff}^{-1} \left[\xi^{-1} - \xi_{eq}^{-1} \right]$$

near-equilibrium limit:

$$\delta \kappa_n = \kappa_n - \kappa_n^{eq}$$

~ small, linearize

$$\partial_\tau \kappa_2 = -2 \tau_{eff}^{-1} a_2 \delta \kappa_2$$

$$\partial_\tau \kappa_3 = -3 \tau_{eff}^{-1} \left[a_2 \delta \kappa_2 + a_3 \delta \kappa_3 \right]$$

$$\partial_\tau \kappa_4 = -4 \tau_{eff}^{-1} \left[a_2 \delta \kappa_2 + a_3 \delta \kappa_3 + a_4 \delta \kappa_4 \right]$$

lower cumulatns relax
back to equilibrium first

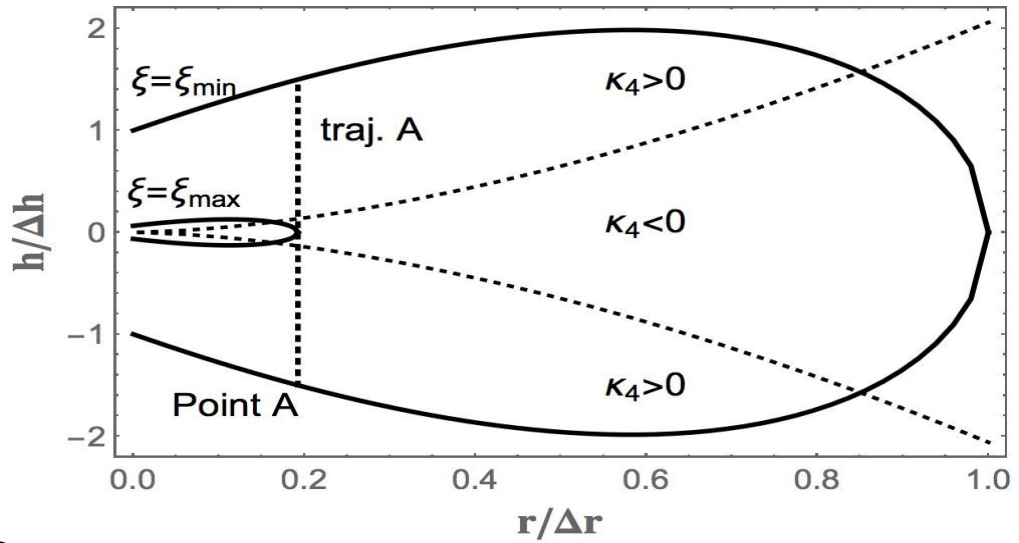
important feature of complete
set of coupled equation

Time evolution: emulating heavy-ion collisions

universal parameters:

3-d Ising universality:

$$\sigma_0(r,h),\ m_\sigma(r,h),\lambda_3(r,h),\lambda_4(r,h)$$



choose critical region in r-h plane through: $\xi_{max}/\xi_{min}=3$

r: reduced temperature
h: magnetic field

model-H dynamical universality:

$$\tau_{eff}=\tau_{rel}\left(\frac{\xi_{eq}}{\xi_{min}}\right)^z,\quad z=3$$

τ_{rel} : relaxation time at outside edge of critical region

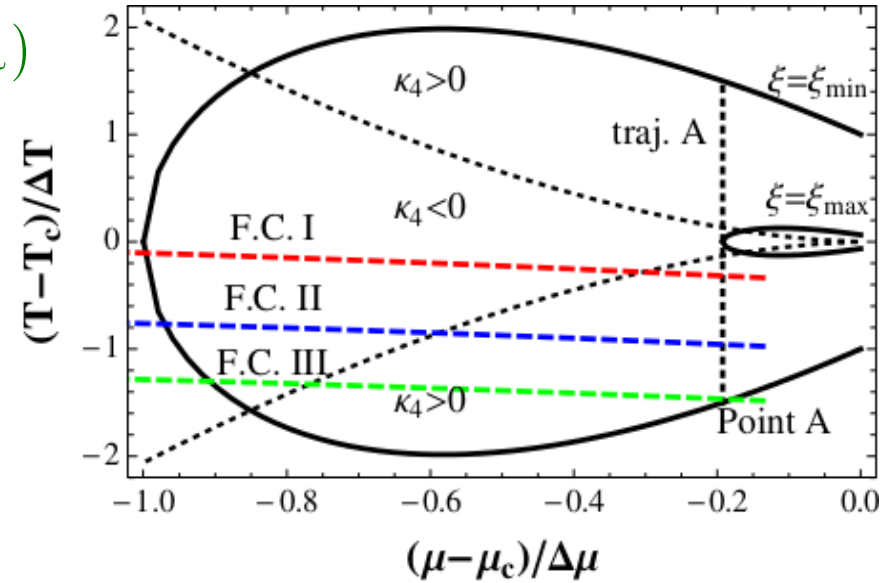
Son, Stephanov: arXiv:hep-ph/0401052

Time evolution: emulating heavy-ion collisions

map to QCD phase diagram: $(r, h) \rightarrow (T, \mu)$

naïve choice:

$$\frac{T - T_c}{\Delta T} = \frac{h}{\Delta h} \qquad \frac{\mu - \mu_c}{\Delta \mu} = -\frac{r}{\Delta r}$$



trajectories: 3-d Hubble-like expansion

$$\frac{T(\tau)}{T_I} = \left(\frac{\tau}{\tau_I}\right)^{3c_s^2} \qquad \frac{V(\tau)}{V_I} = \left(\frac{\tau}{\tau_I}\right)^{3}$$

entropy ~ constant

speed of sound: $c_s^2 = 0.15$

free parameter:

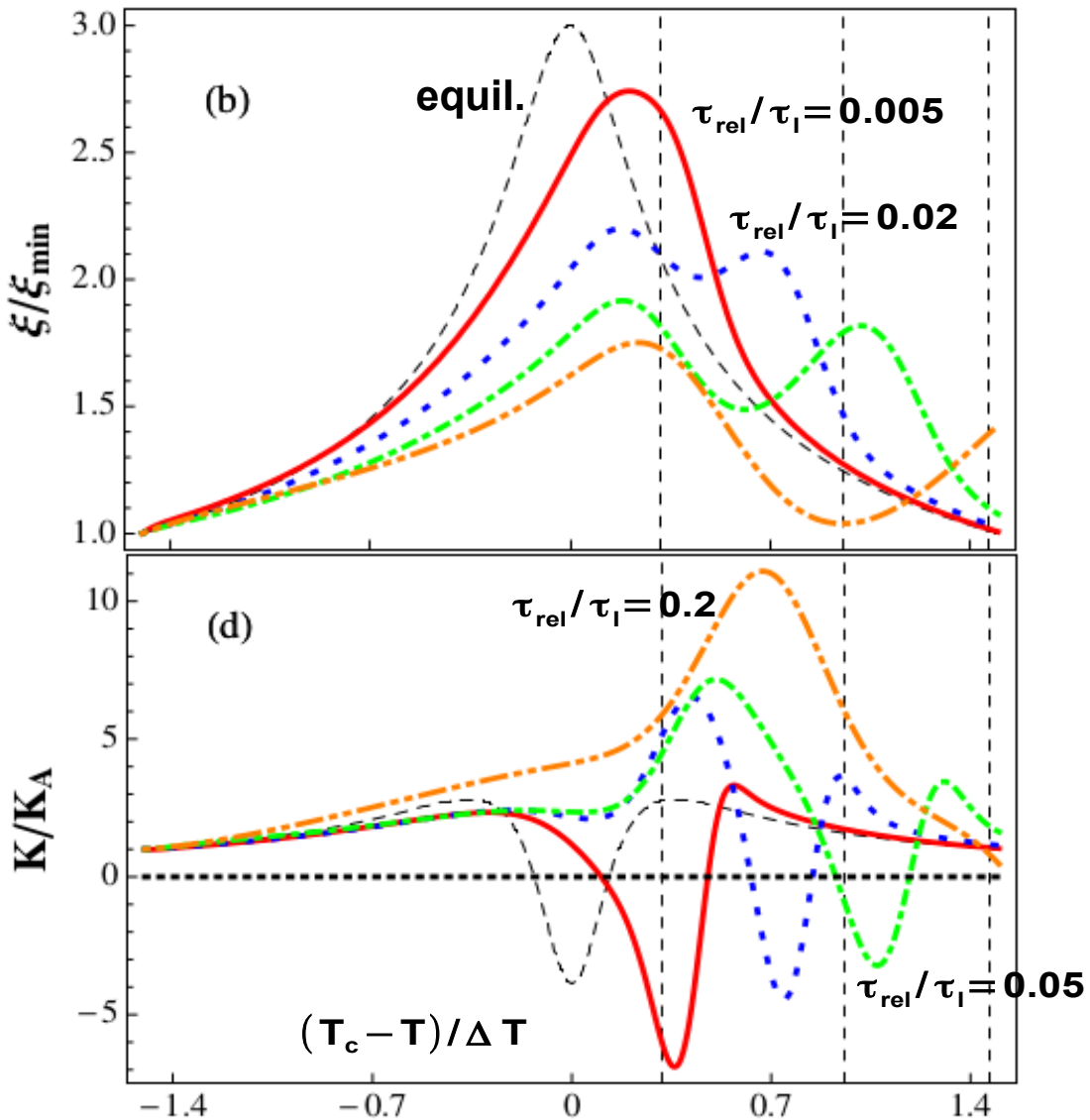
τ_{rel}/τ_I

τ_{rel} : relaxation time for the critical mode at the boundary of critical region

τ_I : time when system enters critical region

ballpark guess: $\tau_{rel} = 1$ fm, $\tau_I = 10$ fm

Cumulants: as time goes by ...



critical slowing down limits growth of correlation length

similar to Berdnikov-Rajagopal

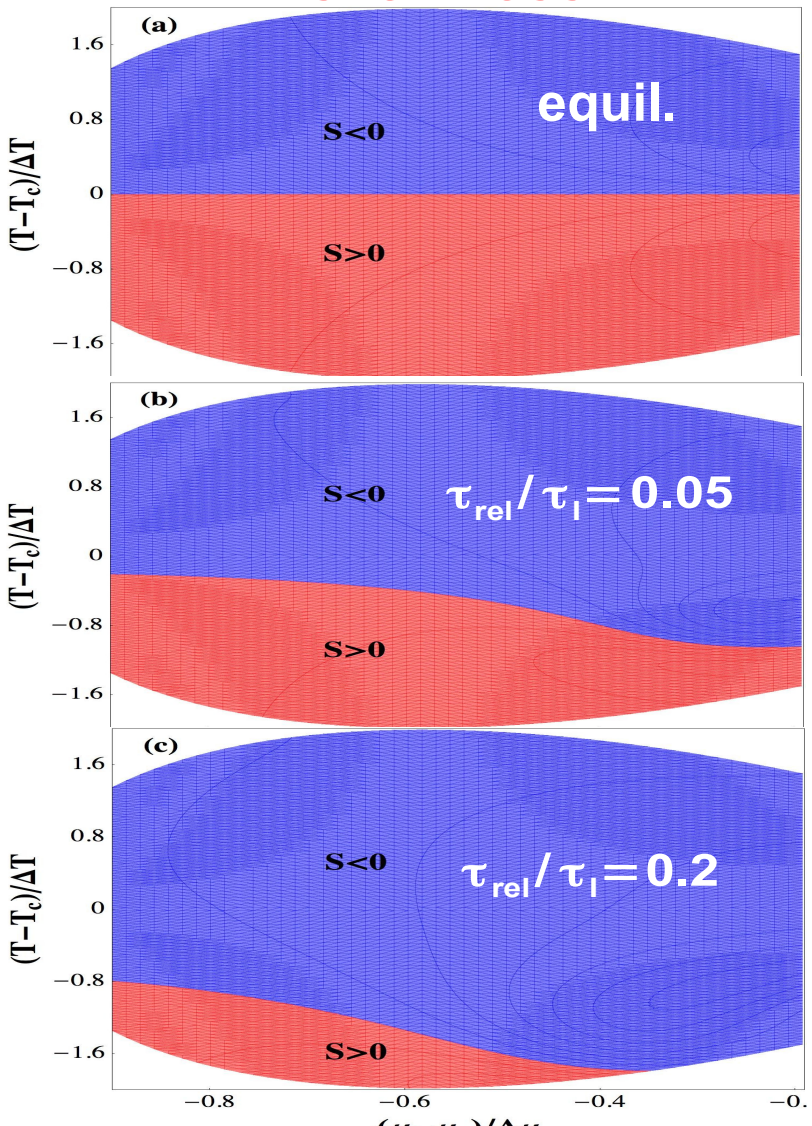
Non-Gaussian cumulants do not follow growth of the correlation length

unlike equilibrium expectation

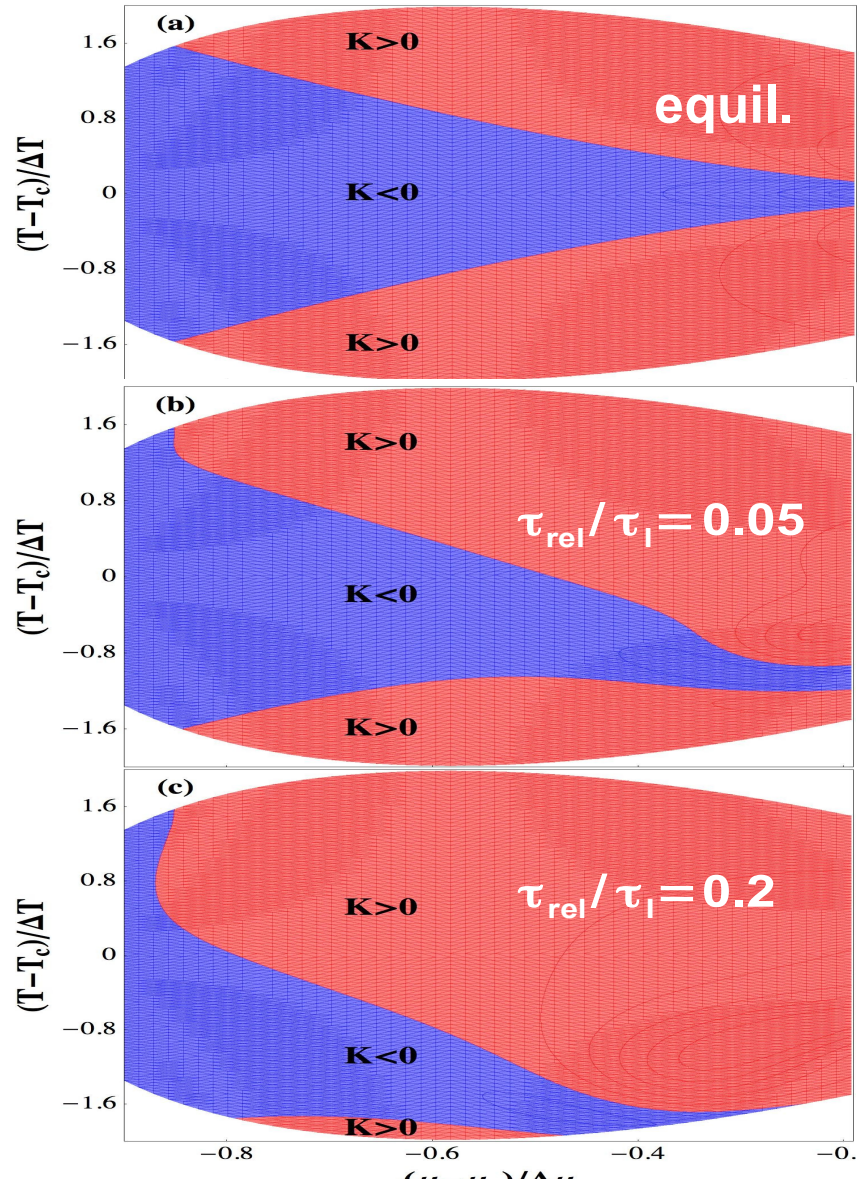
sign can be different from the equilibrium one

Signs of skewness & kurtosis

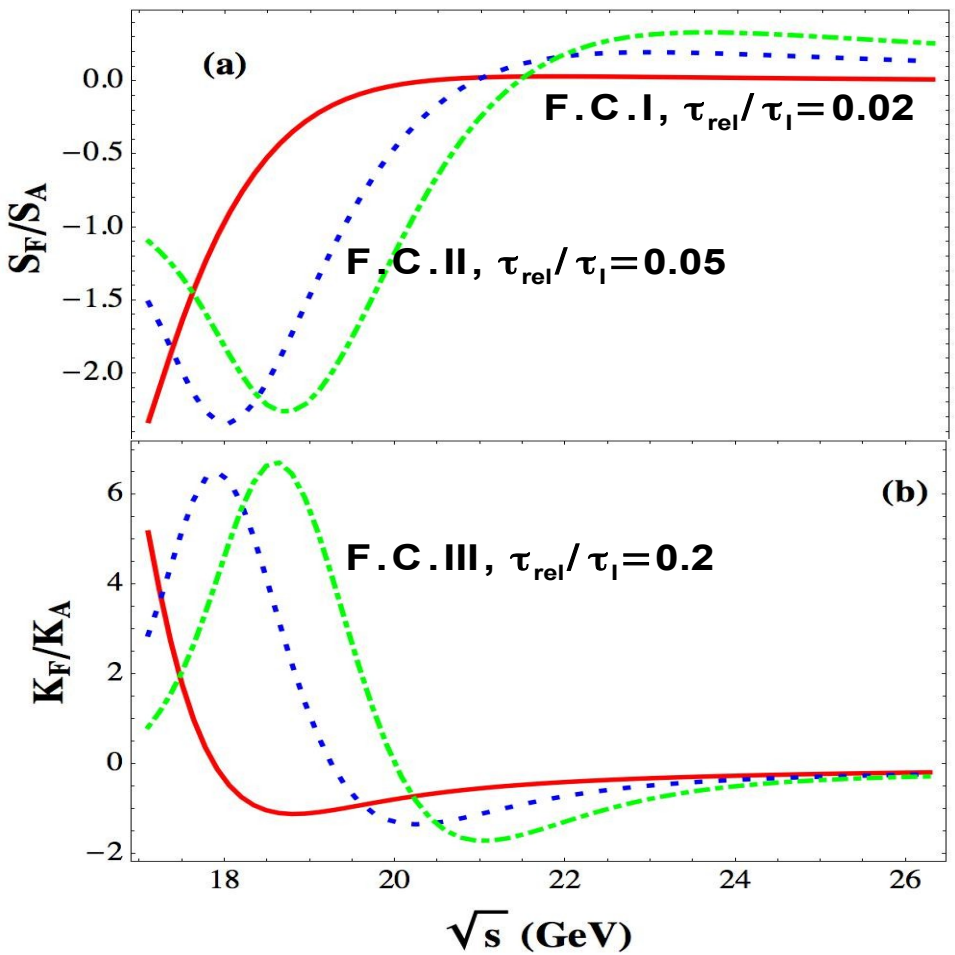
skewness

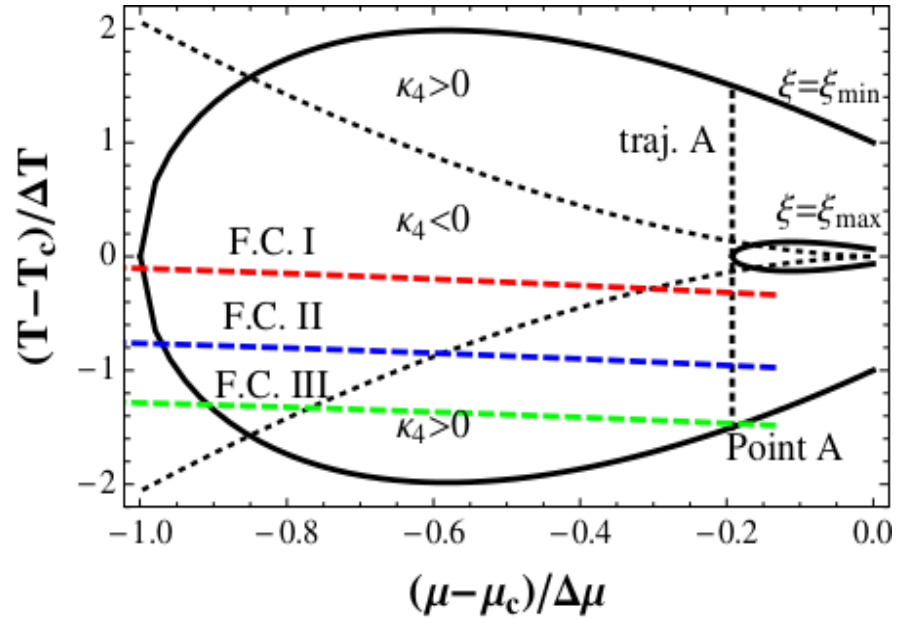


kurtosis



Cumulants along the freeze-out





depending on location of the freeze-out line and value of the relaxation time similar non-monotonic features as a function of $\sqrt{s}$

Summary

critical slowing down limits growth of correlation length

non-Gaussian cumulants do not follow growth of the correlation length

→ equilibrium scaling relations do not hold

sign of non-Gaussian cumulants can be different from equilibrium one