Real Time Evolution of Non-Gaussian Cumulants in the QCD Critical Regime

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based on: arXiv:1506.00645

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Why non-equilibrium?

freeze-out?

non-equilibrium memory effects are required to preserve remnant of critical signatures unless accidental freeze-out near critical point

Stephanov: arXiv:1104.1627
Simple non-equilibrium evolution of cumulants

Ansatz for evolution of correlation length:

$$\frac{\partial \tau}{\partial \tau} \xi^{-1} = -\tau_{\text{eff}}^{-1} \left[ \xi^{-1} - \xi_{\text{eq}}^{-1} \right]$$

with dynamical universality: \( \tau_{\text{eff}} \sim \xi^z \)

rely on equilibrium scaling for non-Gaussian cumulants:

$$k_3^{eq} \sim \xi_{eq}^{9/2}$$
$$k_4^{eq} \sim \xi_{eq}^7$$

➢ do such scalings really hold out of equilibrium?
➢ signs of non-Gaussian cumulants out of equilibrium?
Effective action & scope

free energy density: \( \Omega_0(\sigma) = \frac{1}{2} m_\sigma^2 (\sigma - \sigma_0)^2 + \frac{\lambda_3}{3} (\sigma - \sigma_0)^3 + \frac{\lambda_4}{4} (\sigma - \sigma_0)^4 \)

equilibrium distribution: \( P_0(\sigma) \sim \exp(-V \Omega_0(\sigma)/T) \)

\( \sigma \): critical mode (zero-momentum)
→ linear combination of chiral condensate and baryon current

limit to: scaling regime, but not at the critical point \( L_{\text{micr}} < \xi < L \)

\( \epsilon = \sqrt{\xi^3/V} \ll 1 \)

mass term \( \sim \sigma^2/\xi_{\text{eq}}^2 \)
kinetic term (momentum dependence) \( \sim \sigma^2/L^2 \) \( \leftarrow \) neglected

\( m_\sigma^{-1} \equiv \xi_{\text{eq}} \)
Cumulants

\( M(\tau) = \langle \sigma \rangle \)
\( \kappa_2(\tau) = \langle (\delta \sigma)^2 \rangle \)
\( \kappa_3(\tau) = \langle (\delta \sigma)^3 \rangle \)
\( \kappa_4(\tau) = \langle (\delta \sigma)^4 \rangle - 3 \kappa_2^2(\tau) \)

\( \delta \sigma = \sigma - M(\tau) \)
\( \langle \cdots \rangle = \int d\sigma (\cdots) P(\sigma; \tau) / \int d\sigma P(\sigma; \tau) \)

\( M^{eq} = \sigma_0 \)
\( \kappa_2^{eq} = \frac{\xi_{eq}^2}{V_4} \)
\( \kappa_3^{eq} = -\frac{2\lambda_3}{V_4^2} \xi_{eq}^6 \)
\( \kappa_4^{eq} = -\frac{6}{V_4^3} \left[ 2(\lambda_3 \xi_{eq})^2 - \lambda_4 \right] \)

\( m_0 = \xi_{eq}^{-1} \)
\( \sigma = \tilde{\sigma}_0 T (T \xi_{eq})^{-1/2} \)
\( \lambda_3 = \tilde{\lambda}_3 T (T \xi_{eq})^{-3/2} \)
\( \lambda_4 = \tilde{\lambda}_4 (T \xi_{eq})^{-1} \)

\( M^{eq} \sim \xi_{eq}^{-1/2} \)
\( \kappa_2^{eq} \sim \xi_{eq}^2 \)
\( \kappa_3^{eq} \sim \xi_{eq}^{9/2} \)
\( \kappa_4^{eq} \sim \xi_{eq}^7 \)

Power counting in \( \epsilon \):
\( \frac{\epsilon M^{eq}}{b}, \frac{\kappa_2^{eq}}{b}, \frac{\kappa_3^{eq}}{\epsilon b^3}, \frac{\kappa_4^{eq}}{\epsilon^2 b^4} \sim O(1) \)

\( \epsilon = \sqrt{\frac{\xi_{eq}^3}{V}} \)
\( b = \sqrt{\kappa_2^{eq}} \)
\( V_4 = V / T \)

Assume this power counting holds for non-equilibrium evolution of cumulants in the critical regime and make systematic expansions in \( \epsilon \)
Real time evolution of cumulants

Langevin dynamics: soft critical mode receives small, random kicks from a bath of hard modes

Fokker-Planck equation:

$$\partial_\tau P(\sigma; \tau) = \frac{1}{m_\sigma^2 \tau_{\text{eff}}} \left[ \partial_\sigma \left[ \partial_\sigma \Omega_0(\sigma) + V_4^{-1} \partial_\sigma \right] P(\sigma; \tau) \right]$$

$$\tau_{\text{eff}} : \text{effective relaxation time of the critical modes}$$

$$\tau_{\text{eff}} \sim \xi^z$$

$$z : \text{dynamical critical exponent}$$

$$\partial_\tau \langle f(\sigma) \rangle = -\frac{1}{m_\sigma^2 \tau_{\text{eff}}} \left[ \langle f'(\sigma) \Omega_0'(\sigma) \rangle - V_4^{-1} \langle f''(\sigma) \rangle \right]$$

Time evolution of the cumulatns and systematic expansion in

$$\epsilon = \sqrt{\xi^3 / V}$$
Real time evolution of cumulants: coupled equations

closed set of coupled time evolution equations:

\[
\partial_\tau M = -\tau^{-1}_{\text{eff}} p_1(M) \left[ 1 + O(\epsilon) \right]
\]

\[
\partial_\tau \kappa_n = -n \tau^{-1}_{\text{eff}} p_n(M, \kappa_2, \cdots, \kappa_n) \left[ 1 + O(\epsilon) \right]
\]

\( \epsilon = \sqrt{\xi^3/V} \ll 1 \)

evolution of the higher cumulants couples only to lower ones

\( p_n \rightarrow \text{polynomials} \)
Real time evolution of cumulants: two limits

Gaussian limit: \[ \Omega_0(\sigma) = \frac{1}{2} m^2_o(\sigma - \sigma_0)^2 \quad \kappa_3^{eq} = \kappa_4^{eq} = 0 \]

\[ \partial_\tau \kappa_n = - n \tau^{-1}_{eff} \left[ \kappa_n - \kappa_n^{eq} \right] \]

evolutions decouple, non-Gaussian cumulants are damped, higher cumulants damps faster

for \( n=2 \) reduces to the old Berdnikov-Rajagopal Ansatz:

near-equilibrium limit: \[ \delta \kappa_n = \kappa_n - \kappa_n^{eq} \quad \sim \text{small, linearize} \]

\[ \partial_\tau \kappa_2 = - 2 \tau^{-1}_{eff} a_2 \delta \kappa_2 \]
\[ \partial_\tau \kappa_3 = - 3 \tau^{-1}_{eff} [ a_2 \delta \kappa_2 + a_3 \delta \kappa_3 ] \]
\[ \partial_\tau \kappa_4 = - 4 \tau^{-1}_{eff} [ a_2 \delta \kappa_2 + a_3 \delta \kappa_3 + a_4 \delta \kappa_4 ] \]

lower cumulatns relax back to equilibrium first
important feature of complete set of coupled equation
Time evolution: emulating heavy-ion collisions

universal parameters:

3-d Ising universality:

\( \sigma_0(r, h), m_0(r, h), \lambda_3(r, h), \lambda_4(r, h) \)

choose critical region in r-h plane through: \( \xi_{\text{max}} / \xi_{\text{min}} = 3 \)

model-H dynamical universality:

\[
\tau_{\text{eff}} = \tau_{\text{rel}} \left( \frac{\xi_{\text{eq}}}{\xi_{\text{min}}} \right)^z, \quad z = 3
\]

\( \tau_{\text{rel}} \): relaxation time at outside edge of critical region

r: reduced temperature
h: magnetic field

Time evolution: emulating heavy-ion collisions
map to QCD phase diagram: \((r, h) \rightarrow (T, \mu)\)

naïve choice:
\[
\frac{T - T_c}{\Delta T} = \frac{h}{\Delta h} \quad \frac{\mu - \mu_c}{\Delta \mu} = -\frac{r}{\Delta r}
\]

trajectories: 3-d Hubble-like expansion
\[
\frac{T(\tau)}{T_1} = \left(\frac{\tau}{\tau_1}\right)^3 c_s^2 \quad \frac{V(\tau)}{V_1} = \left(\frac{\tau}{\tau_1}\right)^3
\]

entropy \(\sim\) constant
speed of sound: \(c_s^2 = 0.15\)

free parameter:
\[
\tau_{rel}/\tau_1
\]
\(\tau_{rel}\) : relaxation time for the critical mode
at the boundary of critical region
\(\tau_1\) : time when system enters critical region

ballpark guess: \(\tau_{rel}=1\ \text{fm}, \ \tau_1=10\ \text{fm}\)
Cumulants: as time goes by ...

\[ \tau_{\text{rel}} / \tau_I = 0.005 \]

\[ \tau_{\text{rel}} / \tau_I = 0.02 \]

\[ \tau_{\text{rel}} / \tau_I = 0.05 \]

\[ \tau_{\text{rel}} / \tau_I = 0.2 \]

critical slowing down limits

growth of correlation length

similar to Berdnikov-Rajagopal

Non-Gaussian cumulants

do not follow growth

of the correlation length

unlike equilibrium expectation

sign can be different from

the equilibrium one
Signs of skewness & kurtosis

\[ \frac{\tau_{\text{rel}}}{\tau_I} = 0.05 \]

\[ \frac{\tau_{\text{rel}}}{\tau_I} = 0.2 \]
Cumulants along the freeze-out

Depending on location of the freeze-out line and value of the relaxation time similar non-monotonic features as a function of $\sqrt{s}$.
Summary

Critical slowing down limits growth of correlation length.

Non-Gaussian cumulants do not follow growth of the correlation length → equilibrium scaling relations do not hold.

Sign of non-Gaussian cumulants can be different from equilibrium one.