QUARK MATTER 2015

The XXVth International Conference on Ultrarelativistic Nucleus-Nucleus Collisions

Critical point search from an extended parameter space of lattice QCD at finite temperature and density

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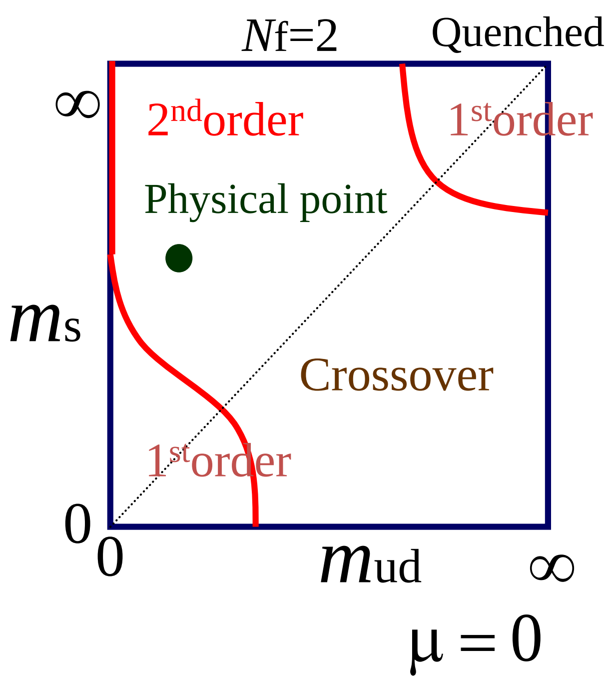
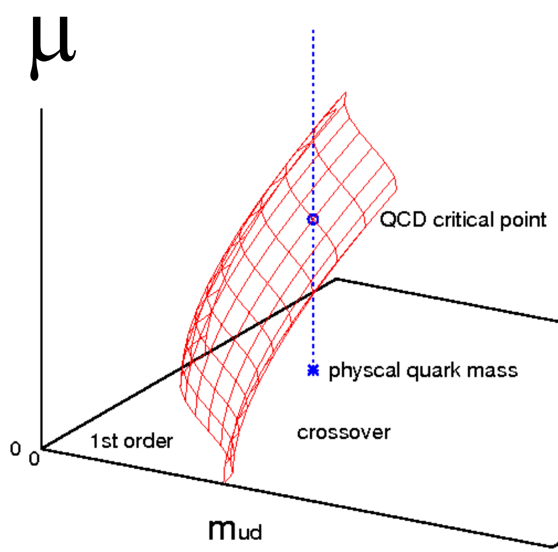
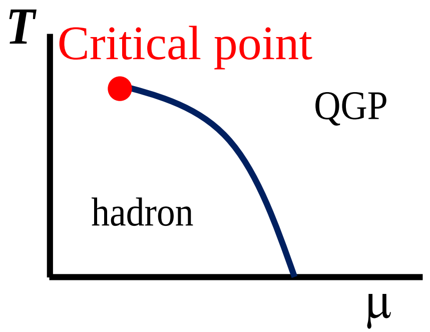
Quark Matter 2015, Kobe, Japan, Sep. 27-Oct. 3, 2015

Quark Mass dependence of QCD phase trantion





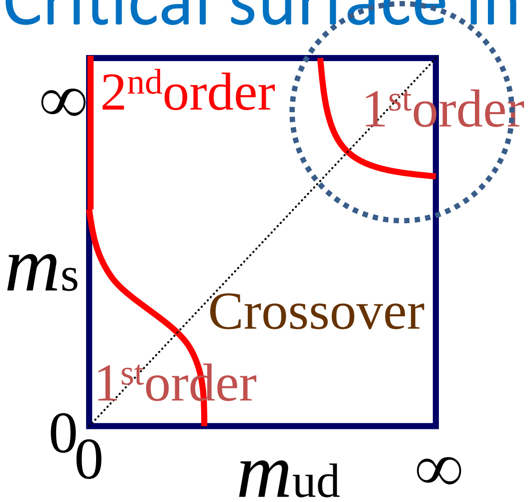


Bielefeld-Swansea Collab., 02, 03

P.de Forcrand & O.Philipsen, 03, 07

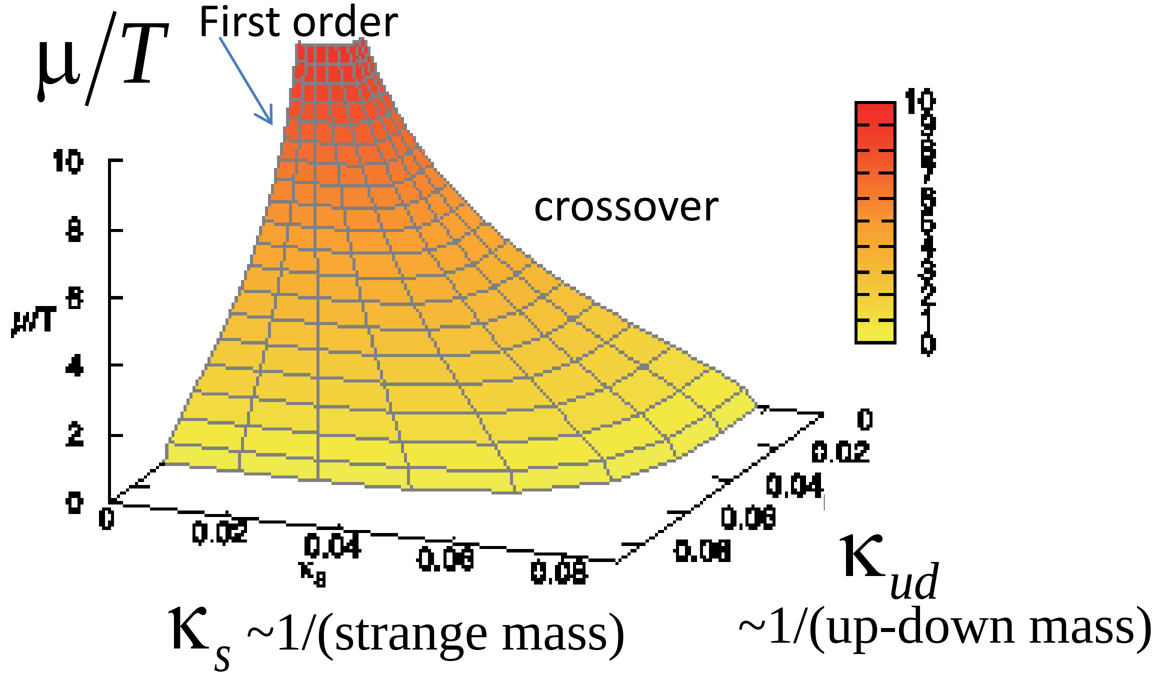
- On the line of physical mass, the crossover at low density ⇨ 1st order transition at high density.
- It is difficult to study the high density region due to the sign problem, but we may find the critical point from the information at low density.

Critical surface in the heavy quark region of (2+1)-flavor QCD



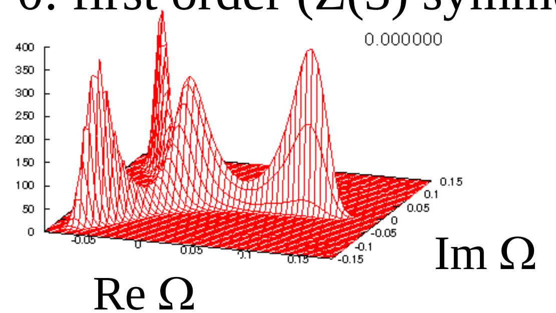
WHOT-QCD Collab., Phys.Rev.D89, 034507(2014)

Critical surface at finite density

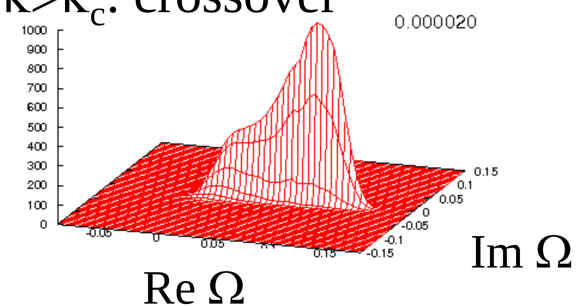


Polyakov loop distribution

κ=0: first order (Z(3) symmetric)



κ>κ_c: crossover

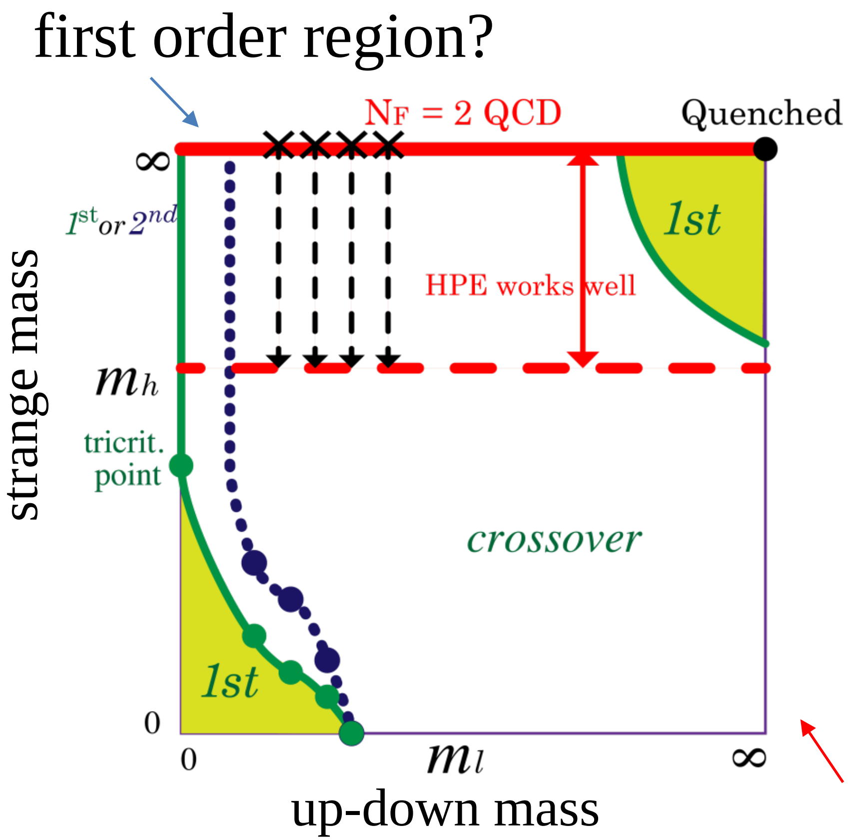


The critical (κ_{ud}, κ_s) satisfies $(N_t = 4)$

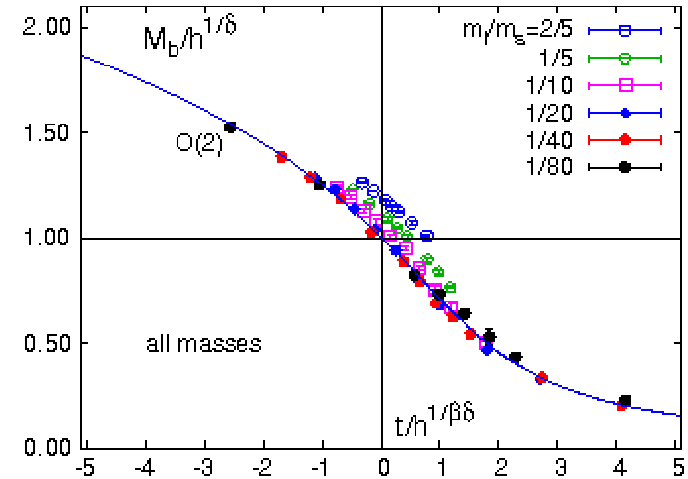
$$2\kappa_{\mathrm{ud\,cp}}^{N_t} \cosh(\mu_{\mathrm{ud}}/T) + \kappa_{\mathrm{s\,cp}}^{N_t} \cosh(\mu_{\mathrm{s}}/T) \approx 4\times 10^{-5}$$

Hopping parameter (κ) expansion was a key point in this analysis.

(2+1)-flavor QCD by Hopping parameter exp.



Scaling plot of the order parameter
BNL-Bielefeld, PRD80,094505(2009)



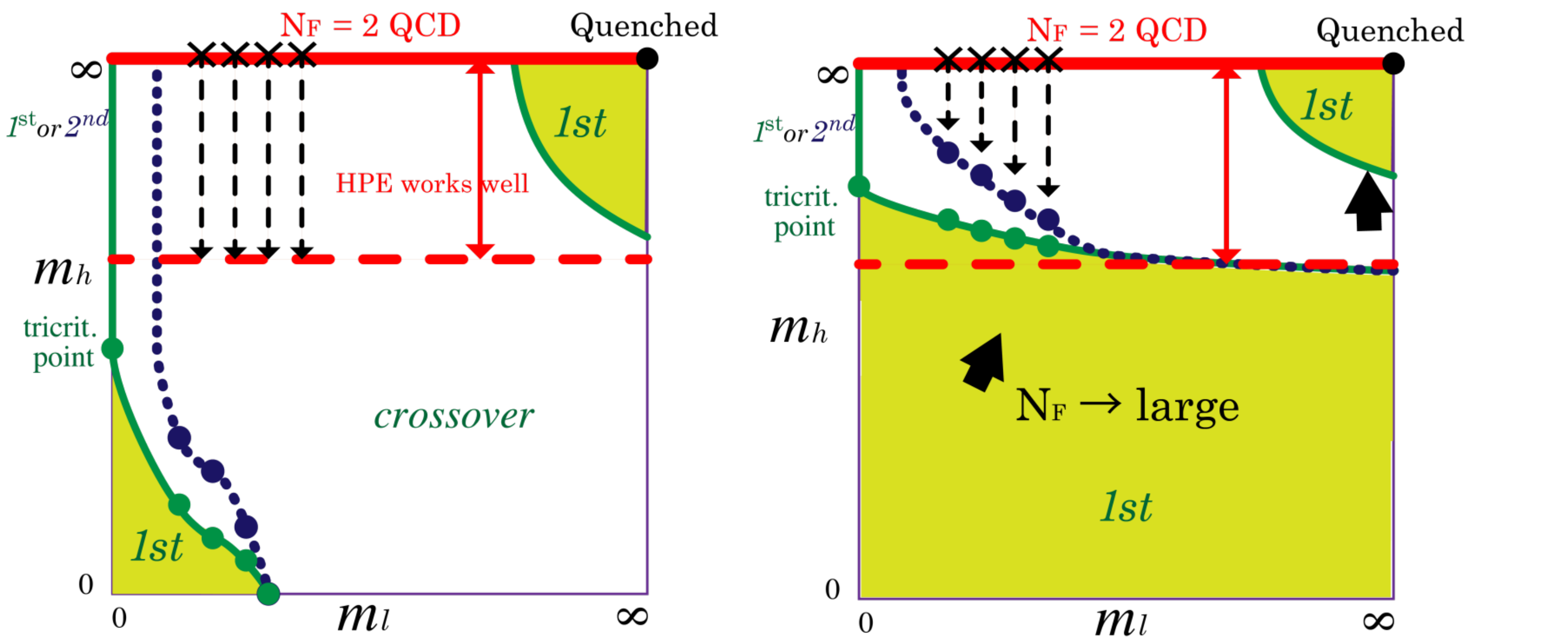
This gives an alternative way to study the massless 2-flavor QCD

HPE is applicable when the strange quark is very heavy.

1. Critical surface in the heavy quark region
2. Nature of chiral transition of massless 2-flavor QCD (m_s=∞)

   1^{st} order or 2^{nd} order? (Long standing problem!)

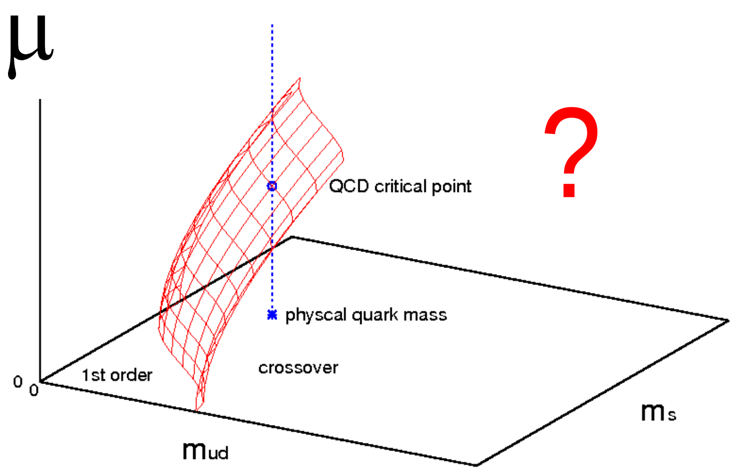
(2+1)-flavor QCD ⟹ (2+Many)- flavor QCD

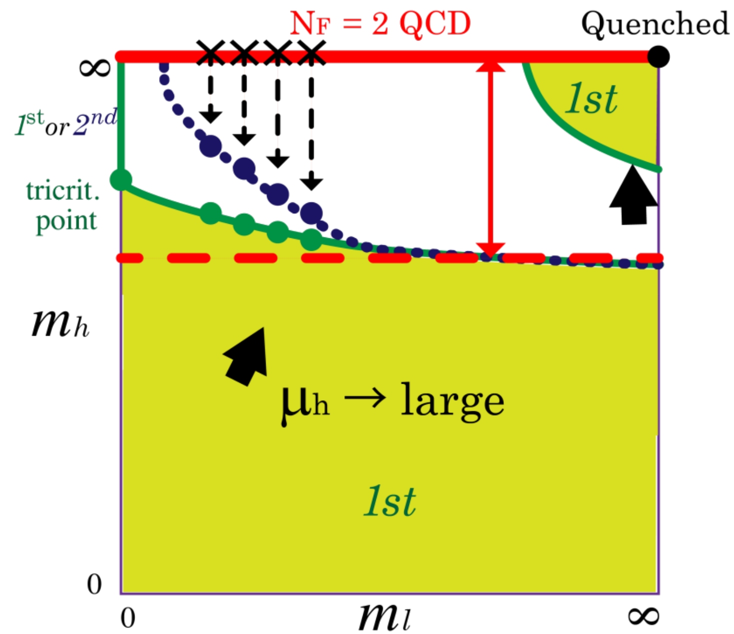


- However, the critical surface in the light quark region is difficult to access at present.
- We study many-flavor QCD with
  2 light quarks and N_f heavy quarks.
- It is known that 1st order region is wider as increasing N_f
  S. Ejiri, N. Yamada, Phys. Rev. Lett. 110, 172001 (2013)

⟹ Easy to find the boundary of 1st order transition region.

At large μ of strange quark





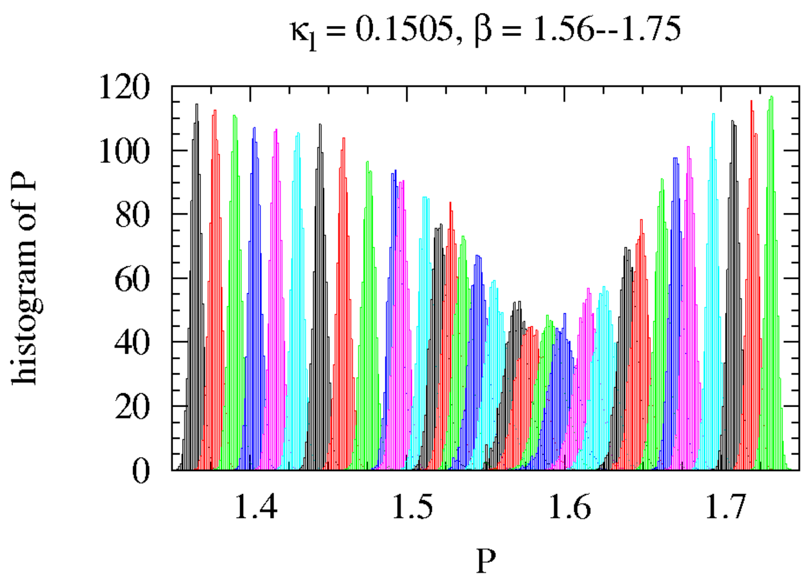
- As we will show, κ_c decreases exponentially as μ_h increases.
- The sign problem of heavy quark determinant is not serious.
  - The strange quark μ is well controlled.
- Then, the hopping parameter expansion works well even for (2+1)-flavor QCD.

Phase structure of (2+many)-flavor QCD using Wilson quark action

- Measuring plaqette distribution function (Histogram)
- Light quark mass dependence
- Chemical potential dependence

Simulations

Iwasaki gauge action +
2-flavor clover -Wilson fermion action,
κ=0.145, 0.1475, 0.150, 0.1505,
m_π/m_ρ = 0.6647, 0.5761, 0.4677, 0.4575,
16^3x4 lattice.



Dynamical heavy quark effect is added by the reweighting method.
det*M*: Hopping parameter expansion

Reweighting method for plaquette distribution function

$$W(P,\beta,m,\mu)\equiv\int DU\delta\left(\hat{P}-P\right)\prod_{f=1}^{N_f}\det M\left(m_f,\mu_f\right)e^{6N_{\rm site}\beta\hat{P}}$$

$$S_g=-6N_{\rm site}\beta\hat{P}$$

$$\left(\beta=6/g^2\right)$$

plaquette P (1x1 Wilson loop for the standard action) $\left(\sim F_{\mu\nu}F^{\mu\nu}\right)$

$$R(P,\beta,\beta_0 m,m_0,\mu)\equiv W(P,\beta,m,\mu)/W(P,\beta_0,m_0,0)$$

(Reweight factor)

$$R(P)=\frac{\left\langle\delta\left(\hat{P}-P\right)e^{6N_{\rm site}(\beta-\beta_0)\hat{P}}\prod_f\frac{\det M\left(m_f,\mu_f\right)}{\det M\left(m_0,0\right)}\right\rangle_{(\beta_0,\mu=0)}}{\left\langle\delta\left(\hat{P}-P\right)\right\rangle_{(\beta_0,\mu=0)}}\equiv\left\langle e^{6N_{\rm site}(\beta-\beta_0)\hat{P}}\prod_f\frac{\det M\left(m_f,\mu_f\right)}{\det M\left(m_0,0\right)}\right\rangle_{P:\text{fixed}}$$

- We perform simulations at some κ_l and do not use reweighting method for κ_l (light quark mass).
- Taylor expansion of the light quark determinant in terms of μ_l
  - $O(\mu_l^2)$: 1st and 2nd terms are computed.
  - Valid at small μ_l/T
- Hopping Parameter(κ_h) Expansion (HPE) for heavy quark determinant.

Reweighting method -heavy quark part-

(Polyakov loop: $\Omega_R, +i\Omega_I$)

- Hopping Parameter Expansion (HPE) for heavy quark mass κ_h

$$N_f \ln\left(\frac{\det M(\kappa,\mu)}{\det M(0,0)}\right) = N_f\left(288 N_{\text{site}}\kappa^4 P + 12\cdot 2^{N_t} N_s^3 \kappa^{N_t} \cosh\left(\frac{\mu}{T}\right)\left(\Omega_R + i\tanh\left(\frac{\mu}{T}\right)\Omega_I\right) + \cdots\right)$$

Complex phase

$$\left(\beta^* = \beta + 48 N_f \kappa_h^4, \quad \bar{h} \equiv h\cosh(\mu_h/T) = 2N_f(2\kappa_h)^{Nt}\cosh(\mu_h/T), \quad \tanh(\mu_h/T)\right)$$

- In the leading order of HPE, there are 3 terms: (P, Ω_R, Ω_I)
- This factor is controlled by only 3 parameters $\left(\beta^*, \bar{h}, \tanh(\mu_h/T)\right)$
- We adjust β^* at the transition temperature.
- Complex phase part is well controlled because $\tanh(\mu_h/T) \leq 1$

First order phase transition (double-peaked distribution)

Effective potential

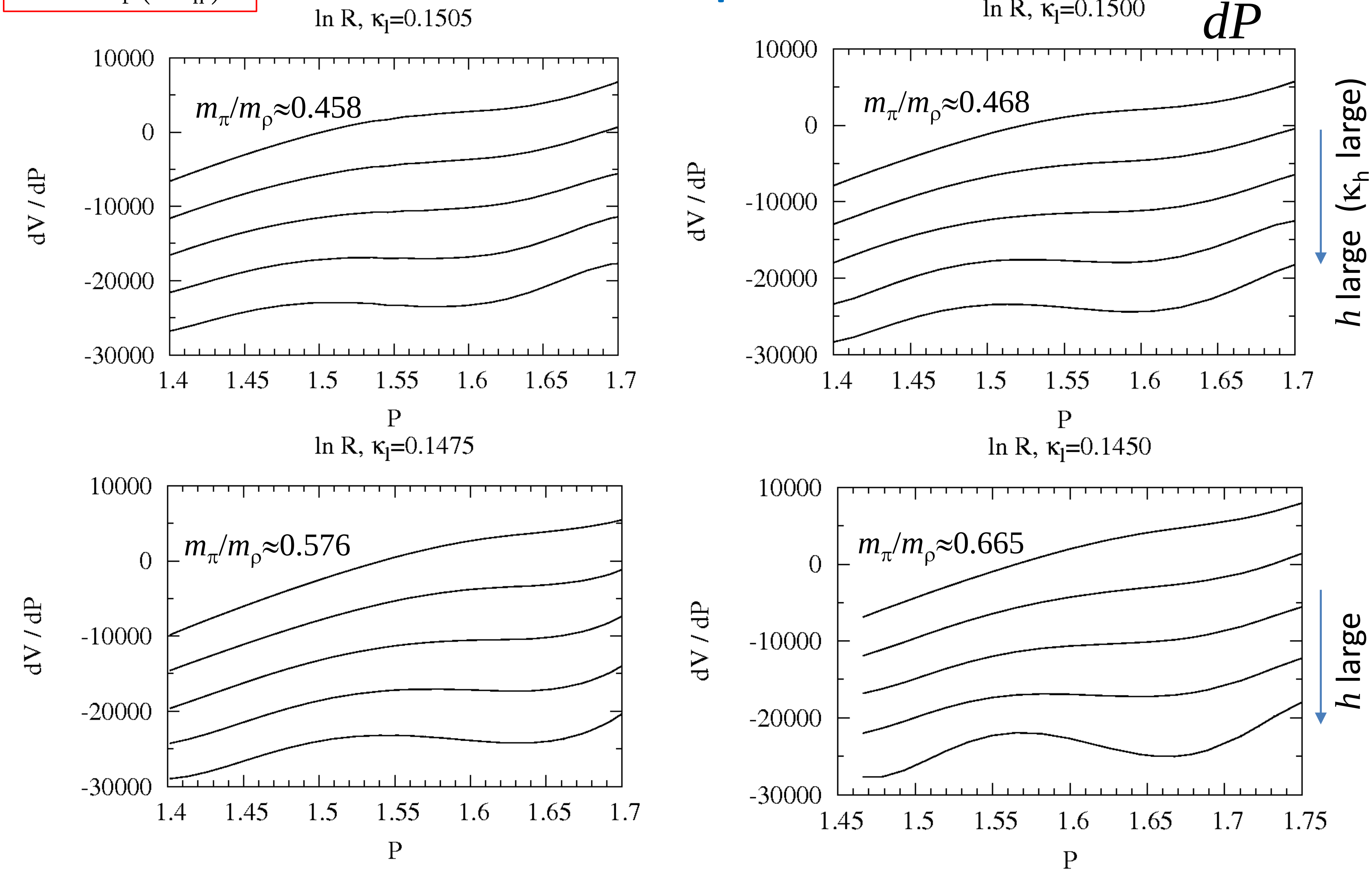
$$V_{\text{eff}}(P) = -\ln W(P)$$

S-shaped function

Negative curvature

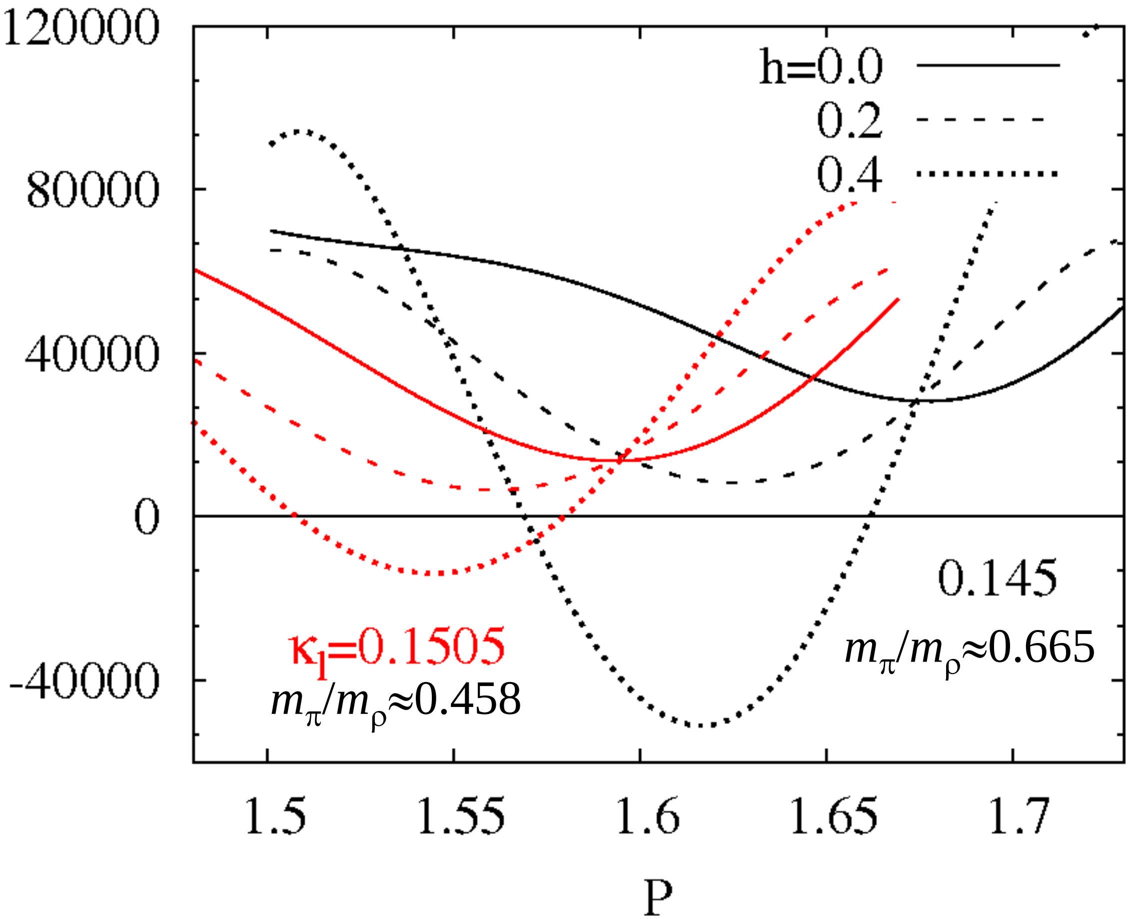
$h = 2N_f(2\kappa_h)^{N_t}$

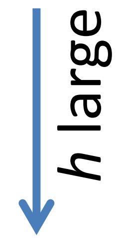
Quark mass dependence of $\dfrac{dV_{\mathrm{eff}}}{dP}$

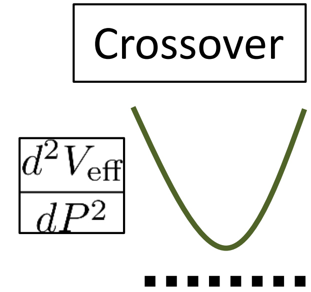


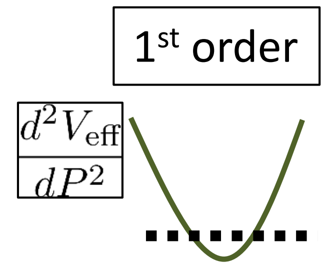
- The derivative of V_{eff} becomes an S-shaped function at large h (κ).
- Critical point: light quark mass dependence is small in this region.

h dependence of curvature $\frac{d^2V_{\mathrm{eff}}}{dP^2}$









$$h = 2N_{\mathrm{f}}\left(2\kappa_{\mathrm{h}}\right)^{N_t}$$

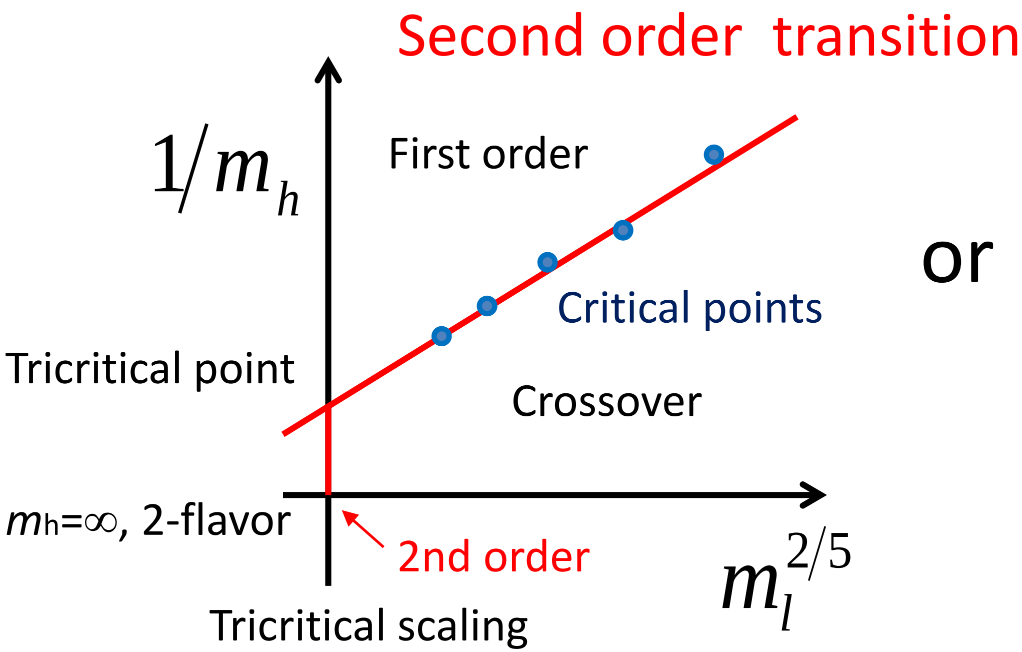
- If there is a region of P where d^2V/dP^2 is negative, phase transition is of 1st order
- d^2V/dP^2 become smaller as increasing h
- We determine h_c at which negative curvature appears

Nature of 2-flavor QCD in the chiral limit (μ=0)

2nd order or 1st order?

Light quark mass (m_l) dependence of the critical line

- Trictitical scaling behavior?
- Is there a first order transition region in 2-flavor QCD?



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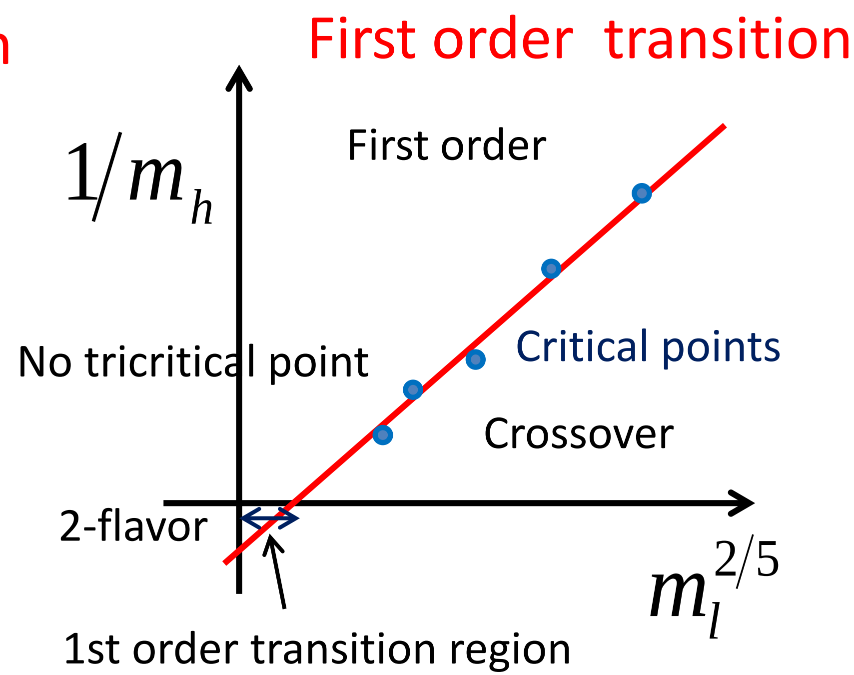
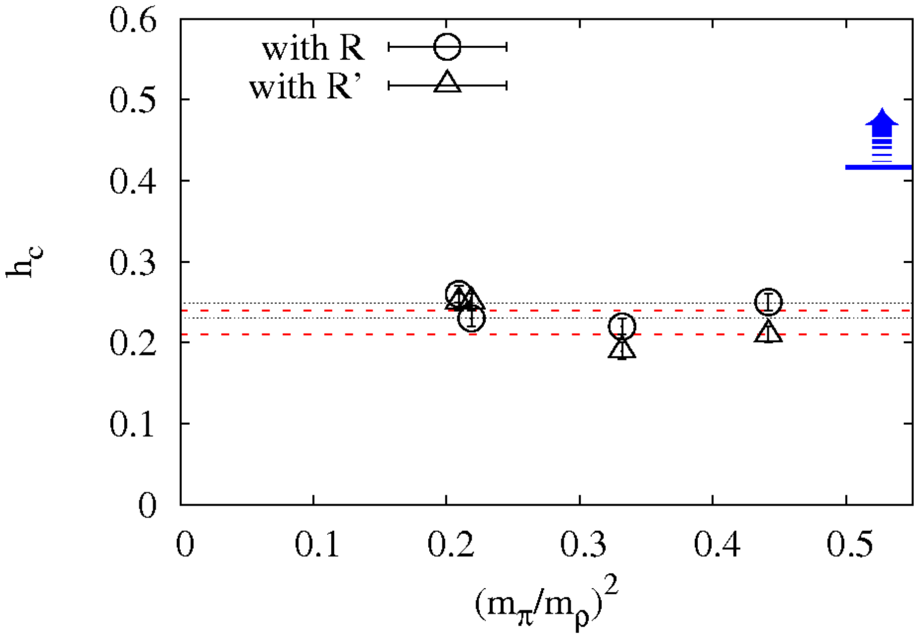


Similar study in QCD with an imaginary chemical potential:
Bonati, D' Elia, de Forcrand, Philipsen, Sanfilippo, arXiv:1311.0473; 1408.5086

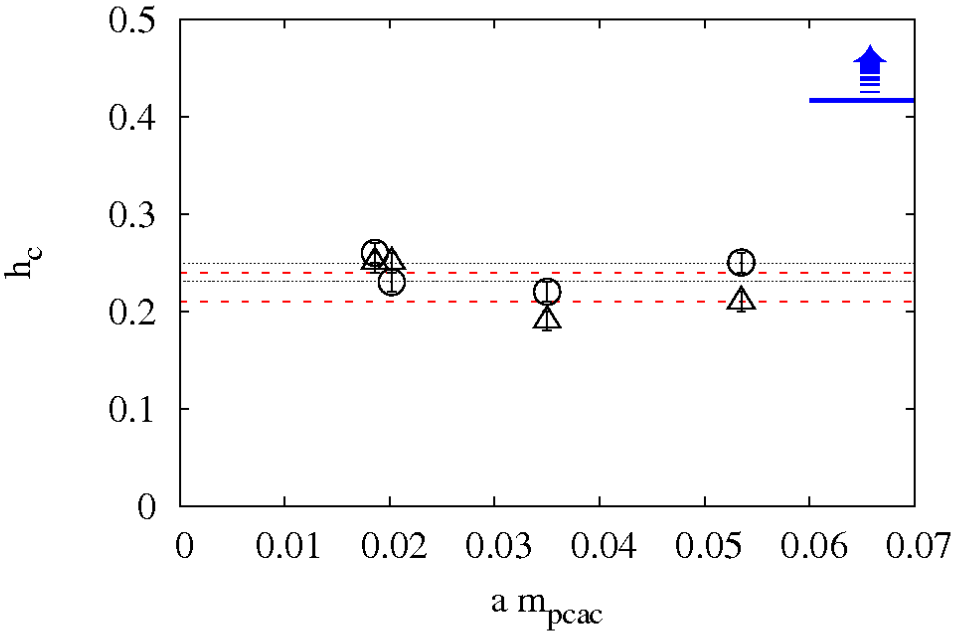
Light quark mass dependence at μ=0

$$h = 2N_f (2\kappa_h)^{N_t}$$ for Wilson quarks

m_p/m_r ratio dependence

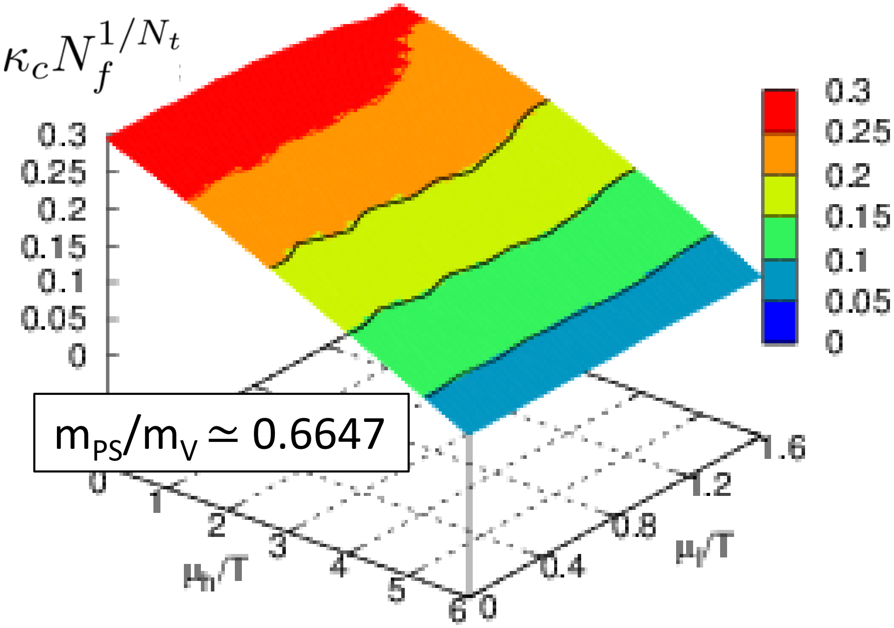


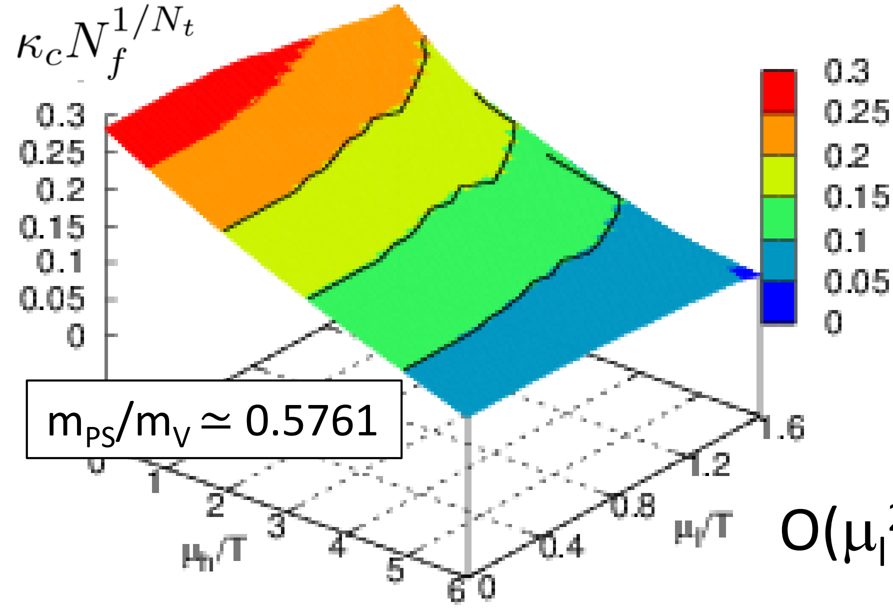
PCAC quark mass dependence



- Critical point: light quark mass dependence is small in the region we investigated.
- The blue lines are the critical point at $m_l = \infty$ (N_f=0+32).
- The first order transition in the massless 2-flavor QCD is not suggested.

Finite μ: (μ_l,μ_h) dependence of $\kappa_c N_f^{1/Nt}$





$O(\mu_l^2)$

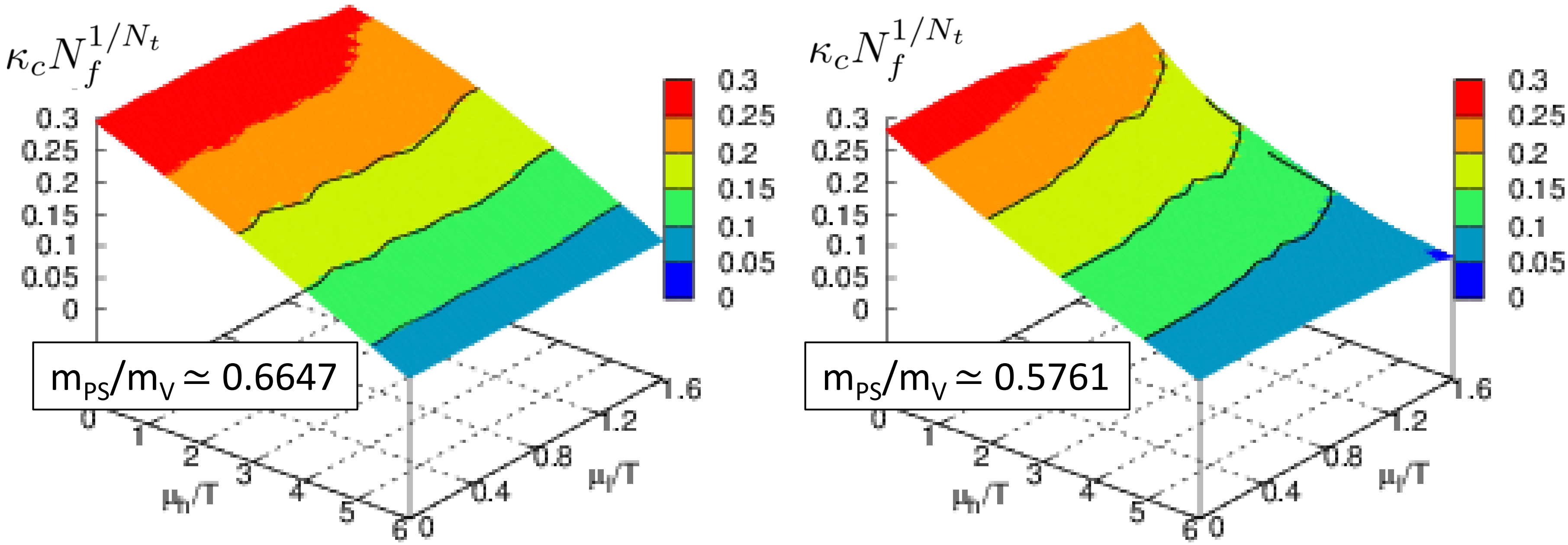
- The reweighting factor is controlled by $\left(\bar{h}, \tanh(\mu_h/T)\right)$
- The dependence of $\tanh(\mu_h/T)$ is only 30%, at most.
- κ_c exponentially decrease as increasing μ_h/T

$$\kappa_h^c = \frac{1}{2}\left(\frac{\bar{h}_c}{2N_f \cosh(\mu_h/T)}\right)^{1/Nt} \approx \frac{1}{2}\left(\frac{\bar{h}_c}{N_f}\right)^{1/Nt} e^{-\frac{\mu_h}{N_t T}}$$

(for large μ_h/T)

($\kappa_c N_f^{1/Nt}$ = 0.25, 0.20, 0.15, 0.10, 0.05, 0.00)

Application range of HPE



- For (2+1)-flavor QCD, the vertical axis is κ_c (N_f=1).
- If we assume HPE is applicable for $\kappa < 0.1$,
- The investigation of the critical surface by HPE is available in highly dense region

  ($\mu_h/T > 5.0$, **light blue** region)

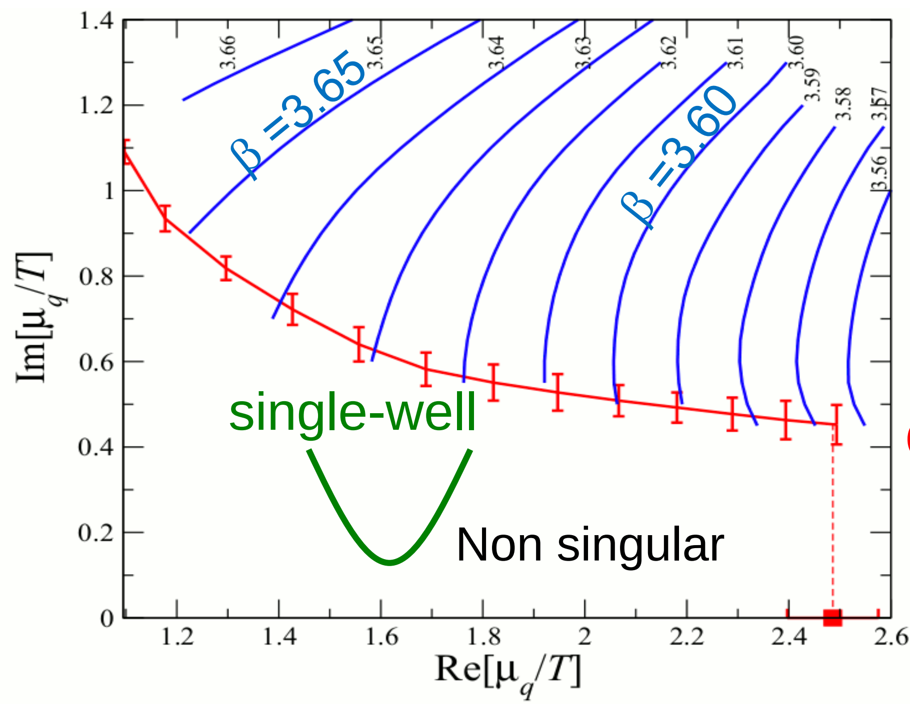
Summary

- We investigated the phase structure of (2+Nf)-flavor QCD.
- Applying the reweighting method, we determine the critical mass of heavy flavors terminating the first order region.
  - The critical mass becomes larger with N_f.
  - The light quark mass dependence the critical heavy quark mass is small in the region we investigated.
  - The first order transition in 2-flavor QCD is not suggested.
  - The first order region becomes wider as increasing μ_l and μ_h.
  - κ_c exponentially decrease as increasing μ_h/T.
    - At large μ_h/T, investigation of the critical surface by HPE is possible even for the case (2+1)-flavor QCD.
- This may be a good approach for the determination of boundary of the first order region in (2+1)-flavor QCD at finite density.

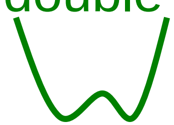
Outlook

- Beyond the hopping parameter expansion for small N_f.
- Complex μ: a double-well potential when Im μ is large.
  – Extrapolation from the complex μ plane to real axis?

Position of double-well potential for each β.



double-well



Lee-Yang zeros

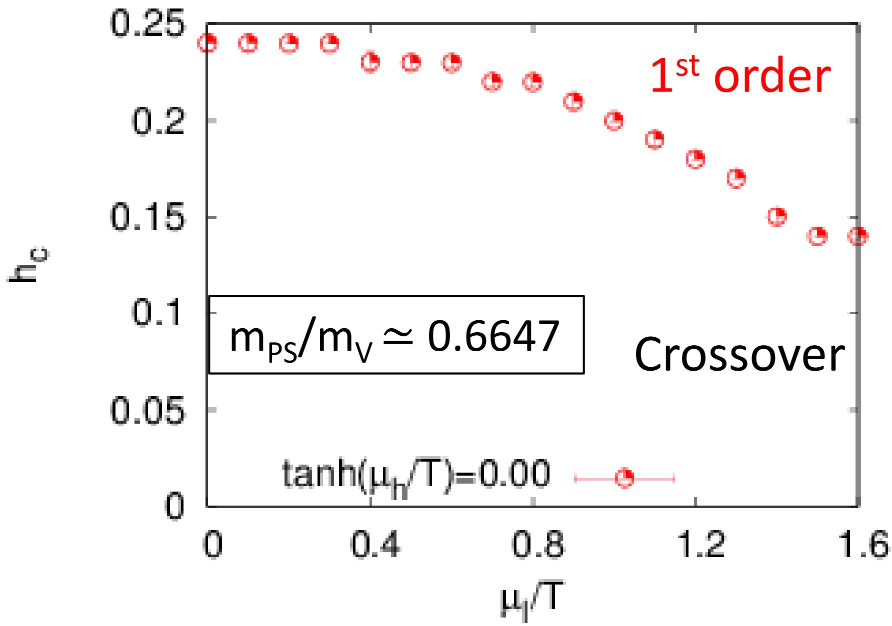
Critical line

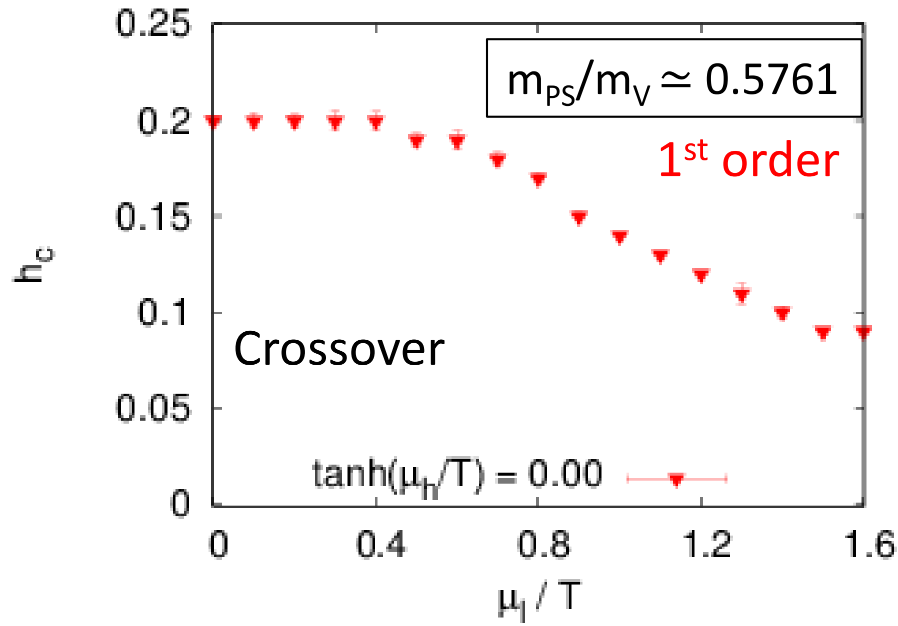
Complex μ ⟺ Distribution function W: complex

$|W(P)|$

N_f=2 p4-staggared, $m_\pi/m_\rho \approx 0.7$
data: Beilefeld-Swansea Collab., PRD71,054508(2005)

μ_l dependence of h_c at μ_h=0





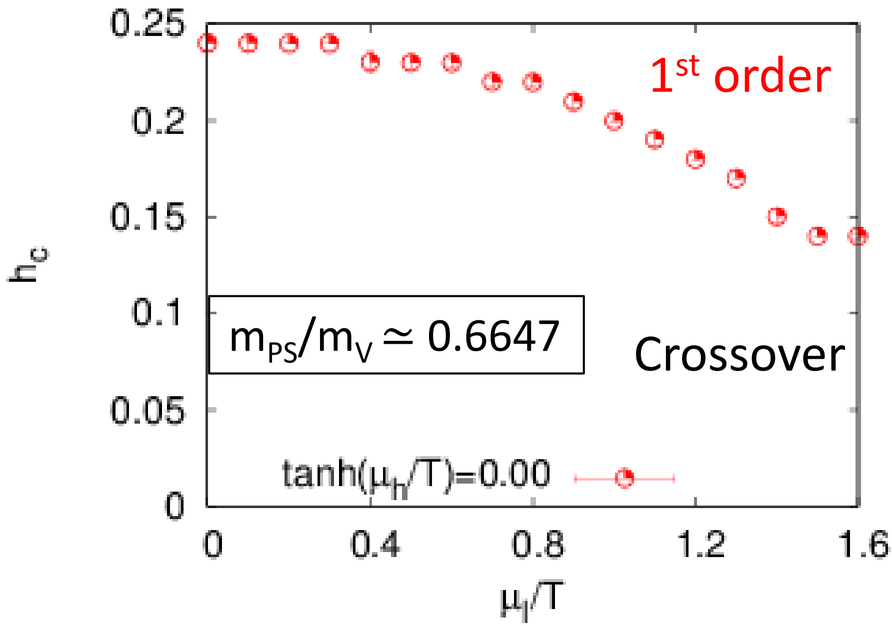
- h_c decrease as increasing μ_l/T
  - κ_c($\sim$1/m_c) decrease as increasing μ_l/T

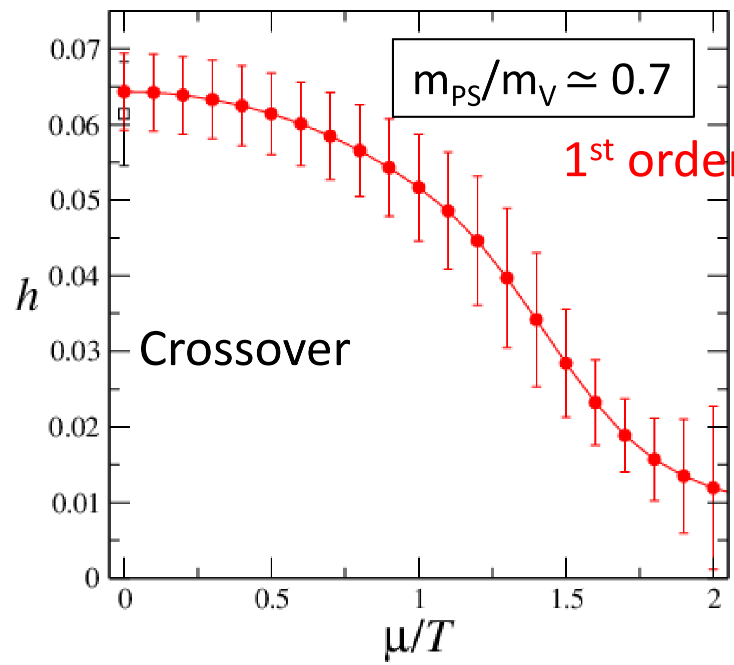
$$h = 2N_f(2\kappa_h)^{N_t}$$

$$\kappa_h^c = \frac{1}{2}\left(\frac{h_c}{2N_f}\right)^{1/N_t}$$

1st order region become wider as increasing μ_l/T

μ_l dependence of h_c at μ_h=0
～Wilson vs. Staggered～

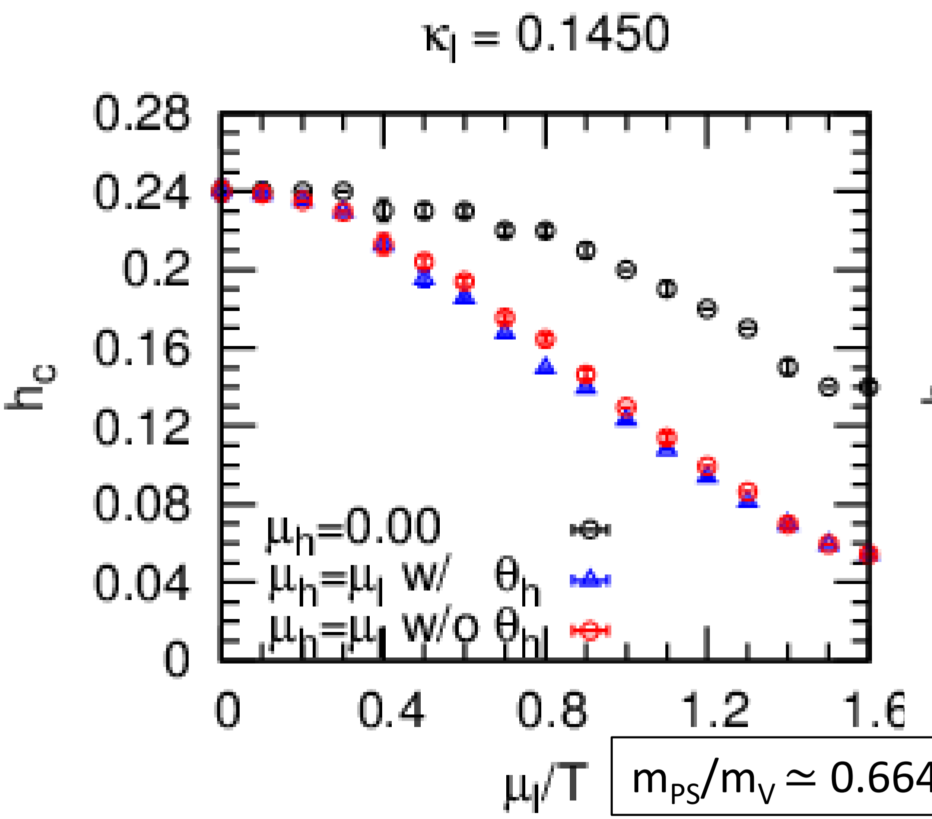




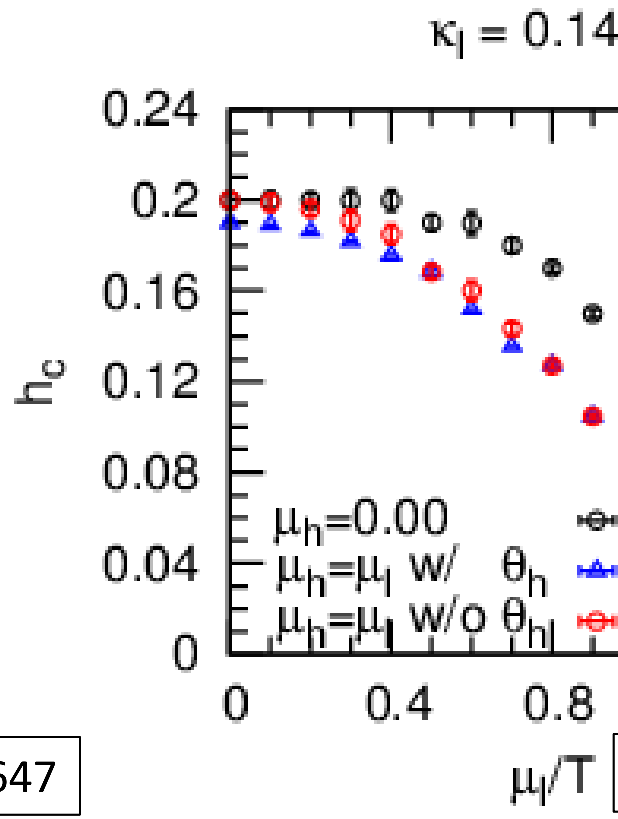
- We compare our results with a previous results obtained by the p4-improved staggered quarks.
- These values of h_c are different because the lattice spacing is coarse. (N_s^3x N_t = 16^3 x 4)

κ_c(～$1/m_c$) decrease as increasing μ_l/T in both results

μ_l dependence of h_c at $\mu_h = \mu_l$



$m_{PS}/m_V \simeq 0.6647$

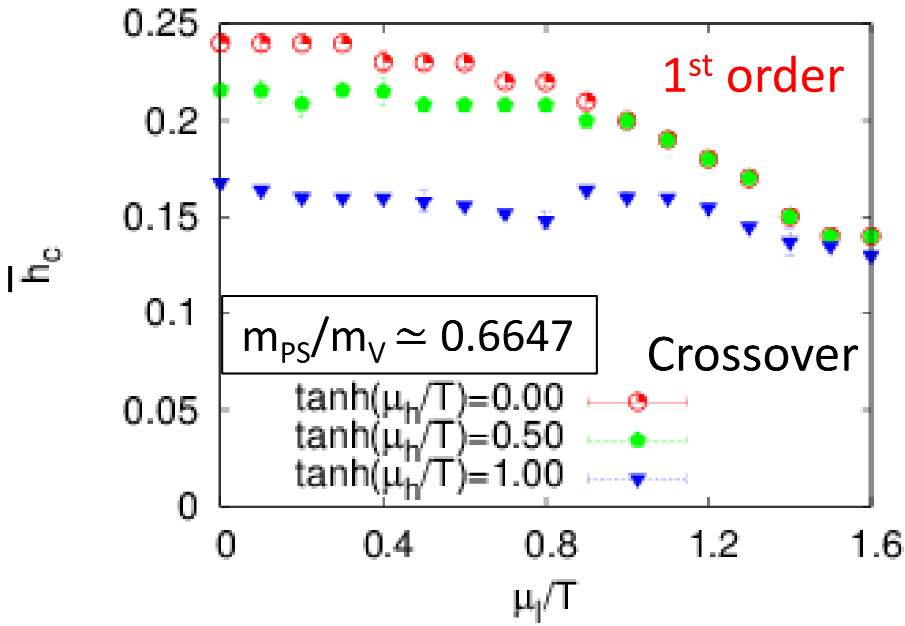


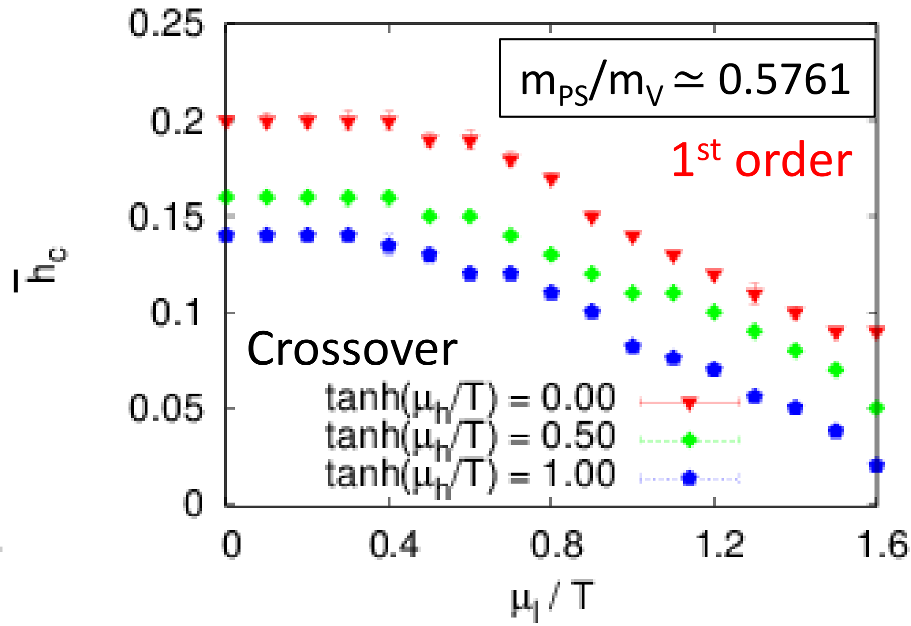
$m_{PS}/m_V \simeq 0.5761$

- h_c is well-approximated by $h_c = \dfrac{\overline{h}_c}{\cosh(\mu_h/T)}$
- Black: $\mu_h = 0$, Blue: $\mu_h = \mu_l$, Red: approximation.
- The reweighting factor is controlled by $\left(\overline{h}, \tanh(\mu_h/T)\right)$

μ_l dependence of $\bar{h}_c$ at $\mu_h \neq 0$

$$\bar{h} \equiv h\cosh(\mu_h/T) = 2N_f(2\kappa_h)^{N_t}\cosh(\mu_h/T)$$





- The reweighting factor is controlled by $(\bar{h}, \tanh(\mu_h/T))$
- $\bar{h}_c$ decrease as increasing (μ_l/T, μ_h/T)
- $\tanh(\mu_h/T)$ dependence is ONLY 30% at most.

($\tanh(\mu_h/T)$=0.0,0.5,1.0 ⇔ μ_h/T=0.00, 0.55, ∞)