LATTICE QCD: BULK AND TRANSPORT PROPERTIES OF QCD MATTER

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Lattice QCD

- Best first principle-tool to extract predictions for the theory of strong interactions in the non-perturbative regime

- Uncertainties:
  - Statistical: finite sample, error $\sim 1/\sqrt{\text{sample size}}$
  - Systematic: finite box size, unphysical quark masses

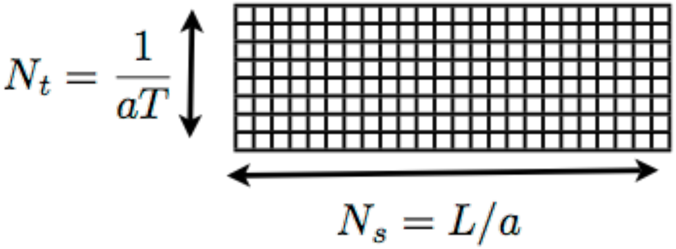
- Given enough computer power, uncertainties can be kept under control

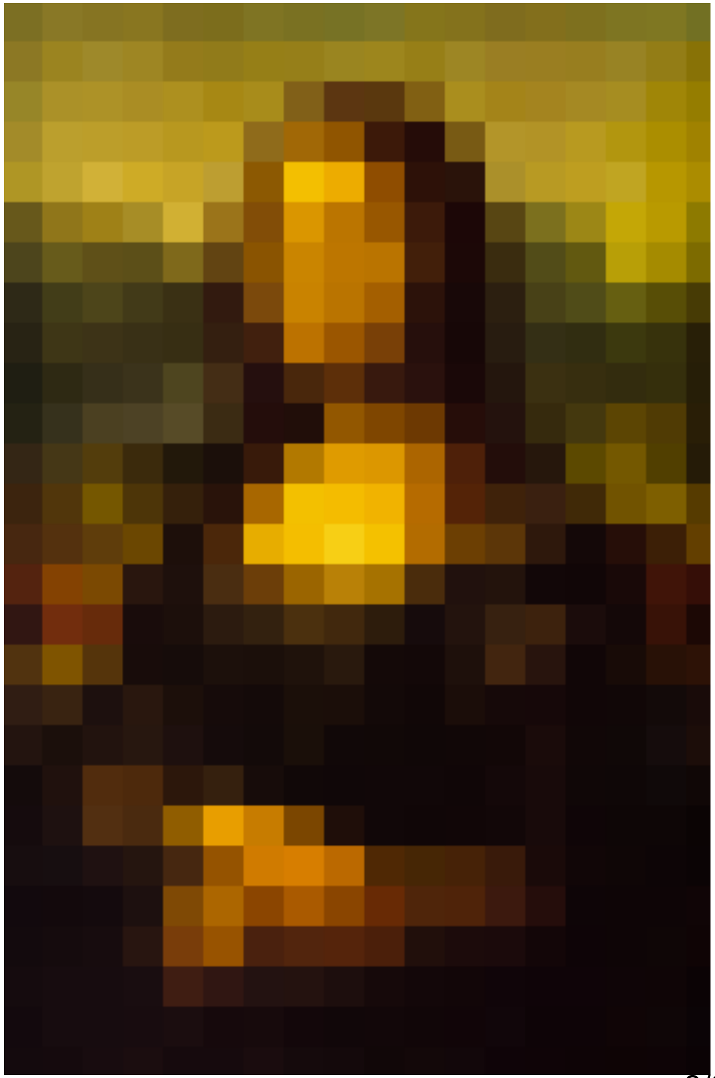
- Results from different groups, adopting different discretizations, converge to consistent results

- Unprecedented level of accuracy in lattice data

Importance of continuum limit

- Lattice action: parametrization used to discretize the Lagrangian of QCD on a space-time grid





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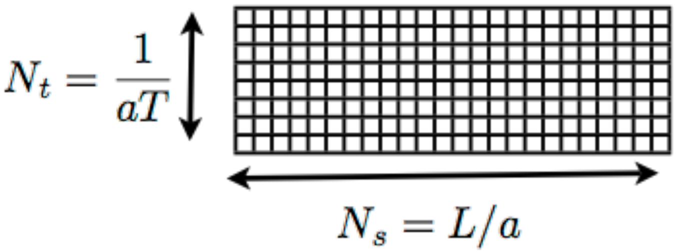
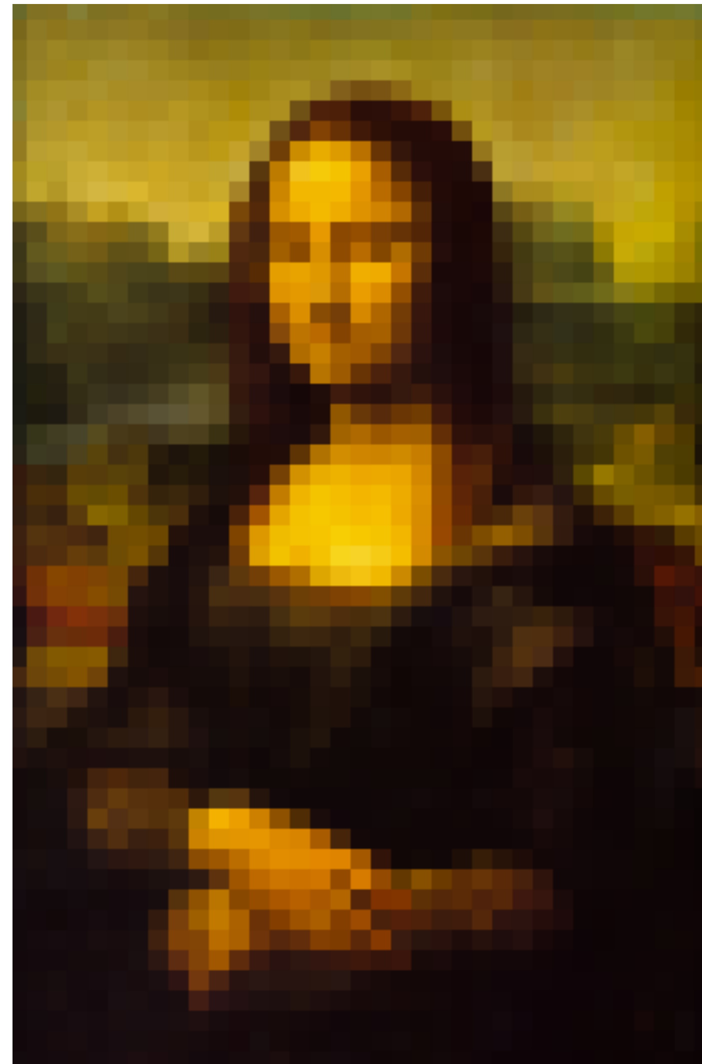


- Repeat the simulations on finer lattices (smaller a ←→ larger N_t)



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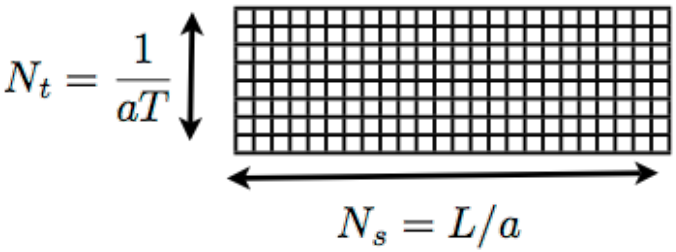

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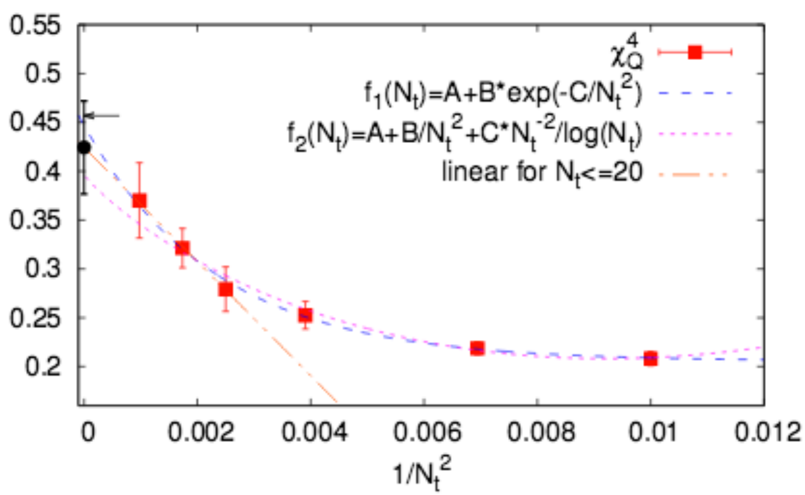

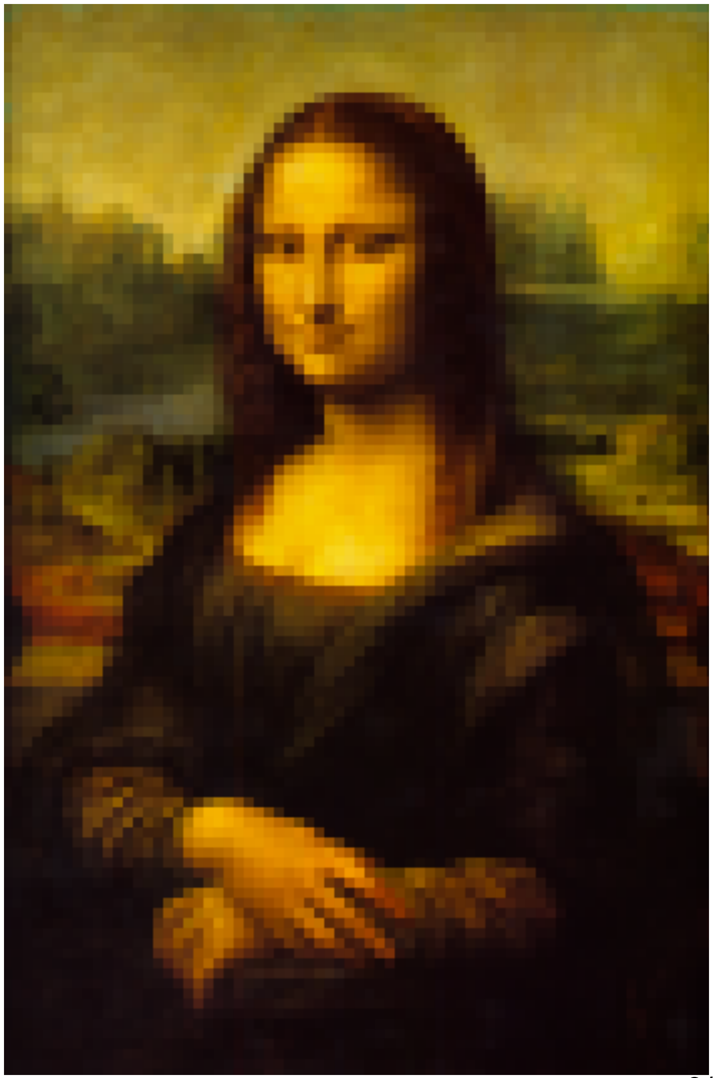

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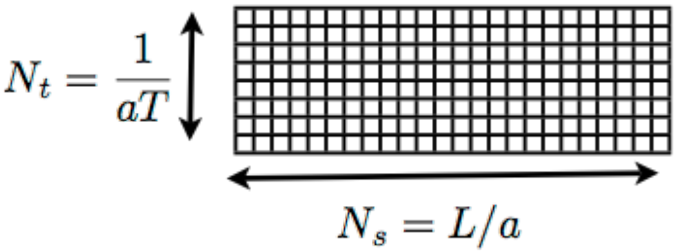

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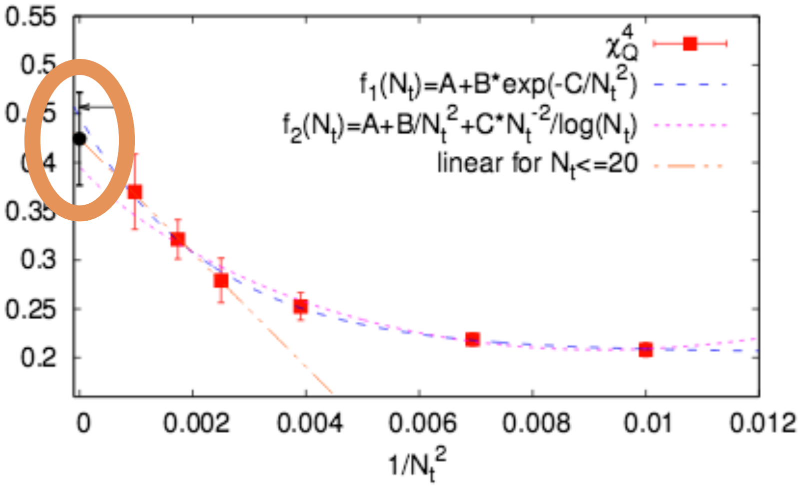
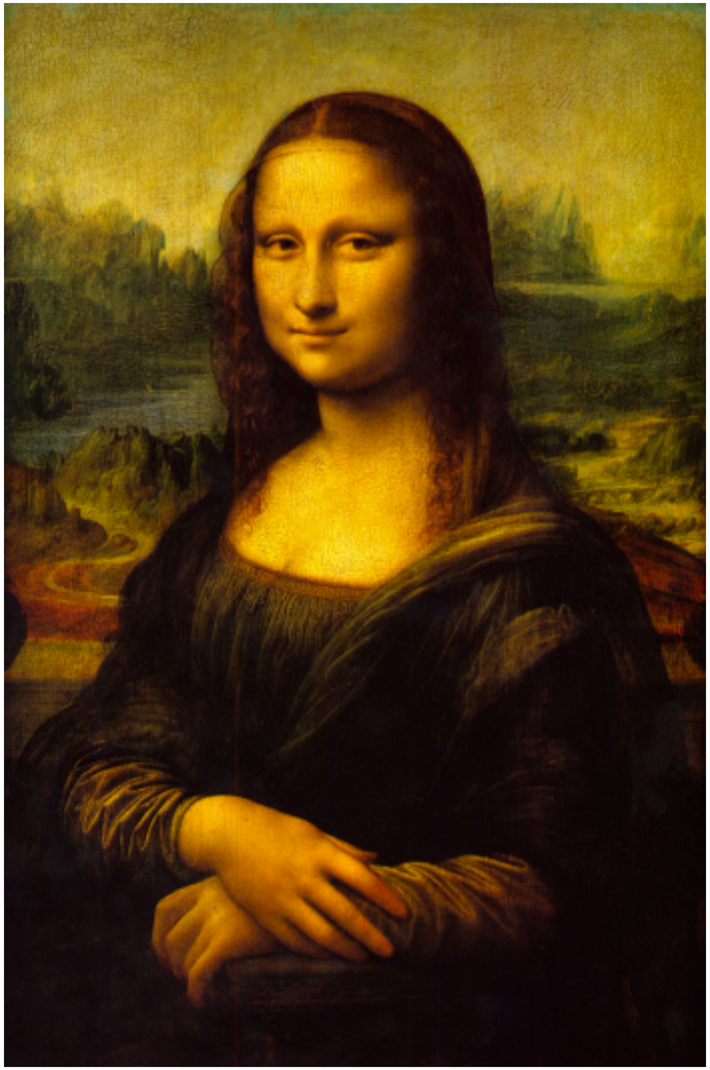
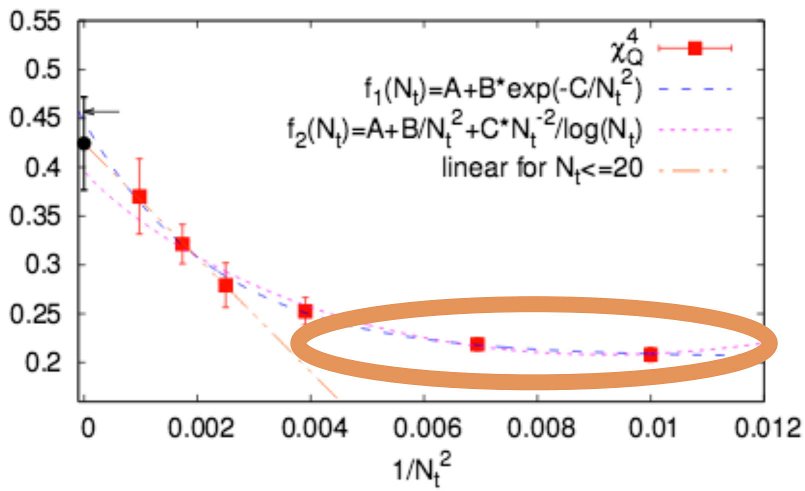




Importance of continuum limit

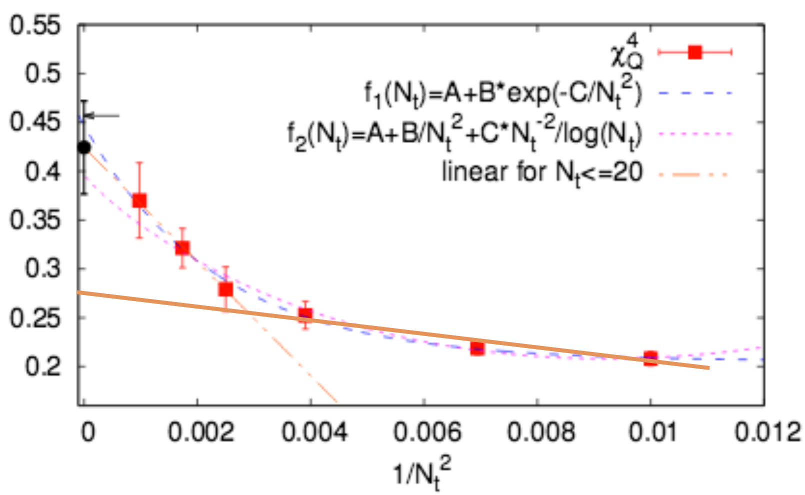
- Observables are affected by discretization effects differently



Importance of continuum limit

- Observables are affected by discretization effects differently

- In quantitative predictions, finite-N_t results can lead to misleading information



Importance of continuum limit

- Observables are affected by discretization effects differently

- In quantitative predictions, finite-N_t results can lead to misleading information

- Message: continuum extrapolated data always preferable

Low temperature phase: HRG model

Dashen, Ma, Bernstein; Prakash, Venugopalan, Karsch, Tawfik, Redlich

- Interacting hadronic matter in the ground state can be well approximated by a non-interacting resonance gas
- The pressure can be written as:

$$p^{HRG}/T^4 = \frac{1}{VT^3}\sum_{i\in\ mesons} \ln \mathcal{Z}^M_{m_i}(T,V,\mu_{X^a}) + \frac{1}{VT^3}\sum_{i\in\ baryons} \ln \mathcal{Z}^B_{m_i}(T,V,\mu_{X^a})$$

where

$$\ln \mathcal{Z}^{M/B}_{m_i} = \mp \frac{V d_i}{2\pi^2}\int_0^\infty dk k^2 \ln(1 \mp z_i e^{-\varepsilon_i/T}) \quad ,$$

with energies $\varepsilon_i = \sqrt{k^2 + m_i^2}$, degeneracy factors d_i and fugacities

$$z_i = \exp\left((\sum_a X_i^a \mu_{X^a})/T \right) \quad .$$

X^a: all possible conserved charges, including the baryon number B, electric charge Q, strangeness S.

- Up to which temperature do we expect agreement with the lattice data?

High temperature limit

- QCD thermodynamics approaches that of a non-interacting, massless quark-gluon gas:

$$\left(\frac{P}{T^4}\right)_{\text{ideal}} = \frac{8\pi^2}{45} + \sum_{f=u,d,s}\left[\frac{7\pi^2}{60} + \frac{1}{2}\left(\frac{\mu_f}{T}\right)^2 + \frac{1}{4\pi^2}\left(\frac{\mu_f}{T}\right)^4\right]$$

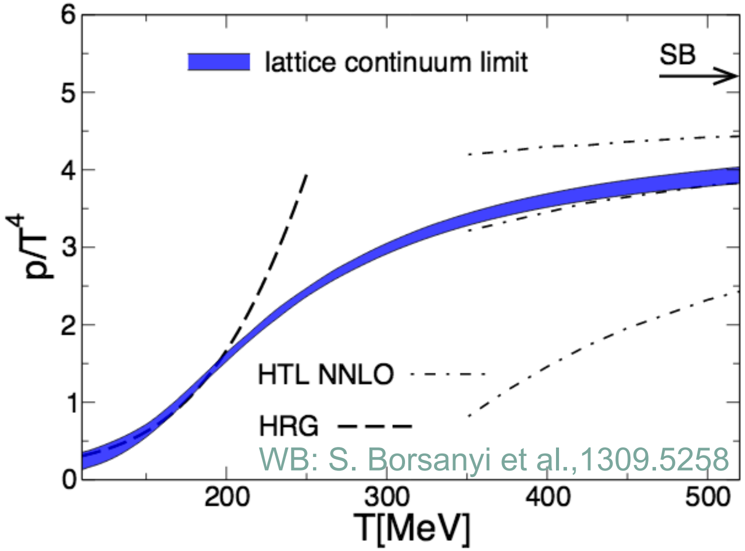
- We can switch on the interaction and systematically expand the observables in series of the coupling *g*

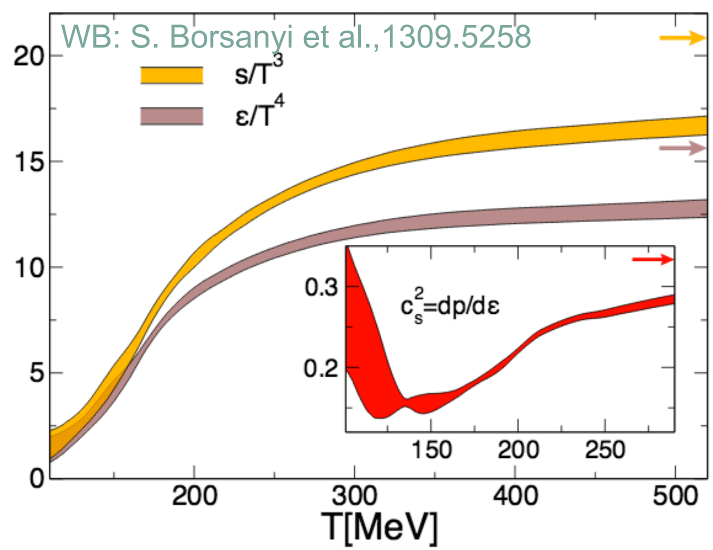
- Resummation of diagrams (HTL) or dimensional reduction are needed, to improve convergence

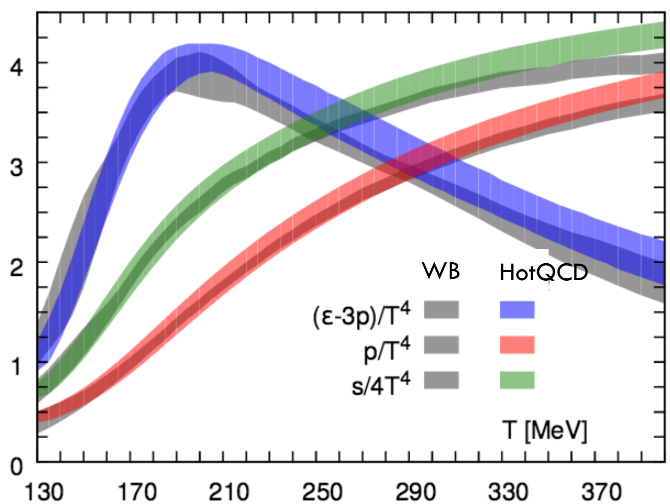
  Braaten, Pisarski (1990); Haque et al. (2014); Hietanen et al (2009)

- At what temperature does perturbation theory break down?

QCD Equation of state at μ_B=0







- EoS available in the continuum limit, with realistic quark masses
- Agreement between stout and HISQ action for all quantities

WB: S. Borsanyi et al., 1309.5258, PLB (2014)
HotQCD: A. Bazavov et al., 1407.6387, PRD (2014)

Sign problem

- The QCD path integral is computed by Monte Carlo algorithms which samples field configurations with a weight proportional to the exponential of the action

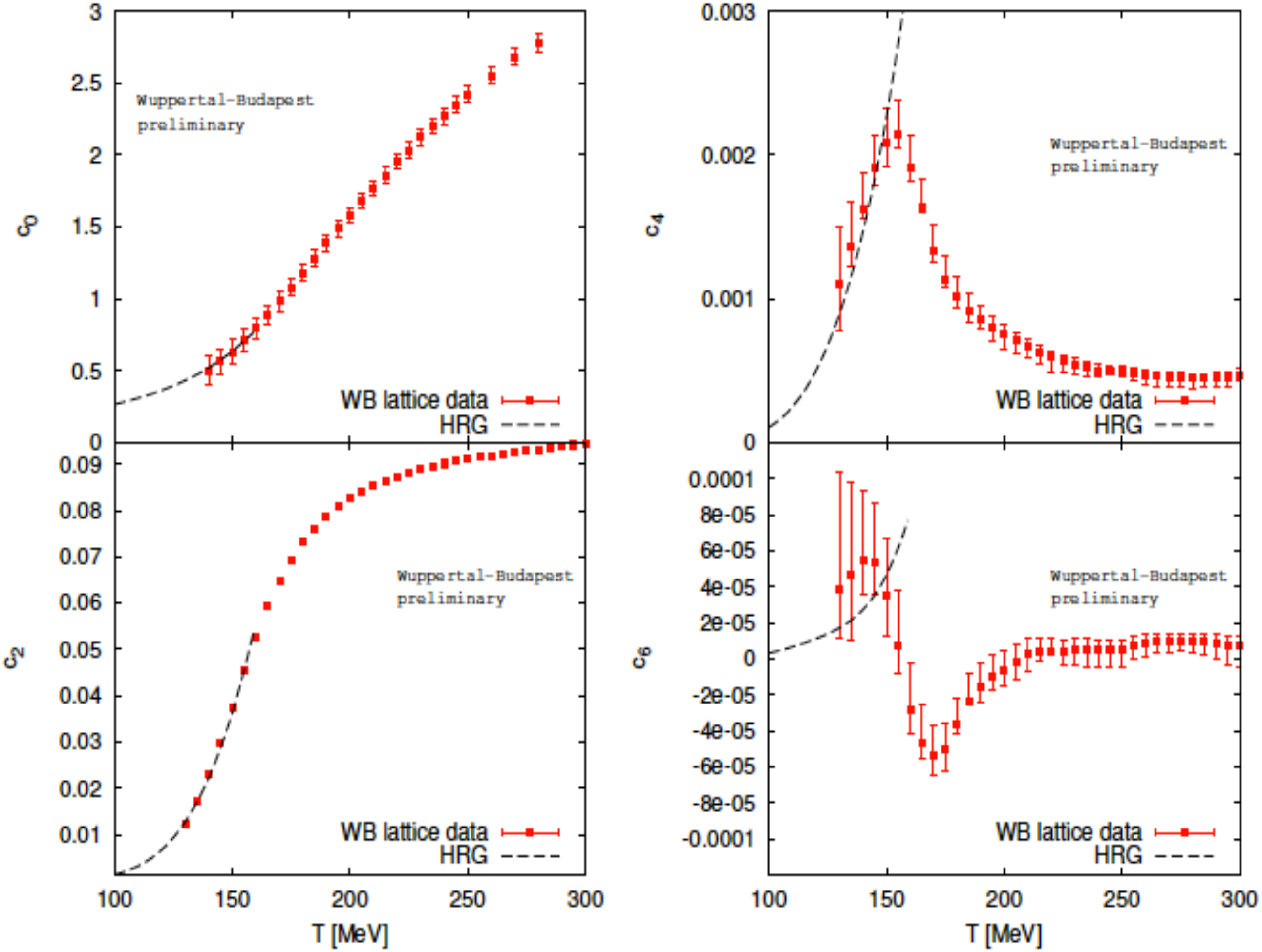
$$Z(\mu_B, T) = \mathrm{Tr}\left(e^{-\frac{H_{QCD} - \mu_B N_B}{T}}\right) = \int \mathcal{D}U e^{-S_G[U]} \det M[U, \mu_B]$$

- $\det M[\mu_B]$ complex → Monte Carlo simulations are not feasible

- We can rely on a few approximate methods, viable for small μ_B/T:
  - Taylor expansion of physical quantities around $\mu=0$ (Bielefeld-Swansea collaboration 2002; R. Gavai, S. Gupta 2003)
  - Reweighting (complex phase moved from the measure to observables) (Barbour et al. 1998; Z. Fodor and S, Katz, 2002)
  - Simulations at imaginary chemical potentials (plus analytic continuation) (Alford, Kapustin, Wilczek, 1999; de Forcrand, Philipsen, 2002; D'Elia, Lombardo 2003)

Equation of state at $\mu_B>0$

Talk by S. Borsanyi on Monday

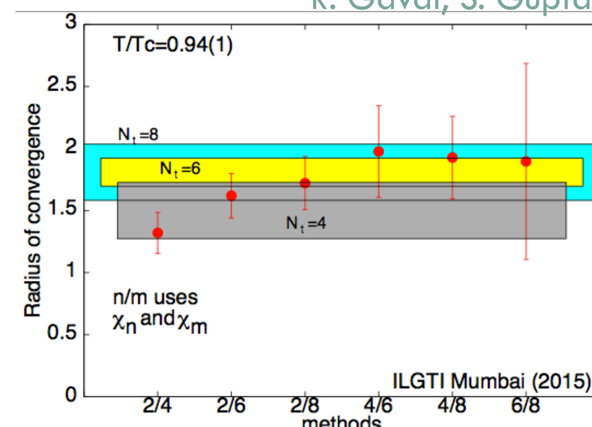


- Expand the pressure in powers of μ_B

$$\frac{p(\mu_B)}{T^4} = c_0 + c_2\left(\frac{\mu_B}{T}\right)^2 + + c_4\left(\frac{\mu_B}{T}\right)^4 + c_6\left(\frac{\mu_B}{T}\right)^6 + \mathcal{O}(\mu_B^8)$$

- Continuum extrapolated results for c_2, c_4, c_6 at the physical mass
- Radius of convergence:

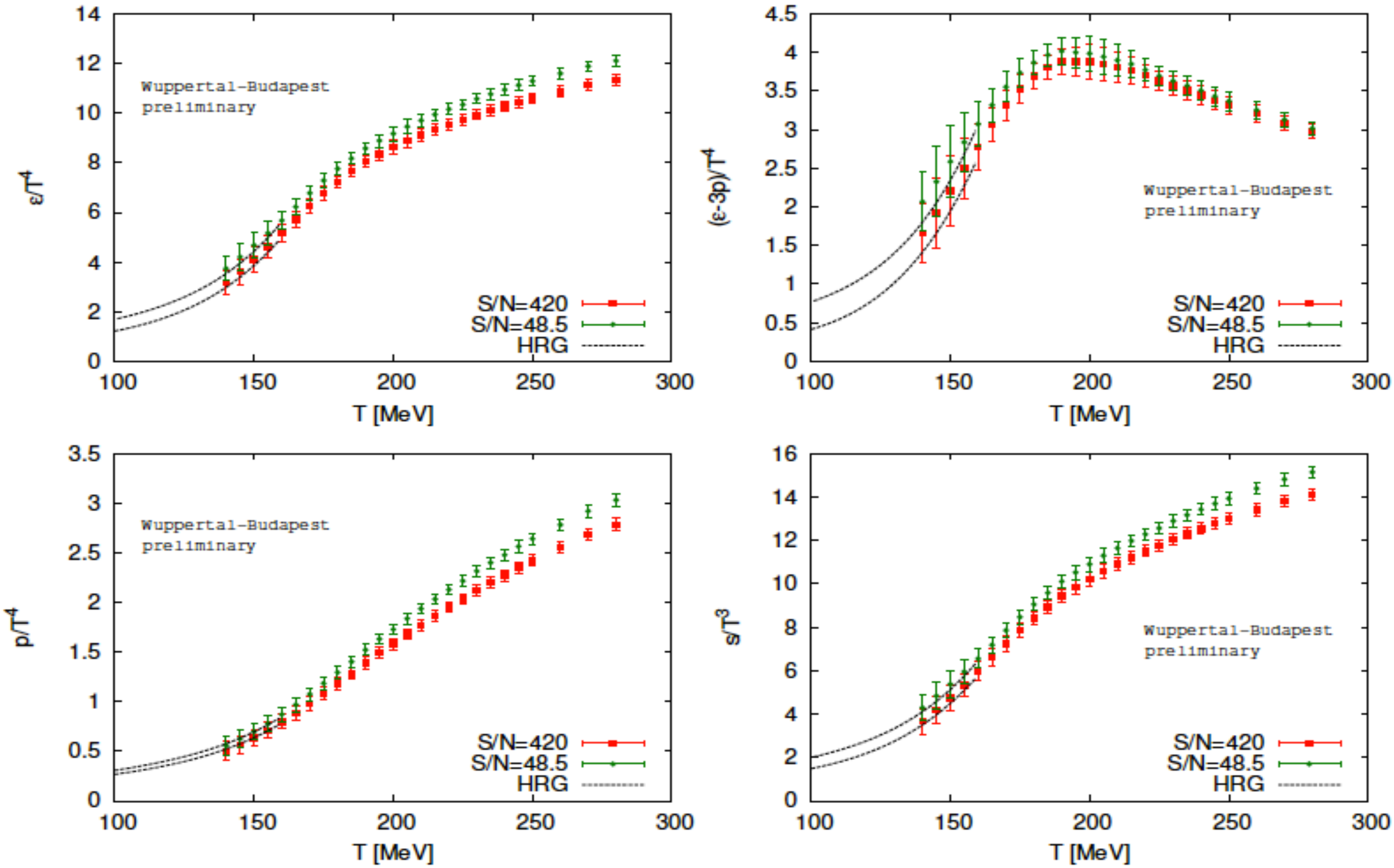
R. Gavai, S. Gupta



WB: S. Borsanyi et al., preliminary

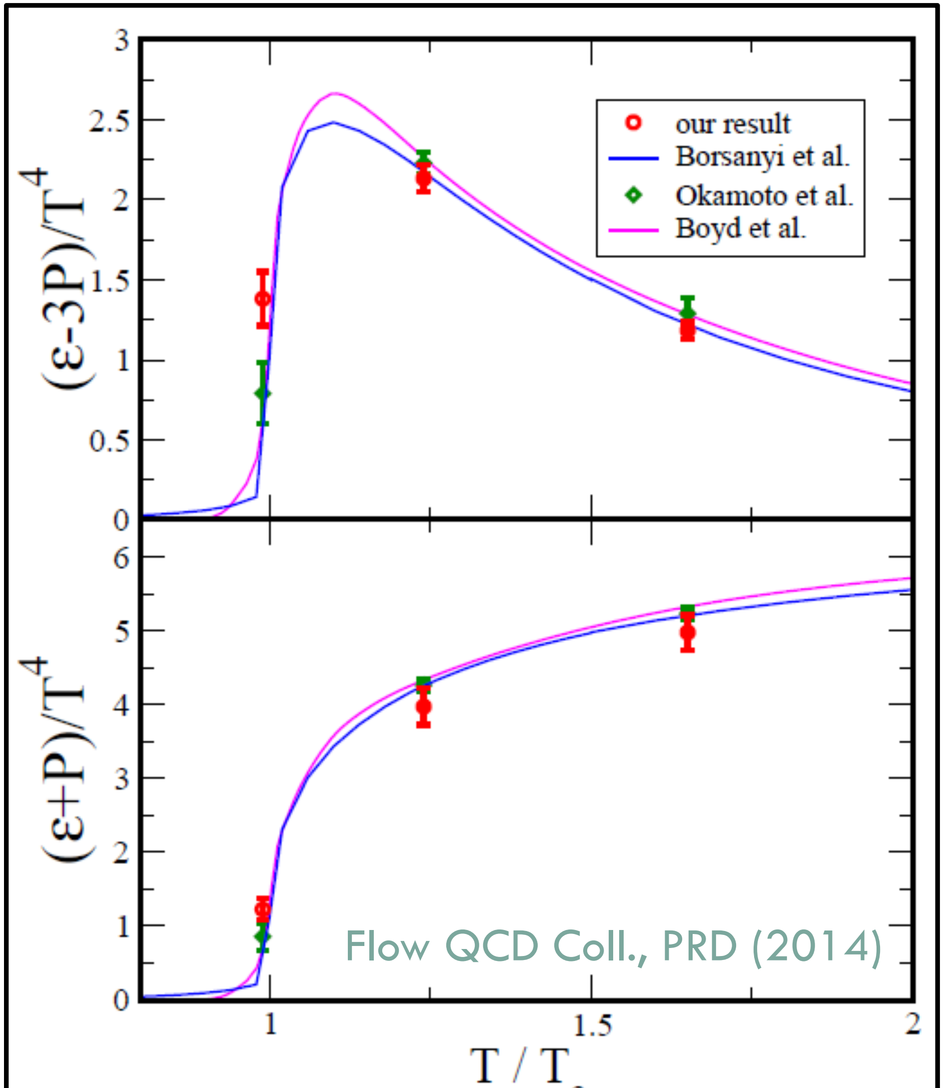
Equation of state at $\mu_B>0$

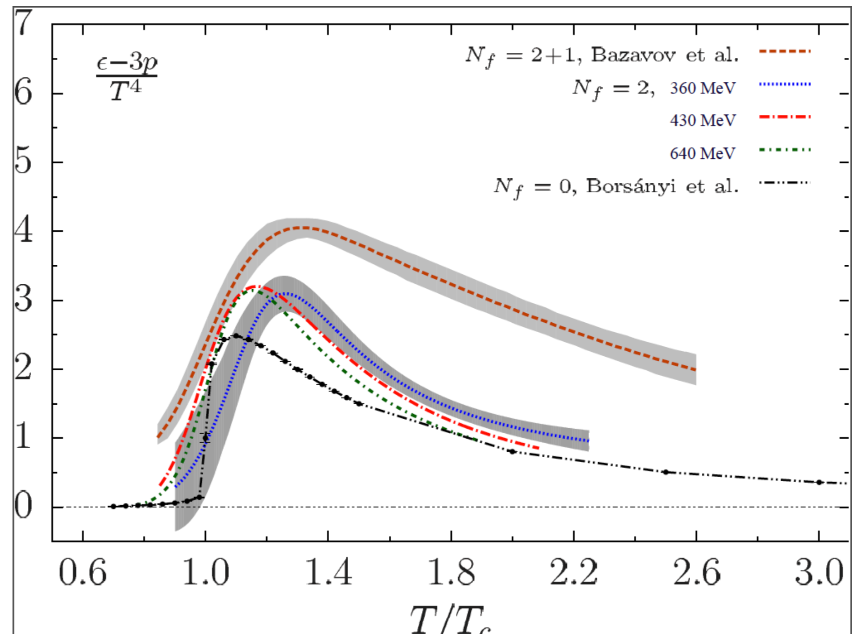
- Calculate the EoS along the constant S/N trajectories



WB: S. Borsanyi et al., premilinary

Alternative methods for thermodynamics

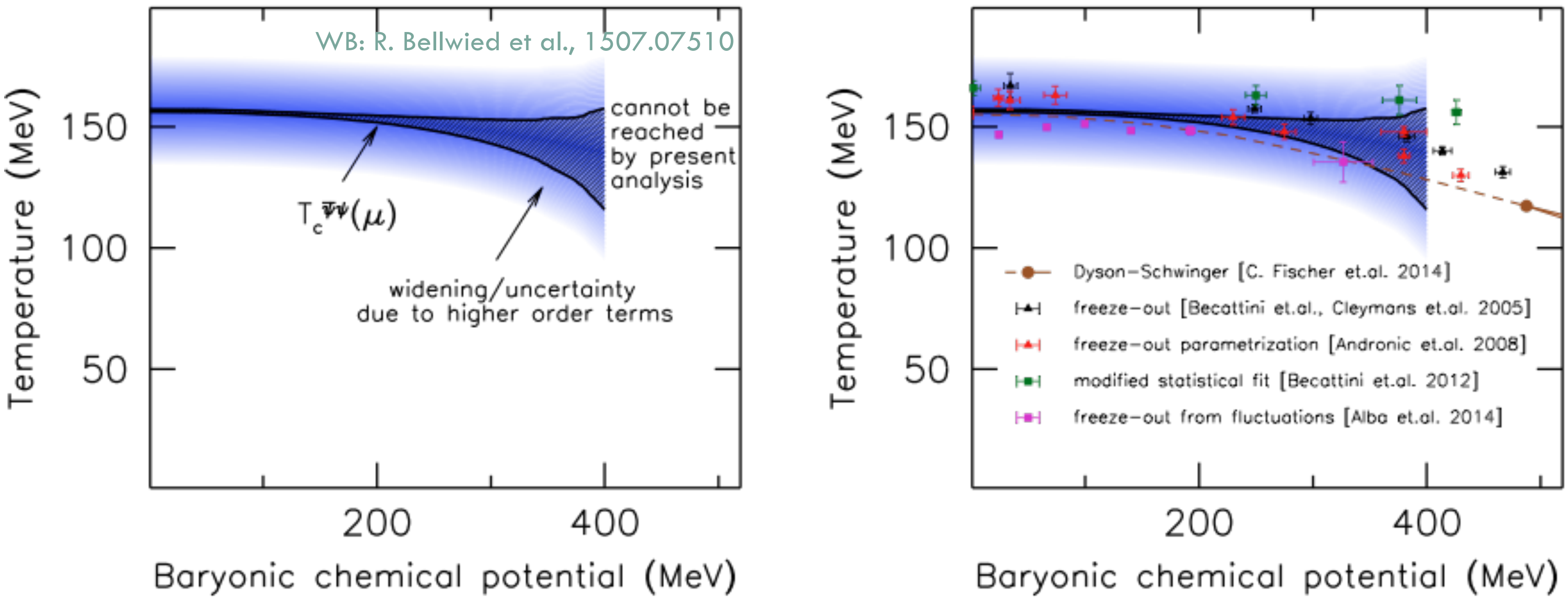




F. Burger et al., PRD (2015)

- Gradient flow: EoS in the quenched approximation
- Twisted mass Wilson fermions: EoS available so far for heavier-than-physical quark masses and N_f=2

QCD phase diagram



Curvature κ defined as:

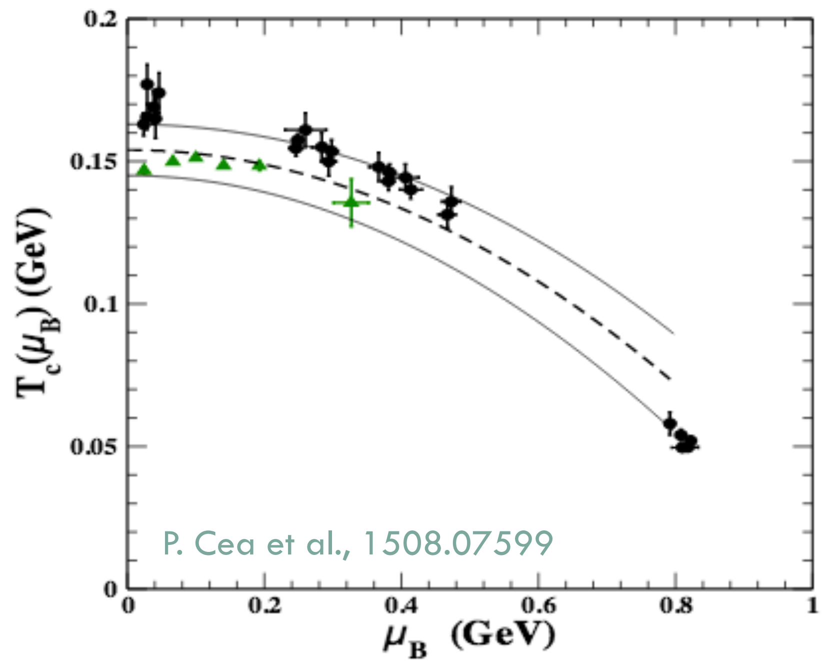
$$\frac{T_c(\mu_B)}{T_c(\mu=0)} = 1 - \kappa\left(\frac{\mu_B}{T_c(\mu_B)}\right)^2 + \lambda\left(\frac{\mu_B}{T_c(\mu_B)}\right)^4 \dots$$

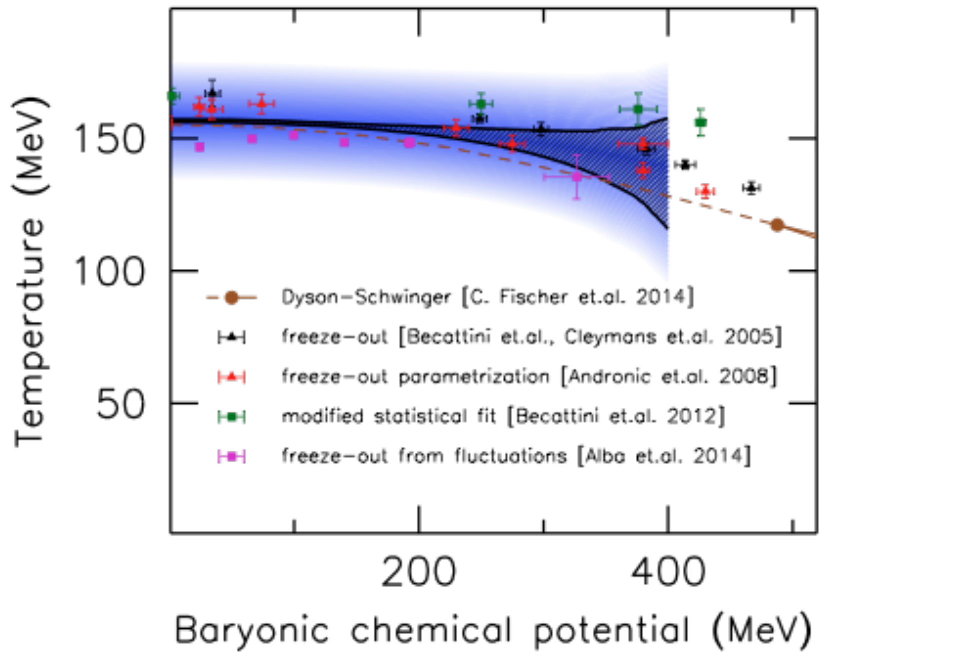
Recent results:

$$\kappa = 0.0149 \pm 0.0021$$

WB: R. Bellwied et al., 1507.07510

QCD phase diagram





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$$\kappa = 0.020(4)$$

P. Cea et al., 1508.07599

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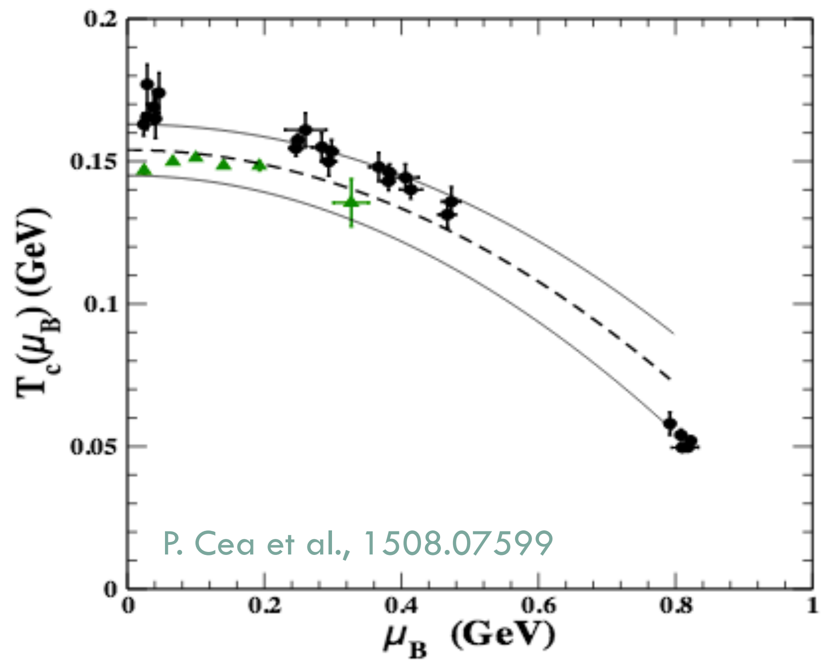

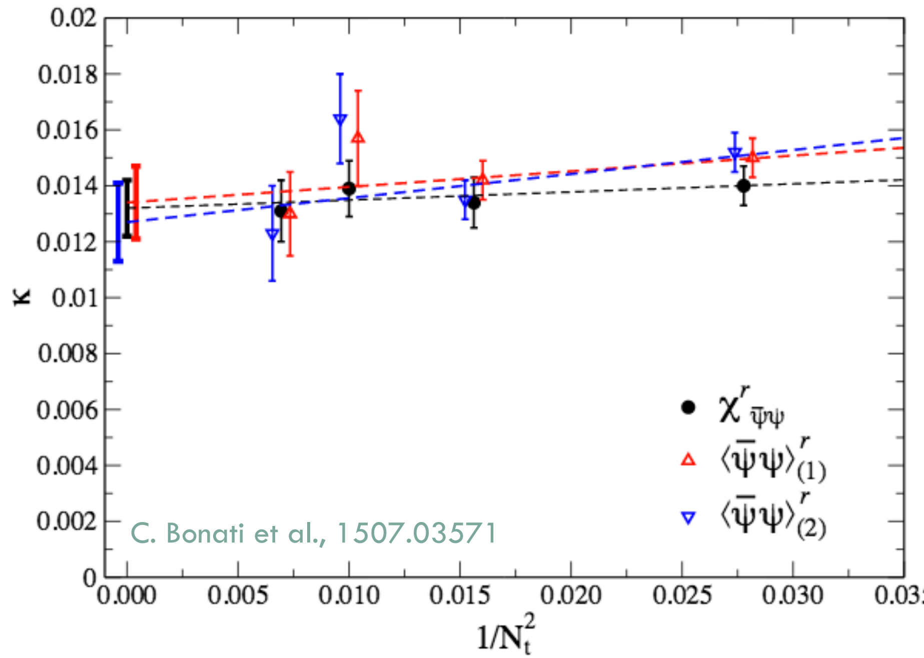

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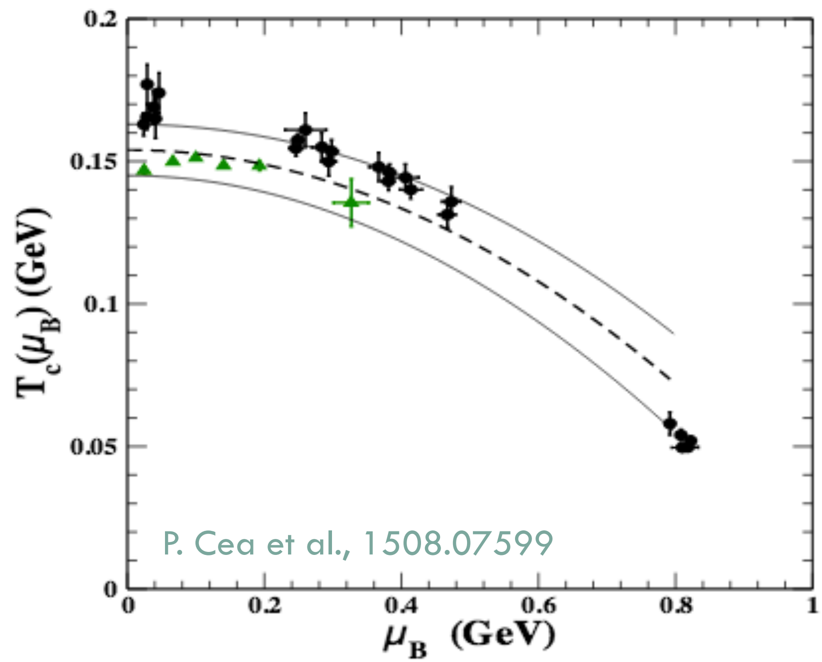
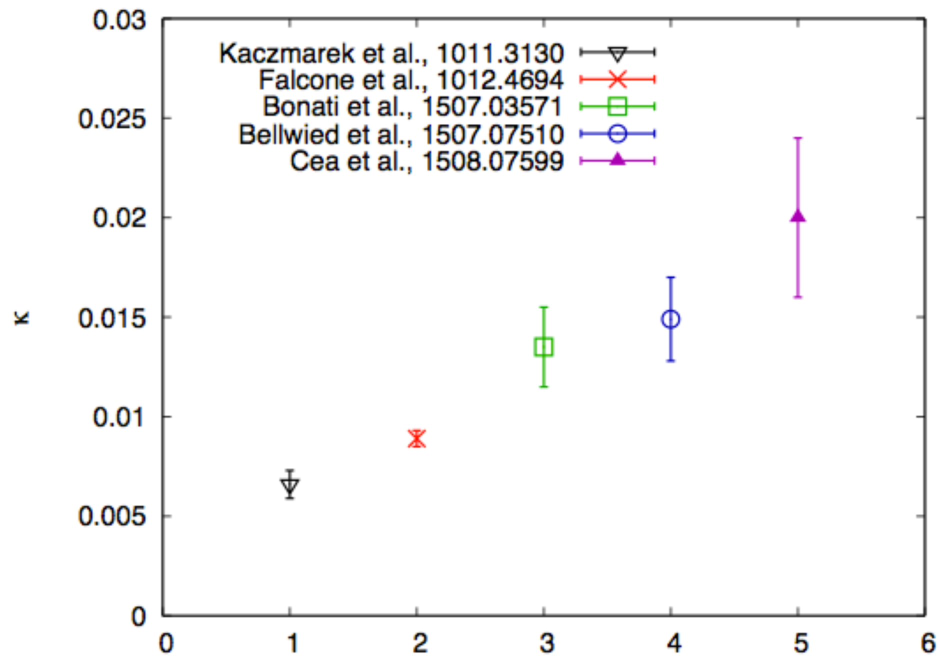
P. Cea et al., 1508.07599

$$\kappa = 0.0135(20)$$

C. Bonati et al., 1507.03571

QCD phase diagram





1. Kaczmarek et al., Nf=2+1, p4 staggered action, Taylor expansion, μ_s=0, N_t=8
2. Falcone et al., Nf=2+1, p4 staggered action, analytic continuation, $\mu_s=\mu_u=\mu_d$, N_t=4
3. Bonati et al., Nf=2+1,stout staggered action, analytic continuation, μ_s=0, continuum extrapolated
4. Bellwied et al. (WB), Nf=2+1, 4stout staggered action, analytic continuation, $\langle n_s \rangle$=0, cont. extrap.
5. Cea et al., Nf=2+1, HISQ staggered action, analytic continuation, $\mu_s=\mu_u=\mu_d$, cont. extrapolated

Fluctuations of conserved charges

- Definition:

$$\chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n} p/T^4}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}.$$

- Relationship between chemical potentials:

$$\begin{aligned}
\mu_u &= \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q; \\
\mu_d &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q; \\
\mu_s &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S.
\end{aligned}$$

- They can be calculated on the lattice and compared to experiment

Connection to experiment

- Fluctuations of conserved charges are the cumulants of their event-by-event distribution

$$\text{mean}: M = \chi_1 \qquad \text{variance}: \sigma^2 = \chi_2$$

$$\text{skewness}: S = \chi_3/\chi_2^{3/2} \qquad \text{kurtosis}: \kappa = \chi_4/\chi_2^2$$

$$S\sigma = \chi_3/\chi_2 \qquad \kappa\sigma^2 = \chi_4/\chi_2$$

$$M/\sigma^2 = \chi_1/\chi_2 \qquad S\sigma^3/M = \chi_3/\chi_1$$

- Lattice QCD results are functions of temperature and chemical potential
  - By comparing lattice results and experimental measurement we can extract the freeze-out parameters from first principles

F. Karsch: Centr. Eur. J. Phys. (2012)

Things to keep in mind

- Effects due to volume variation because of finite centrality bin width
  - Experimentally corrected by centrality-bin-width correction method

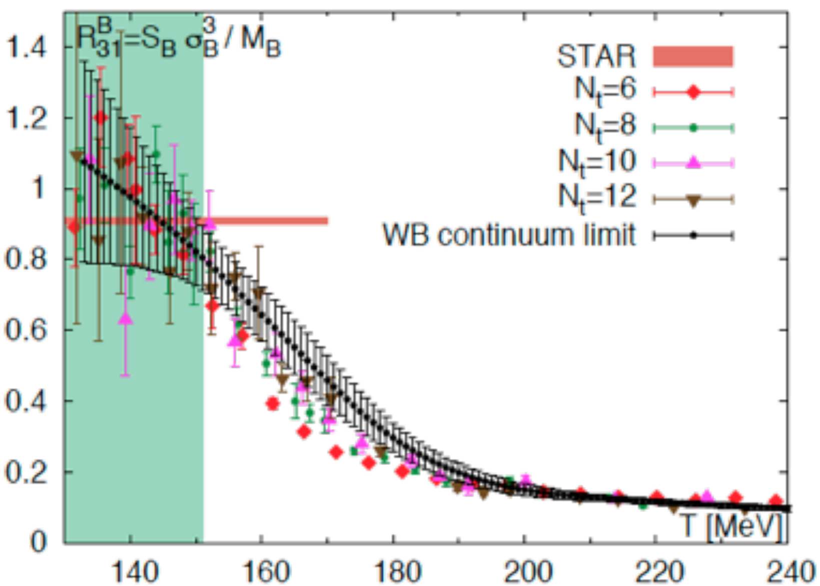
  V. Skokov et al., PRC (2013)
- Finite reconstruction efficiency
  - Experimentally corrected based on binomial distribution A.Bzdak,V.Koch, PRC (2012)
- Spallation protons
  - Experimentally removed with proper cuts in p_T
- Canonical vs Gran Canonical ensemble
  - Experimental cuts in the kinematics and acceptance V. Koch, S. Jeon, PRL (2000)
- Proton multiplicity distributions vs baryon number fluctuations
  - Recipes for treating proton fluctuations

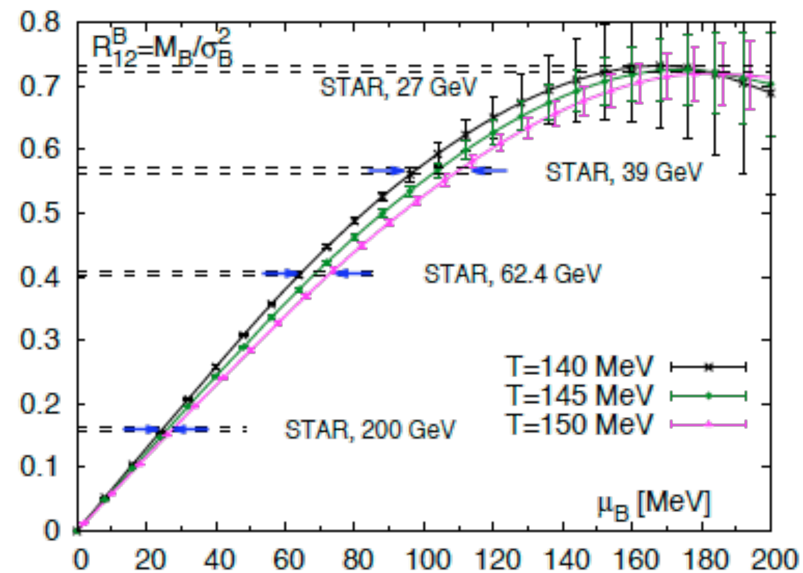
  M. Asakawa and M. Kitazawa, PRC(2012), M. Nahrgang et al., 1402.1238
- Final-state interactions in the hadronic phase
  - Consistency between different charges = fundamental test

J.Steinheimer et al., PRL (2013)

Freeze-out parameters from B fluctuations

- Thermometer: $\dfrac{\chi_3^B(T,\mu_B)}{\chi_1^B(T,\mu_B)}$ $=S_B\sigma_B^3/M_B$ Baryometer: $\dfrac{\chi_1^B(T,\mu_B)}{\chi_2^B(T,\mu_B)}$ $=\sigma_B^2/M_B$





WB: S. Borsanyi et al., PRL (2014)
STAR collaboration, PRL (2014)

- Upper limit: $T_f \leq 151\pm4$ MeV
- Consistency between freeze-out chemical potential from electric charge and baryon number is found.

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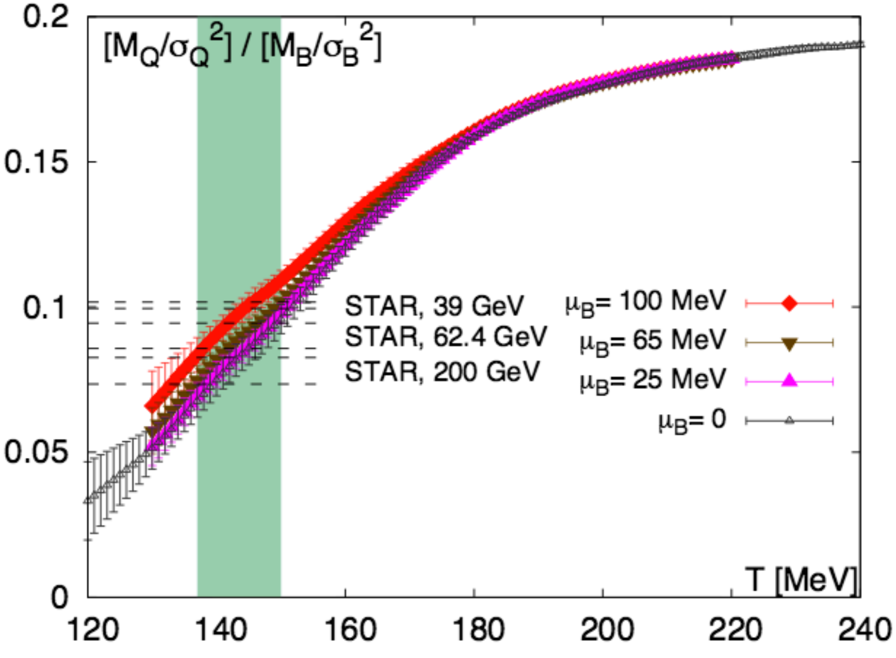

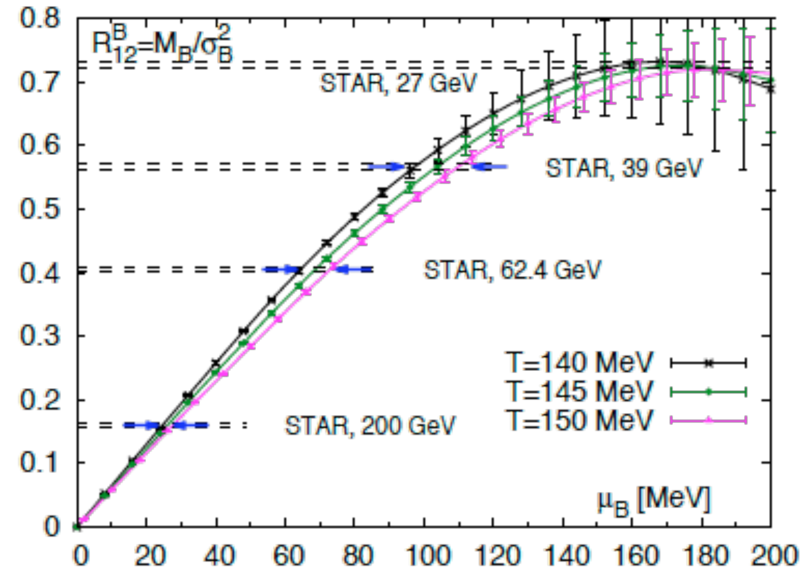

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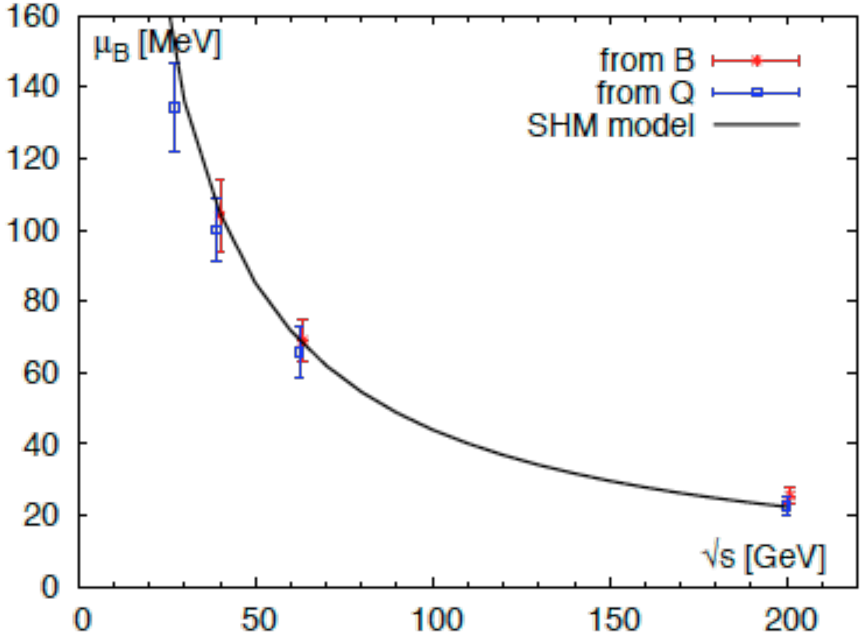

$\sqrt{s}[GeV]$	μ_B^f [MeV] (from B)	μ_B^f [MeV] (from Q)
200	25.8±2.7	22.8±2.6
62.4	69.7±6.4	66.6±7.9
39	105±11	101±10
27	-	136±13.8

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Curvature of the freeze-out line

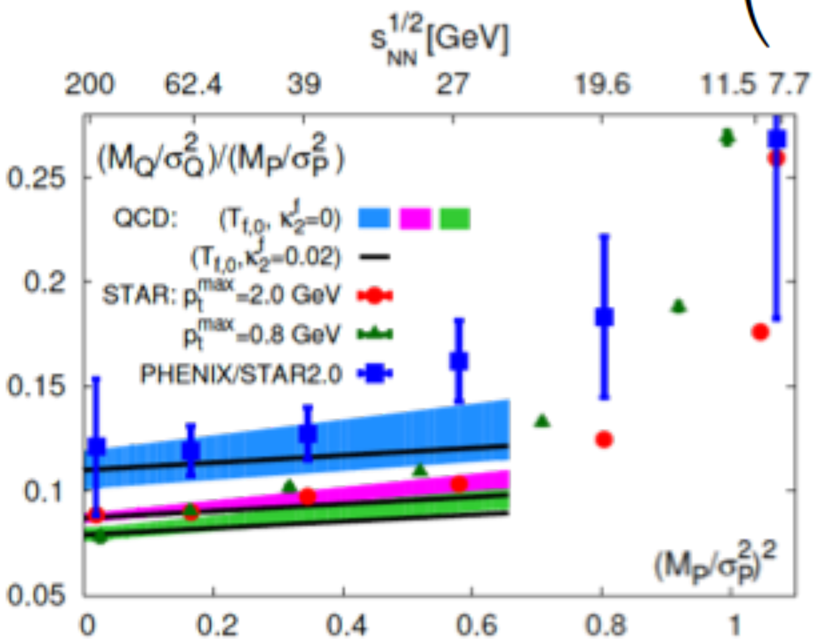
- Parametrization of the freeze-out line:

Talk by F. Karsch on Monday

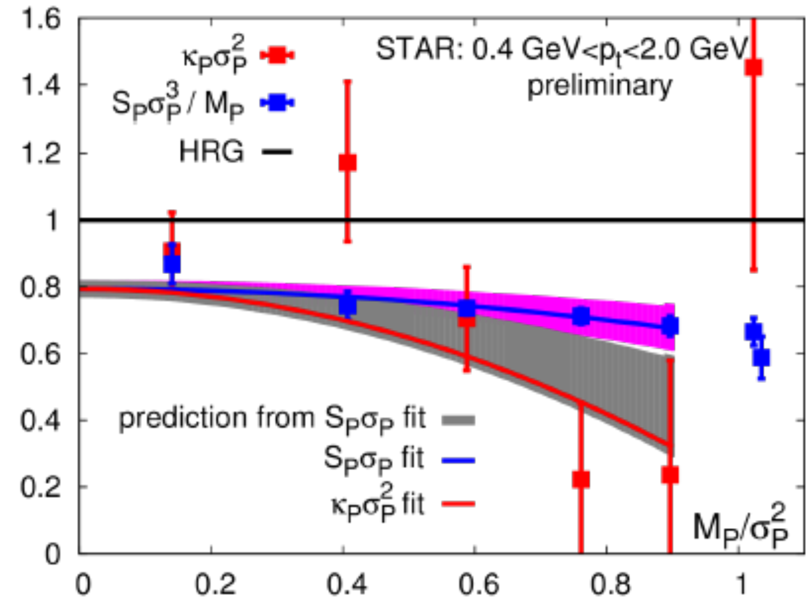
$$T_f(\mu_B) = T_{f,0}\left(1 - \kappa_2^f \bar{\mu}_B^2 - \kappa_4^f \bar{\mu}_B^4\right)$$

- Taylor expansion of the "ratio of ratios" $R_{12}^{QB} = [M_Q/\sigma_Q^2]/[M_B/\sigma_B^2]$

$$R_{12}^{QB} = R_{12}^{QB,0} + \left(R_{12}^{QB,2} - \kappa_2^f T_{f,0} \left.\frac{\mathrm{d}R_{12}^{QB,0}}{\mathrm{d}T}\right|_{T_{f,0}}\right)\hat{\mu}_B^2$$



A. Bazavov et al., 1509.05786
STAR0.8: PRL (2013)



STAR2.0: X. Luo, PoS CPOD 2014
PHENIX: 1506.07834

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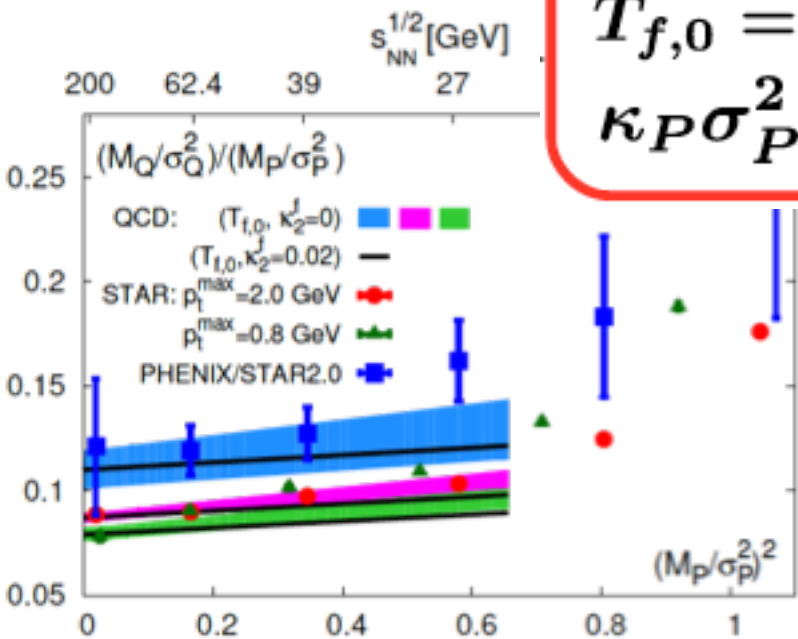
- Taylor expansion of the "ratio of ratios" $R_{12}^{QB} = [M_Q/\sigma_Q^2]/[M_B/\sigma_B^2]$

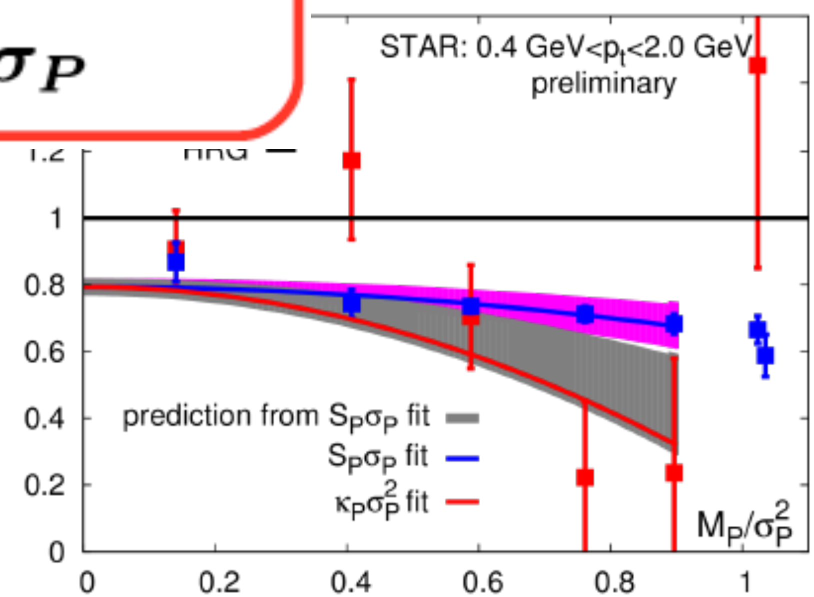
$$R_{12}^{QB} = \ldots \Big) \hat{\mu}_B^2$$

$$\kappa_2^f < 0.011$$

$$T_{f,0} = (147 \pm 2)\ \mathrm{MeV}$$

$$\kappa_P \sigma_P^2 < S_P \sigma_P$$





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Freeze-out line from first principles

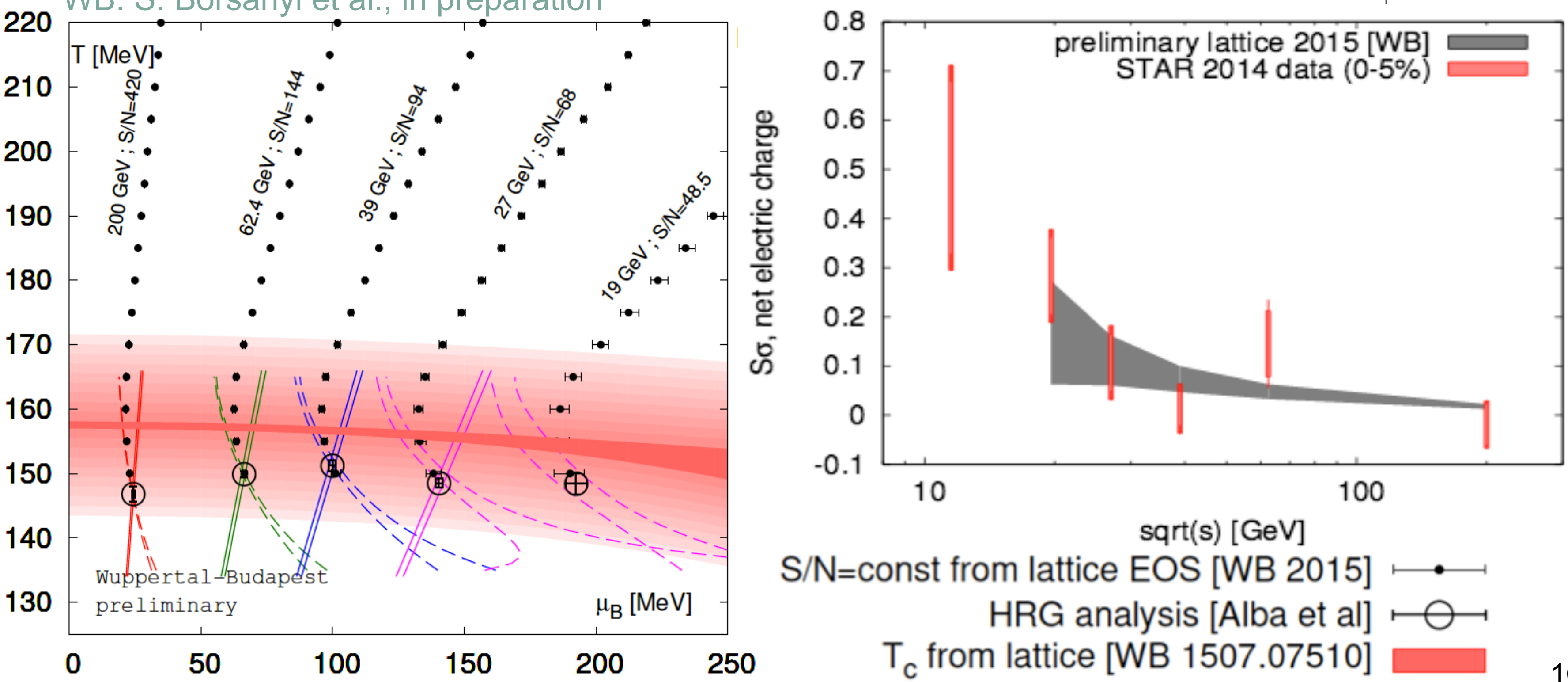
Talk by S. Borsanyi on Monday

- Use T- and μ_B-dependence of R_{12}^Q and R_{12}^B for a combined fit:

$$R_{12}^Q(T,\mu_B)=\frac{\chi_1^Q(T,\mu_B)}{\chi_2^Q(T,\mu_B)}=\frac{\chi_{11}^{QB}(T,0)+\chi_2^Q(T,0)q_1(T)+\chi_{11}^{QS}(T,0)s_1(T)}{\chi_2^Q(T,0)}\frac{\mu_B}{T}+\mathcal{O}(\mu_B^3).$$

$$R_{12}^B(T,\mu_B)=\frac{\chi_1^B(T,\mu_B)}{\chi_2^B(T,\mu_B)}=\frac{\chi_2^B(T,0)+\chi_{11}^{BQ}(T,0)q_1(T)+\chi_{11}^{BS}(T,0)s_1(T)}{\chi_2^B(T,0)}\frac{\mu_B}{T}+\mathcal{O}(\mu_B^3)$$

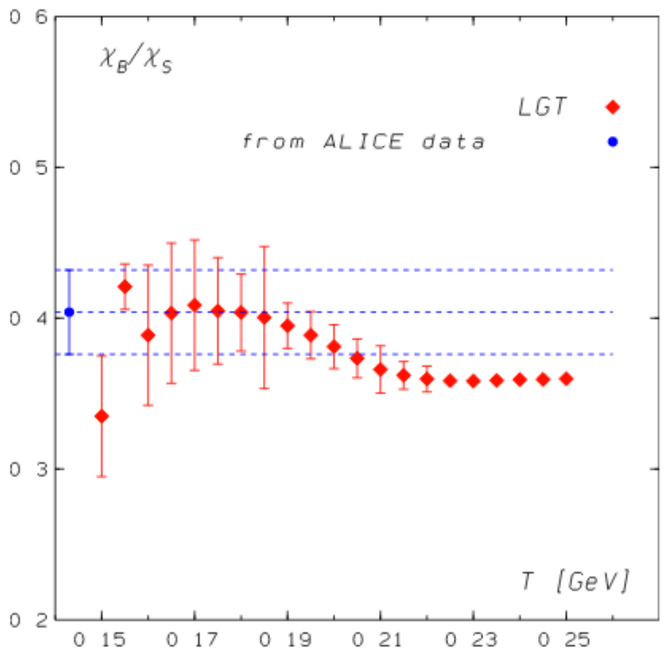
WB: S. Borsanyi et al., in preparation

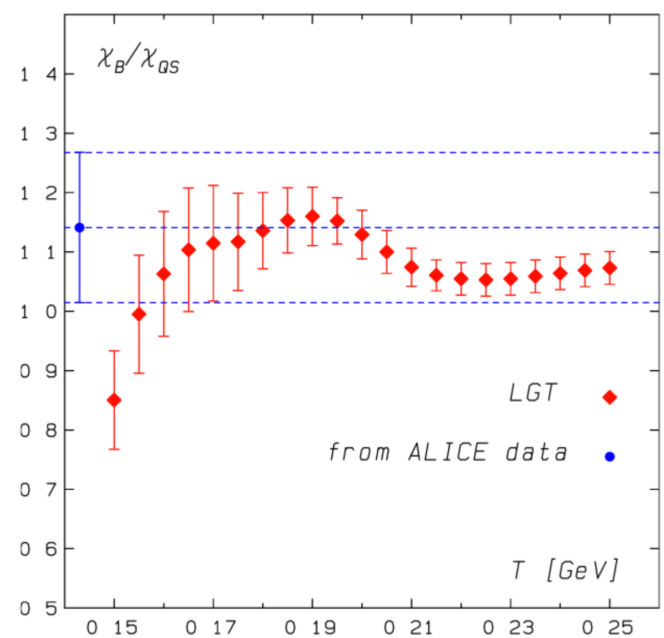


Initial analysis of LHC data

Talk by A. Kalweit on Monday

- Fluctuation data not yet available
- Assuming Skellam distribution, can use yields: $\hat{\chi}_N = \frac{1}{VT^3}\left(\langle N_q \rangle + \langle N_{-q} \rangle\right)$



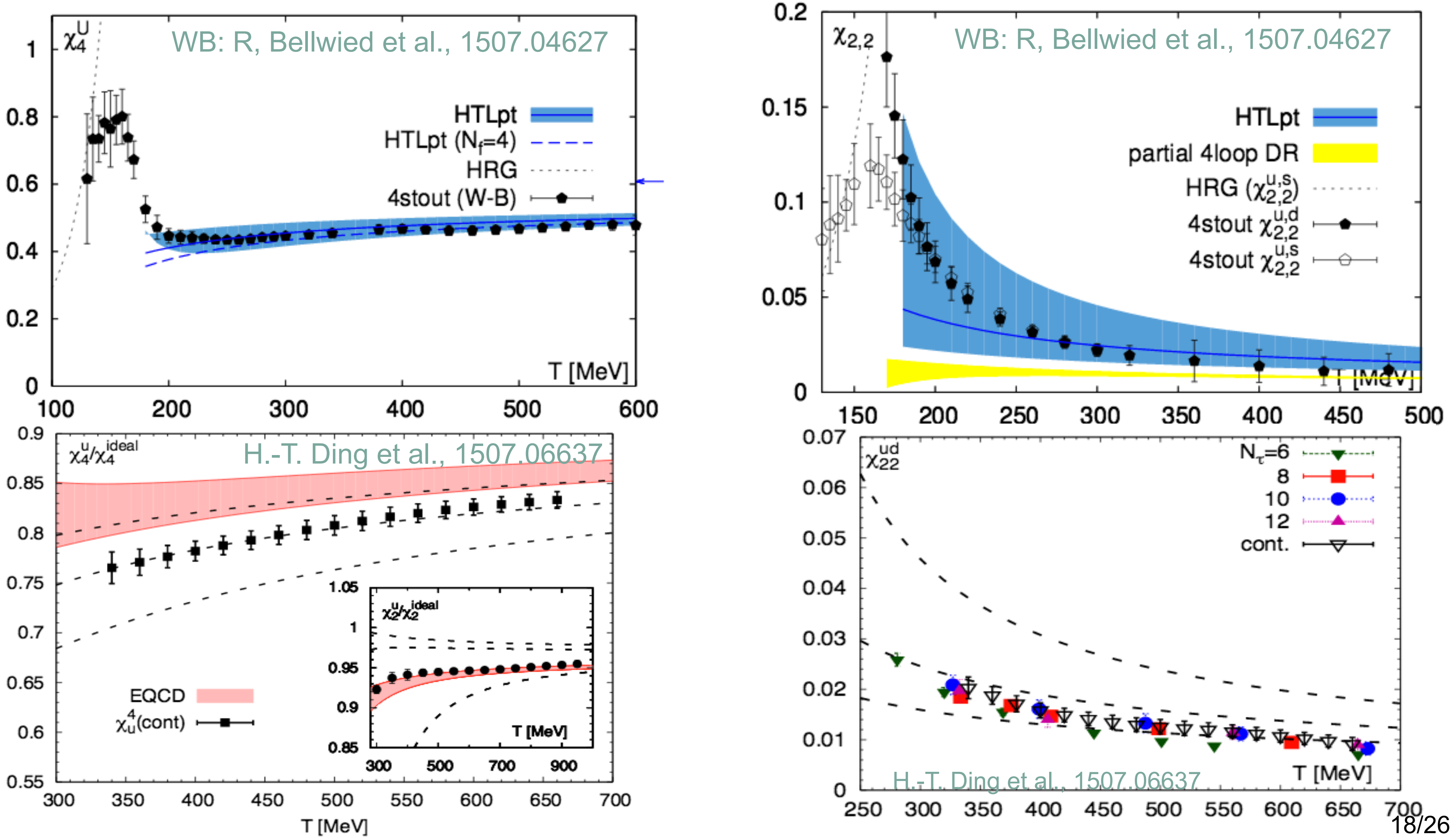


P. Braun-Munzinger et al., PLB (2015)

- Slightly higher temperature than at RHIC: (150<T_f<163) MeV
- Looking forward to fluctuation measurements at the LHC

Fluctuations at high temperatures

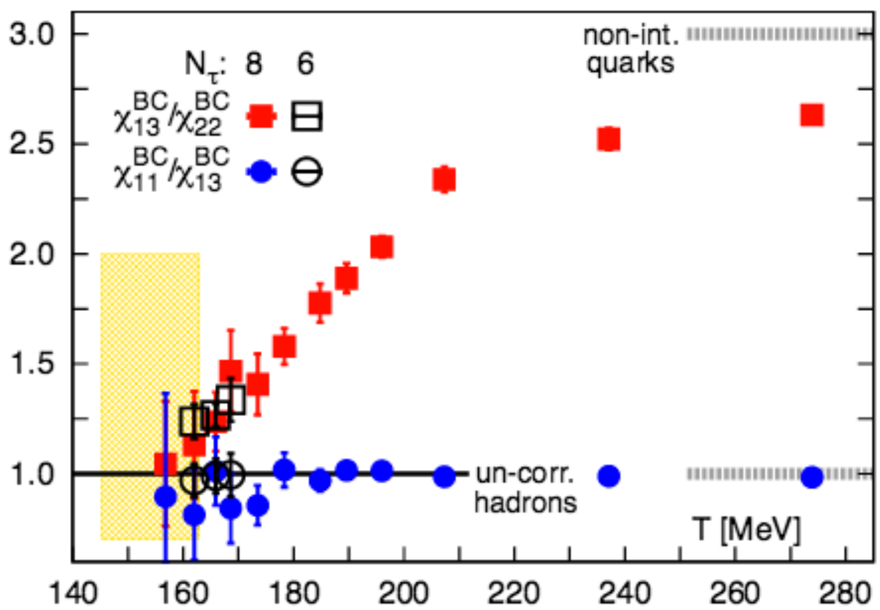
HTL: N. Haque et al., JHEP (2014); DR: S. Mogliacci et al., JHEP (2013)



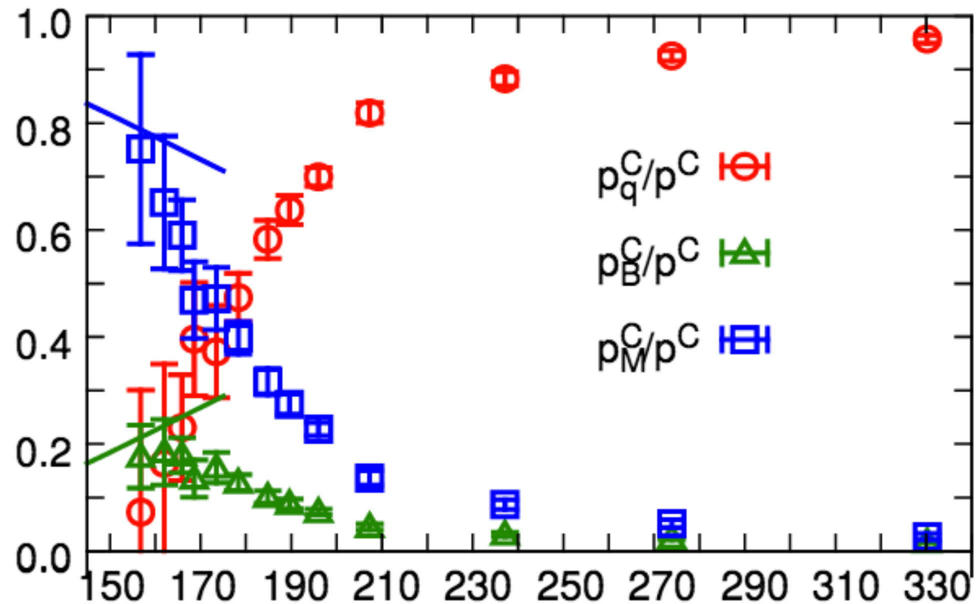
Degrees of freedom from fluctuations

Talk by P. Petreczky on Monday

- Onset of deconfinement for charm quarks:



A. Bazavov et al., PLB (2014)



S. Mukherjee, P. Petreczky, S. Sharma 1509.08887

- Partial meson and baryon pressures described by HRG at T_C and dominate the charm pressure then drop gradually. Charm quark only dominant dof at T>200 MeV

Transport properties

- Matter in the region (1-2)T_c is highly non-perturbative
- Significant modifications of its transport properties
- Common problem:
  - Transport properties can be explored through the analysis of certain correlation functions:

$$G_H(\tau,\vec{p},T) = \int_0^\infty \frac{d\omega}{2\pi}\, \rho_H(\omega,\vec{p},T)\, \frac{\cosh(\omega(\tau-1/2T))}{\sinh(\omega/2T)} \quad = \int d^3x\, e^{i\mathbf{px}} \langle J^\alpha(0,0) J^{\beta\dagger}(\tau,\mathbf{x})\rangle$$

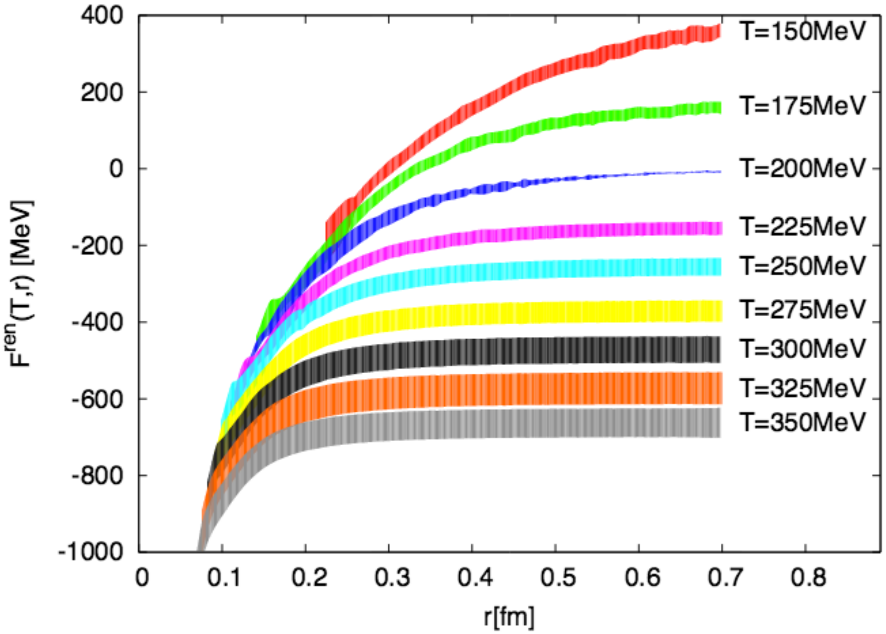
  - Challenge: integrate over discrete set of lattice points in τ direction
  - Use inversion methods like Maximum Entropy Method or modeling the spectral function at low frequencies

Quarkonia properties

- Three main approaches:
  - Potential models with heavy quark potential calculated on the lattice
    - Solve Schroedinger’s equation for the bound state two-point function
  - Extract spectral functions from Euclidean temporal correlators
  - Study spatial correlation functions of quarkonia and their in-medium screening properties

Inter-quark potential

- Static quark-antiquark free-energy

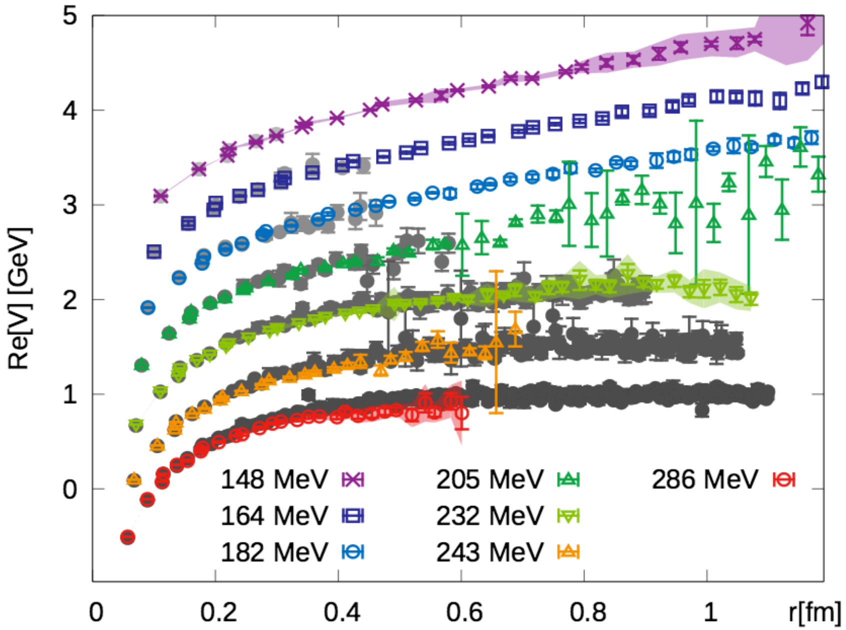


Borsanyi et al. JHEP(2015)

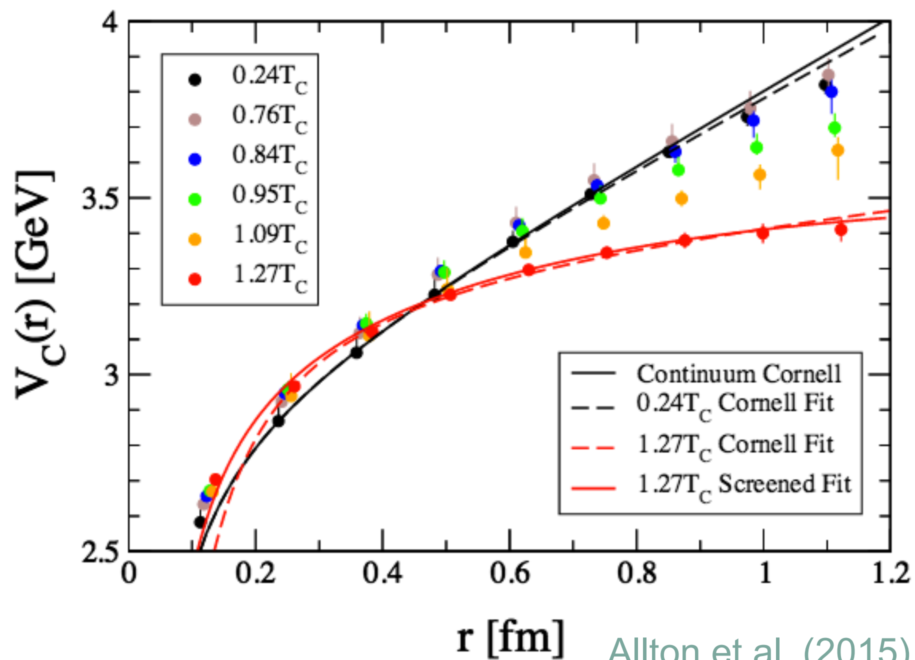
- Continuum extrapolated result with N_f=2+1 flavors at the physical mass

Inter-quark potential

- Quark-antiquark potential in N_f=2+1 QCD



Burnier et al. (2014)



Allton et al. (2015)

- Real part of the complex potential lies close to the color singlet free energy
- Central potential: combination of pseudoscalar and vector potentials:

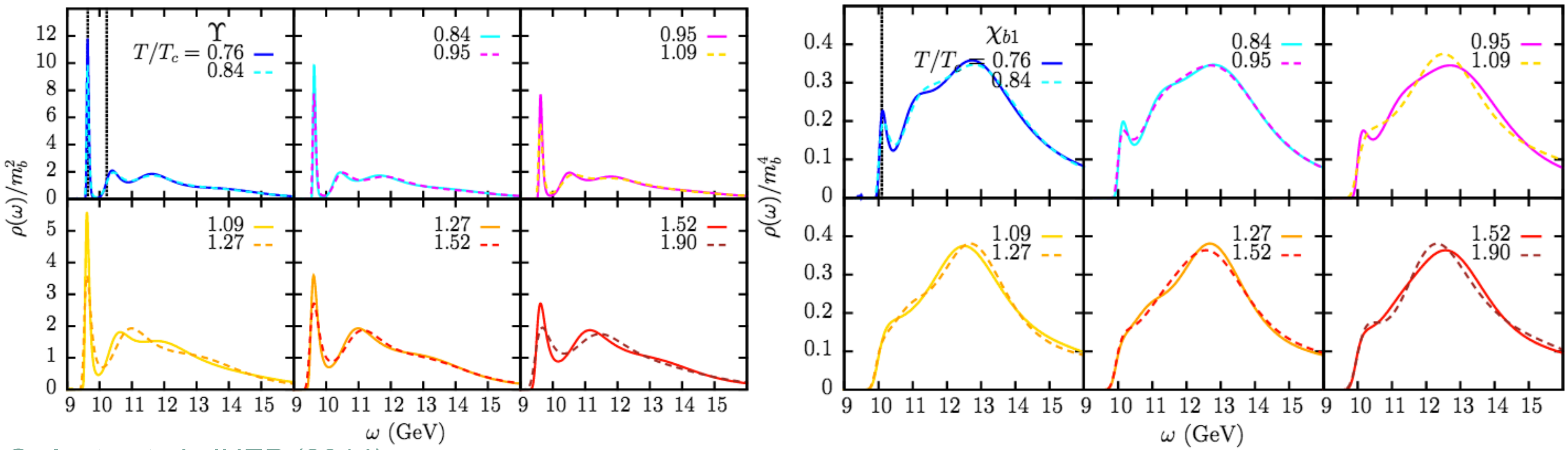
$$V_C(\mathbf{r}) = \frac{1}{4}V_{PS} + \frac{3}{4}V_V$$

Quarkonia spectral functions

- Charmonium spectral functions in quenched approximation and preliminary studies with dynamical quarks yield consistent results: all charmonium states are dissociated for $T \gtrsim 1.5 T_c$

H. Ding et al., PRD (2012)
G. Aarts et al., PRD (2007)
WB: S. Borsanyi et al., JHEP (2014)

- Bottomonium (N_f=2+1, m_π=400 MeV), MEM:



G. Aarts et al. JHEP (2014)

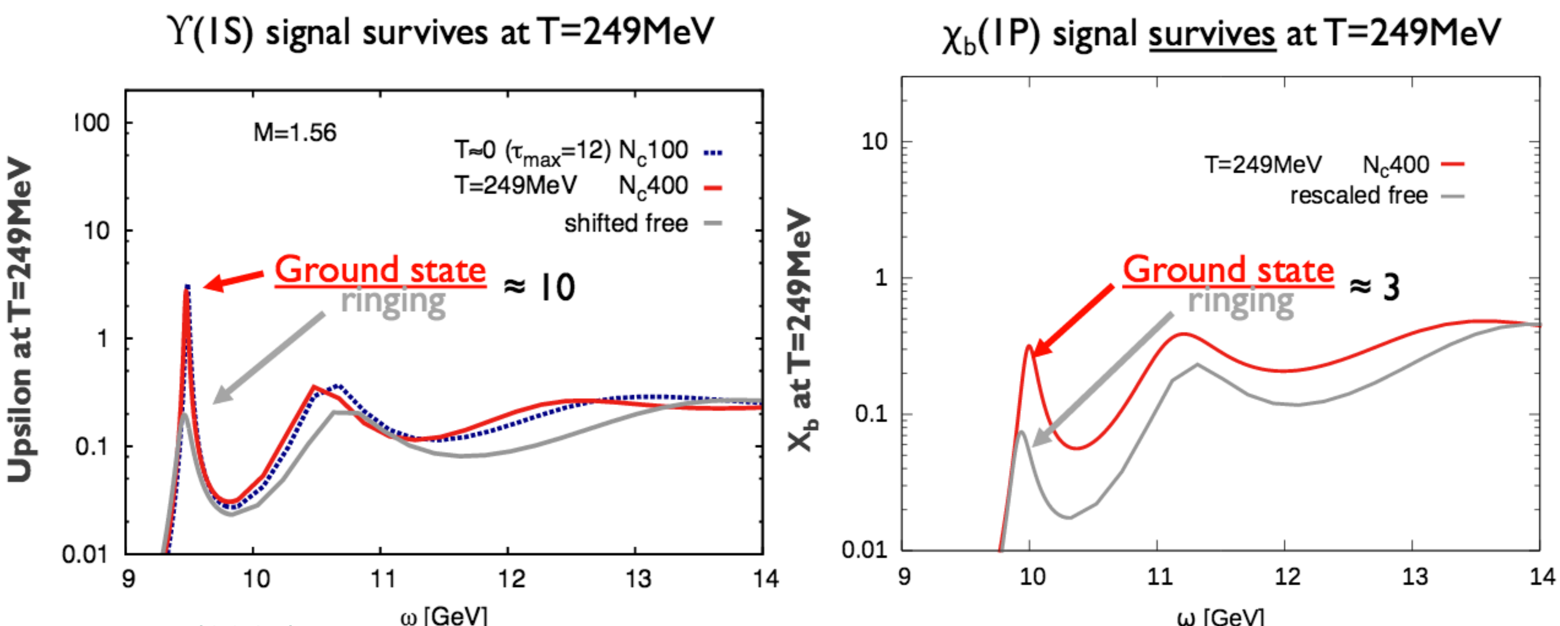
- S-wave ground state survives up to 1.9 T_c, P-wave ground state melts just above T_c

Quarkonia spectral functions

- Charmonium spectral functions in quenched approximation and preliminary studies with dynamical quarks yield consistent results: all charmonium states are dissociated for $T \gtrsim 1.5 T_c$

H. Ding et al., PRD (2012)
G. Aarts et al., PRD (2007)
WB: S. Borsanyi et al., JHEP (2014)

- Bottomonium (N_f=2+1, m_π=160 MeV), Bayesian method:



S. Kim et al. PRD (2015)

- S-wave ground state and P-wave ground state survive up to T~250 MeV

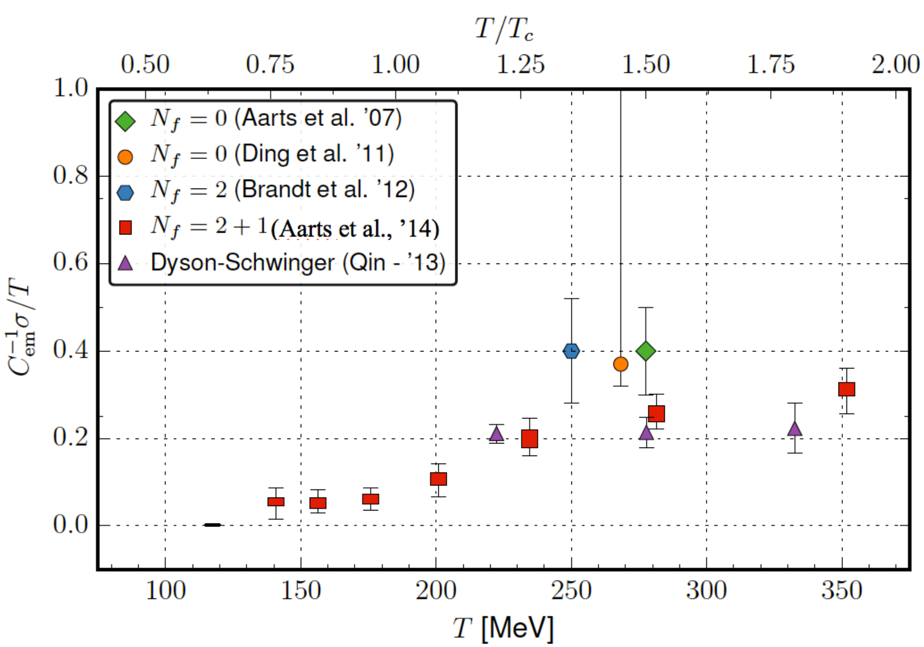
Talk by A. Rothkopf on Tuesday

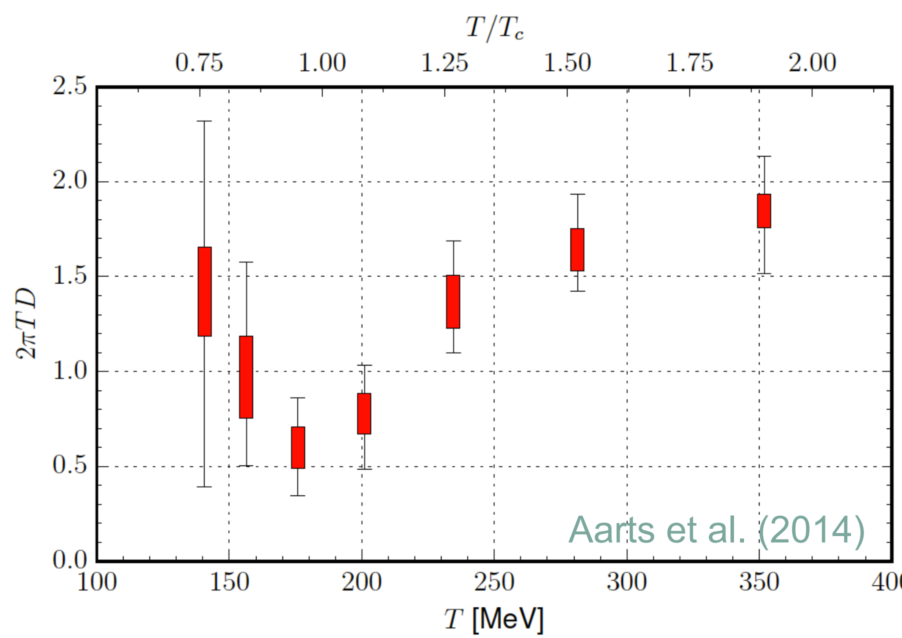
Electric conductivity and charge diffusion

- Definitions:

$$\sigma = \frac{C_{em}}{6} \lim_{\omega\to 0} \lim_{\mathbf{p}\to 0} \sum_{i=1}^{3} \frac{\rho^{ii}(\omega, \mathbf{p}, T)}{\omega}$$

$$D_Q = \sigma / \chi_2^Q$$

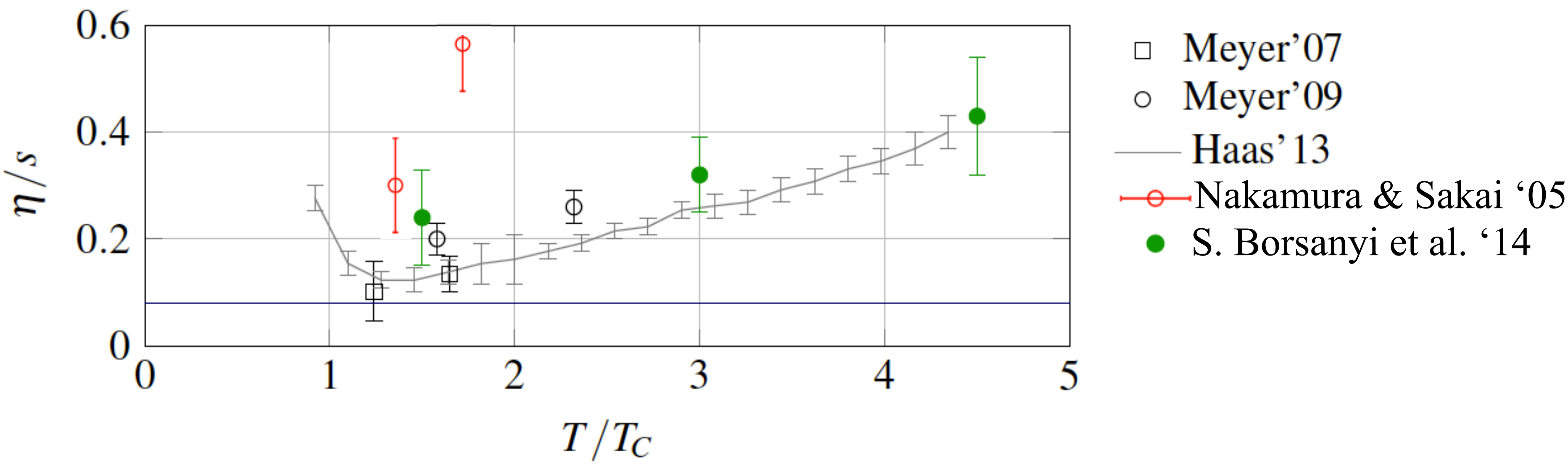




- Electric conductivity measures the response of the medium to small perturbations induced by an electromagnetic field

Viscosity

- Shear viscosity in the pure gauge sector of QCD

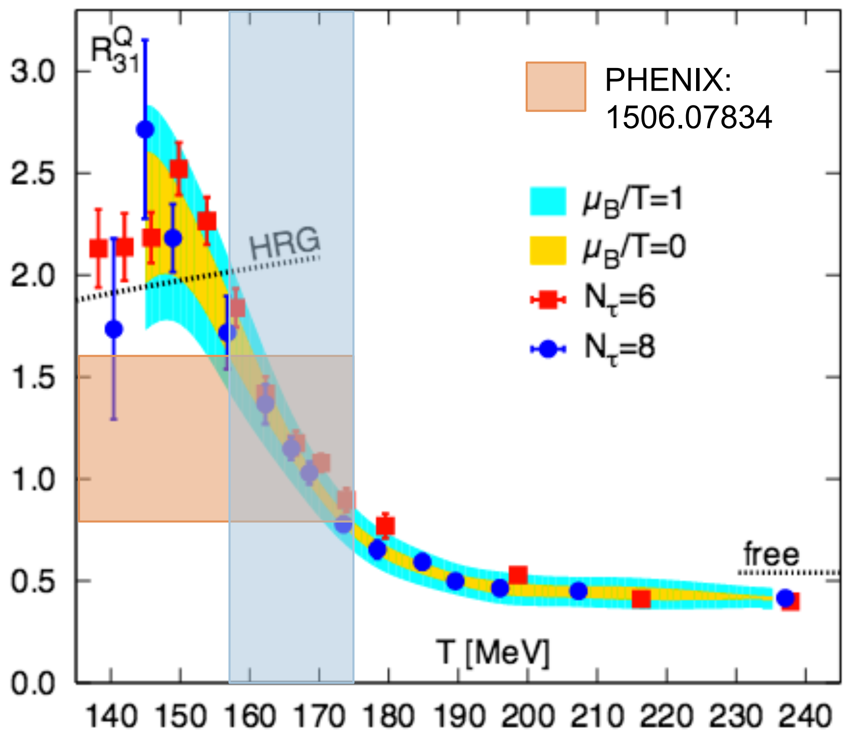


- Challenge: very low signal-to noise ratio for the Euclidean energy-momentum correlator

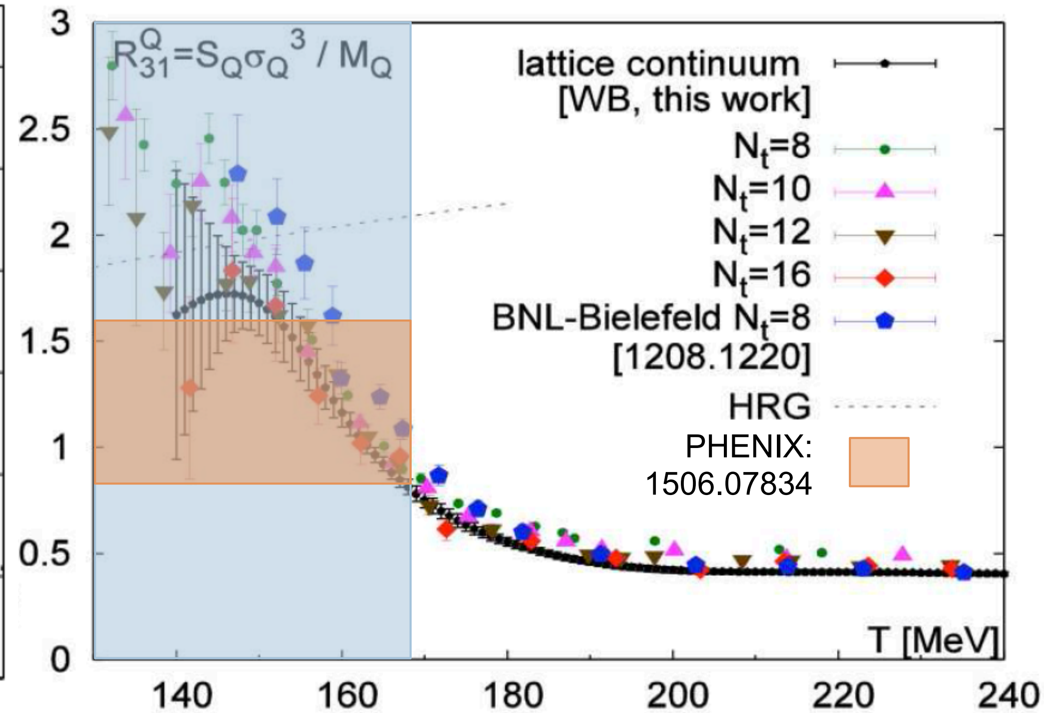
Conclusions

- Unprecedented precision in lattice QCD data allows a direct comparison to experiment for the first time
- QCD thermodynamics at μ_B=0 can be simulated with high accuracy
- Extensions to finite density are under control up to $O(\mu_B^6)$
- Challenges for the near future
  - Sign problem
  - Real-time dynamics

Freeze-out parameters from Q fluctuations



A. Bazavov et al. (2014)



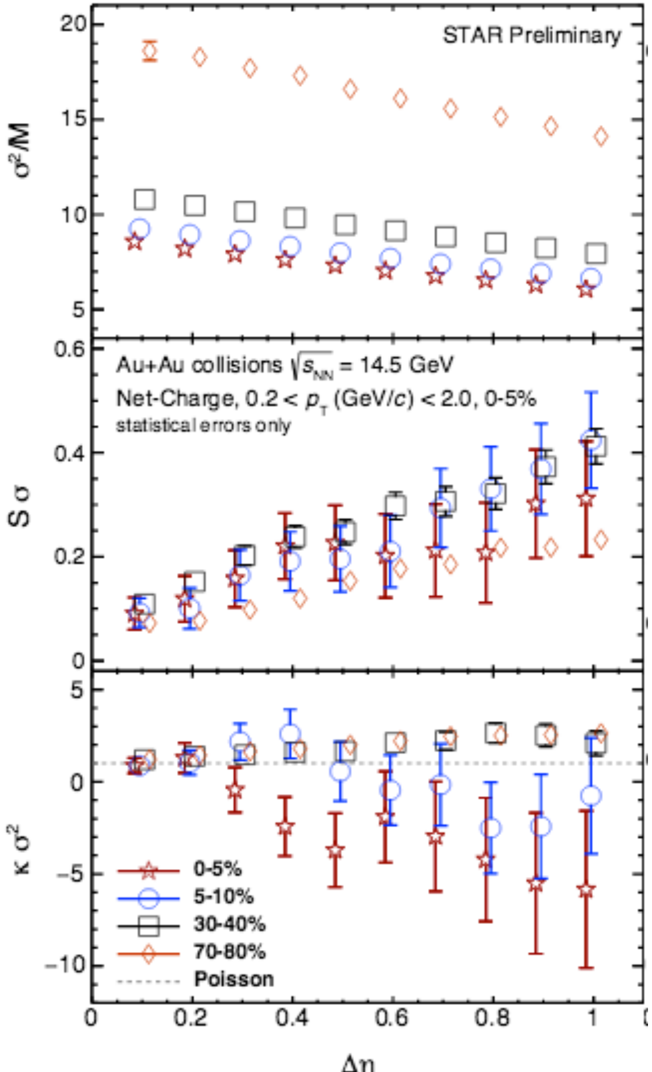
WB: Borsanyi et al. PRL (2013)

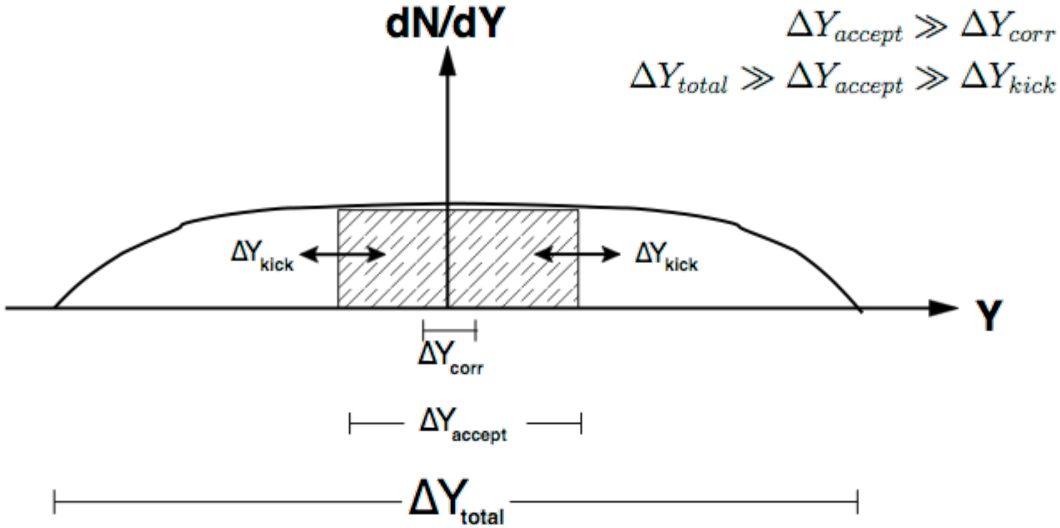
- Studies in HRG model: the different momentum cuts between STAR and PHENIX are responsible for more than 30% of their difference

F. Karsch et al., 1508.02614

- Using continuum extrapolated lattice data, lower values for T_f are found

Effects of kinematic cuts





V. Koch, 0810.2520

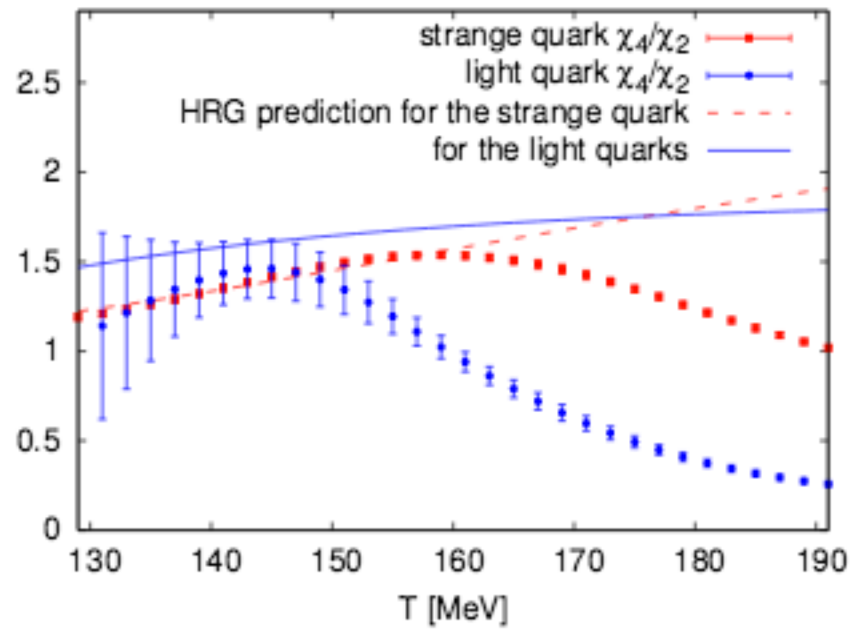
- Rapidity dependence of moments needs to be studied for $1<\Delta\eta<2$
- Difference in kinematic cuts between STAR and PHENIX leads to a 5% difference in T_f

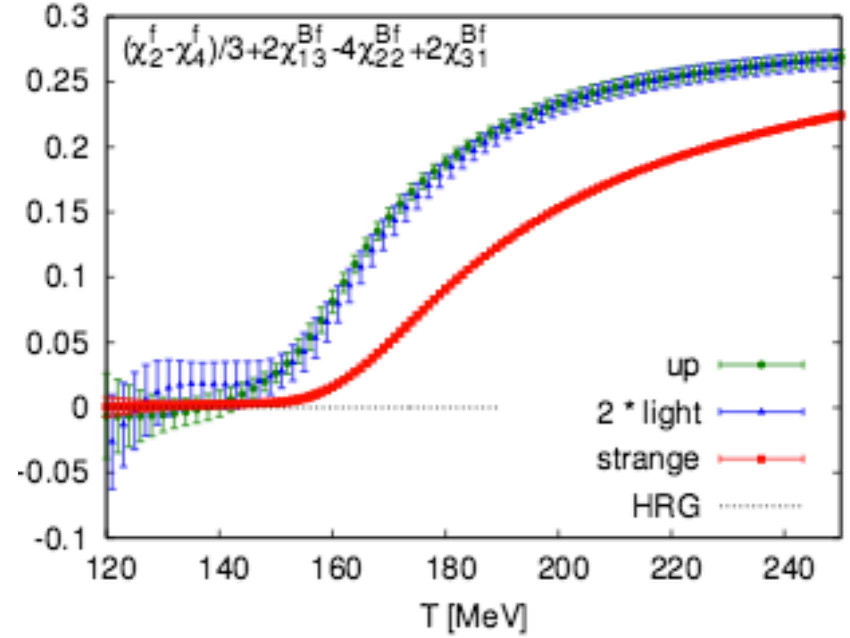
Talk by J. Thaeder on Monday

Talk by F. Karsch on Monday

Strangeness fluctuations

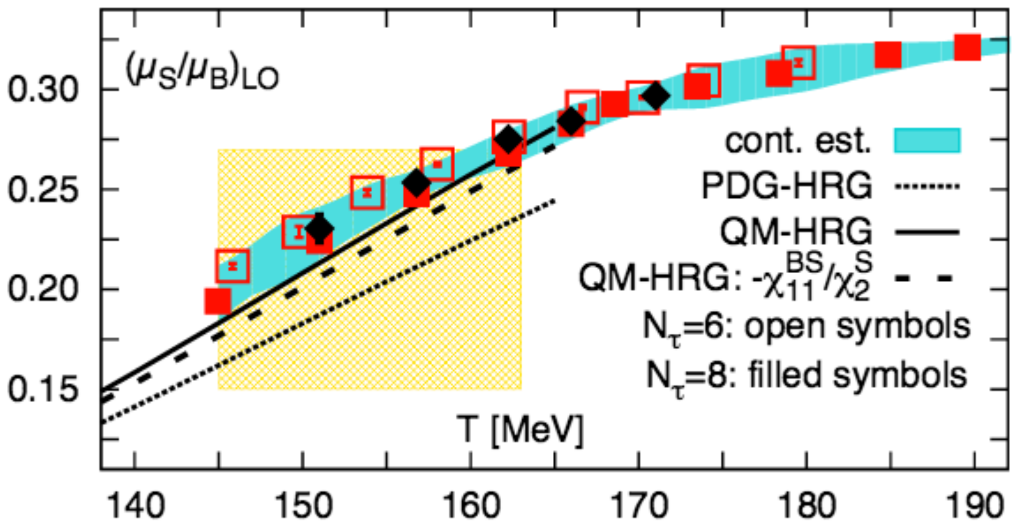
WB: R. Bellwied et al, PRL (2013)



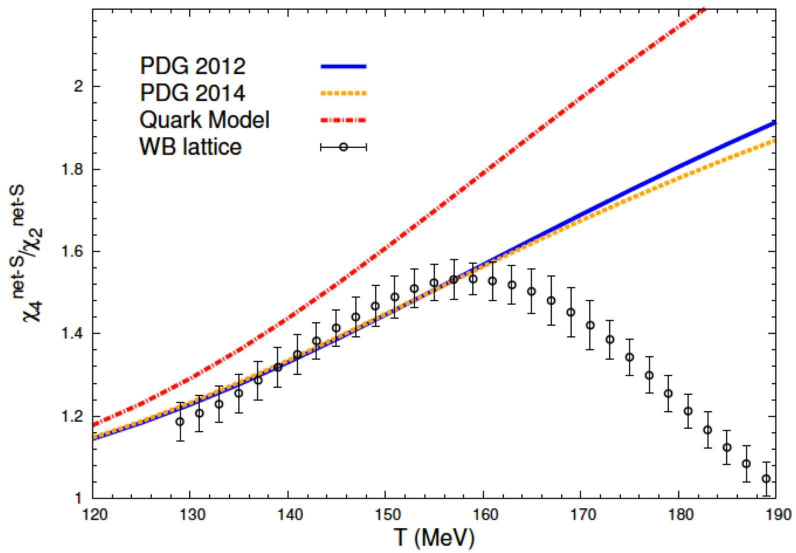


- Lattice data hint at possible flavor-dependence in transition temperature
- Possibility of strange bound-states above T_c?

Additional strange hadrons



A. Bazavov et al., PRL (2014)

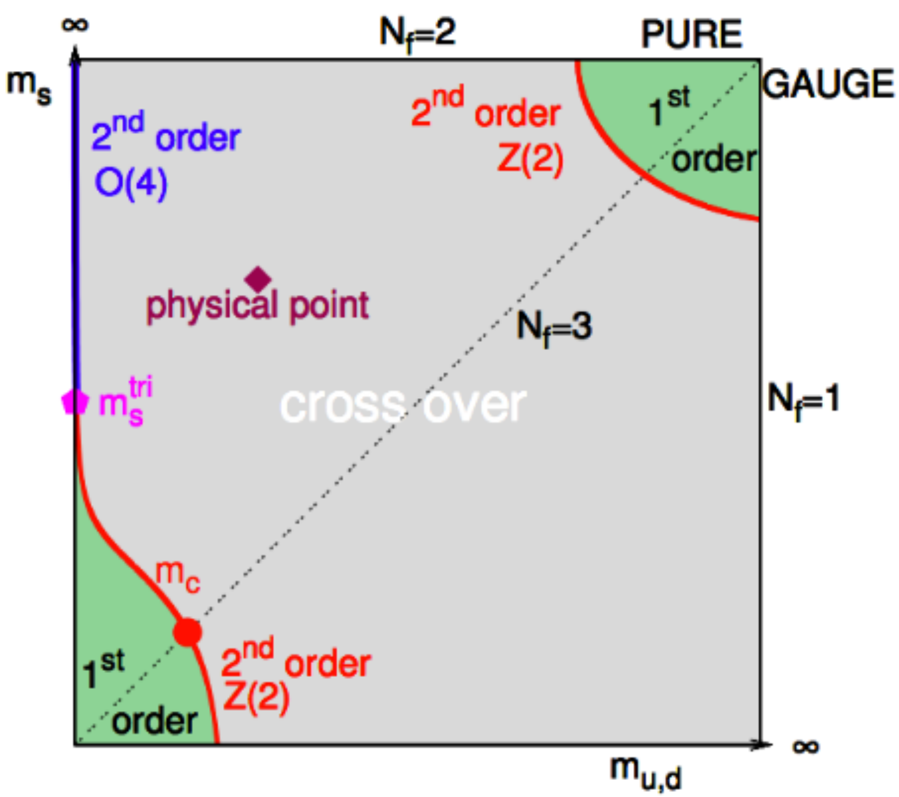


Poster by P. Alba

- Discrepancy between lattice and HRG for μ_S/μ_B can be understood by introducing higher mass states predicted by the Quark Model
- Discrepancy between QM predictions and lattice data for χ_4^S/χ_2^S needs to be understood
- Their effect on freeze-out conditions needs to be investigated taking into account their decay feed-down into stable states

P. M. Lo et al., 1507.06398

Columbia plot



- Pure gauge theory: Tc=294(2) MeV
  Francis et al., 1503.05652
- N_f=2 QCD at $m_\pi > m_\pi^{phys}$:
  - O(a) improved Wilson, N_t=16
  - m_π=295 MeV T_c=211(5) MeV
  - m_π=220 MeV T_c=193(7) MeV

  Brandt et al., 1310.8326
  - Twisted-mass QCD

  - m_π=333 MeV T_c=180(12) MeV

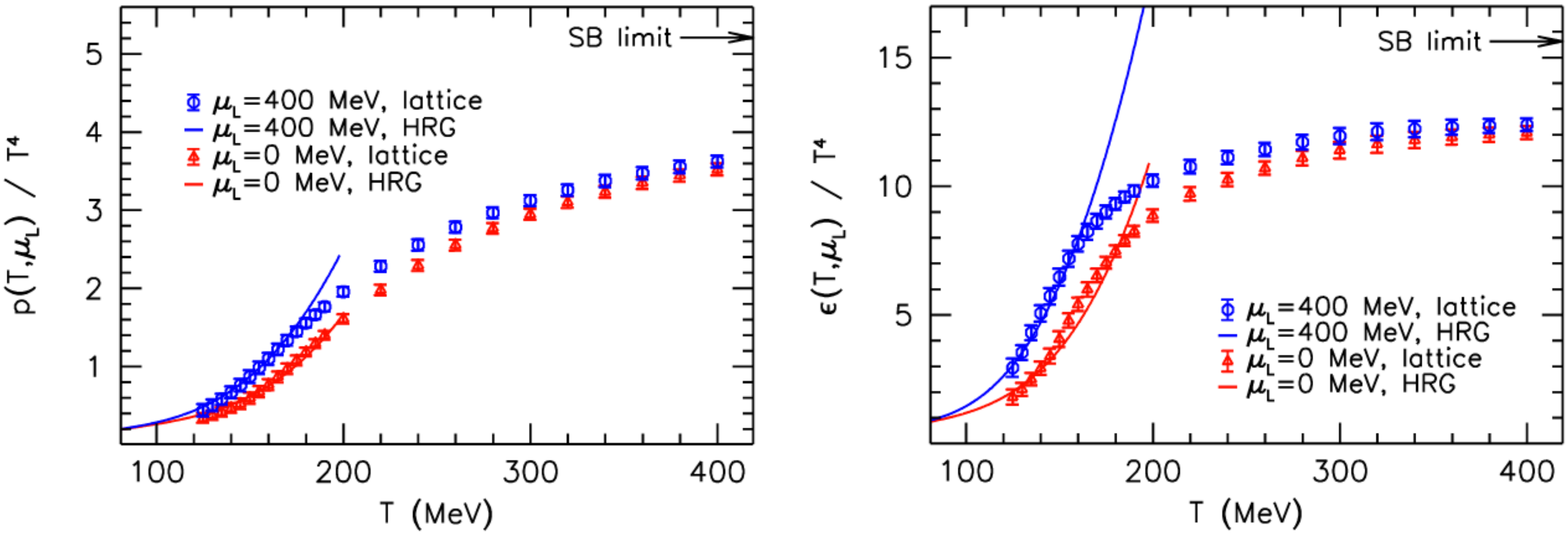
  Burger et al., 1412.6748
- N_f=2+1 O(a) improved Wilson
  - Continuum results

  Borsanyi et al., 1504.03676
- HISQ action, N_t=6, no sign of 1^{st} order phase transition at m_π=80 MeV
  HotQCD, 1312.0119, 1302.5740

Equation of state at $\mu_B>0$

- Expand the pressure in powers of μ_B (or $\mu_L=3/2(\mu_u+\mu_d)$)

$$\frac{p(T,\{\mu_i\})}{T^4} = \frac{p(T,\{0\})}{T^4} + \frac{1}{2}\sum_{i,j}\frac{\mu_i\mu_j}{T^2}\chi_2^{ij} \quad \text{with} \quad \chi_2^{ij} \equiv \frac{T}{V}\frac{1}{T^2}\frac{\partial^2 \log \mathcal{Z}}{\partial\mu_i\partial\mu_j}\bigg|_{\mu_i=\mu_j=0}$$



S. Borsanyi et al., JHEP (2012)

- Continuum extrapolated results at the physical mass