The QCD Critical Point and Related Observables

Marlene Nahrgang

October 1st, 2015

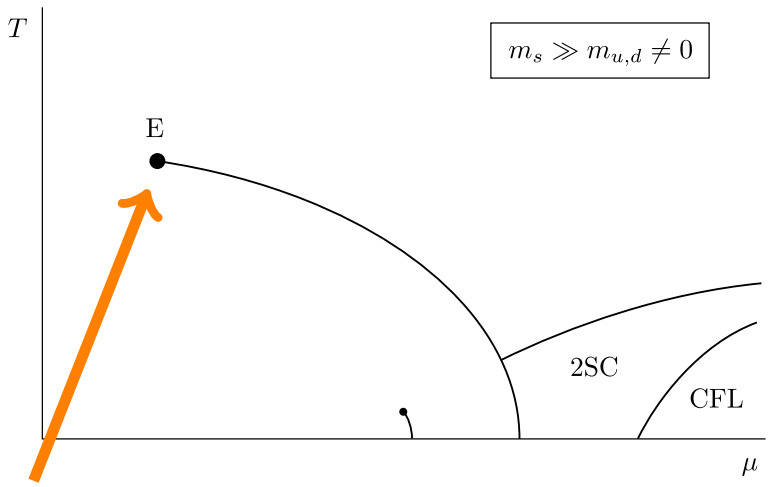
Quark Matter 2015, Kobe, Japan

Duke UNIVERSITY

DAAD Deutscher Akademischer Austauschdienst
German Academic Exchange Service

Ideas about the QCD phase diagram

K. Rajagopal, F. Wilczek, In: At the frontier of particle physics 3 (2000)



Critical End Point, "... which can be found in heavy-ion collision experiments"

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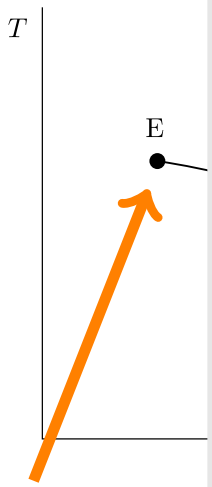
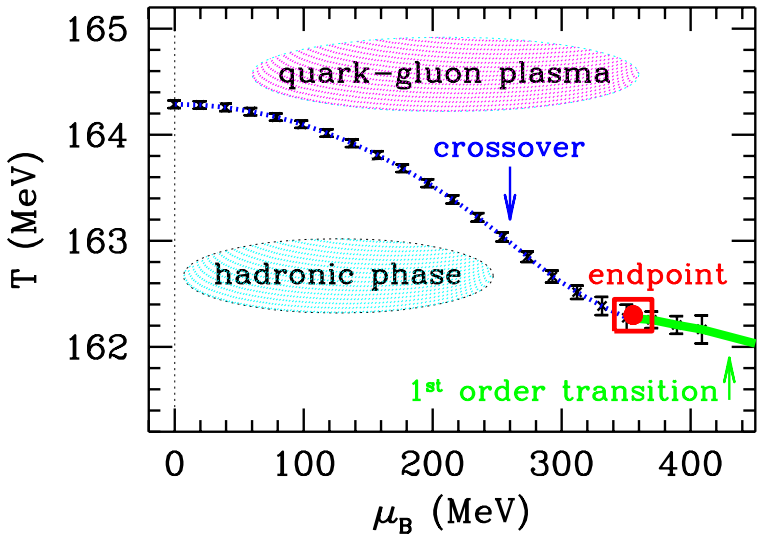


Critical End
in heavy

Z. Fodor, S. Katz, JHEP04 204 (2004)



"Most importantly one has to extrapolate to the continuum limit."

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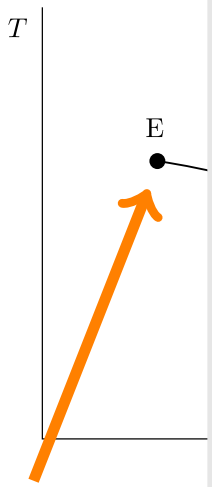

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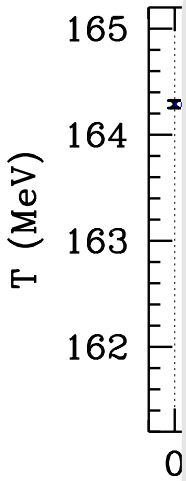

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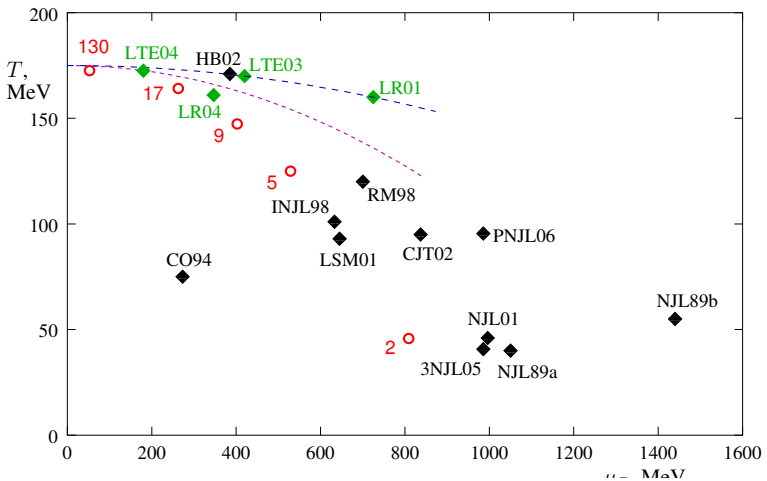
M. Stephanov, PoS LAT2006 (2006)



"...predictions for the location of the QCD critical point..."

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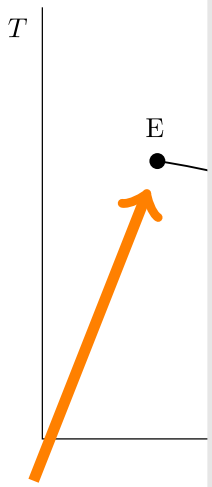

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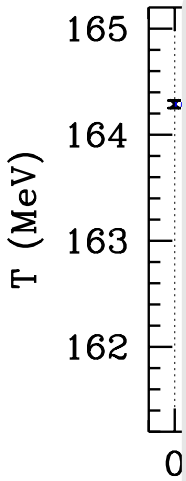

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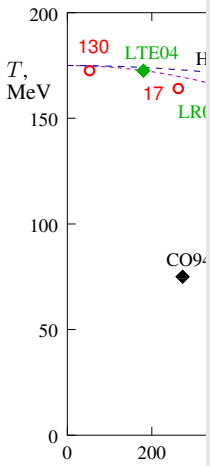

"...pr
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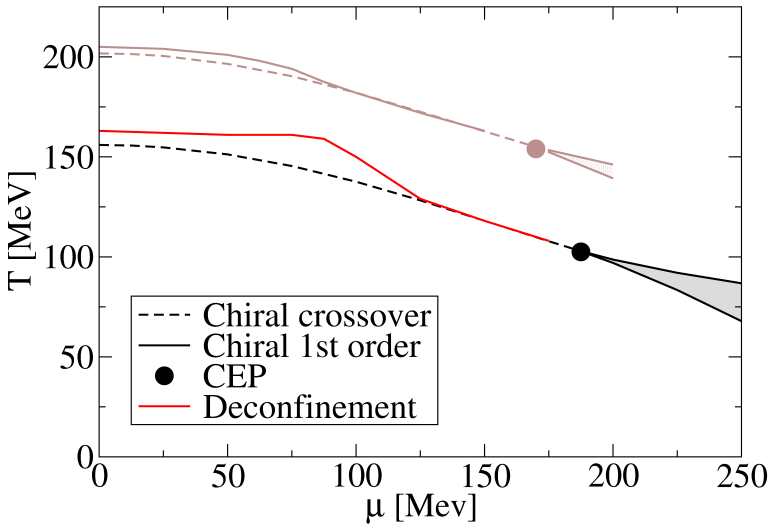
C. Fischer, J. Luecker, PLB718 (2013)



"We ... find a potential critical endpoint..."

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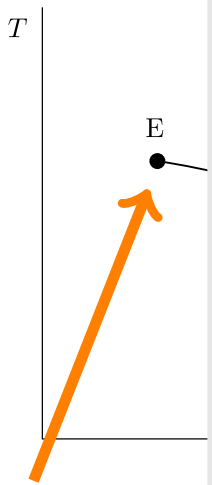

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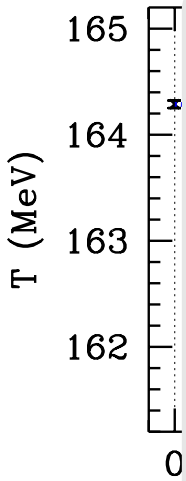

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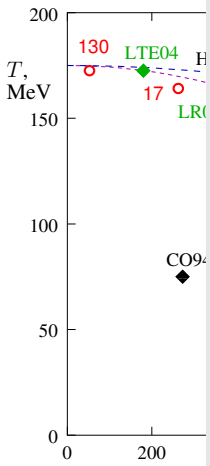

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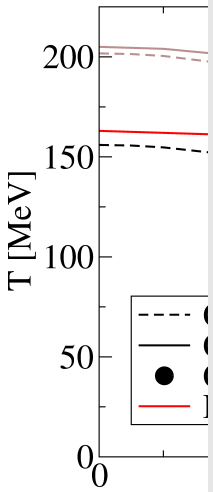
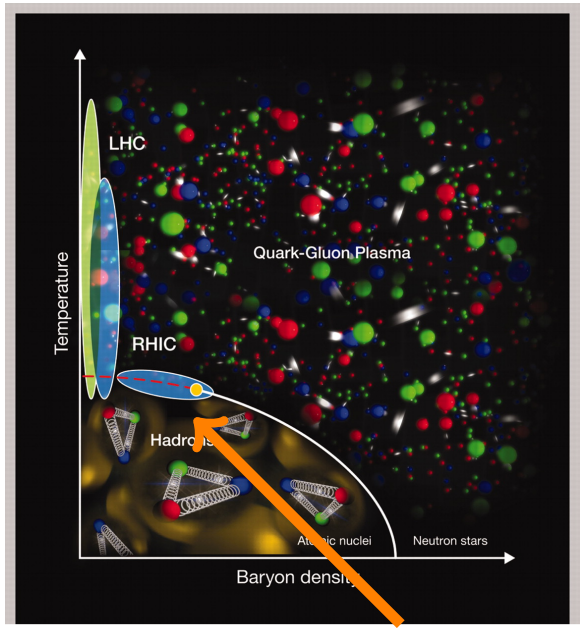


"We ... find

B. Jacak, B. Müller Science 337 (2012)



"There may be a critical point
in the phase diagram... "

The QCD phase diagram and heavy-ion collisions

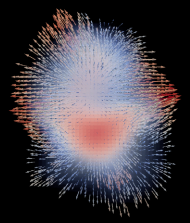
highly dynamical

short times

small volume

inhomogeneous

by H. Petersen



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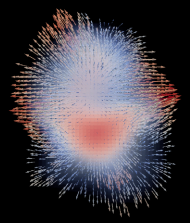

Heavy-ion collisions

QCD thermodynamics & critical point

homogeneous

infinite

long lived

static

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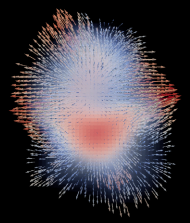

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Dynamical modeling

QCD thermodynamics & critical point

homogeneous

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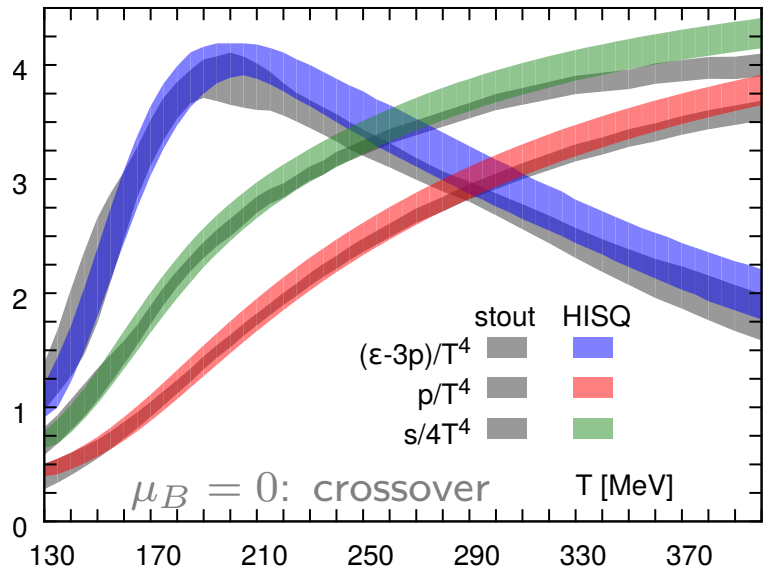
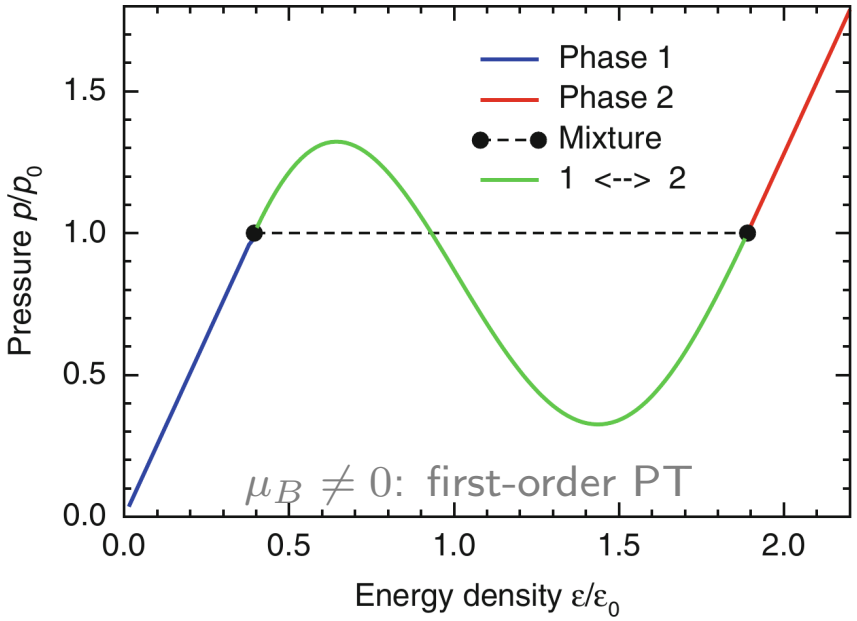
Phase transitions: the equation of state

HotQCD Coll. PRD90 (2014); Wuppertal-Budapest Coll. PLB730 (2014)



CBM physics book, Lecture Notes in Physics 814 (2011)



- Thermodynamic quantities change characteristically at the phase transition.
- Speed of sound $c_s^2 = (\partial p/\partial e)_S \rightarrow$ minimum around a crossover
  - $\Rightarrow$ vanishes at the first-order PT
- Compressibility $\kappa_S = -1/V(\partial V/\partial p)_S \rightarrow$ maximum around a crossover
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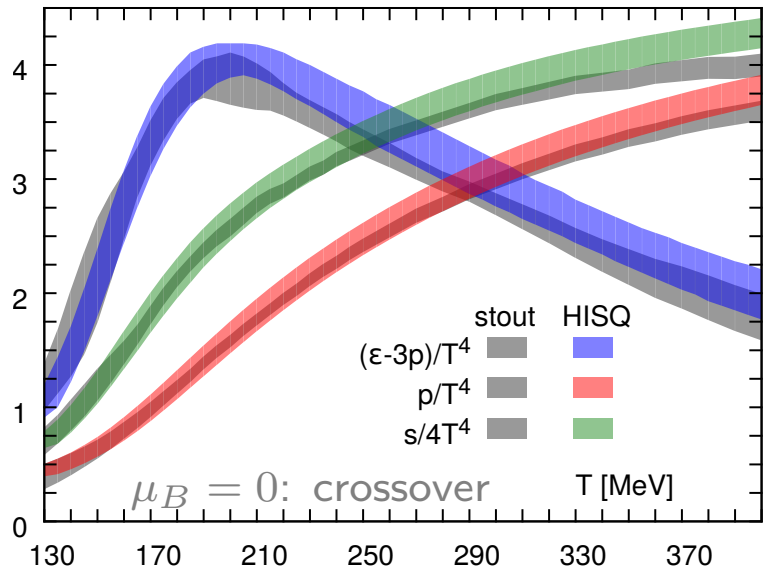

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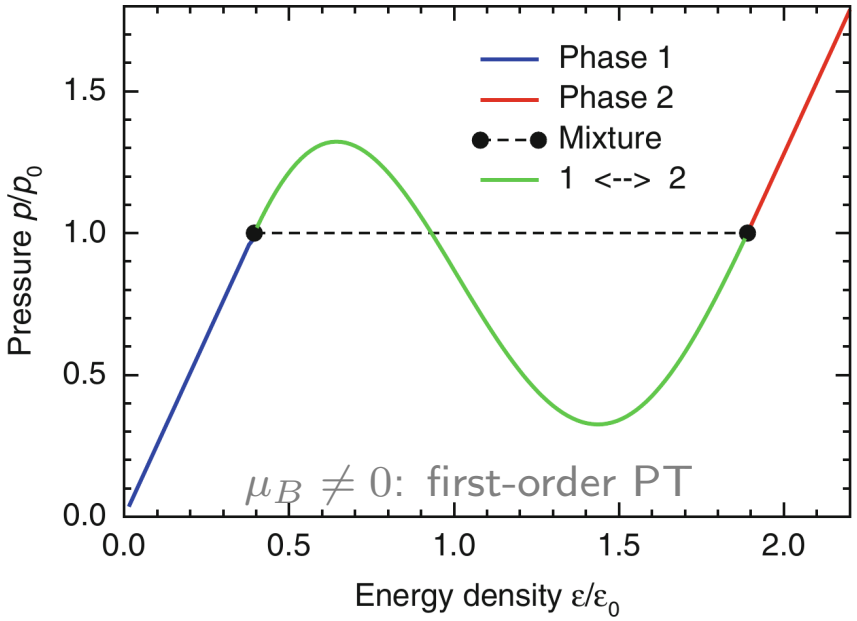

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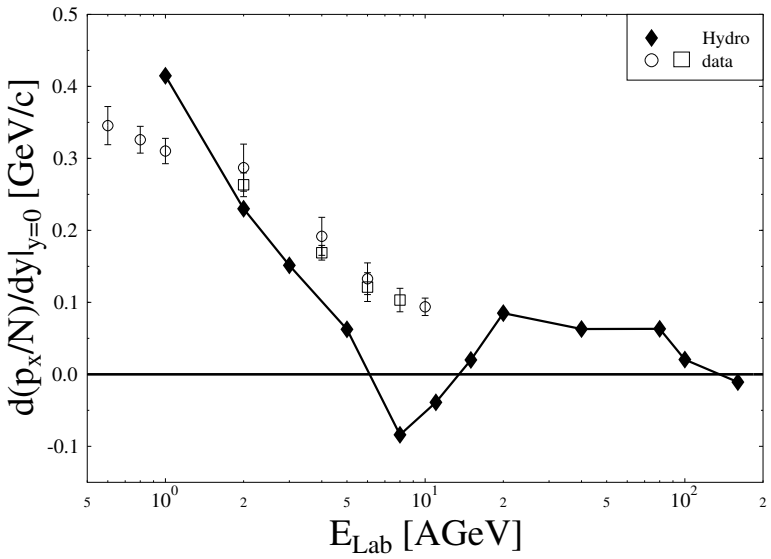
"softest point"
anomaly in the pressure

Phase transitions: the equation of state

- Modeling a phase transition dynamically is simple!
- Need to know the equation of state and transport coefficients $\Rightarrow$ fluid dynamics!



H. Stöcker, NPA780 (2005)

- A pronounced minimum in the slope of the directed flow v_1 is observed in a first-order phase transition.

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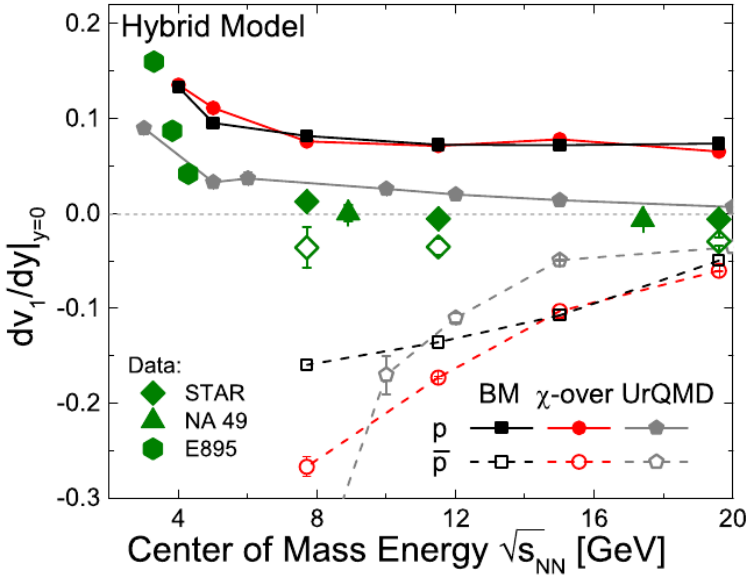

J. Steinheimer, J. Auvinen, H. Petersen, M. Bleicher, H. Stöcker, PRC89 (2014)

- A pronounced minimum in the slope of the directed flow v_1 is not observed in a first-order phase transition?
- In dynamical simulations: no clear sensitivity on a phase transition in the equation of state yet...

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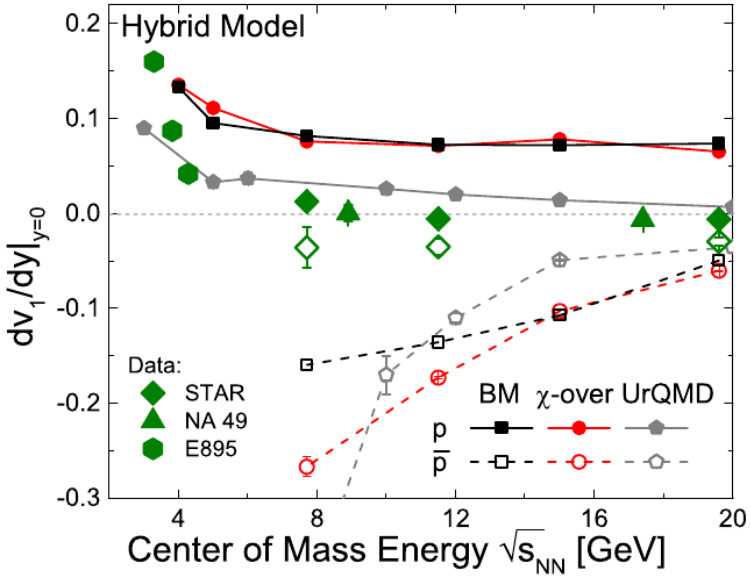

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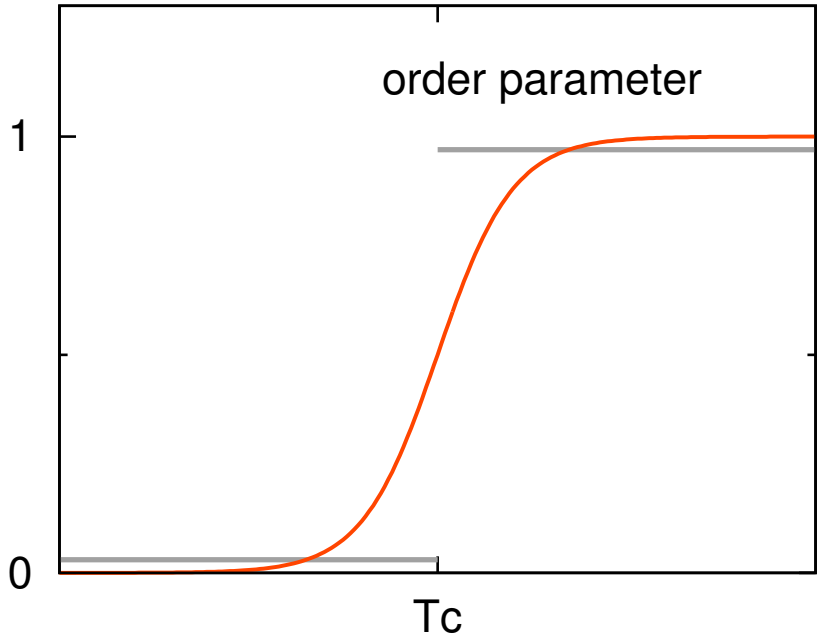
Fluctuations matter at the phase transition!

Phase transitions: order parameter & derivatives

- An order parameter changes characteristically at the phase transition - discontinuously or continuously.



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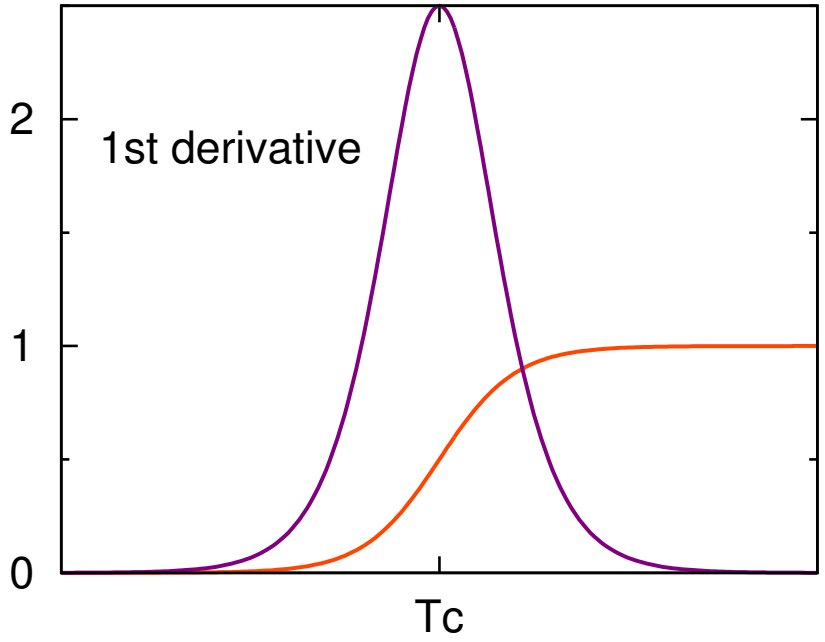

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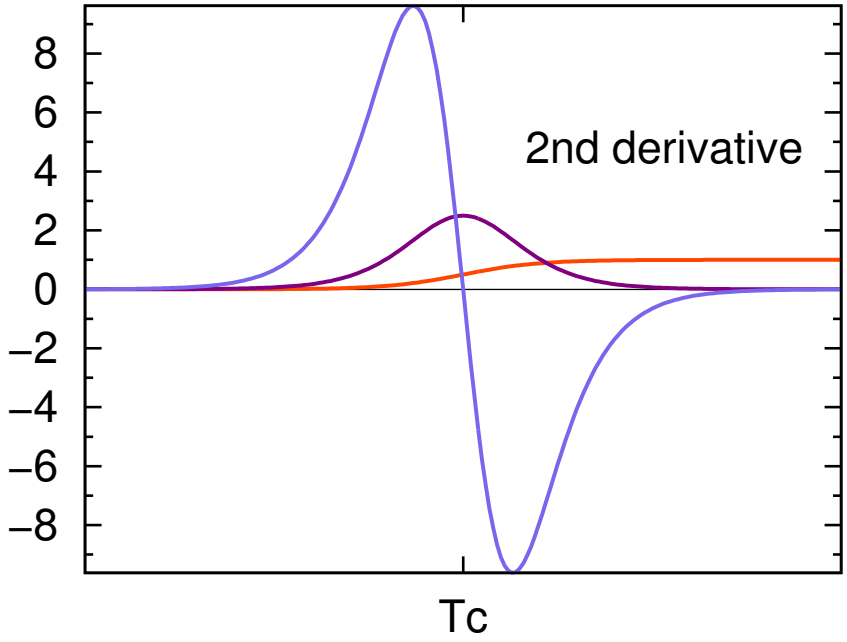

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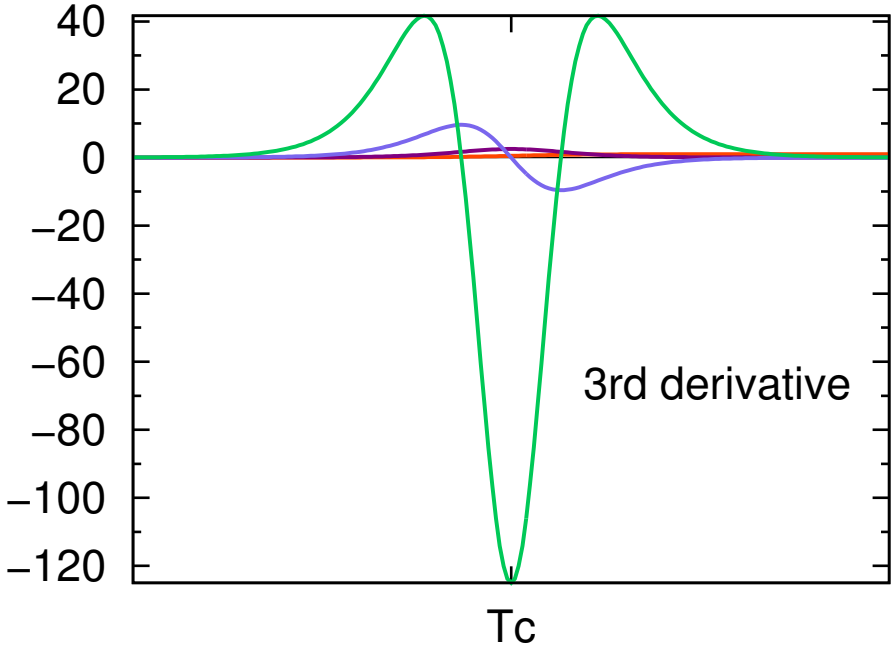

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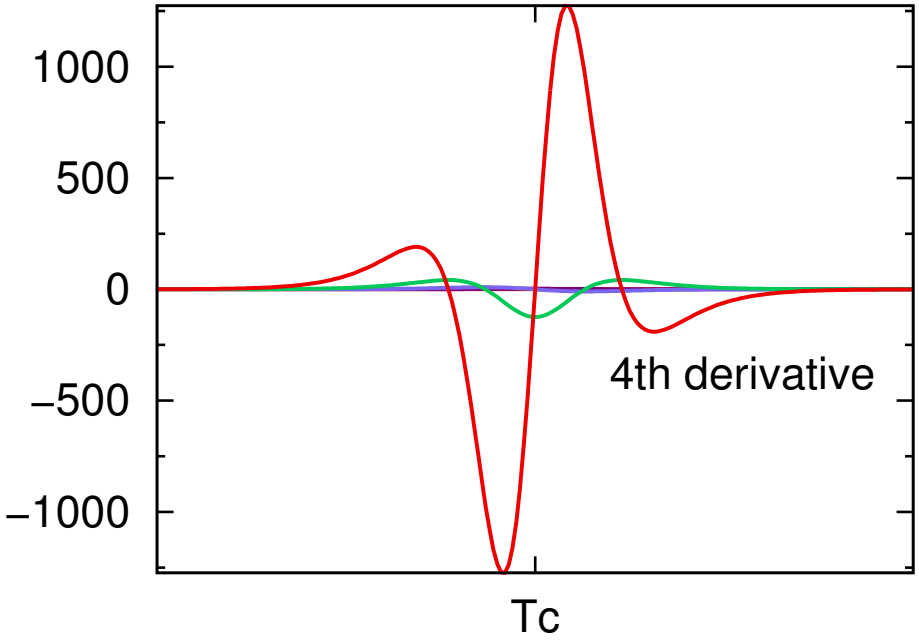

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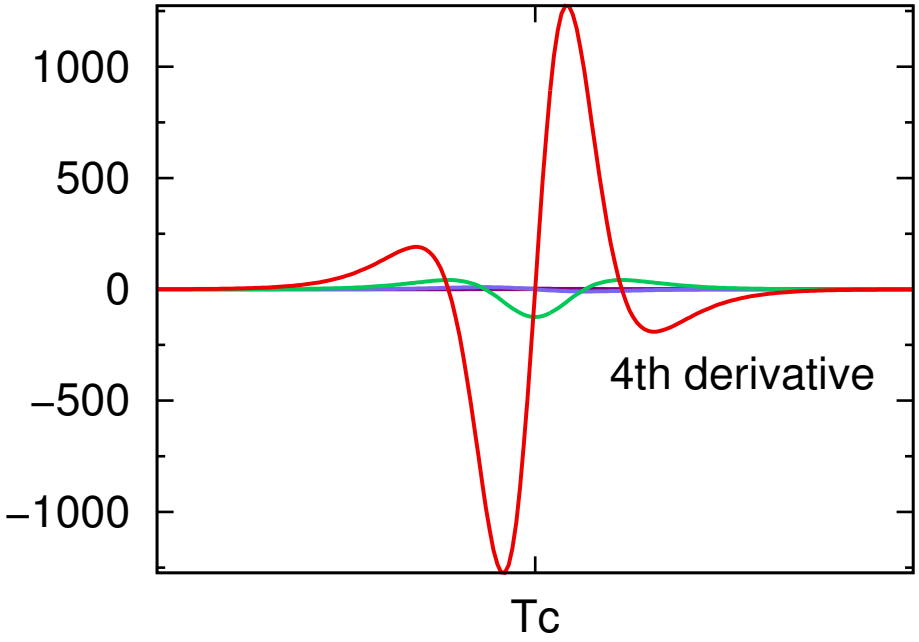

- Derivatives reveal more details!

Derivatives of thermodynamic quantities are related to fluctuations!

What are fluctuation observables?

- Susceptibilities $\chi_n = \left.\frac{\partial^n (P/T^4)}{\partial(\mu/T)^n}\right|_T$ relate to fluctuations in multiplicity

$$\chi_1 = \frac{1}{VT^3}\langle N\rangle\,, \quad \chi_2 = \frac{1}{VT^3}\langle(\Delta N)^2\rangle\,, \quad \chi_3 = \frac{1}{VT^3}\langle(\Delta N)^3\rangle\,,$$

$$\chi_4 = \frac{1}{VT^3}\langle(\Delta N)^4\rangle_c \equiv \frac{1}{VT^3}\left(\langle(\Delta N)^4\rangle - 3\langle(\Delta N)^2\rangle^2\right)\,.$$

- To zeroth-order in volume fluctuations:

$$\frac{\chi_2}{\chi_1} = \frac{\sigma^2}{M} \qquad \frac{\chi_3}{\chi_2} = S\sigma \qquad \frac{\chi_4}{\chi_2} = \kappa\sigma^2$$

variance Skewness Kurtosis

- M, σ^2, S and κ are obtained from measured event-by-event multiplicity distributions.

STAR Coll. PRL112 (2014), PRL113 (2014); PHENIX Coll. arxiv:1506.07834

Non-critical effects on fluctuation observables

- Limited acceptance & detector efficiency. A. Bzdak, V. Koch, PRC86 (2012); PRC91 (2015)
- Isospin randomization. M. Kitazawa, M. Asakawa, PRC85, PRC86 (2012)
- Volume fluctuations V. Skokov, B. Friman, K. Redlich, PRC88 (2013)
  ($\rightarrow$ strongly intensive measures).
  E. Sangaline, arxiv:1505.00261; M. Gorenstein, M. Gazdzicki, PRC84 (2011)
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$\Rightarrow$ These effects are or can be included in microscopic transport models, e.g. UrQMD, (P)HSD, or hybrid models = valuable baseline studies!

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Need to be well understood!

Phase transitions: fluctuations

Critical point

- Universal behavior of the long-wavelength modes.
- Correlation length diverges $\xi \to \infty$.
- Fluctuations of the critical mode σ diverge.
- Higher moments more sensitive to ξ:

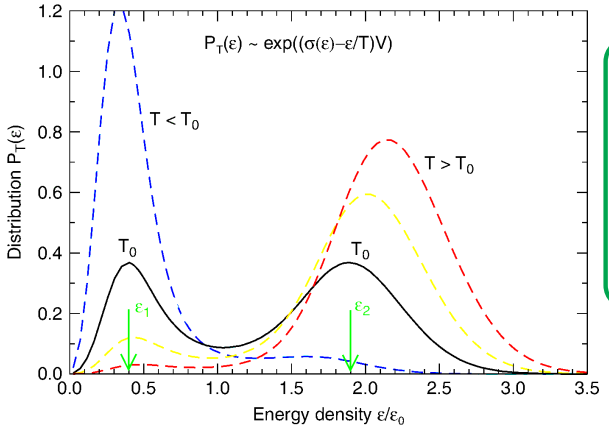
$$\langle \Delta\sigma^2 \rangle \propto \xi^2 \,, \quad \langle \Delta\sigma^3 \rangle \propto \xi^{9/2}$$

$$\langle \Delta\sigma^4 \rangle_c \propto \xi^7 \,.$$

- For QCD: parameters from 3d Ising universality class.
- Relaxation time $\tau_{\rm rel} \propto \xi^z$ diverges $\Rightarrow$ critical slowing down!

First-order phase transition

- Coexistence of two stable thermodynamic phases.
- Metastable states above and below $T_c \Rightarrow$ supercooling and -heating.



CBM physics book

- Nucleation & spinodal decomposition $\Rightarrow$ domain formation.

P. Hohenberg, B. Halperin, RMP49 (1977); T. Hatsuda, T. Kunihiro, PRL55 (1985); L Csernai, I Mishustin, PRL74 (1995); M. Stephanov, K. Rajagopal, E. Shuryak, PRL81 (1998), PRD60 (1999); S. Jeon, V. Koch, PRL83 (1999); B. Berdnikov and K. Rajagopal, PRD61 (2000); Y. Hatta, T. Ikeda, PRD67 (2003); M. Stephanov, PRL102 (2009); J. Randrup, PRC79 (2009), PRC82 (2010); M. Stephanov, PRL107 (2011)

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LARGE fluctuations in equilibrium

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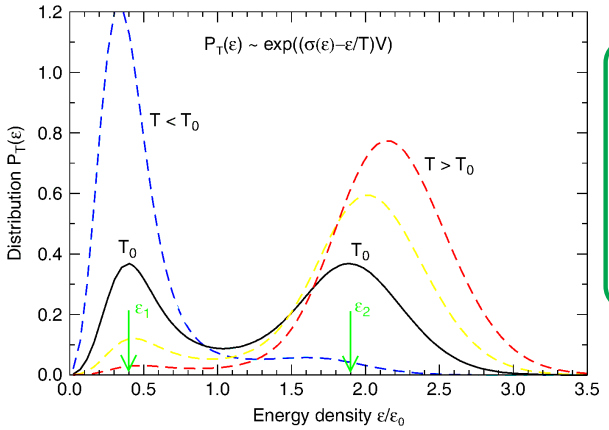

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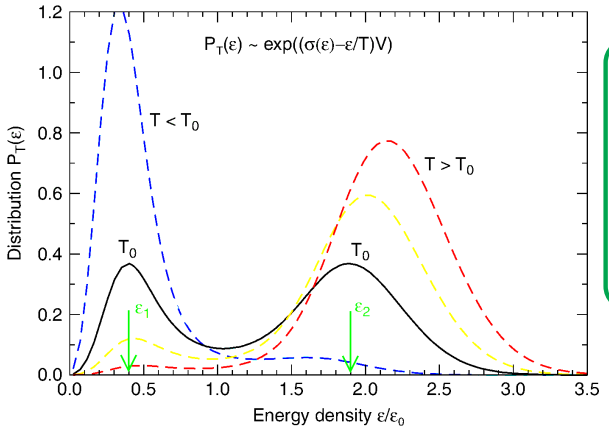

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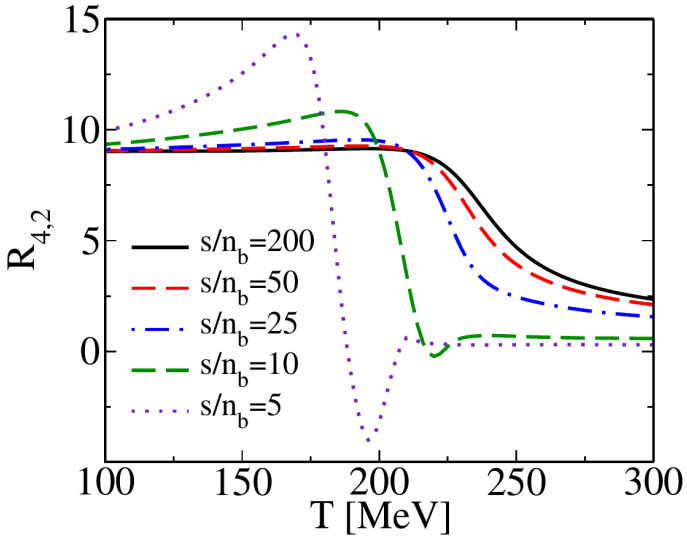
Critical fluctuations in QCD effective models

- Excellent opportunity to study critical fluctuations in conserved-charge densities at finite μ_B.

V. Skokov, B. Friman, K. Redlich, PRC83 (2011)



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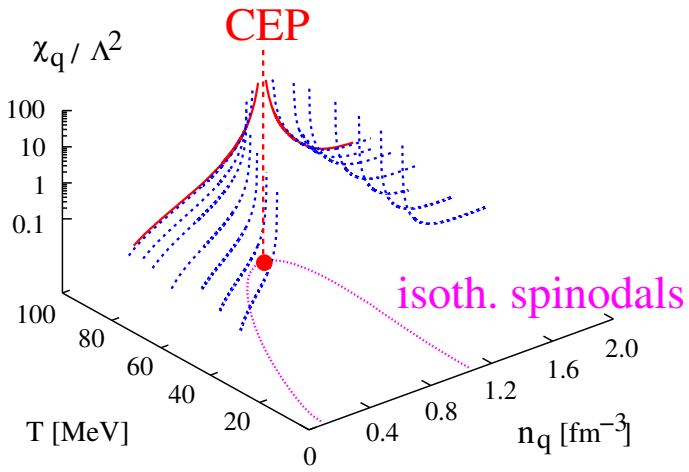

C. Sasaki, K. Redlich, PRD77 (2008)

Divergence of fluctuations along the spinodal lines.

C. Ratti, M. Thaler, W. Weise, PRD73 (2006); B.-J. Schaefer, J. Pawlowski, J. Wambach, PRD76 (2007); C. Sasaki, B. Friman, K. Redlich, PRD75 (2007); C. Ratti, S. Roessner, W. Weise, PLB649 (2007); K. Fukushima, PRD77 (2008); E. Nakano, B.-J. Schaefer, B. Stokic, B. Friman, K. Redlich, PLB682 (2010); T. Herbst, J. Pawlowski, B.-J. Schaefer, PLB696 (2011); K. Morita, V. Skokov, B. Friman, K. Redlich, EPJC74 (2014)

Critical fluctuations in QCD effective models

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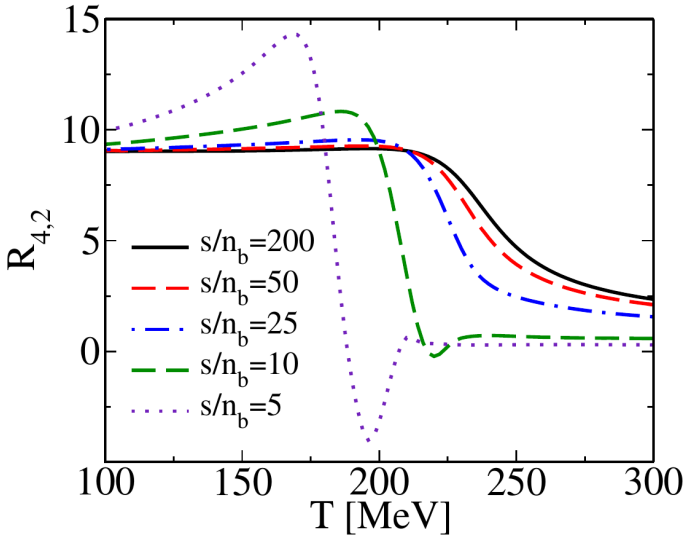

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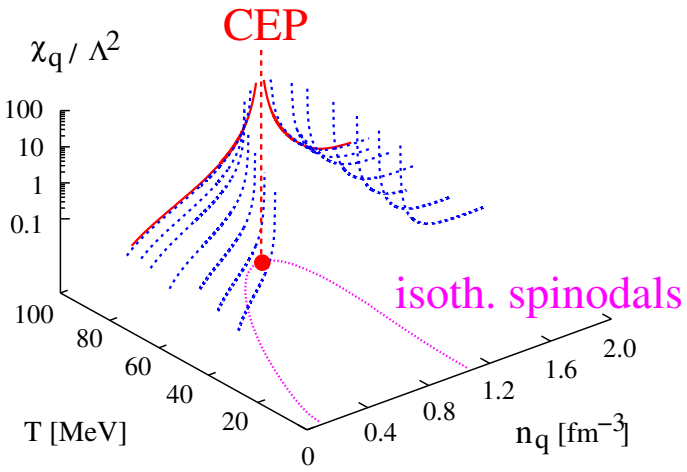

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Clear signals for the phase transition in effective models!

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Critical net-proton fluctuations - phenomenology

IDEA: couple order parameter to measurable particles: $g_p \bar{p} \sigma p$

M. Stephanov, K. Rajagopal, E. Shuryak, PRL81 (1998), PRD60 (1999); C. Athanasiou, K. Rajagopal, M. Stephanov, PRD82 (2010)

- Mass change in a Hadron Resonance Gas: $m_h \to m_h + g_h \Delta\sigma$.
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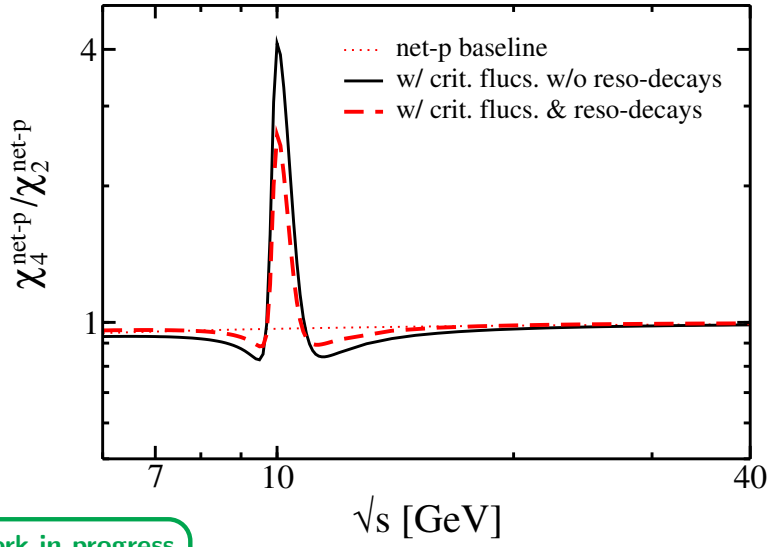
- Fluctuations in net-protons at chemical freeze-out.
- Critical fluctuations are reduced but survive when resonance decays are included!



M. Bluhm, MN, work in progress

- Particle emission during Cooper-Frye freeze-out over a hypersurface from fluid dynamical evolution. H. Song, L. Jiang, work in progress

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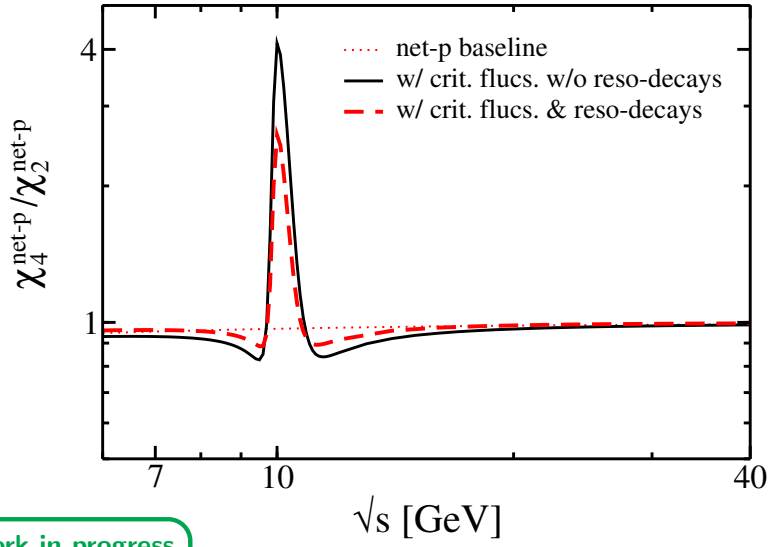

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Still no dynamical fluctuations...

Toward dynamics: memory effects

S. Mukherjee, R. Venugopalan, Y. Yin, PRC92 (2015)

IDEA: real-time evolution of non-Gaussian cumulants in the scaling regime, where

$$L_{\rm micro} \ll \xi \ll L_{\rm sys}$$

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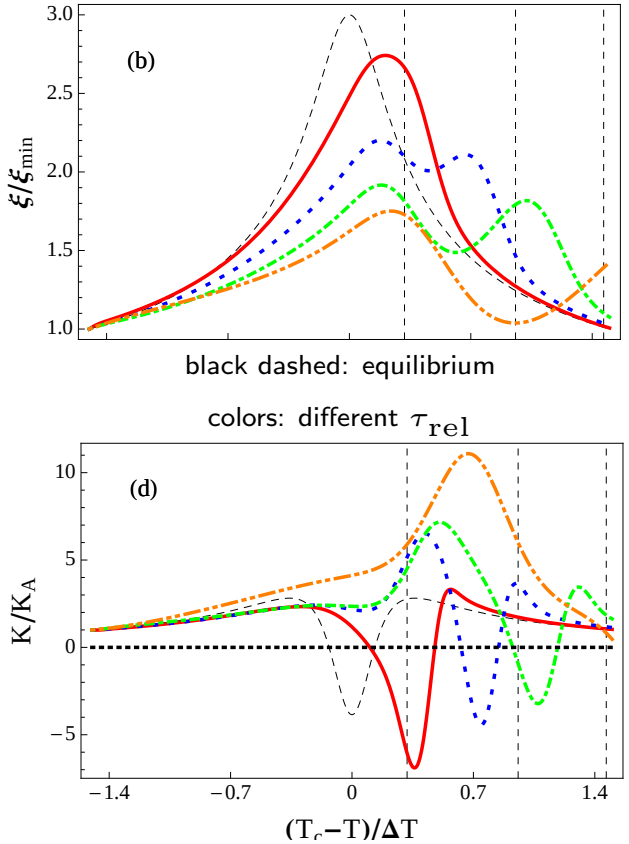
- Magnitude and sign can be different in non-equilibrium compared to equilibrium expectations!
- Different trajectories, chemical freeze-out conditions and $\tau_{\rm rel}$ can give similar results.



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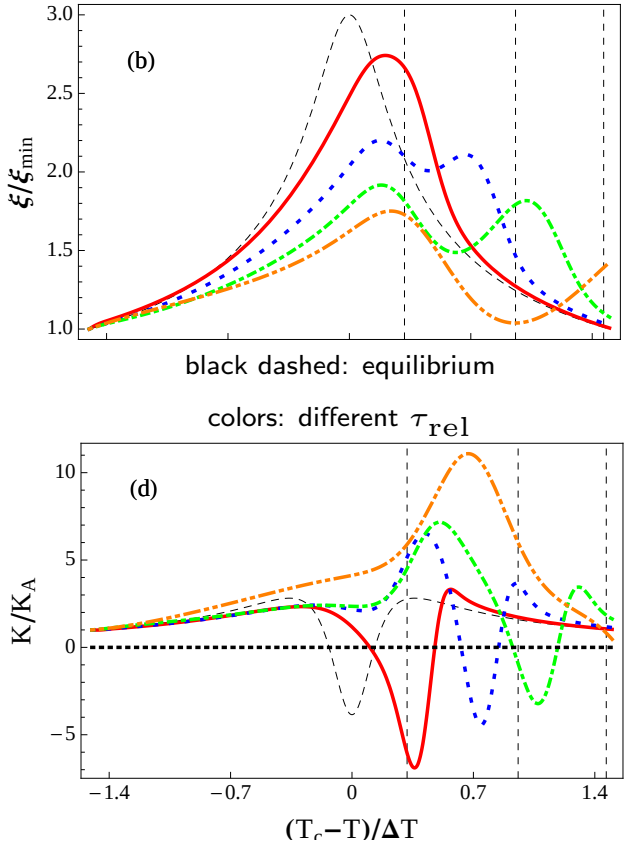


black dashed: equilibrium

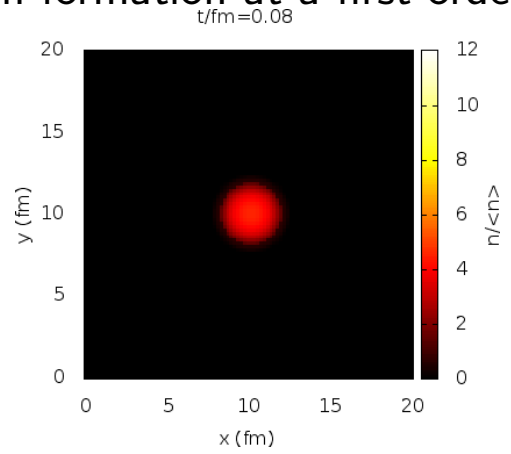
colors: different $\tau_{\rm rel}$

Needs dynamical space-time evolution!

Dynamical modeling of fluctuations

IDEA: explicit propagation of order parameters coupled to QGP evolution.

domain formation at a first-order PT:



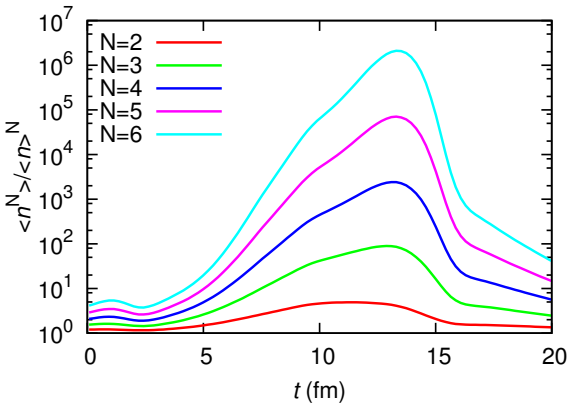
- Relaxation equation for order parameter:

$$\partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma} + g\rho_s + \eta \partial_t \sigma = \xi$$

- Interaction from effective (P)QM model.
- Fluctuations due to noise ξ.
- Coupling to fluid dynamical expansion:

$$\partial_\mu T^{\mu\nu}_{\mathrm{q}} = S^\nu = -\partial_\mu T^{\mu\nu}_\sigma \,, \quad \partial_\mu N^\mu_{\mathrm{q}} = 0$$

- Stochastic source term $\Rightarrow$ dynamical evolution of fluctuations!



Nonequilibrium chiral fluid dynamics (NχFD)

MN, S. Leupold, I. Mishustin, C. Herold, M. Bleicher, PRC84 (2011); PLB711 (2012); JPG40 (2013), C. Herold, MN, I. Mishustin, M. Bleicher PRC87 (2013); NPA925 (2014); C. Herold, MN, Y. Yan, C. Kobdaj JPG41 (2014) & work in progress

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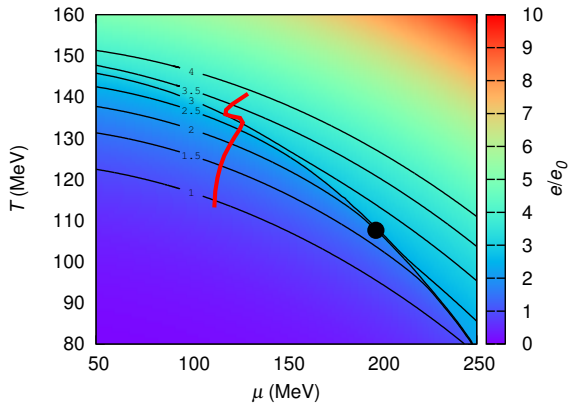
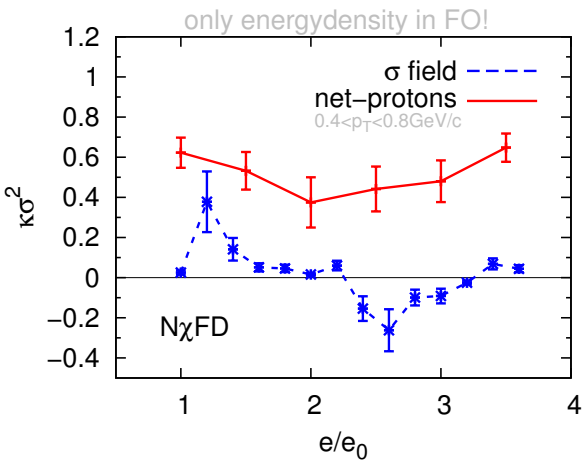
Event-by-event fluctuations on a trajectory near the critical point:





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Fluid dynamical fluctuations

Conventional fluid dynamics propagates thermal averages of the energy density, pressure, velocities, charge densities, etc.

However, ...

- ... already in equilibrium there are thermal fluctuations
- ... the fast processes, which lead to local equilibration also lead to noise!

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Y. Minami, T. Kunihiro, PTP122 (2010); P. Kovtun, G. Moore, P. Romatschke, PRD84 (2011); J. Kapusta, B. Müller, M. Stephanov PRC**85** (2012); C. Chafin, T. Schäfer, PRA87 (2013); P. Romatschke, R. Young, PRA87 (2013); P. Kovtun, G. Moore, P. Romatschke, JHEP1407 (2014); C. Young, J. Kapusta, C. Gale, S. Jeon, B. Schenke, PRC91 (2015)

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Important at the critical point: true critical mode is a fluid dynamical density!

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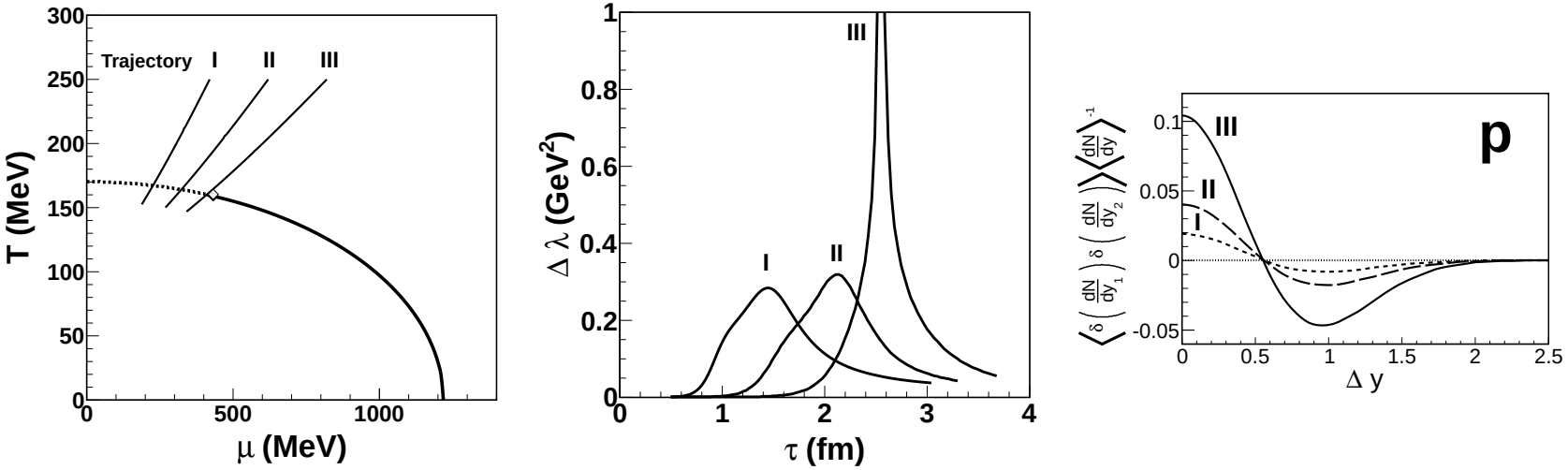
Fluid dynamical fluctuations

Bjorken expansion example with a critical point:

J. Kapusta, J. Torres-Rincon PRC86 (2012)



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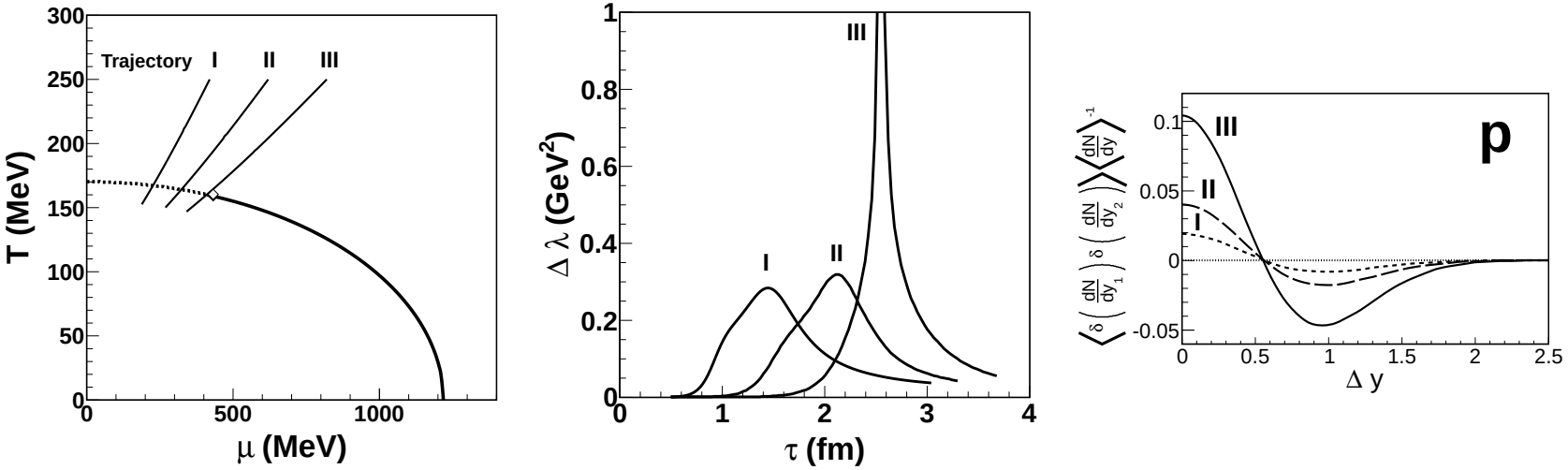

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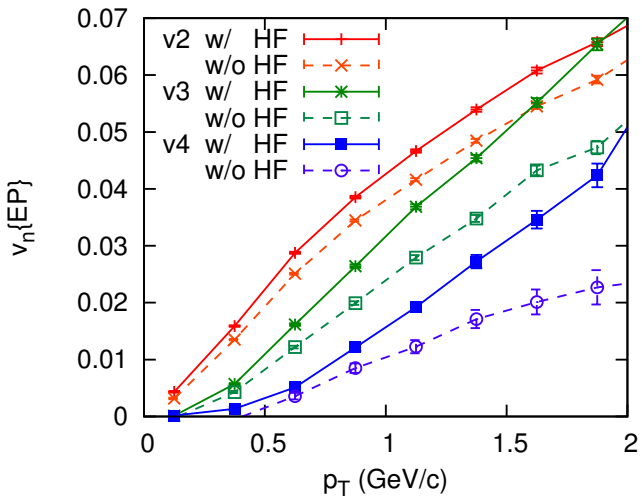
How to implement in a $3+1$d relativistic causal fluid dynamical evolution?

Fluid dynamical fluctuations

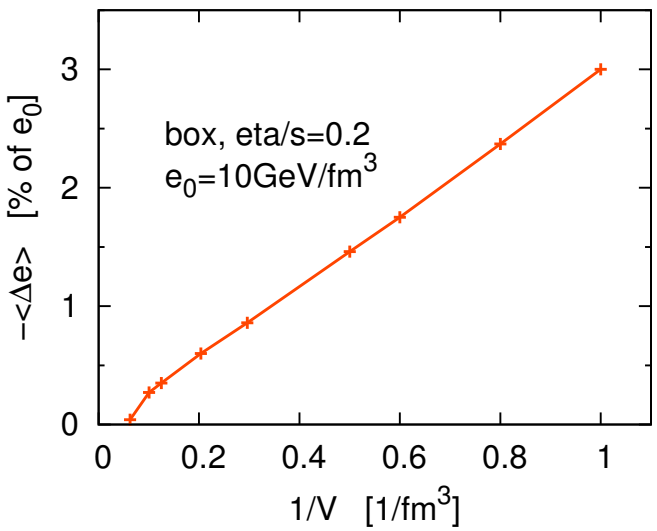
$$\partial_\mu T^{\mu\nu} = \partial_\mu \left(T^{\mu\nu}_{\rm eq} + \Delta T^{\mu\nu}_{\rm visc} + \Xi^{\mu\nu} \right) = 0$$

- Enhancement of flow due to additional fluctuations?



talk by K. Murase, T. Hirano; arxiv:1304.3243

- Important check: equilibrium expectations for fluctuations and nonlinear effects.



MN, M. Bluhm, Y. Karpenko, T. Schäfer, S. Bass, work in progress

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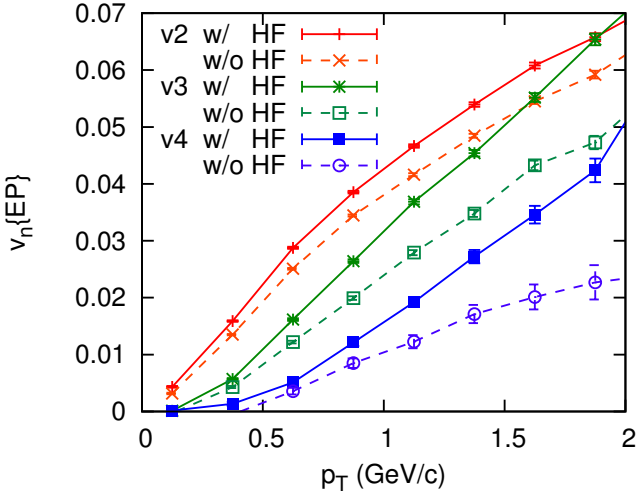

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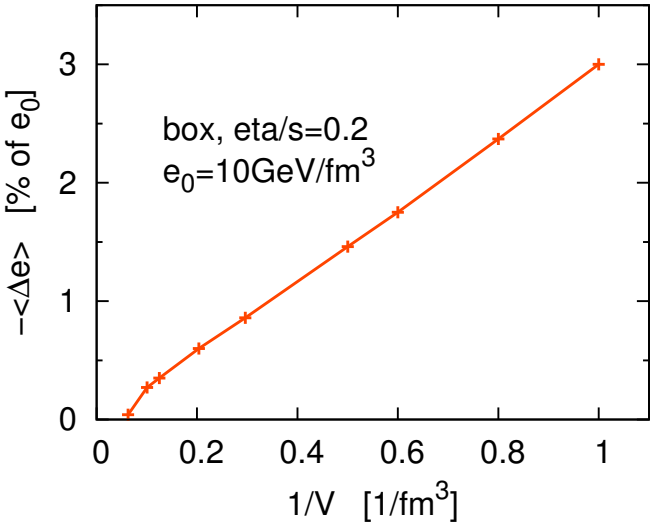

MN, M. Bluhm, Y. Karpenko, T. Schäfer, S. Bass, work in progress

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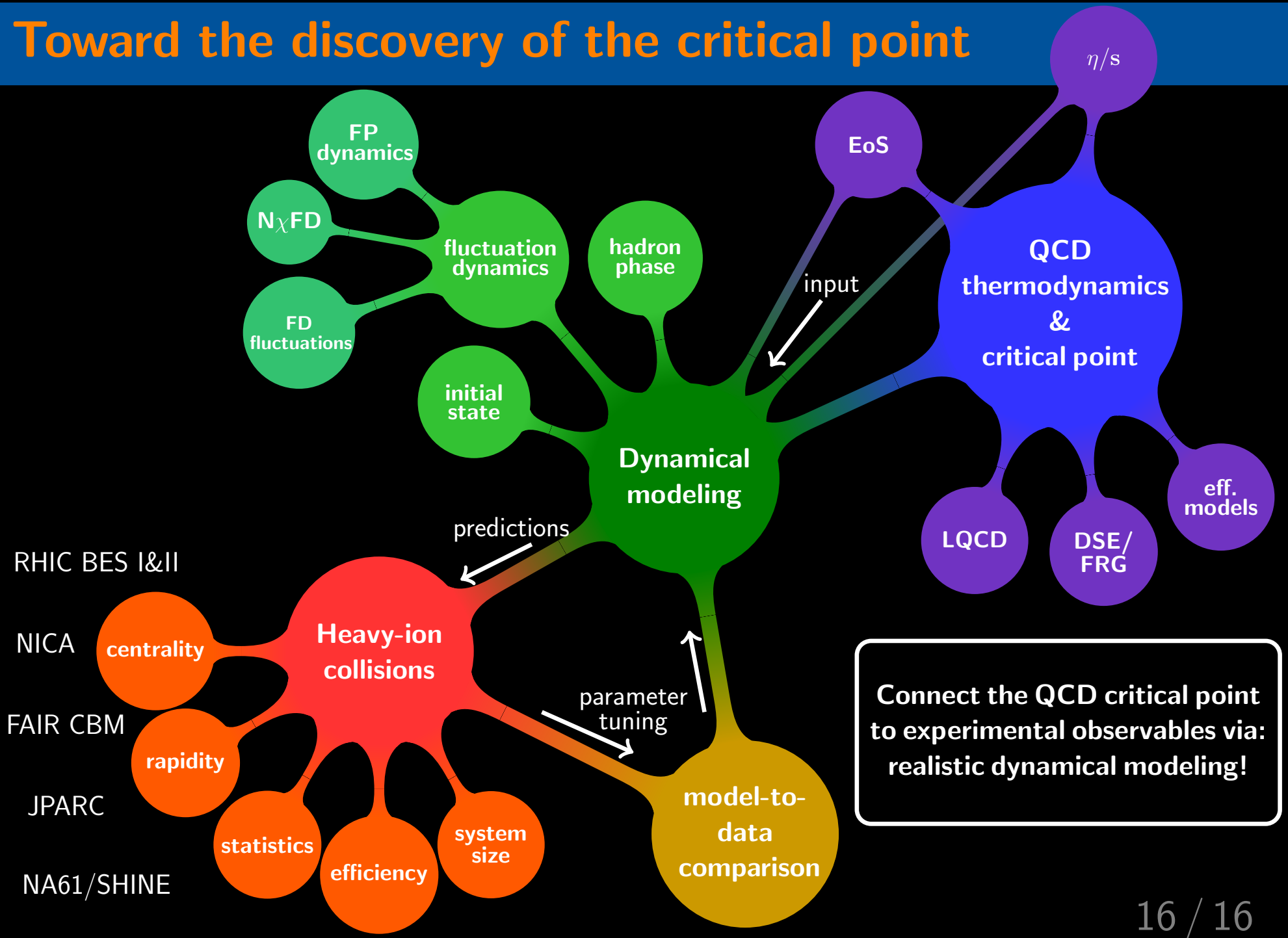
Future: include net-baryon densities!

Toward the discovery of the critical point



Connect the QCD critical point to experimental observables via: realistic dynamical modeling!

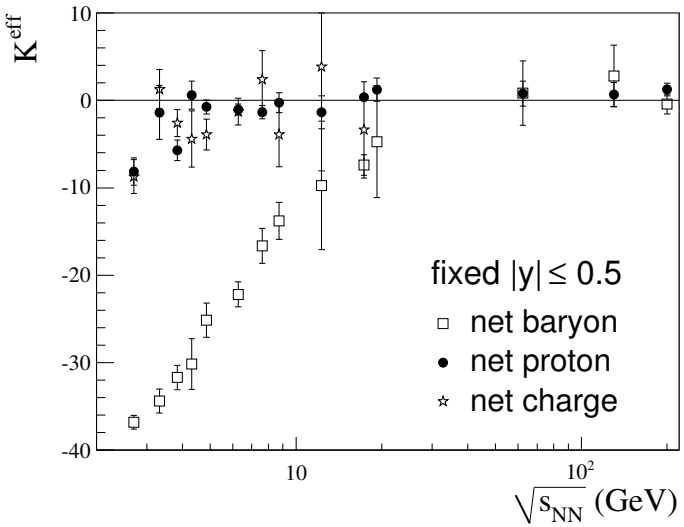
backup

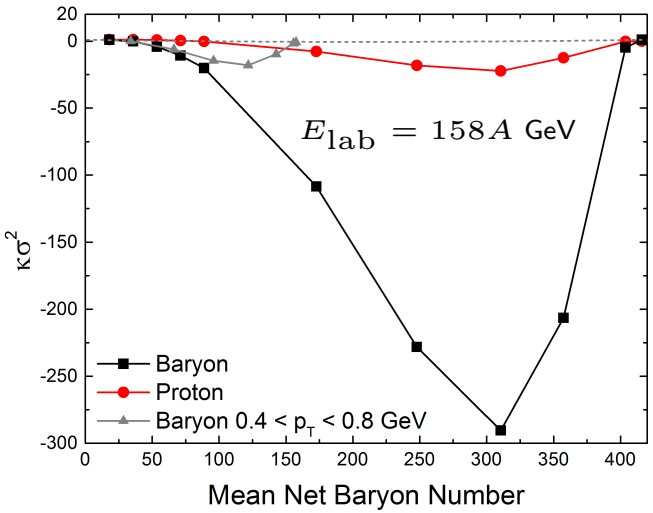
Non-critical effects on fluctuation observables

- Global net-baryon number conservation.

  MN, T. Schuster, M. Mitrovski, R. Stock, M. Bleicher, EPJC72 (2012); A. Bzdak, V. Koch, V. Skokov, PRC87 (2013)

- In a microscopic transport model the microcanonical nature of individual scatterings is preserved.
- Strongly negative kurtosis of net-baryon number due to global conservation and volume fluctuations.
- Net-proton fluctuations follow this trend slightly.





by J. Steinheimer

MN, T. Schuster, M. Mitrovski, R. Stock, M. Bleicher, EPJC72 (2012)

Nonequilibrium correlation length

Phenomenological equation: $\frac{\mathrm{d}}{\mathrm{d}t} m_\sigma(t) = -\Gamma[m_\sigma(t)] \left(m_\sigma(t) - \frac{1}{\xi_{\mathrm{eq}}(t)} \right)$
with input from the dynamical universality class $\Rightarrow \xi \sim 1.5 - 2.5$ fm

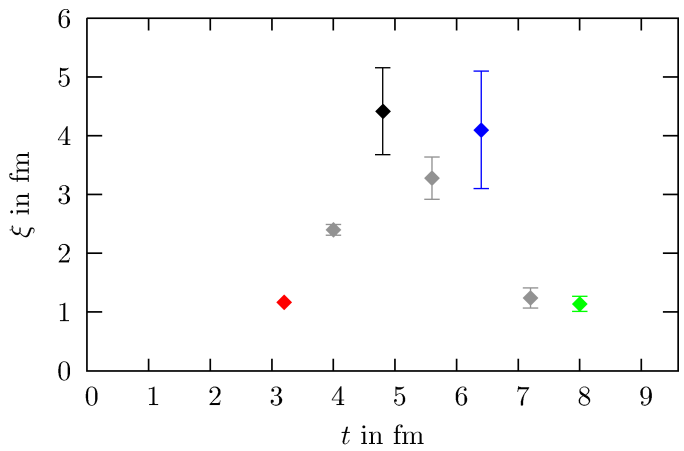
B. Berdnikov and K. Rajagopal, PRD 61 (2000)

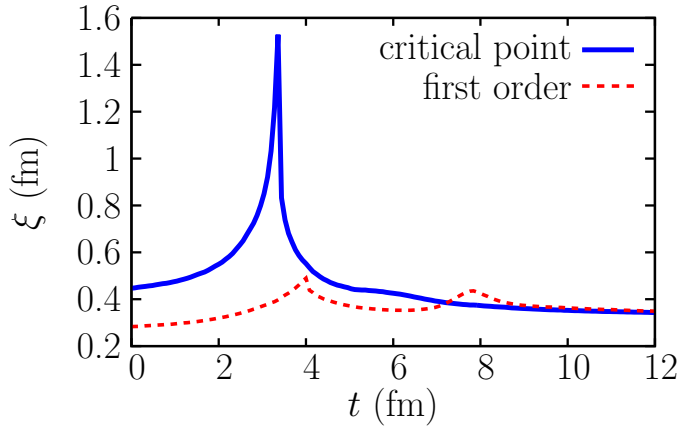
$$G(r) = \int \mathrm{d}^3x \mathrm{d}^3y \langle \sigma(x) - \sigma_0 \rangle \langle \sigma(y) - \sigma_0 \rangle$$
$$\sim \exp(-r/\xi)$$

Assume σ_0 is the volume averaged field.

From the curvature of V_{eff}:

$$\langle \xi^2 \rangle = \langle 1/m_\sigma^2 \rangle = \left\langle \left(\frac{\mathrm{d}^2 V_{\mathrm{eff}}}{\mathrm{d}\sigma^2} \right)^{-1} \right\rangle$$





C. Herold, MN, I. Mishustin, M. Bleicher PRC87 (2013)

Definition of ξ in inhomogeneous systems involves averaging!

$\Rightarrow$ Similar magnitude of $\xi \sim 1.5 - 3$ fm!

Finite-size scaling

In the scaling regime $L_{\mathrm{micro}} \ll \xi \ll L$:

- Finite-size scaling for any intensive thermodynamic quantitiy X with an algebraic singularity at the critical point

$$X_L(T) \propto L^{\gamma/\nu}$$

equilibrium critical exponents:

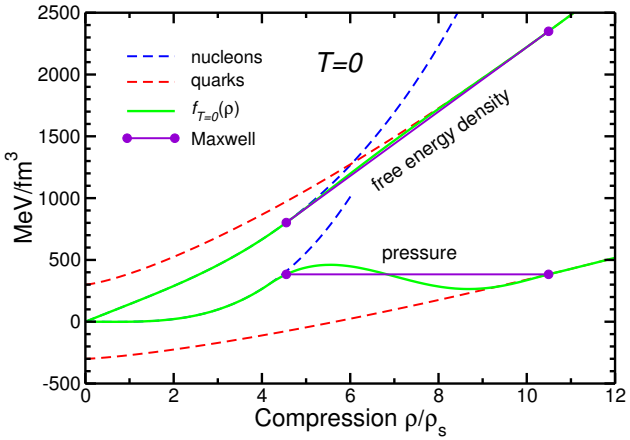
$$\xi_\infty(T) \propto t^{-\nu}\,, \quad X_\infty(T) \propto t^{-\gamma}$$

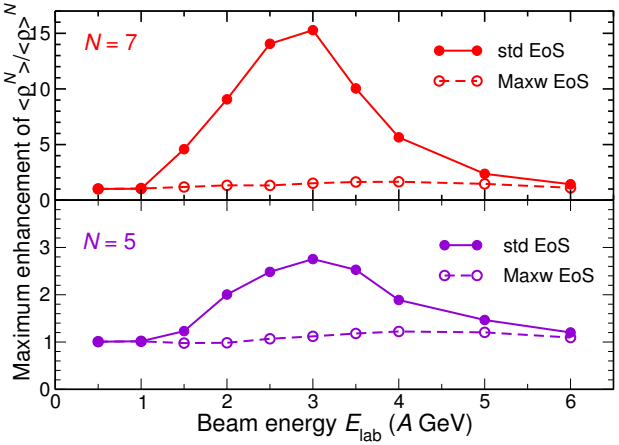
- Position of the peak is shifted: $\Delta t_L = (T_c - T_{c,L})/T_c \propto L^{-\lambda}$
- Vary the system size via centrality, species of nuclei: Can finite-size scaling be seen in observables?
- Expanding system size, freeze-out of fluctuations, critical slowing down... $\Rightarrow$ need dynamical models!

E. Fraga, L. Palhares, P. Sorensen, PRC84 (2011); R. Lacey, PRL114 (2015)

Amplification of initial fluctuations at a FOPT

- Nonequilibrium construction of the EoS from QGP and hadronic matter:





- Significant amplification of **initial** density irregularities.
- But: no clear signals after final hadronic phase.

J. Steinheimer, J. Randrup, PRL 109 (2012), PRC 87 (2013)

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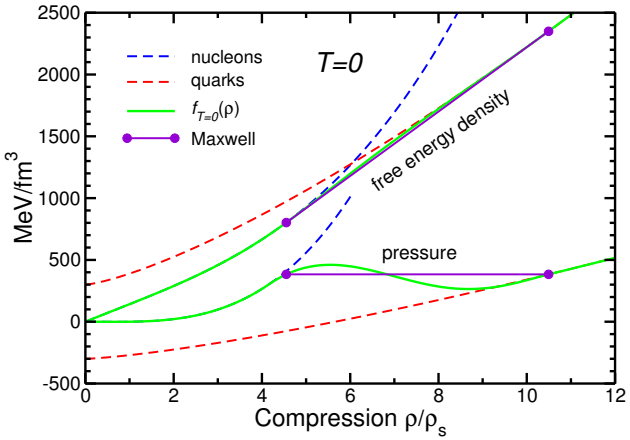
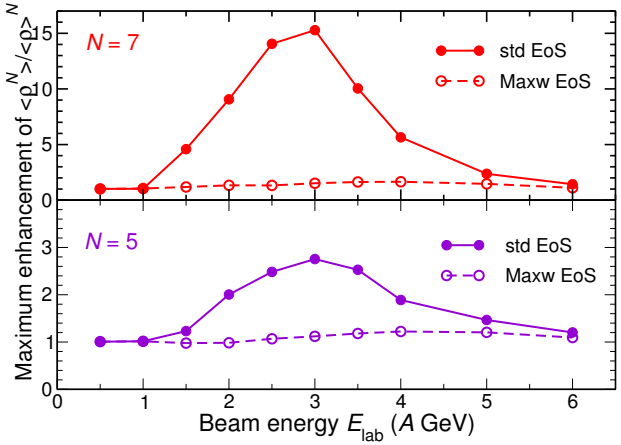
- Nonequilibrium construction of the EoS from QGP and hadronic matter:



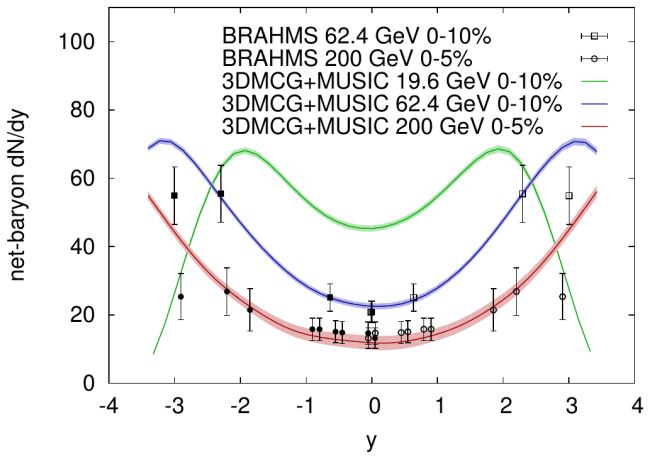


- Significant amplification of **initial** density irregularities.
- But: no clear signals after final hadronic phase.

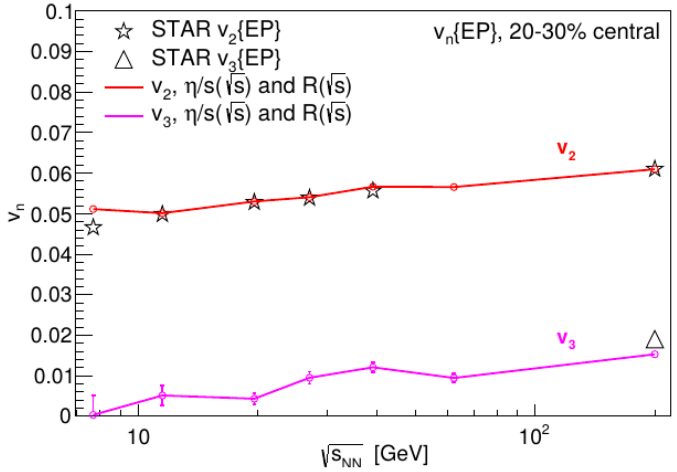
J. Steinheimer, J. Randrup, PRL 109 (2012), PRC 87 (2013)

Deterministic evolution $\Rightarrow$ no dynamical fluctuations

Dynamical modeling at finite μ_B



talk by B. Schenke



talk by Y. Karpenko

- Inclusion of net-baryon diffusion into fluid dynamical simulations:
  - Baryon dissipation.
  - Baryon-shear and baryon-bulk coupling terms.
  - Out-of-equilibrium δf corrections.
- Initial state and initial baryon stopping $\Rightarrow$ explore net-baryon rapidity correlations and fluctuations!
- Is there a fluid dynamical phase at high-baryon densities.
- Importance of correct description of the hadronic phase.
- Bulk viscosity needs to be included.

Equation of state + transport coefficients at finite μ_{B}

Equation of state:

- lattice QCD: cont.-extr. in Taylor expansion up to $\mathcal{O}(\mu^2)$

  Wuppertal-Budapest Coll., JHEP1208 (2012)

  up to $\mathcal{O}(\mu^4)$ not cont.-extr. BNL-Bielefeld-CCNU-Coll., NPA931 (2014)

- effective models: e. g. from

  V. Dexheimer, S. Schramm, PRC81 (2010); M. Hempel, V. Dexheimer, S. Schramm, I. Iosilevskiy PRC88 (2013); J. Steinheimer, V. Dexheimer, H. Petersen, M. Bleicher, S. Schramm, H. Stoecker, PRC81 (2010)

- from 3d Ising model

  C. Nonaka, M. Asakawa PRC71 (2005); M. Bluhm, B. Kampfer CPOD 2006

Transport coefficients:

- η/s in DQPM, reproduces lattice at $\mu_B = 0$, crossover

  H. Berrehrah et al., arxiv:1412.1017

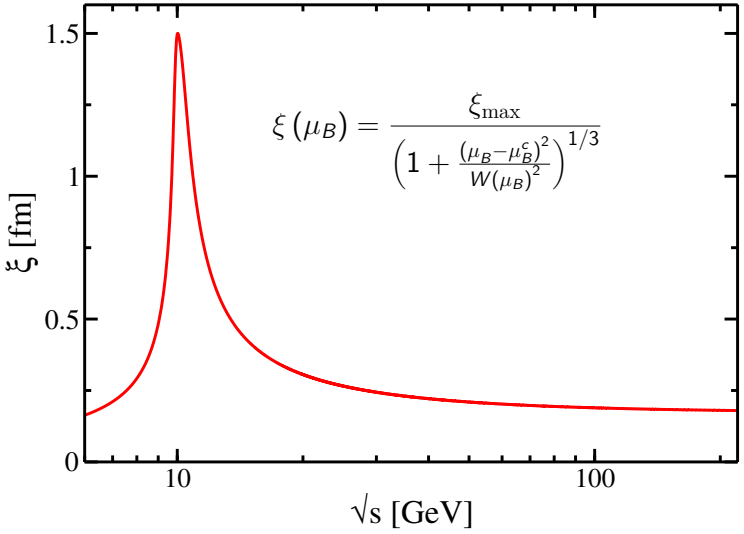
- ζ: universal properties in vicinity of critical point, $\zeta \to \infty$ in $Z(2)$

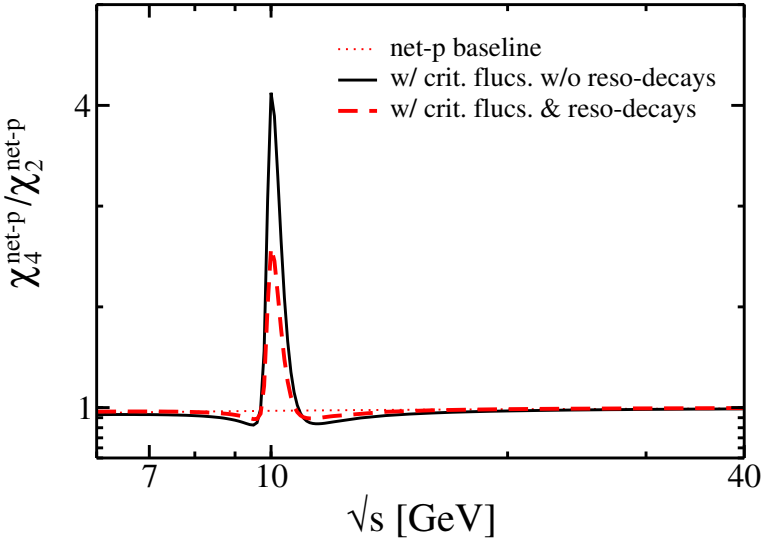
  F. Karsch, D. Kharzeev, K. Tuchin, PLB663 (2008)

- from transport model calculations talk by Y. Karpenko

HRG + critical fluctuations

M. Bluhm, MN, work in progress





- Model for correlation length from

  C. Athanasiou, K. Rajagopal, M. Stephanov, PRD82 (2010)

- Coupling of resonances to the σ-field: $g_{R\sigma} = \frac{m_R}{m_p}\,(3-|S_R|)\,\frac{g_{p\sigma}}{3}$.
- Additional parameters $\tilde{\lambda}_3$, $\tilde{\lambda}_4$ should depend on $\sqrt{s}$.