

Femtoscscopy in Relativistic Heavy Ion Collisions

Experimental Overview in the 3rd Decade *

Mike Lisa
Ohio State University



* or billion seconds

Outline

Broad topic! – Broad overview

- Data systematics & lessons: heavy ion decades 1 & 2
- Data systematics & lessons: heavy ion decade 3
- Puzzles & the most important open questions

Not discussed

- $N > 2$ correlations, coherence: ALICE PRC89 024911 (2014); ALICE PLB739 139 (2014)
- extraction of scattering parameters: STAR Nature (2015)
- imaging PHENIX PRL98 132301 (2007), PRL103 142301 (2009)
NA49 PLB685 41 (2010); STAR PRC88 034906 (2013)
- d+A, p+A (mentioned) ALICE PRC91 034906 (2015); PHENIX NPA931 (2014)
- asymmetry from non-id correlations: no recent (published) results

Decades 1 & 2 (Time & energy)

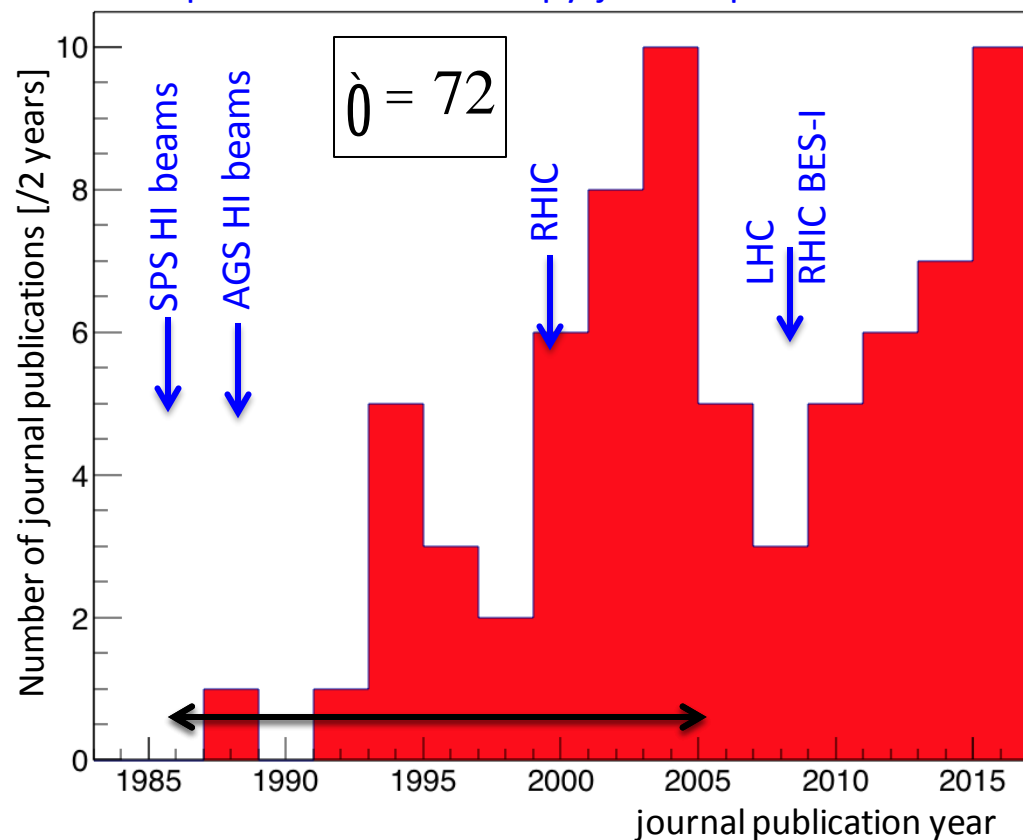
Ann. Rev. Nucl. Part. Sci. 55:357-402 (2005)

FEMTOSCOPY IN RELATIVISTIC HEAVY ION COLLISIONS: Two Decades of Progress

MAL, S. Pratt, R. Soltz, U. Wiedemann

$$\sqrt{s_{NN}} = 2.2 \text{ TeV}$$

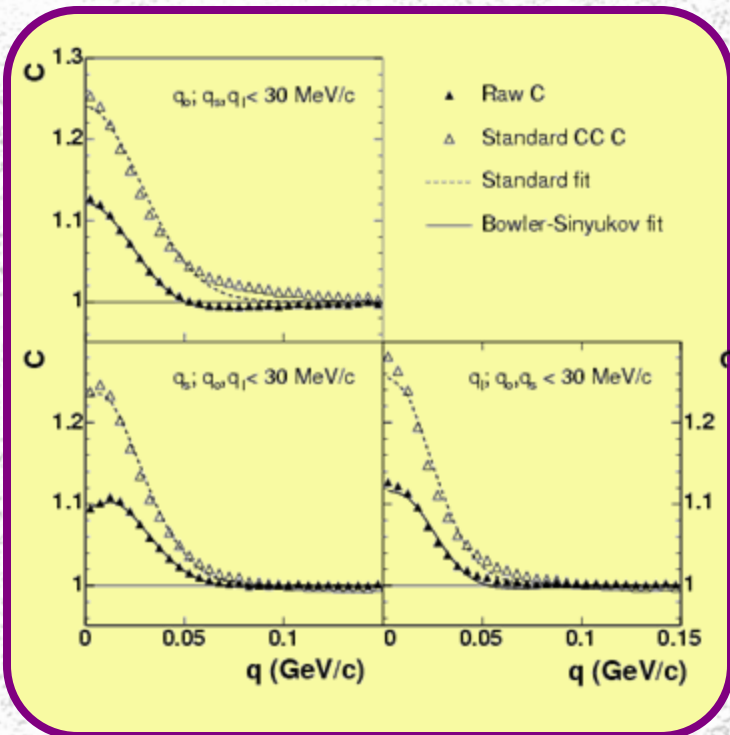
Experimental femtoscopy journal publications



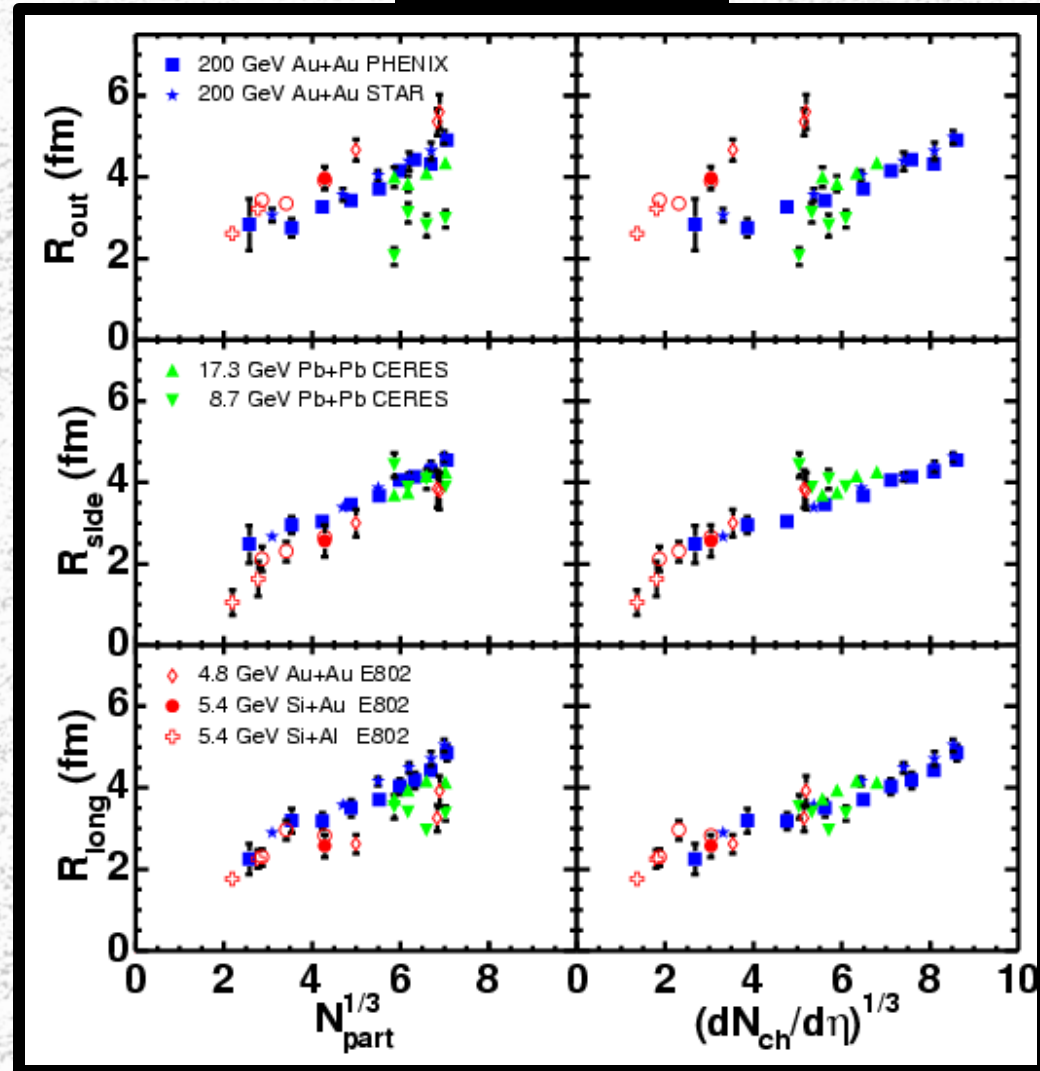
Decades 1 & 2 (Time & energy)

First two decades (time & energy)

- high-stats pion systematics
 - N_{ch} , p_T , ϕ , (γ)



RHIC SPS AGS

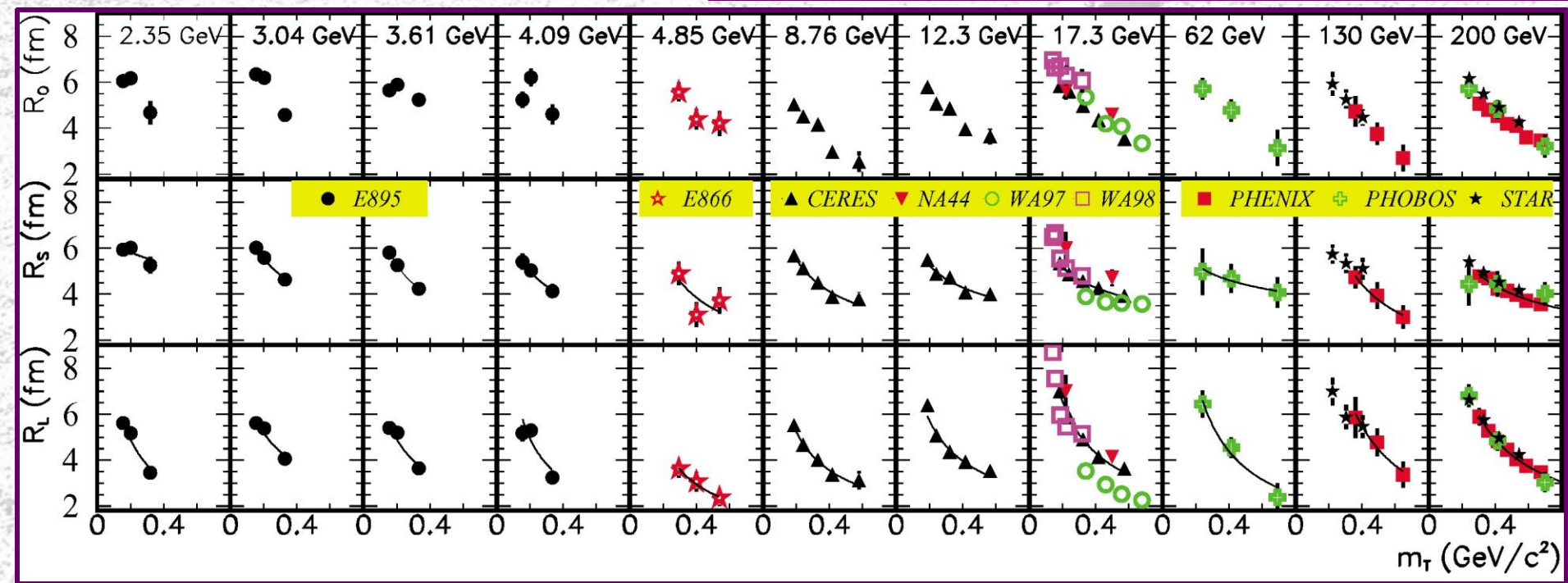
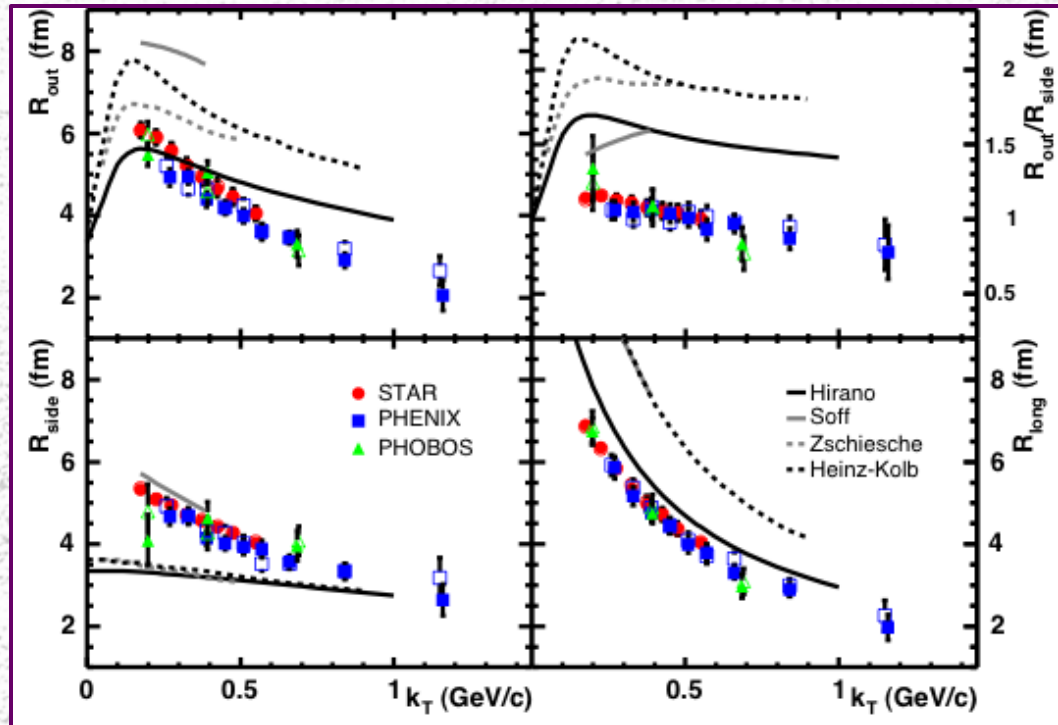


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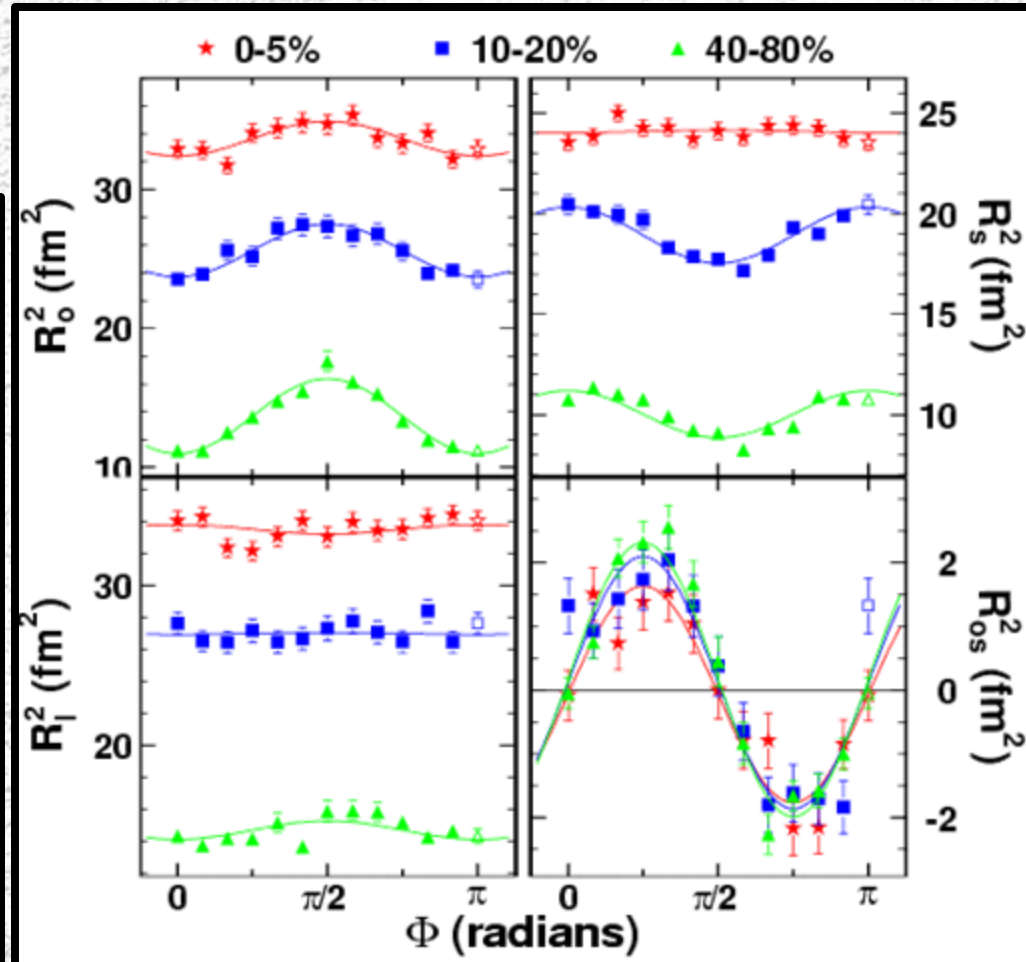
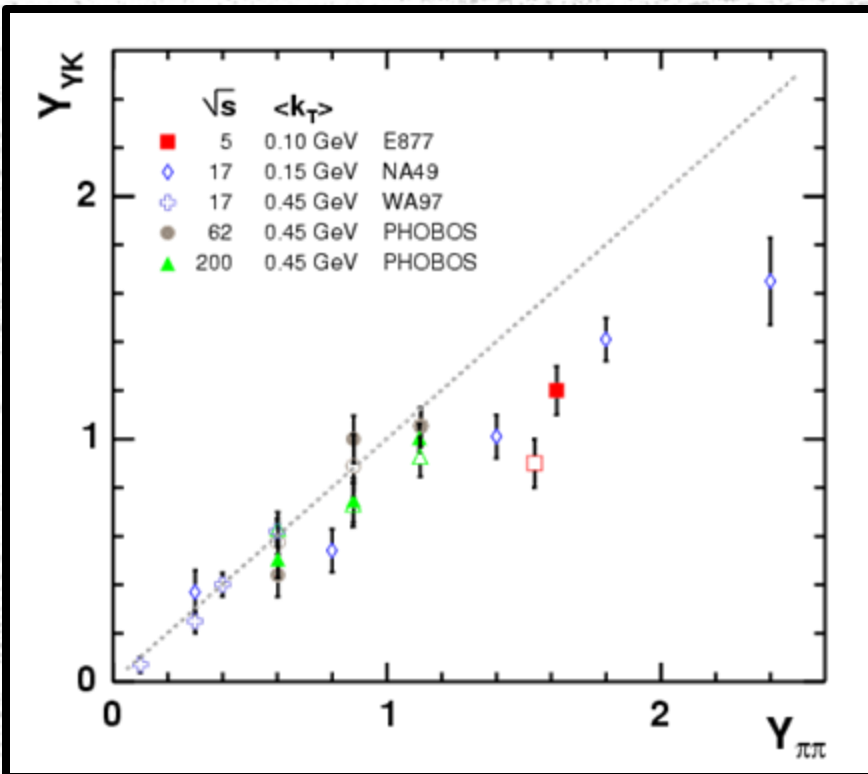


Decades 1 & 2 (Time & energy)

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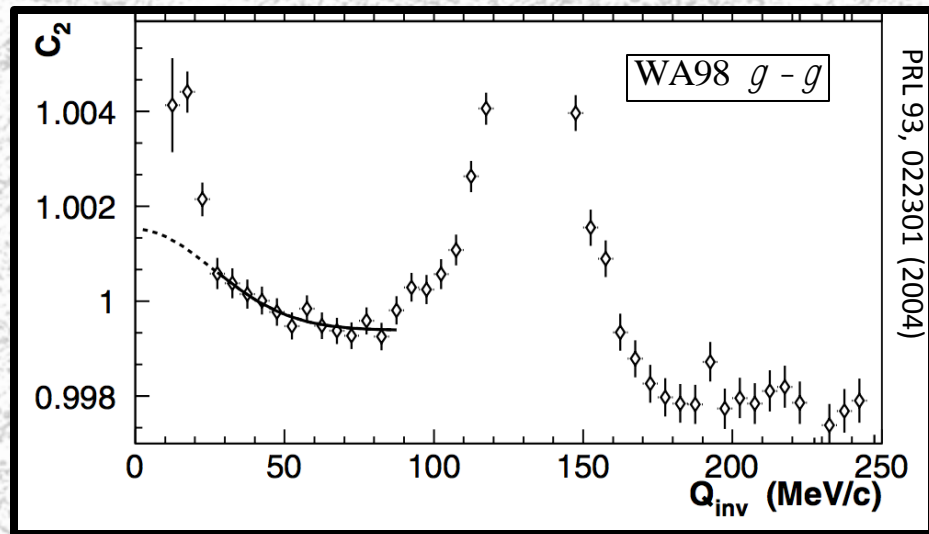
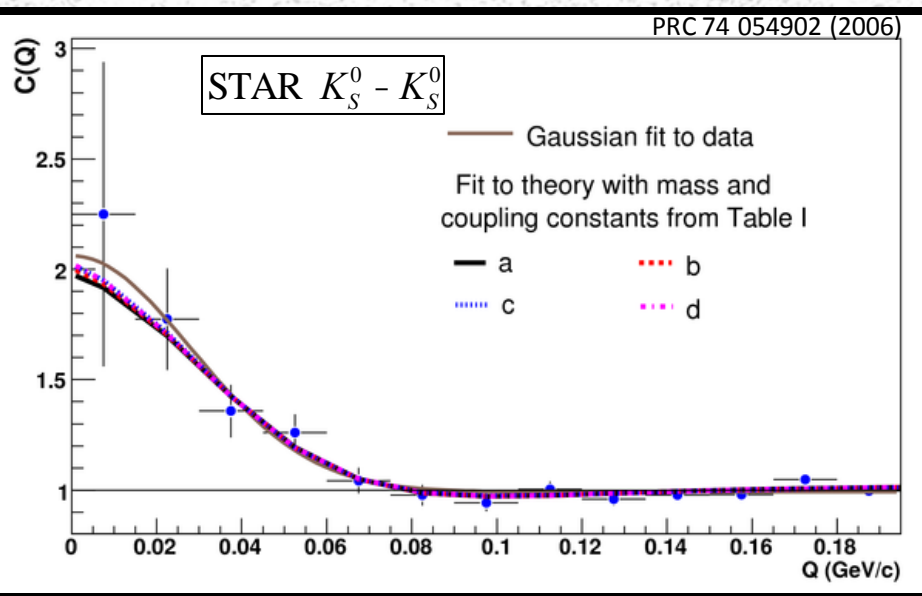
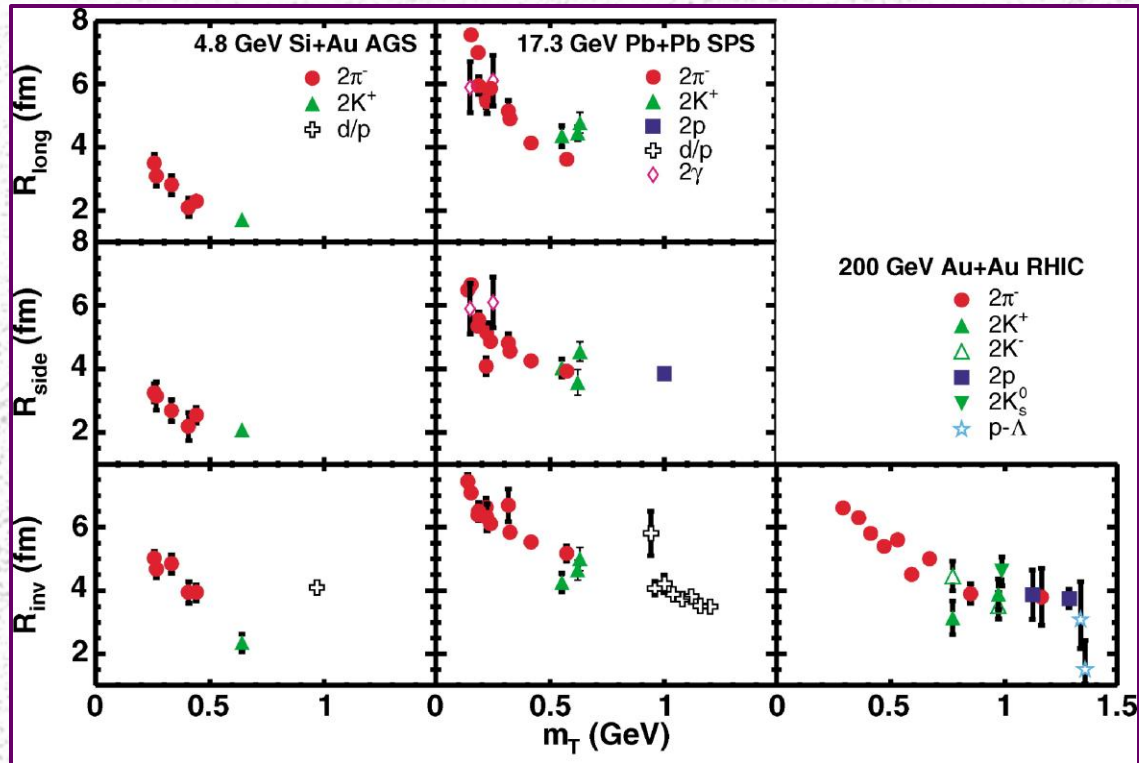
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Decades 1 & 2 (Time & energy)

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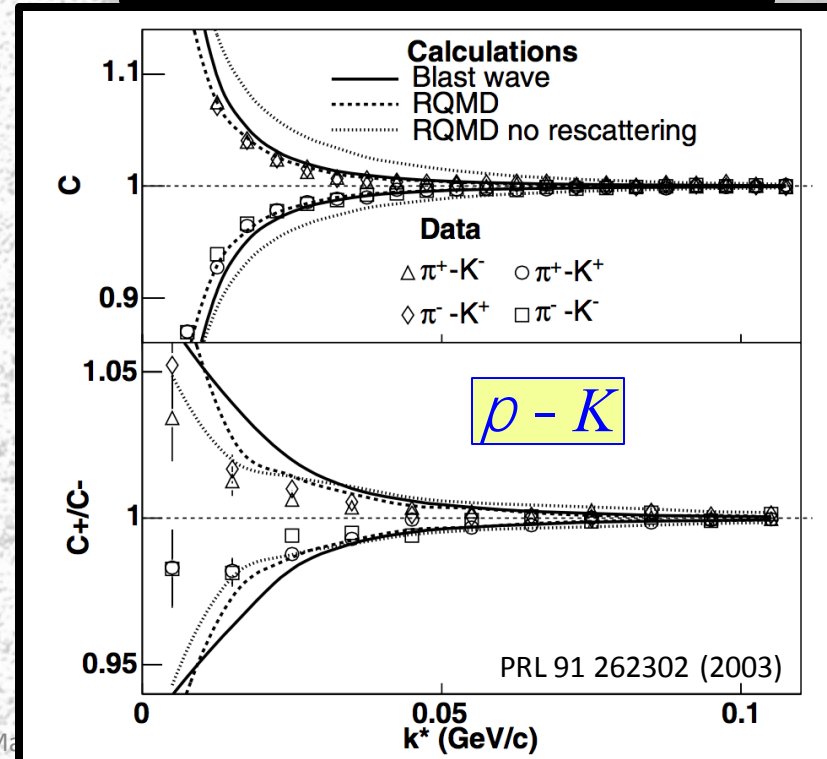
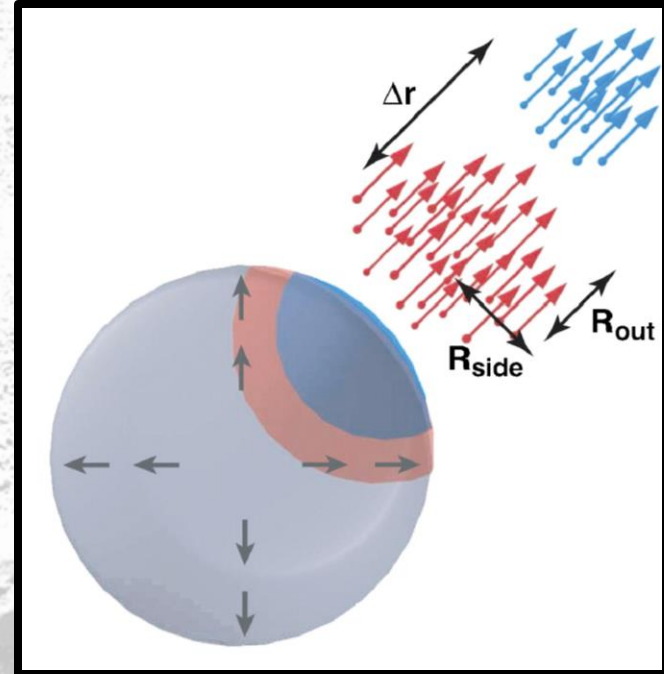
- high-stats pion systematics
 - N_{ch} , p_T , ϕ , $\langle \gamma \rangle$
- other species
 - K^+ , K^- , K^0 , gamma, p, Lambda
 - nucleon coalescence



Decades 1 & 2 (Time & energy)

First two decades (time & energy)

- high-stats pion systematics
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- other species
 - K^+ , K^- , K^0 , gamma, p, Lambda
 - nucleon coalescence
- non-id correlations
 - pi+pi-, p-Lambda, k-pi
 - (p-pi) (Xi-pi) (K-p) (p-pbar)
 - **multi-dimensional**
→ **unique information**



Decades 1 & 2 (Time & energy)

First two decades (time & energy)

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- non-id correlations
 - $\pi^+\pi^-$, p -Lambda, k - π
 - $(p-\pi)$ $(\Xi-\pi)$ $(K-p)$ $(p-pbar)$
 - **multi-dimensional**
→ **unique information**
- extracting interaction parameters
- Source Imaging
- 3-pion correlations: chaoticity

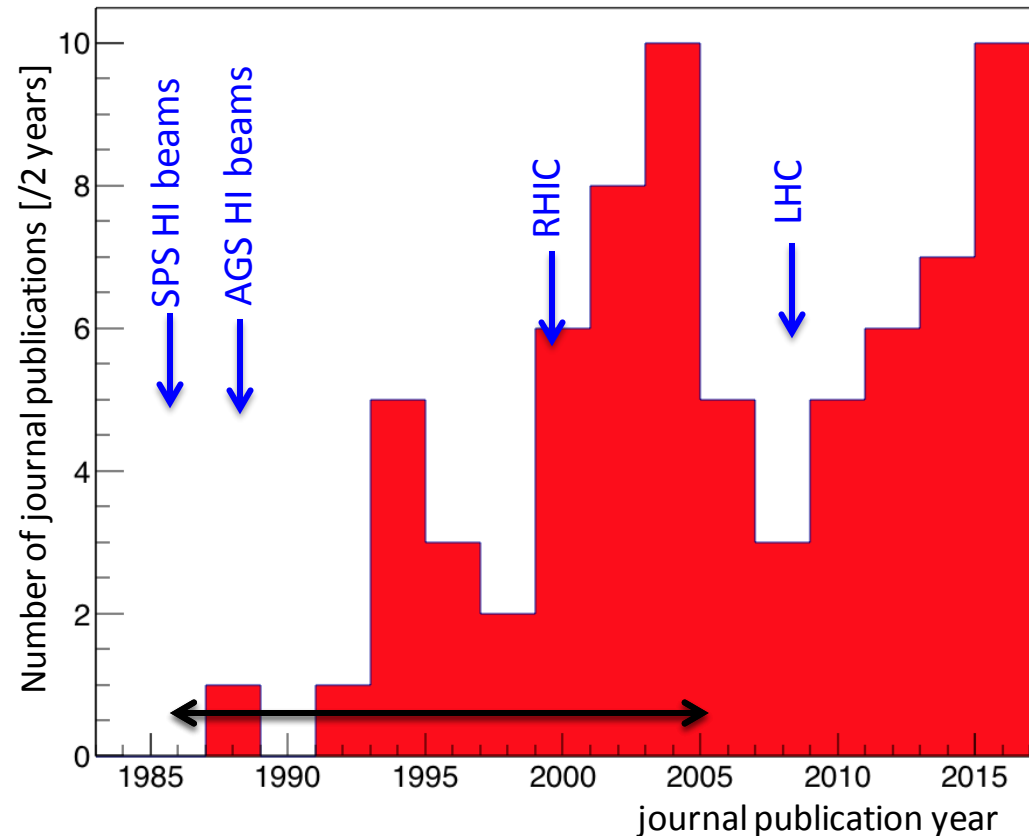
So, what did the first two decades teach us?

Ann. Rev. Nucl. Part. Sci. 55:357-402 (2005)

FEMTOSCOPY IN RELATIVISTIC HEAVY ION COLLISIONS: Two Decades of Progress


MAL, S. Pratt, R. Soltz, U. Wiedemann

Experimental femtoscopy journal publications



The big items

- The “system” *is* a system.
- There is massive explosive radial flow, in all HI collisions
- The spatial **sub**structure of the flow is hydro-like
- There is not a large latent heat (not strongly first-order) @ top RHIC
- pre-equilibrium flow...
- The source at midrapidity is approximately boost invariant



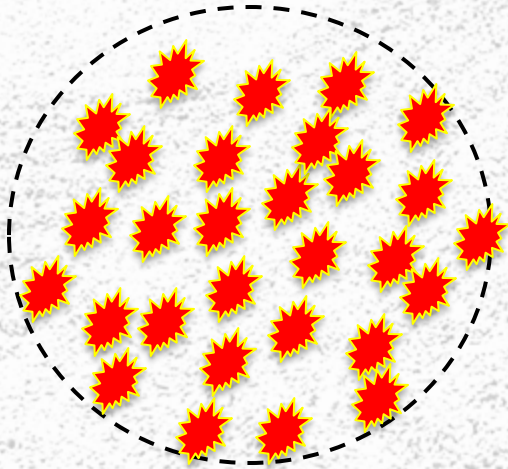
Thanks for the news flash.
I already knew that...

How we know what we “know”

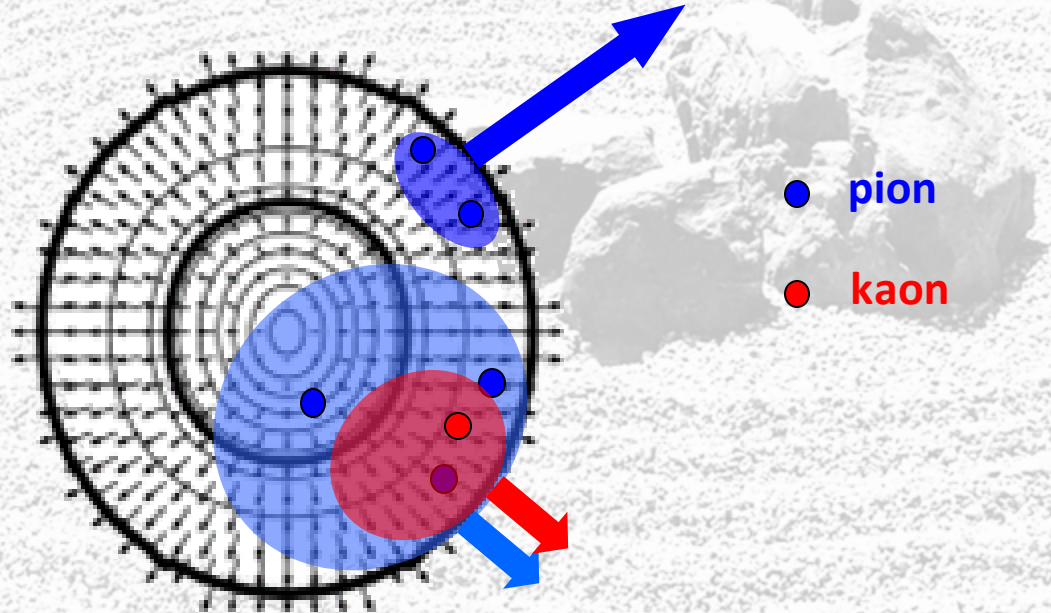
HI collisions create a “collective system of self-interacting matter”

How do we know that a central collision* creates a collective system?

- thermo-chemical fits? p_T spectra?



A collection of nearby, **independent** p+p collisions. **Not “matter”**



Matter is characterized by fields of bulk properties



* citing v_n systematics from an incompletely understood lumpy initial state, requiring very rapid thermalization via unclear mechanisms, is not compelling, if we cannot understand the zeroth order effects of simpler central collisions.

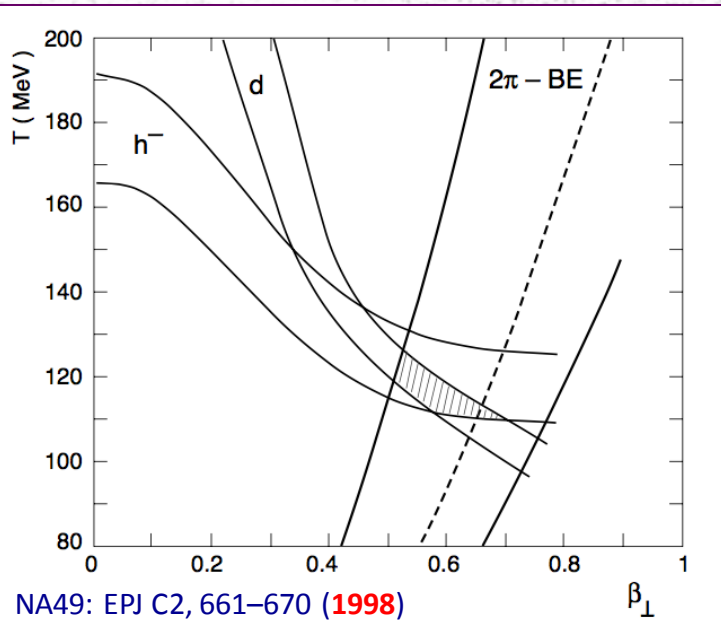
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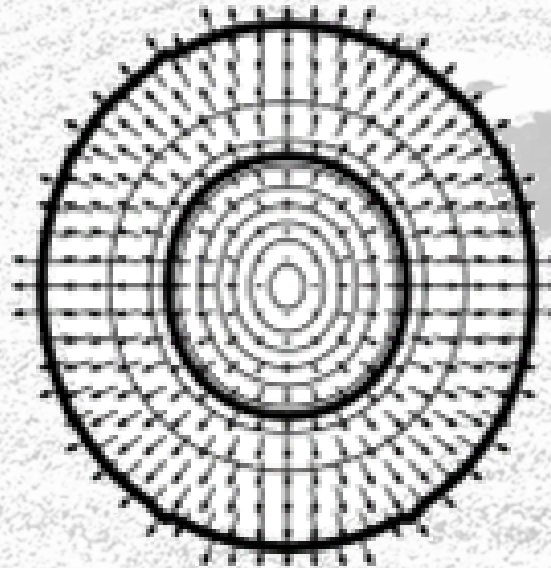
- thermo-chemical fits? p_T spectra?

Constraining freezeout configuration



NA49: EPJ C2, 661–670 (1998)

c.f. all Blast-wave incarnations



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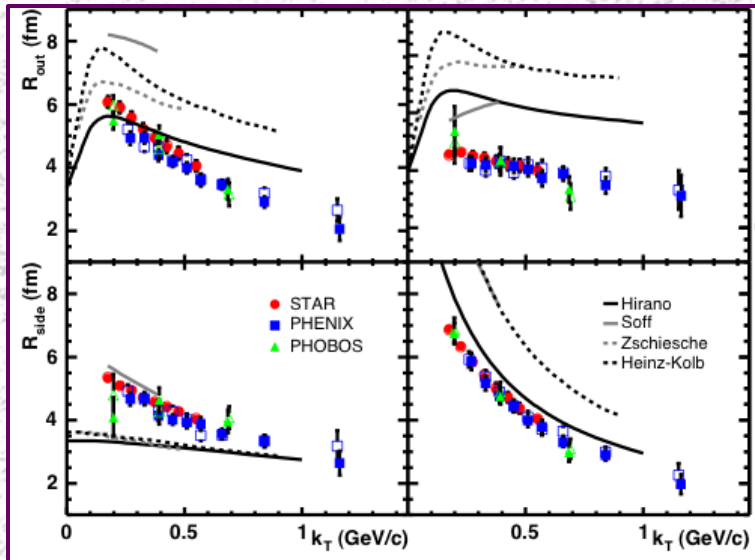
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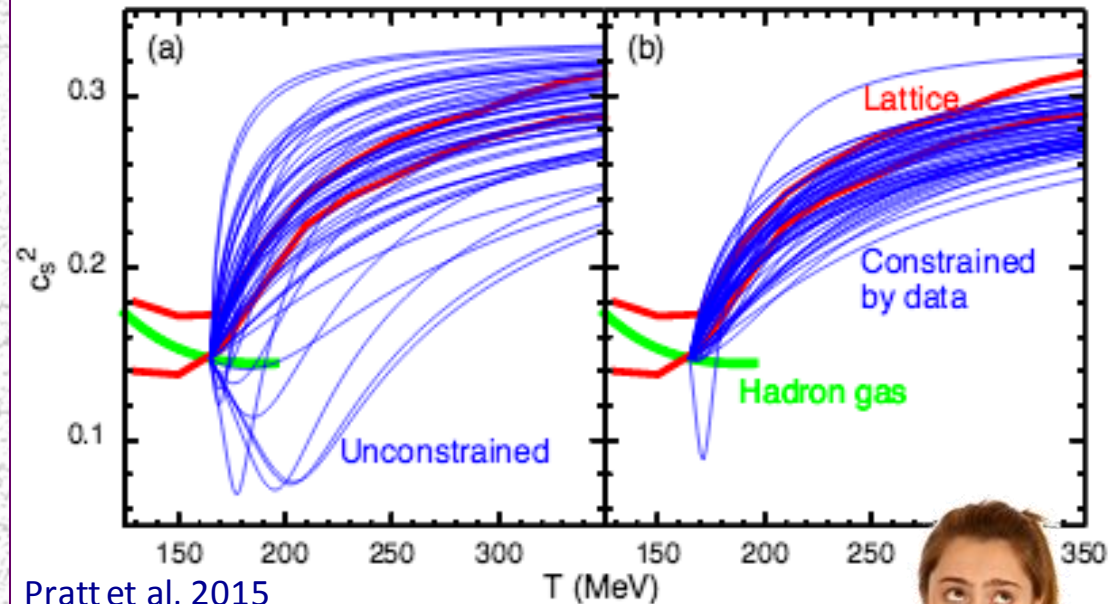
How do we know that a central collision* creates a collective system?

- thermo-chemical fits? p_T spectra?

Constraining hydro/transport



Constraining Equation of State femtoscopy the most constraining data



Pratt et al, 2015

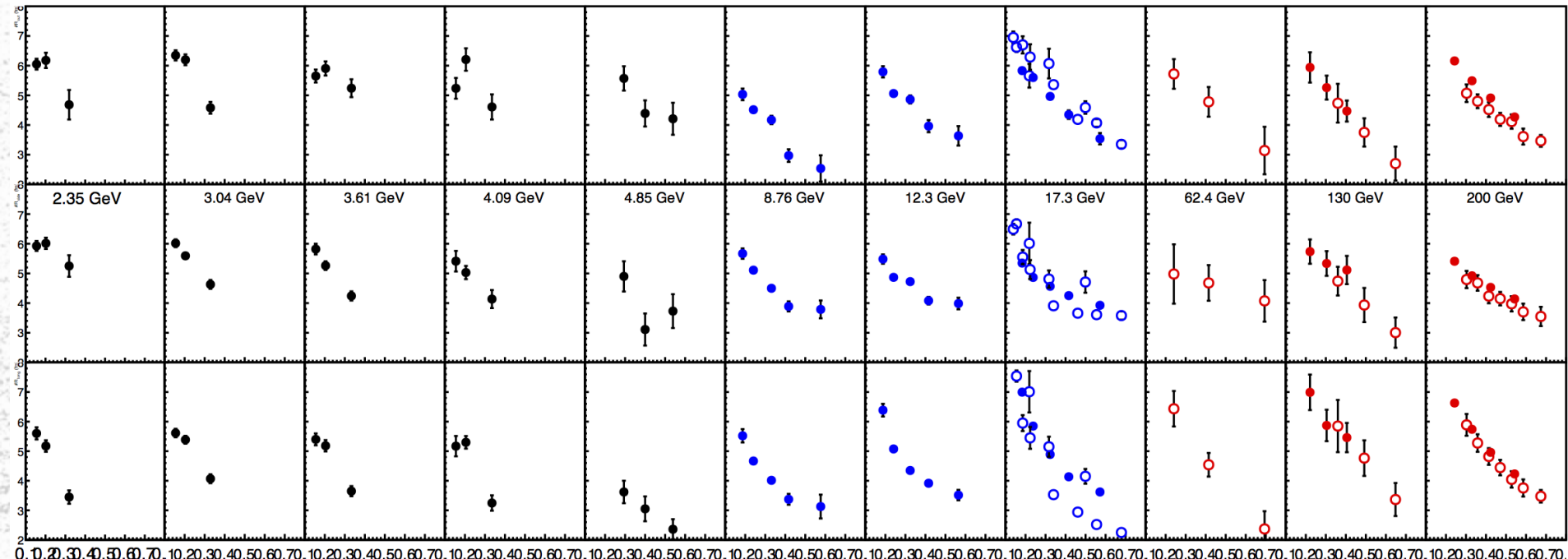
confronting models with data



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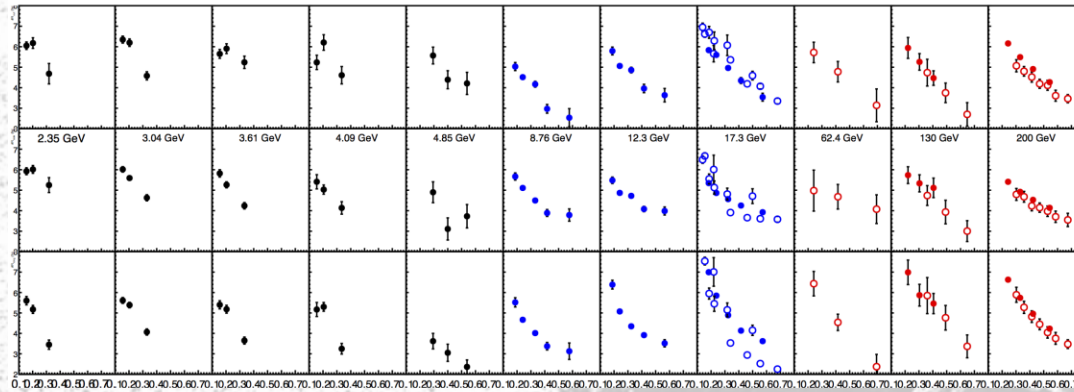
Decade 3

1985-2005 : dynamic range in root(s) = 10^2

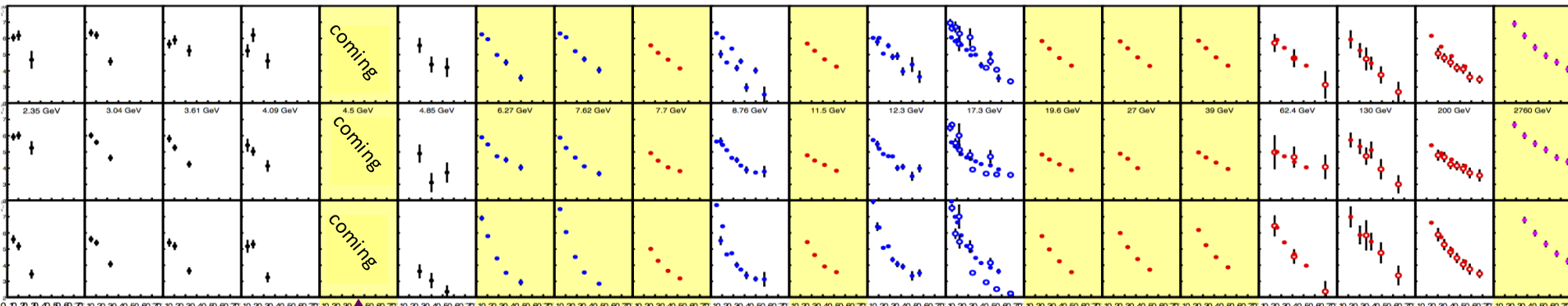


Decade 3

← 1985-2005 : dynamic range in \sqrt{s} = 10^2 →



← 1985-2015 : dynamic range in \sqrt{s} = 10^3 →



STAR
Fixed Target

NA49 Scan

RHIC BES

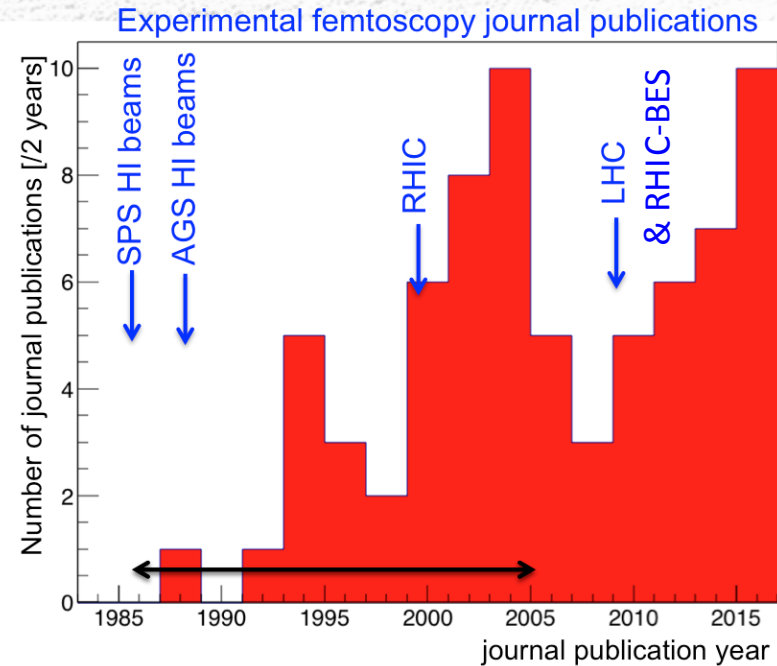
ALICE

Decade 3

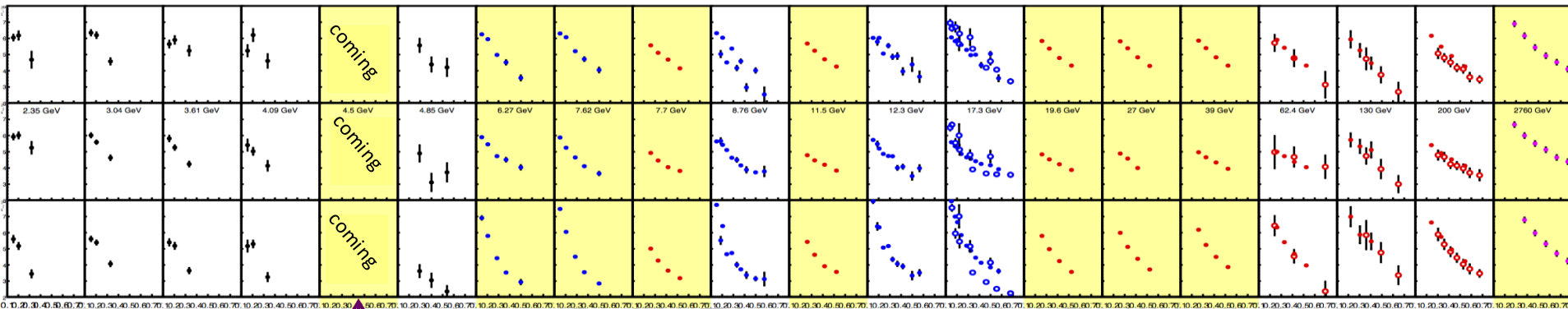
- 43% (31/72) of papers published after 2006

	SPS	RHIC	LHC
A+A	4	10	6
p+p		1	9
p(d)+A		1	2

- Tech advances: residual correlations, SHD – no time ☹️
- New species combinations
- N>2 correlations
- Important new systematics**
- focus on pp “a la HIC”**



1985-2015 : dynamic range in root(s) = 10^3



STAR
Fixed Target

NA49 Scan

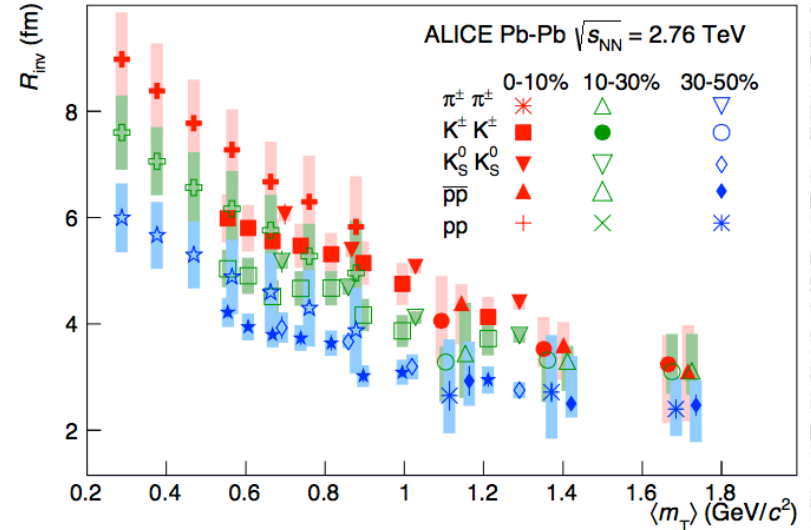
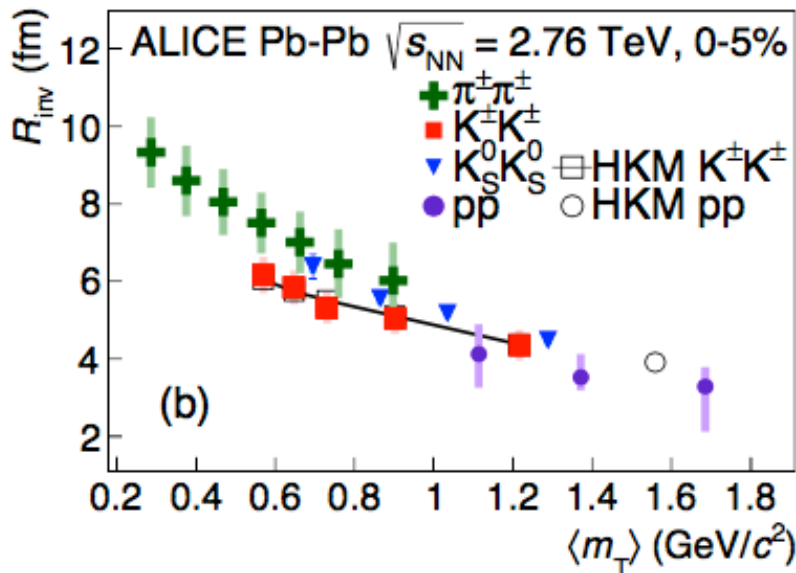
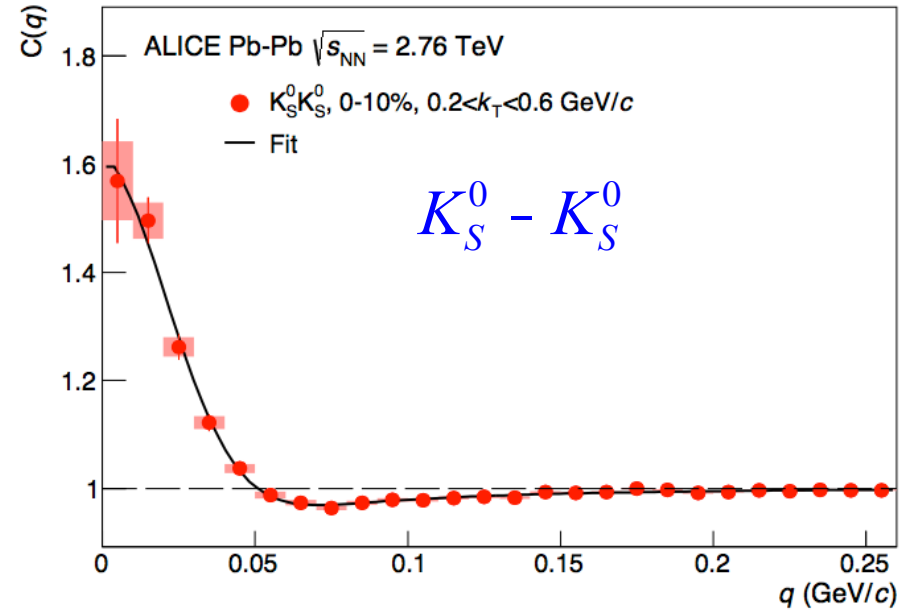
RHIC BES

ALICE

New species combos (& issues)

LHC: $\rho^\pm - \rho^\pm, K^\pm - K^\pm, K_S^0 - K_S^0, p - p, \bar{p} - \bar{p}$

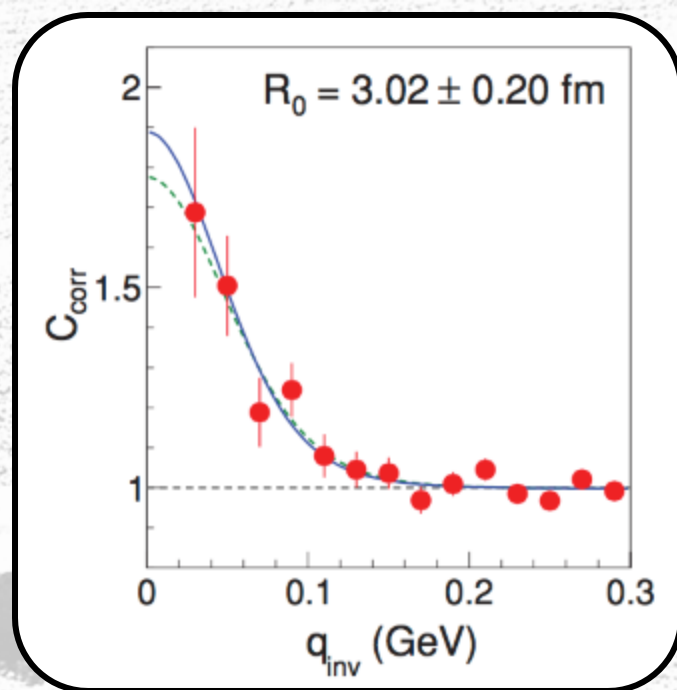
ALICE arxiv 1506.07884 (2015)



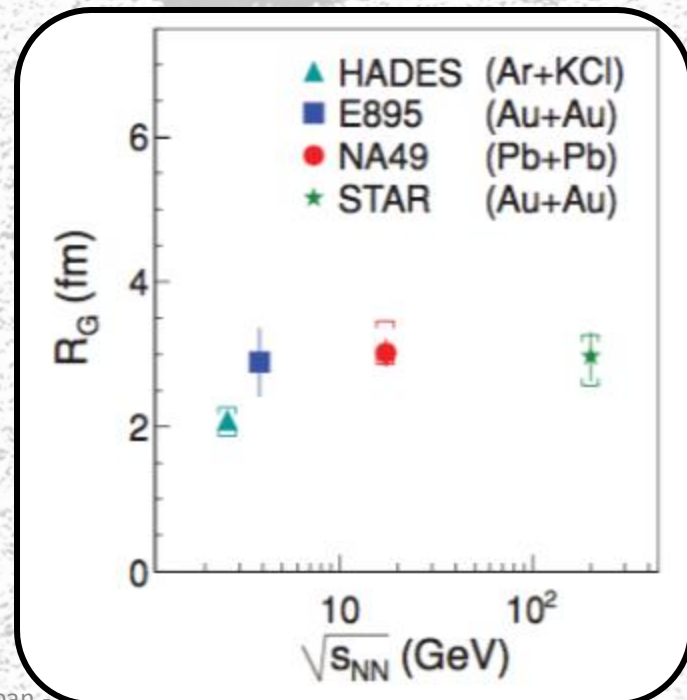
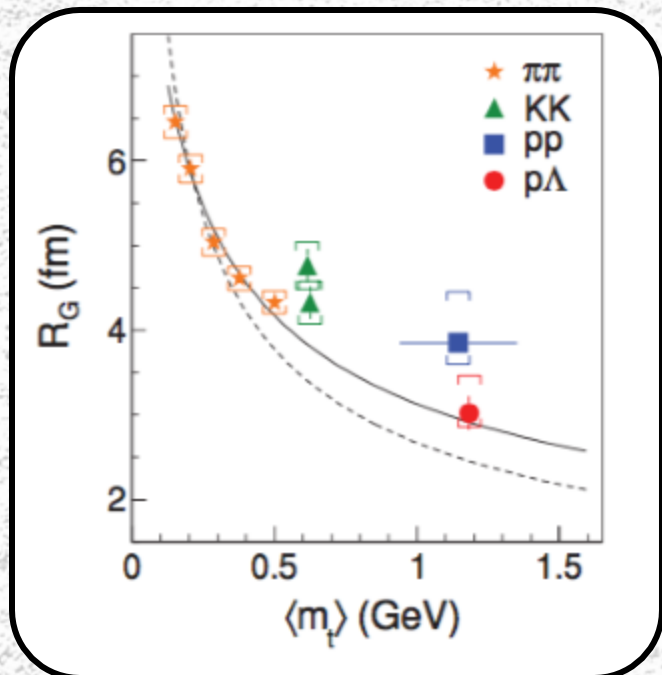
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LHC: $p^\pm - p^\pm, K^\pm - K^\pm, K_S^0 - K_S^0, p - p, \bar{p} - \bar{p}$

SPS: $p^\pm - p^\pm, K^\pm - K^\pm, p - p, p - \text{L}$



NA49 PRC 2011



New species combos (& issues)

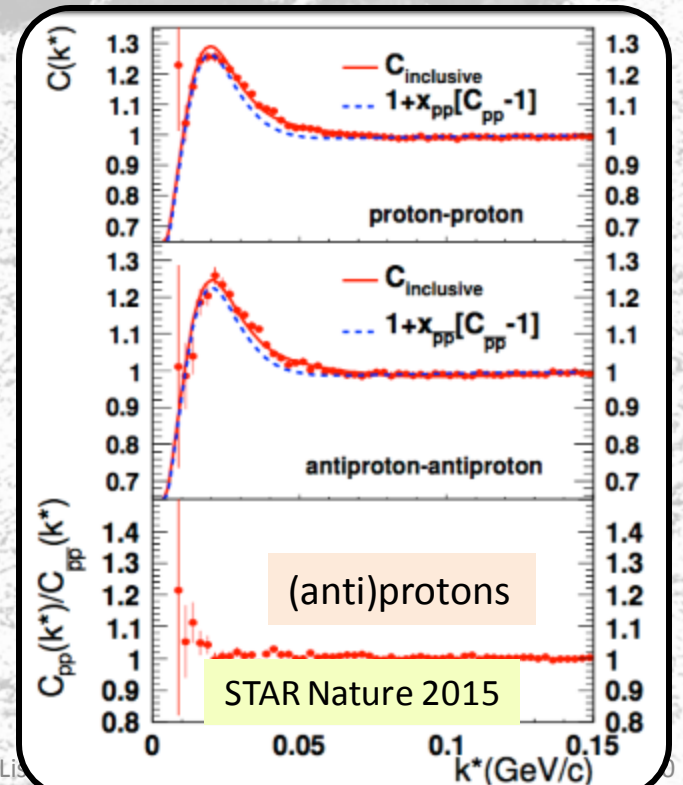
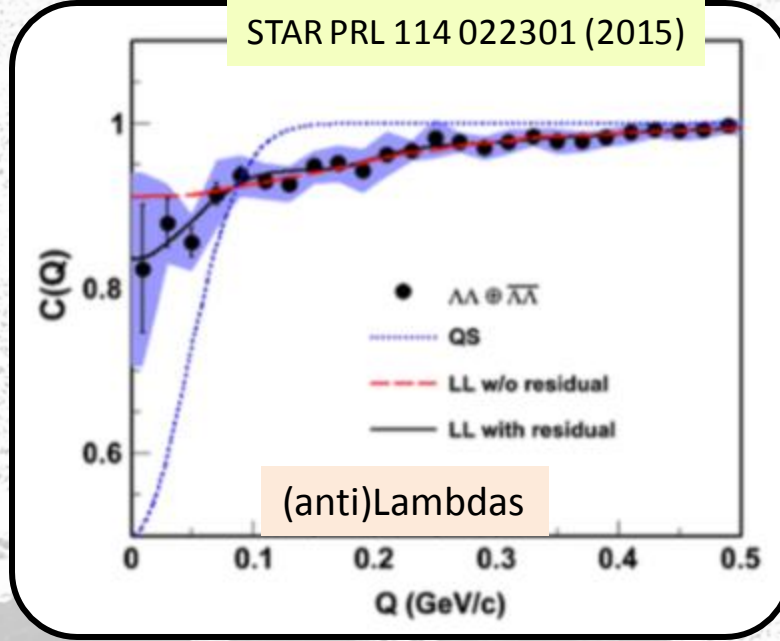
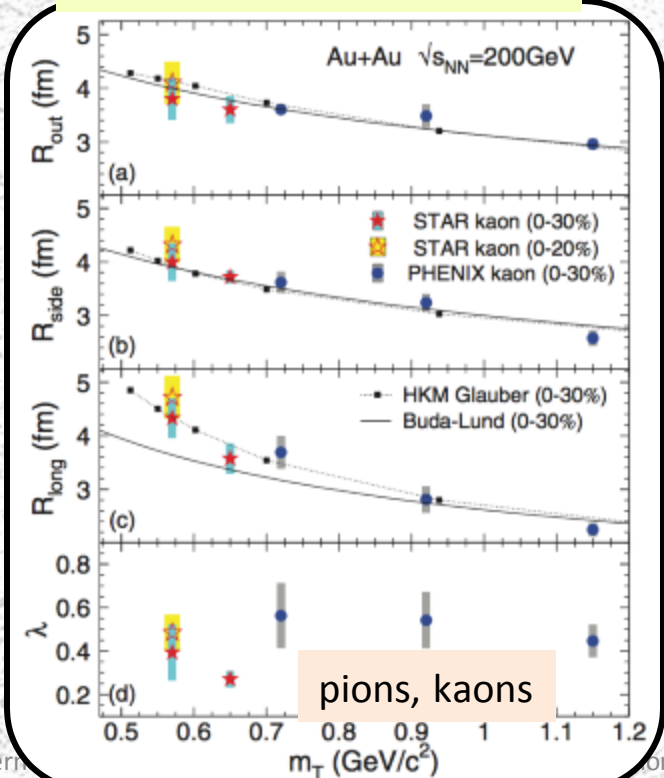
LHC: $\rho^\pm - \rho^\pm, K^\pm - K^\pm, K_S^0 - K_S^0, p - p, \bar{p} - \bar{p}$

SPS: $\rho^\pm - \rho^\pm, K^\pm - K^\pm, p - p, p - \mathbb{L}$

RHIC: $\rho^\pm - \rho^\pm, K^\pm - K^\pm, K_S^0 - K_S^0, p - p, \bar{p} - \bar{p}, \mathbb{L} - \mathbb{L},$
 $K^\pm - \rho^\pm, p - \mathbb{L}, p - \bar{\mathbb{L}}$

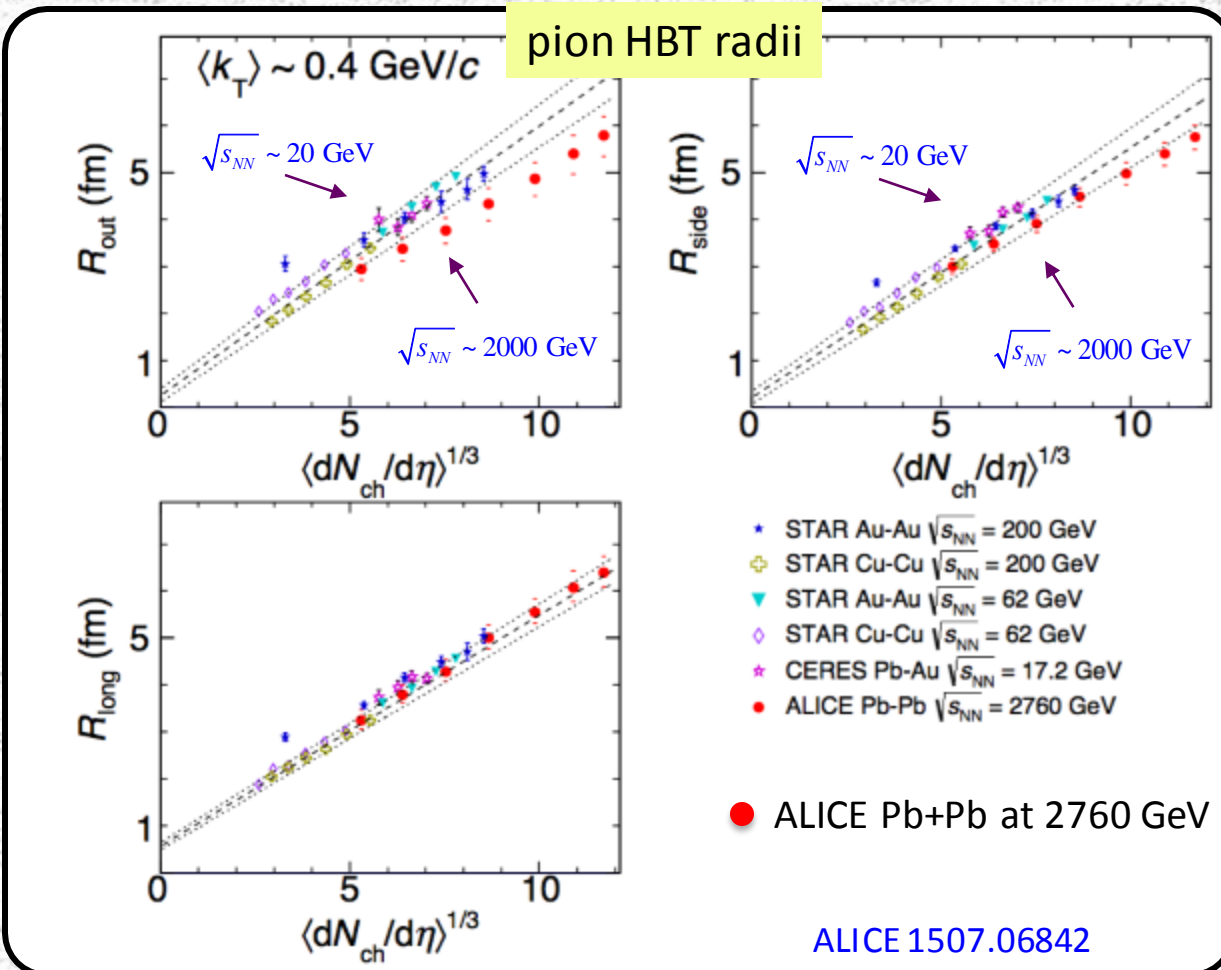
PHENIX PRL 103 (142301 (2009)

STAR PRC 88 034906 (2013)



Decade 3 broad message

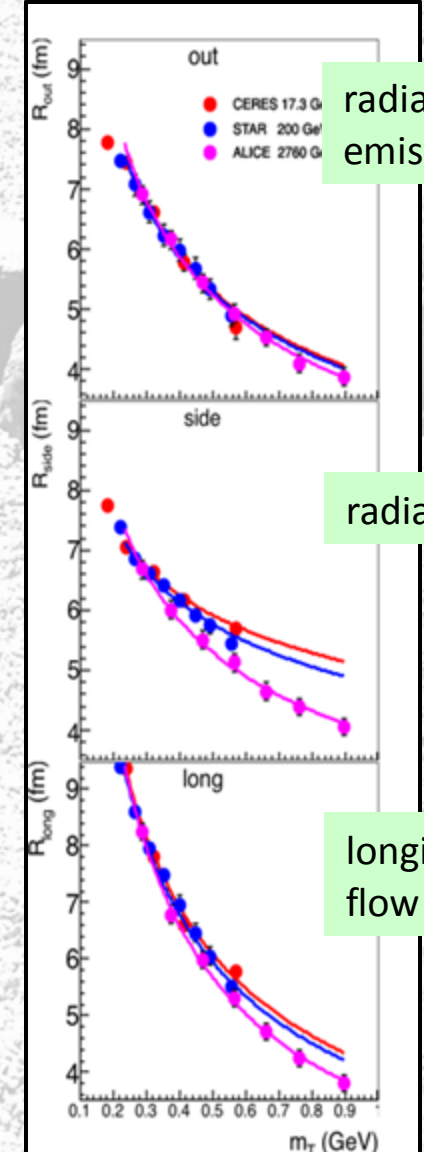
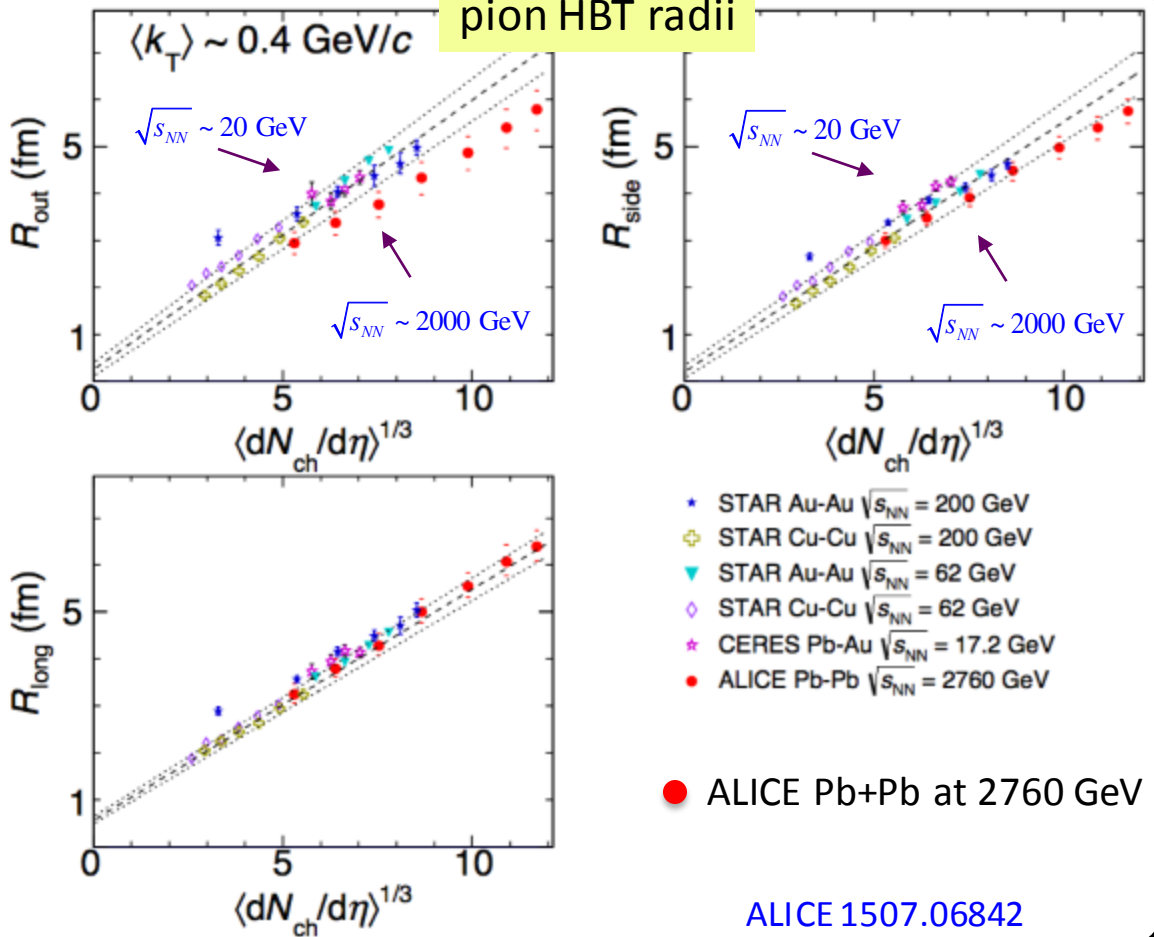
- New energies/species/techniques, higher statistics → reinforce earlier systematics
 - space and time scale \sim multiplicity^{1/3}



Decade 3 broad message

- New energies/species/techniques, higher statistics → reinforce earlier systematics
 - space and time scale \sim multiplicity^{1/3}
 - flow-induced falloff in m_T .
 - larger at LHC

pion HBT radii



radial flow & emission time

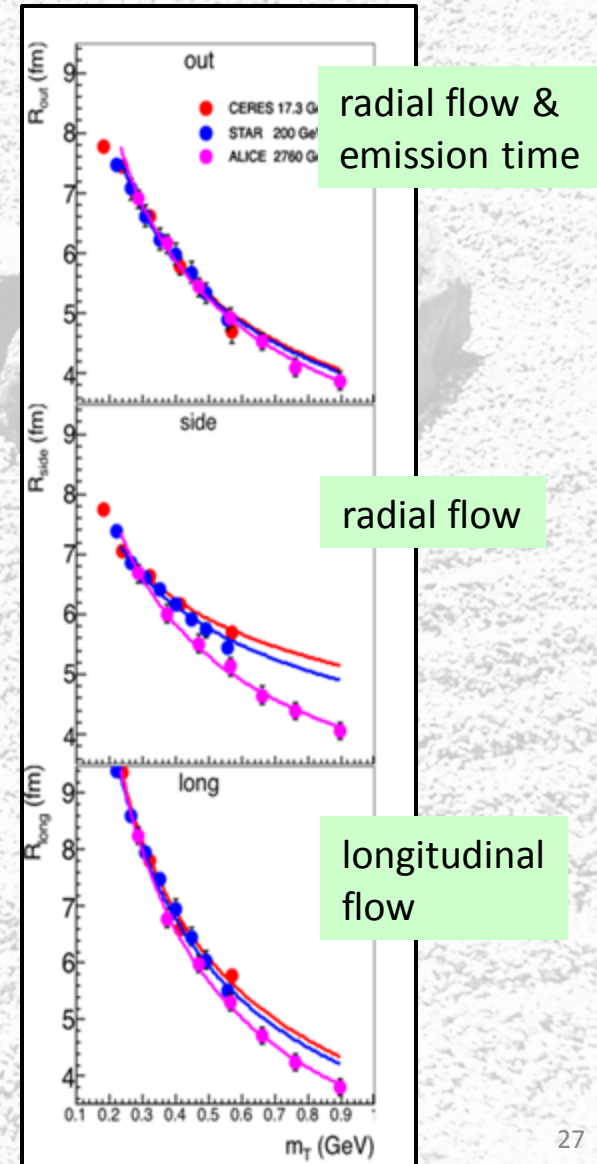
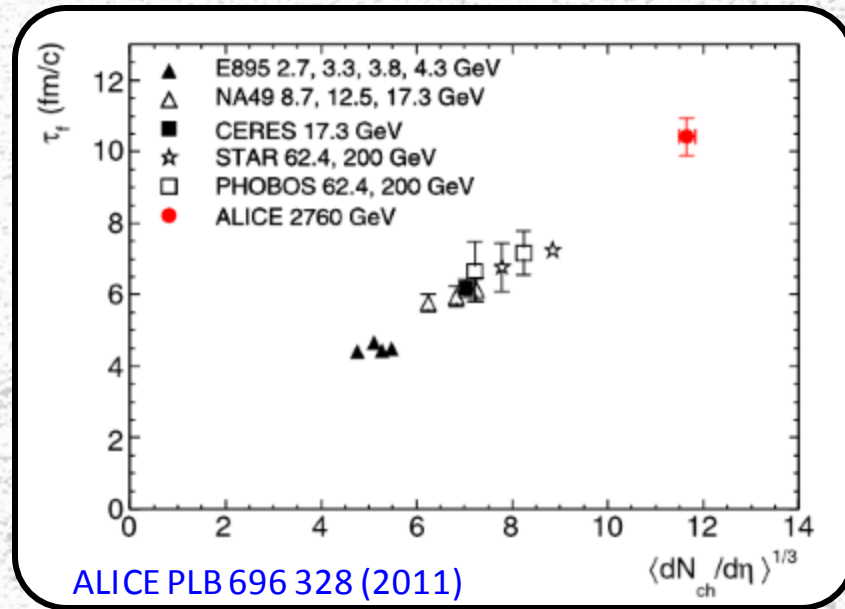
radial flow

longitudinal flow

Decade 3 broad message

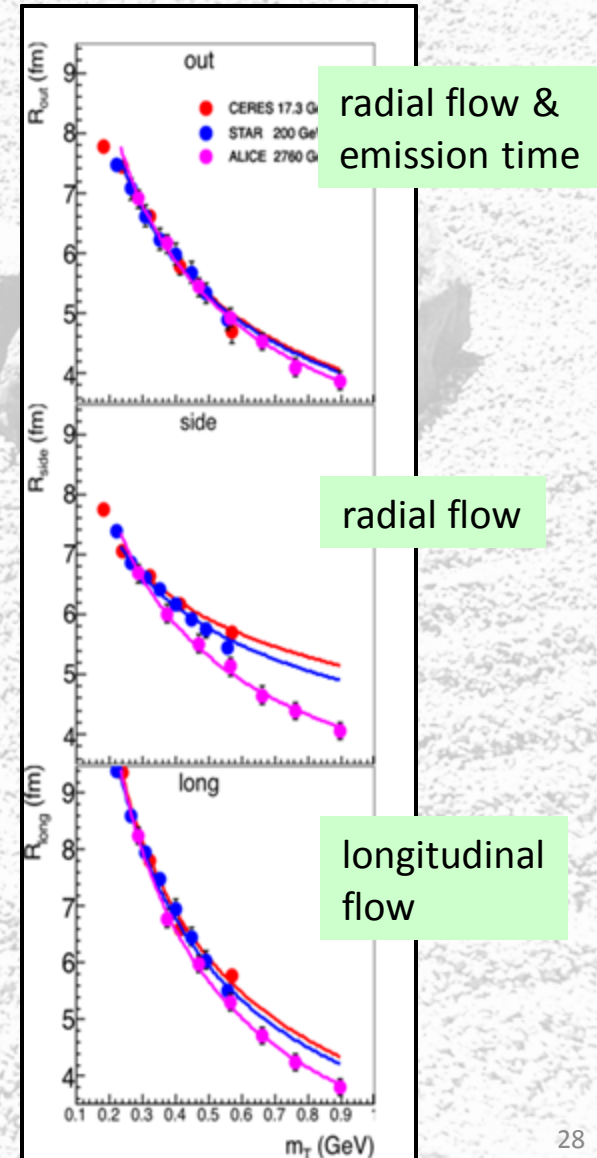
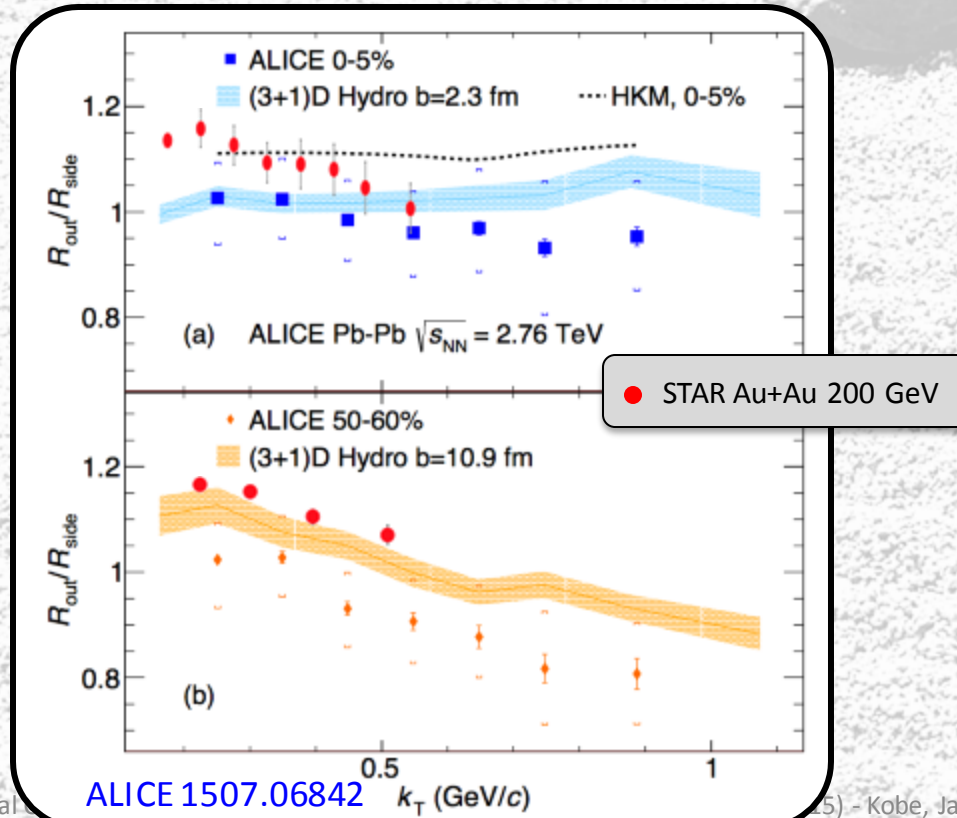
- New energies/species/techniques, higher statistics → reinforce earlier systematics
 - space and time scale \sim multiplicity^{1/3}
 - flow-induced falloff in m_T
 - larger at LHC
 - no sudden jumps in timescales (based on R_L and asHBT)

$$R_l^2 \gg t_0^2 \frac{T}{m_T} \frac{K_2(m_T/T)}{K_1(m_T/T)}$$



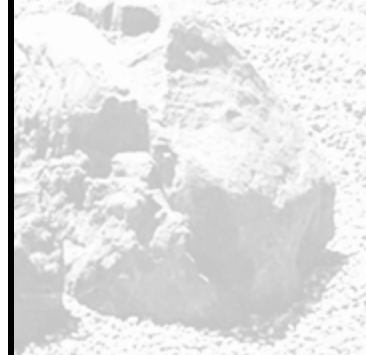
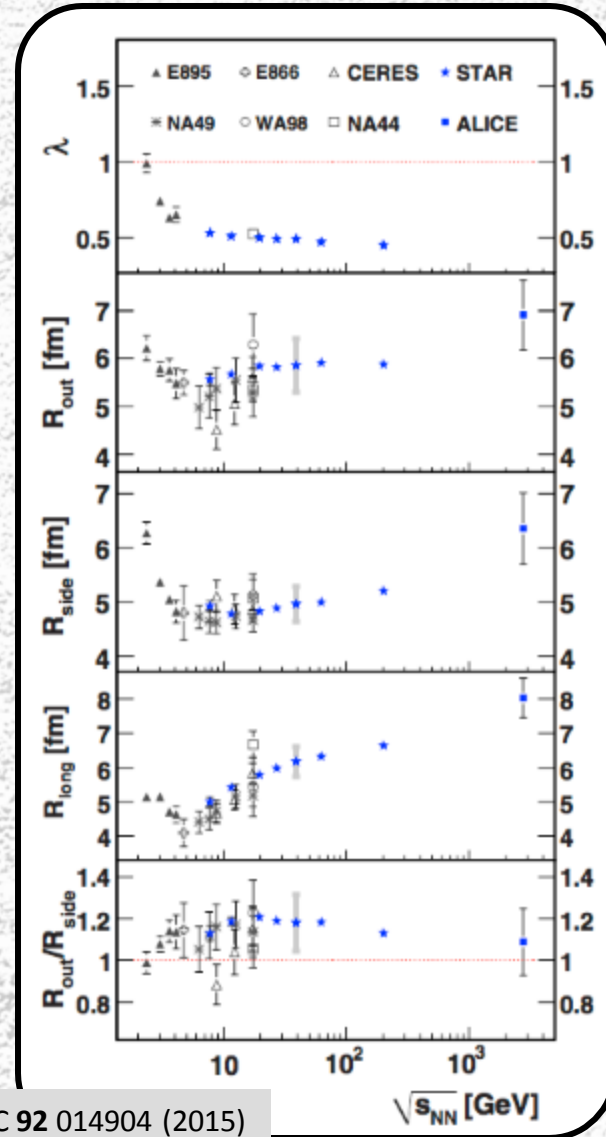
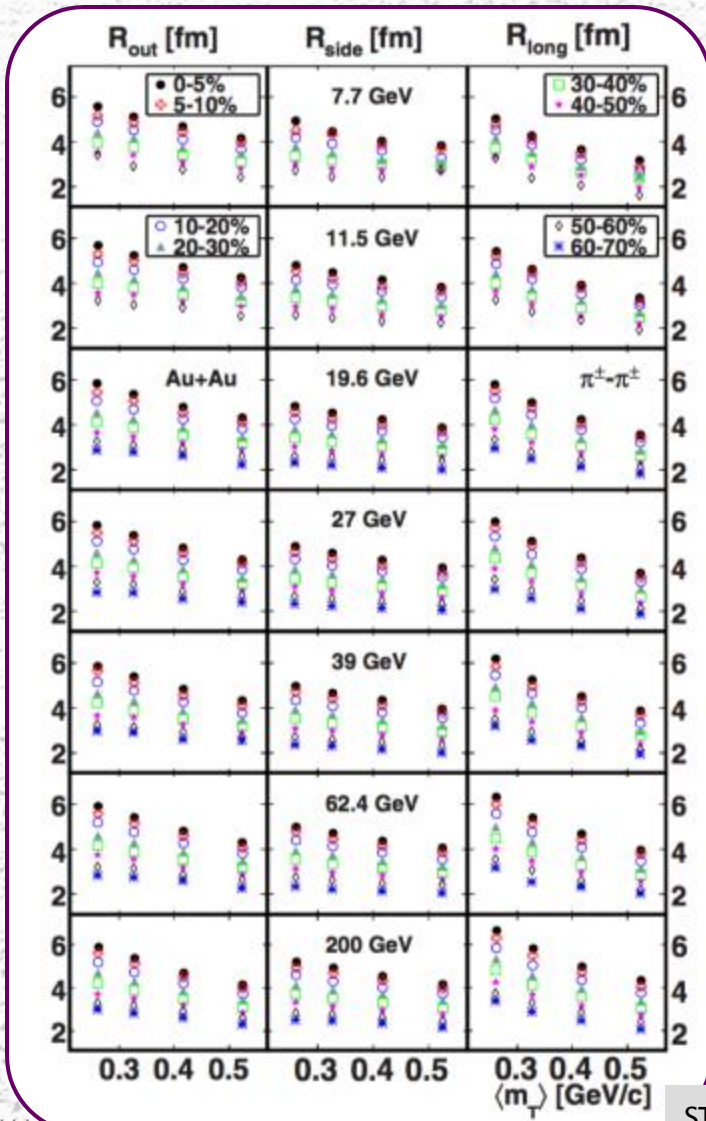
Decade 3 broad message

- New energies/species/techniques, higher statistics → reinforce earlier systematics
 - space and time scale \sim multiplicity^{1/3}
 - flow-induced falloff in m_T
 - larger at LHC
 - no sudden jumps in timescales (based on R_L and asHBT)
 - but sometimes important effects are small
 - ideally, study with *common* systematics



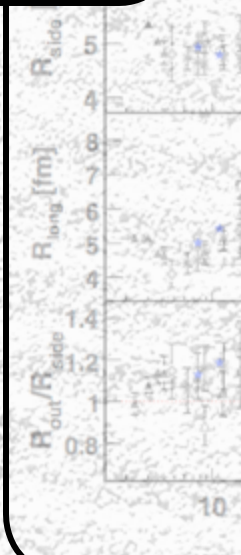
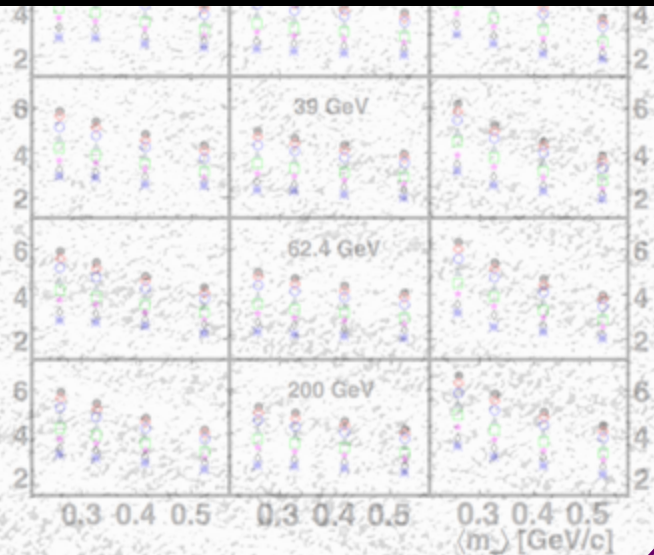
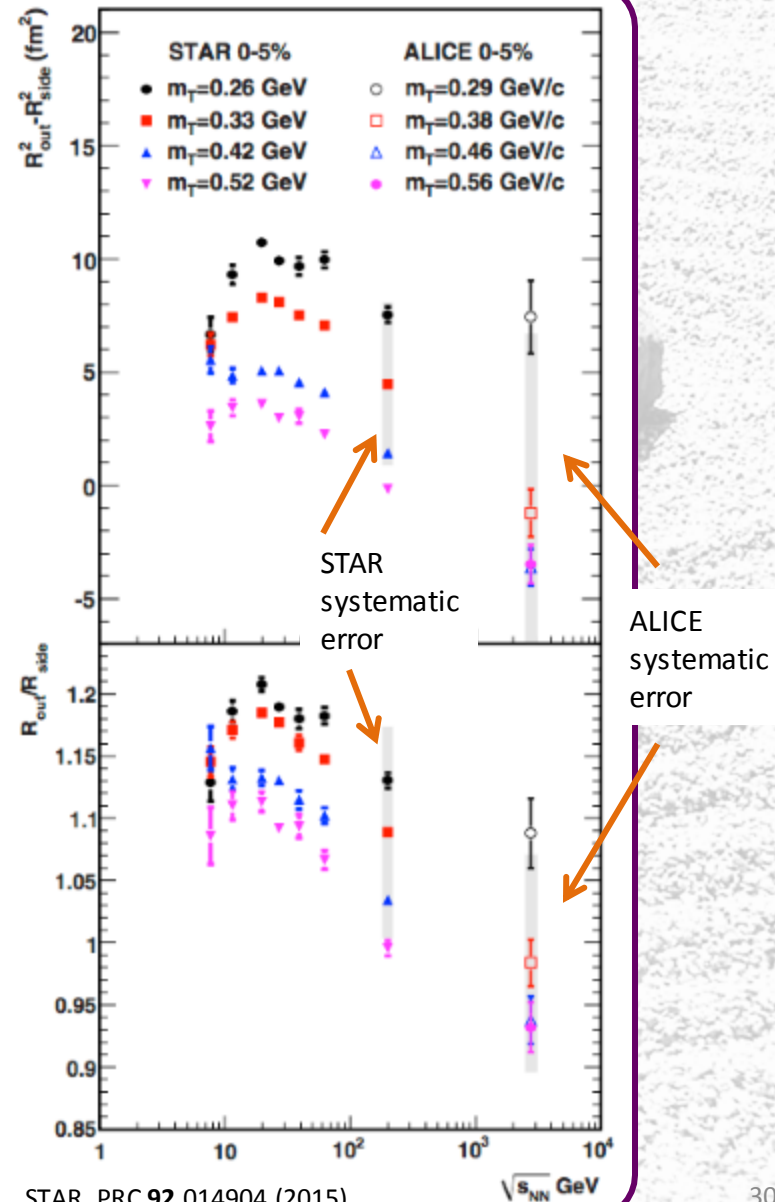
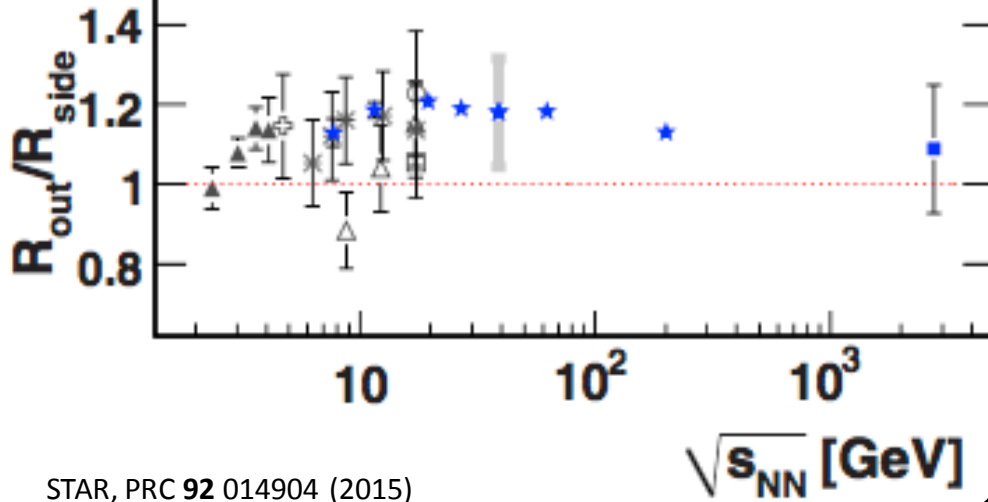
Systematic studies with ONE detector

- NA49, CERES, STAR: scans with common techniques, detectors
 - STAR: **broad scan and fixed acceptance**



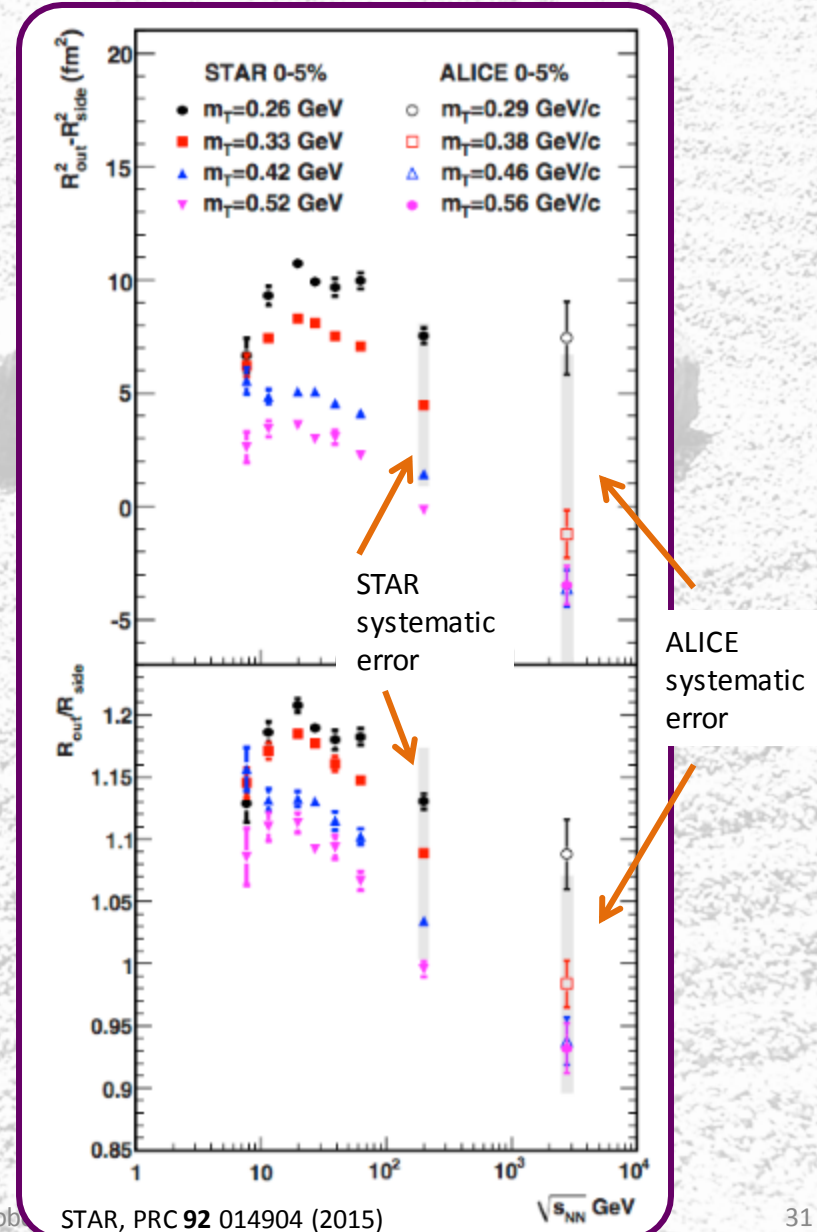
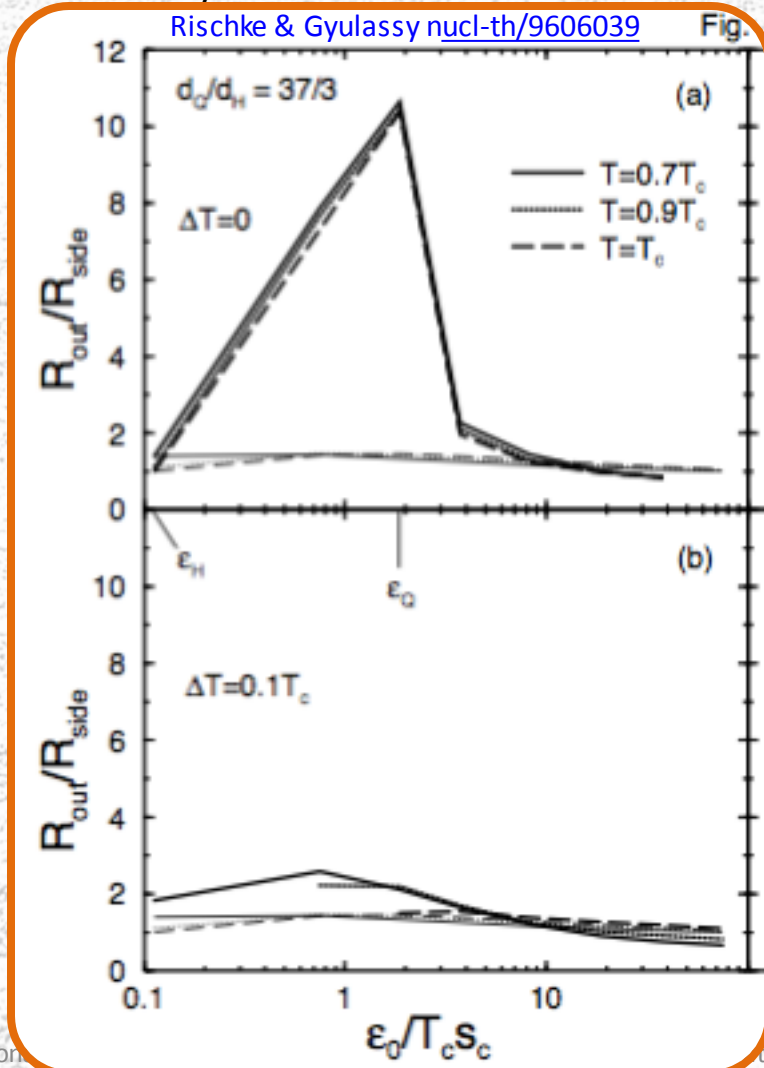
Systematic studies with fixed acceptance

- NA49, CERES, STAR: scans with common techniques, detectors
 - STAR: **broad** scan and fixed **acceptance**

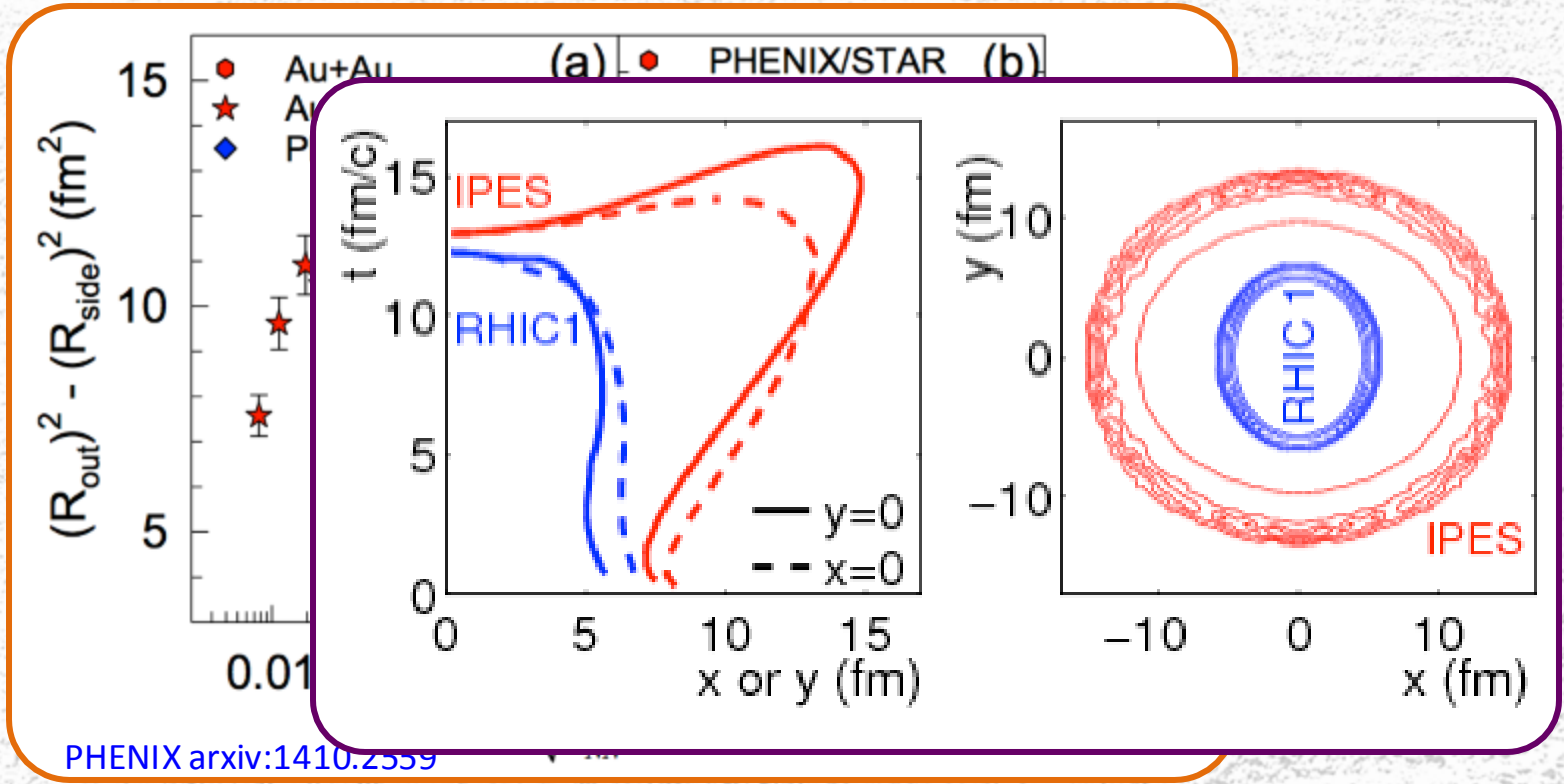


Systematic studies with fixed acceptance

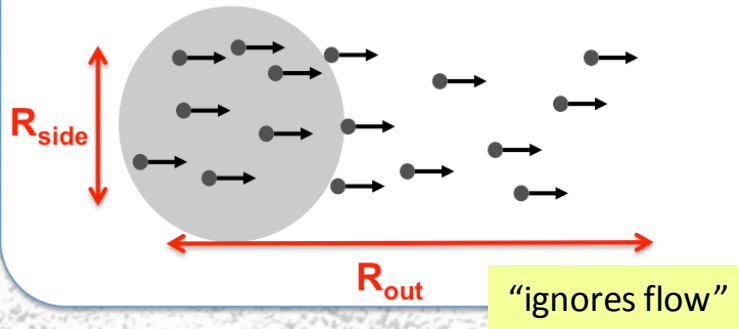
- The signal of latent heat, promised as a “sure thing” to be shown at QM 2001
 - **only 14.5 years late!**
 - more subtle than hoped
 - revealed only in detailed *scan*.



Elaborations...



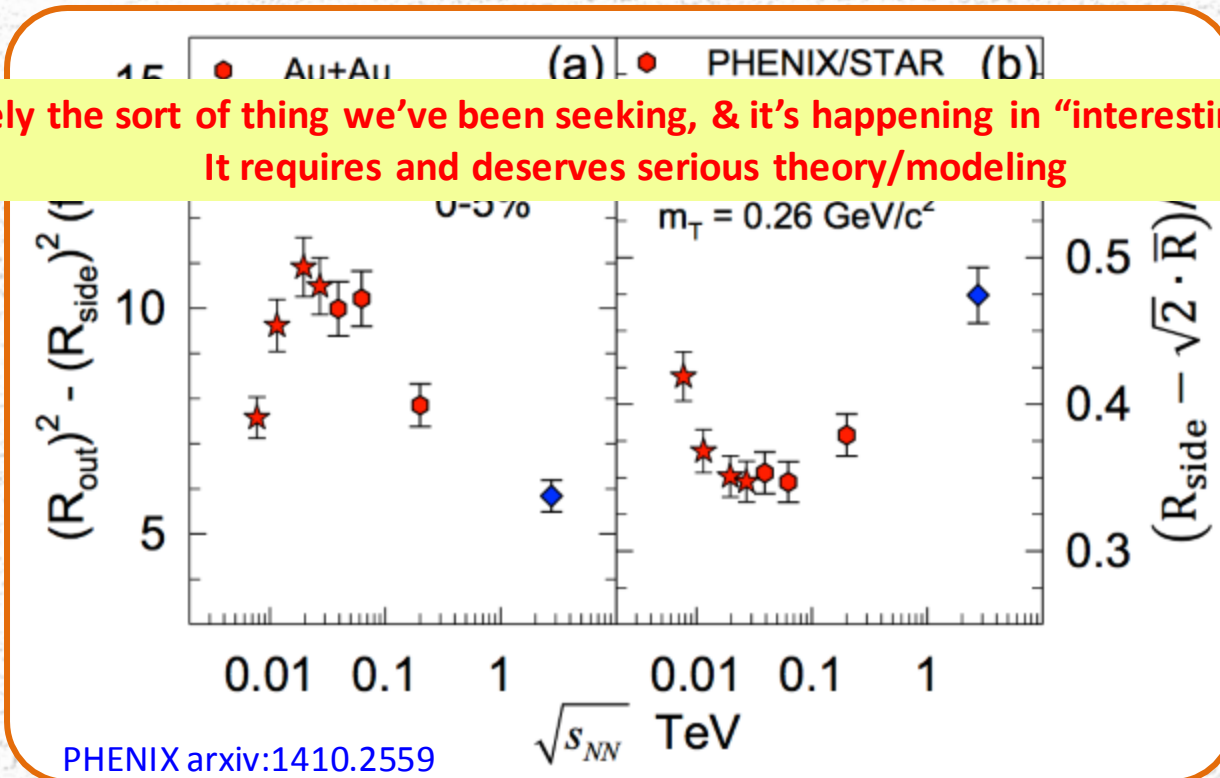
“Burning log” Bertsch, Pratt, Rischke, Gyulassy...



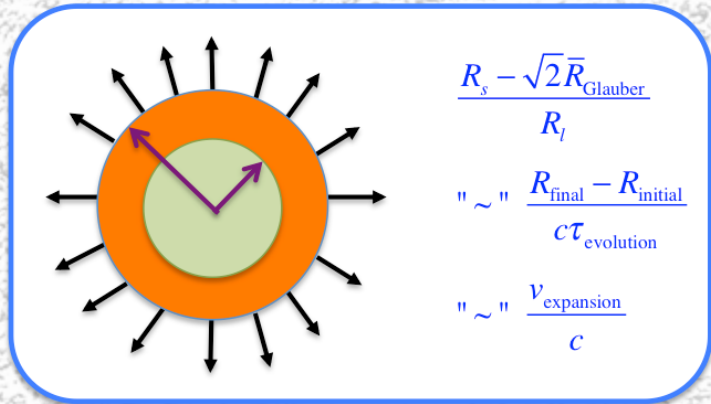
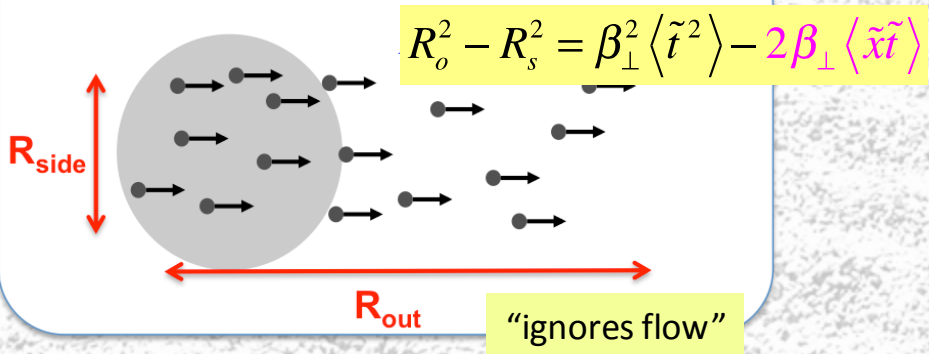
$$R_o^2 - R_s^2 = \beta_{\perp}^2 \langle \tilde{t}^2 \rangle - 2\beta_{\perp} \langle \tilde{x}\tilde{t} \rangle$$

Elaborations...

This is precisely the sort of thing we've been seeking, & it's happening in "interesting" BES energies. It requires and deserves serious theory/modeling



"Burning log" Bertsch, Pratt, Rischke, Gyulassy...



Softening? Critical point??

More than just (!) softening? – finite size scaling

- at CP, susceptibilities diverge for *infinite* system
 - delta function
- for *finite* system (generic)
 - no divergence, just peaking
 - shifted peak position
 - broadened peaks

specifically:

height: $C_T^{\max}(V) \sim L^{g/n}$

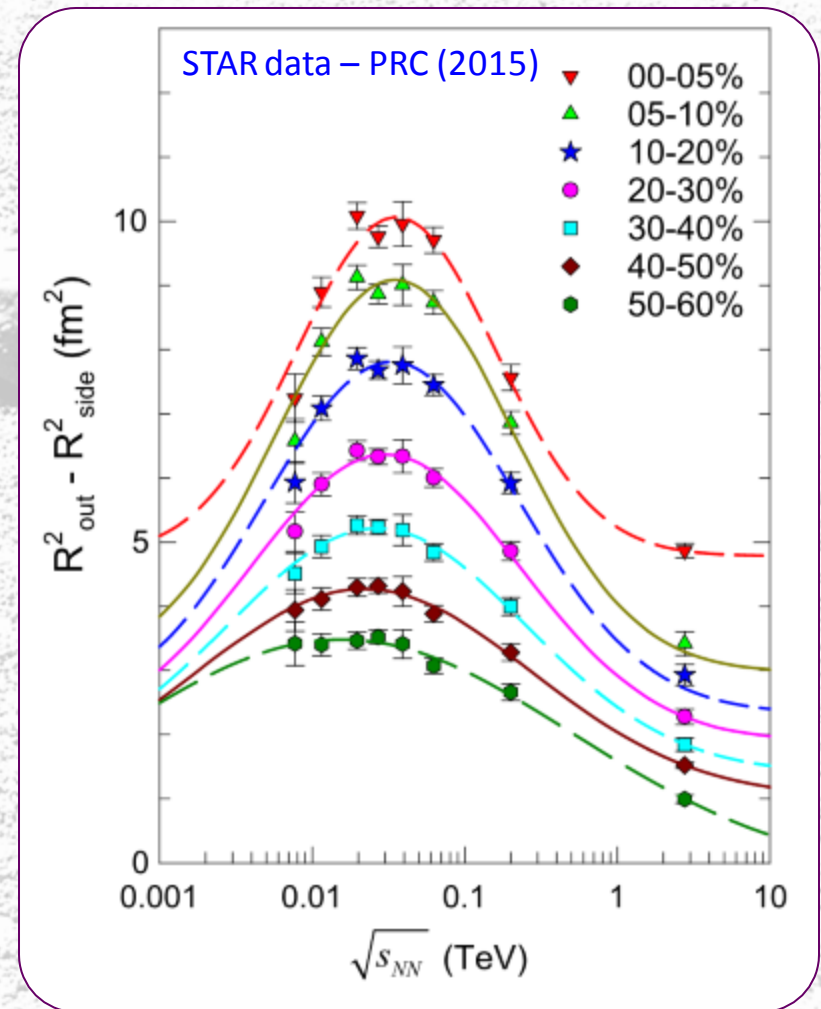
width: $dT(V) \sim L^{-1/n}$

position: $t_T(V) \sim T^{CEP}(V) - T^{CEP}(\infty) \sim L^{-1/n}$

hypothesis

$$R_o^2 - R_s^2 \leftrightarrow C_T$$

$$\bar{R}_{\text{Glauber}} \leftrightarrow L$$



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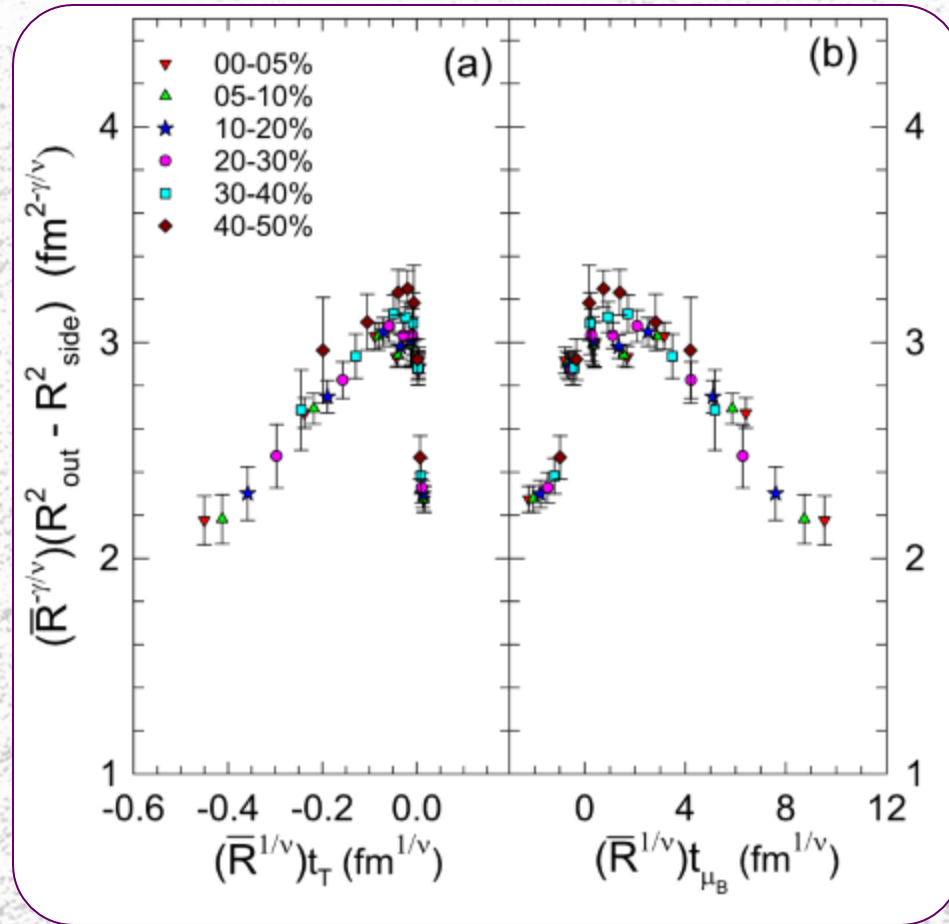
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position: $t_T(V) \sim T^{CEP}(V) - T^{CEP}(\infty) \sim L^{-1/n}$

hypothesis

$R_o^2 - R_s^2 \leftrightarrow C_T$ ← reasonable?

$\bar{R}_{\text{Glauber}} \leftrightarrow L$



$T^{CEP} \sim 165 \text{ MeV}$ $m_B^{CEP} \sim 95 \text{ MeV}$

$g=1.15$ $n=0.67$ (3D Ising)

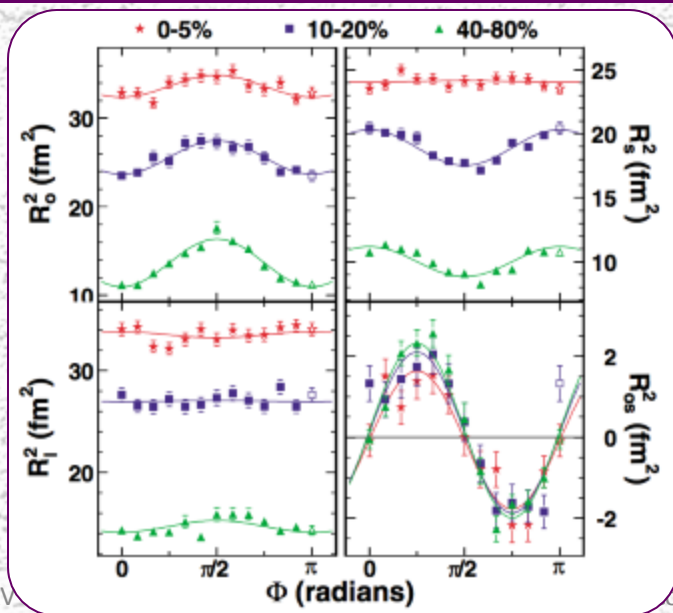
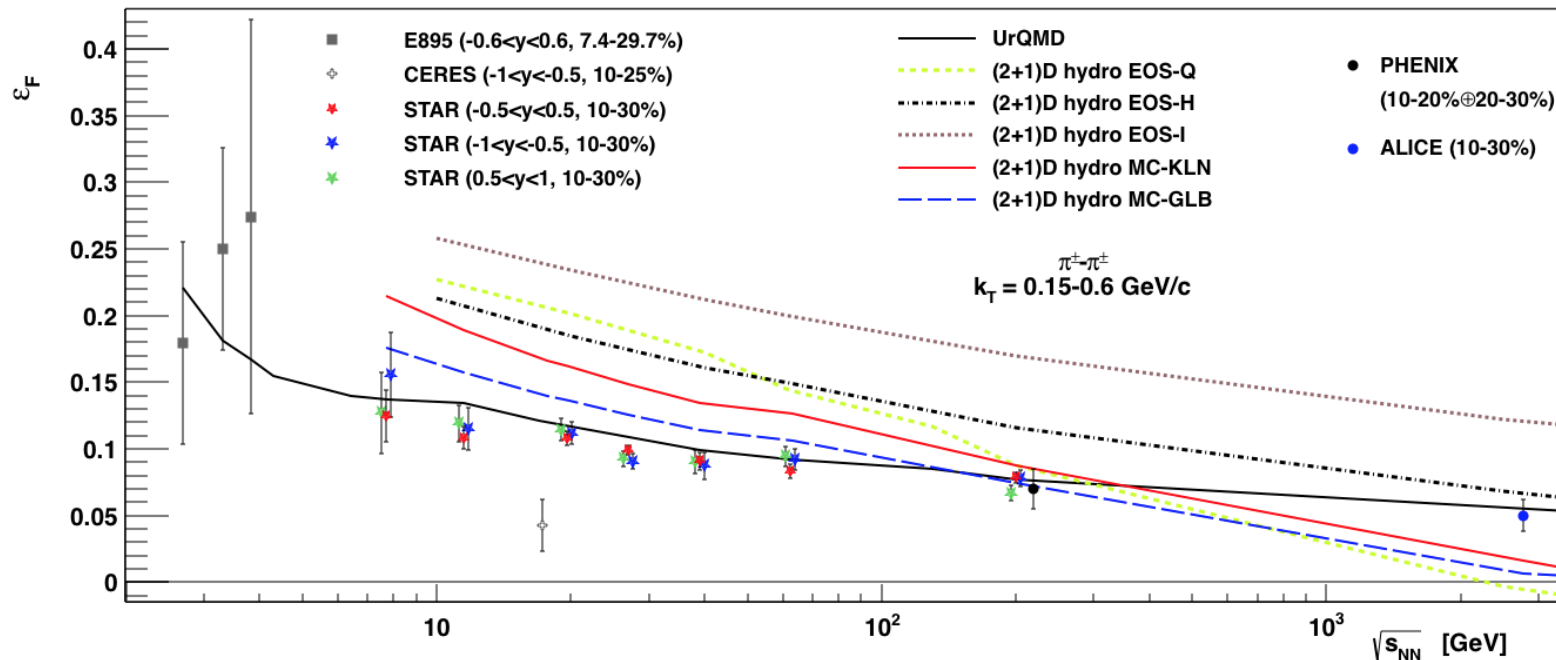
← excluded by Lattice?

Dynamic *shape* evolution

STAR PRC92 014904

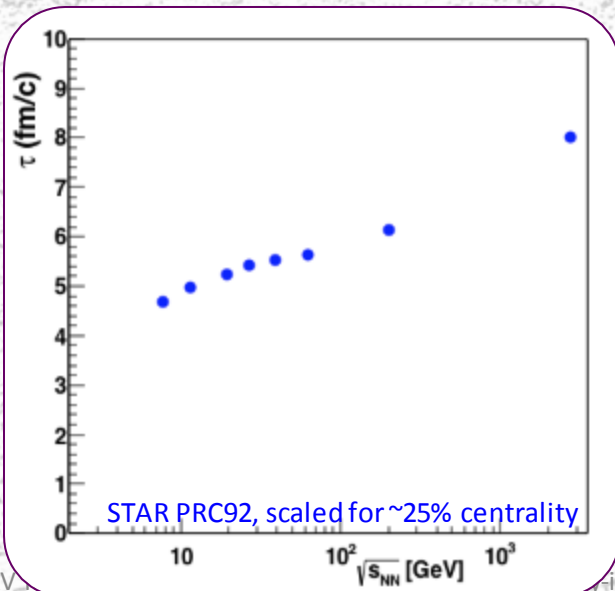
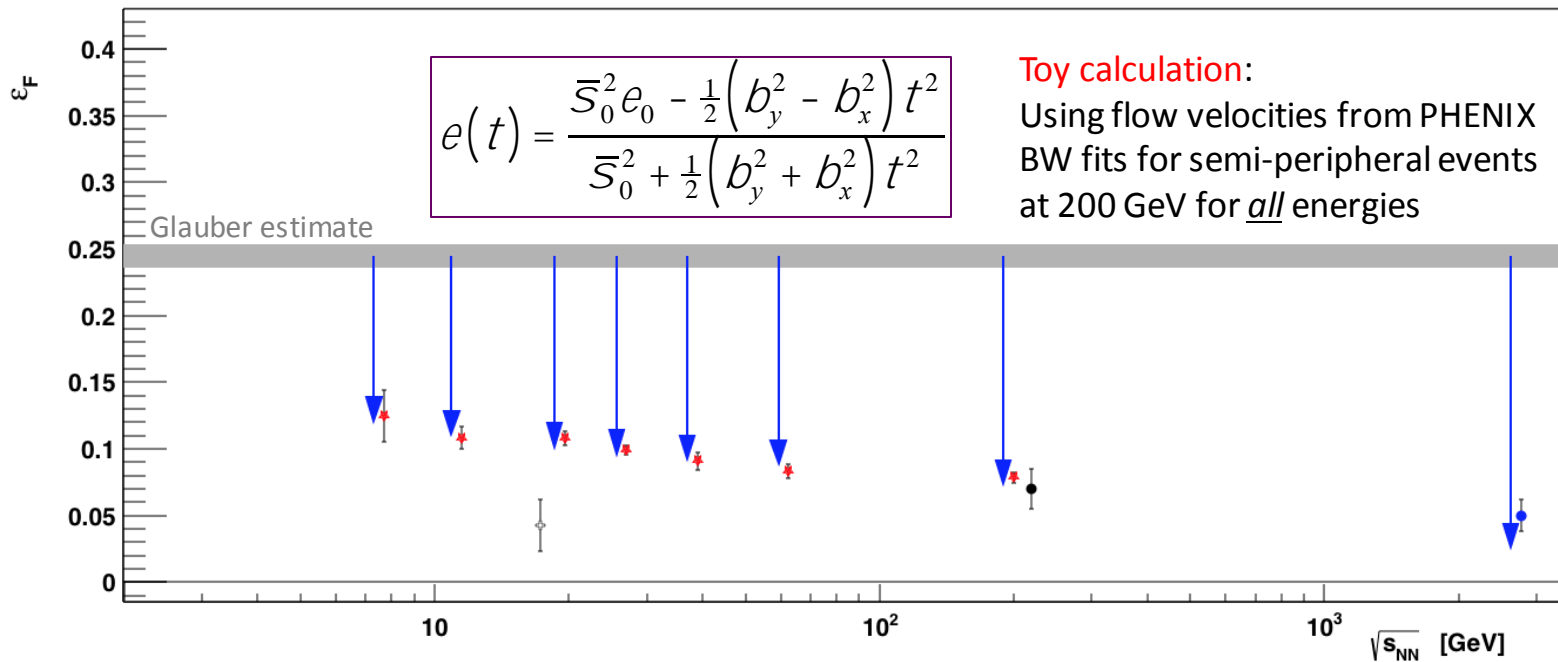
PHENIX
PRL 112 222301

ALICE
NPA 931 1088



- HBT radii relative to 2nd-order plane reveal f.o. shape
- Sensitive to model parameters (EoS, init. cond., eta/s)
- Full excitation function in decade 3
- All collider experiments consistent with common curve...
...which is UrQMD (?!)

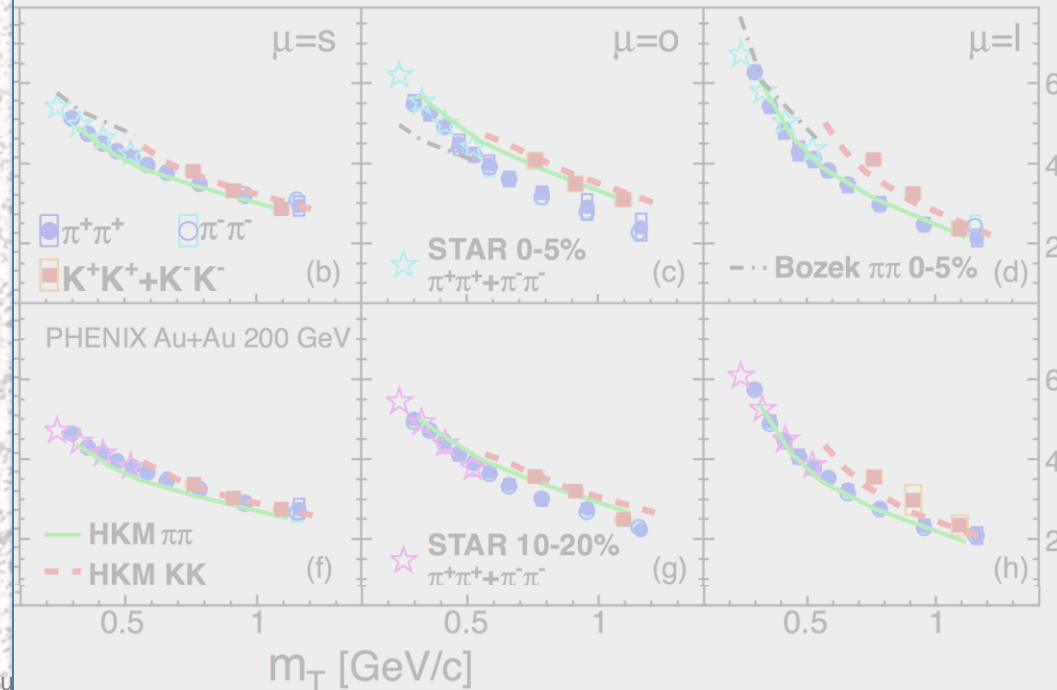
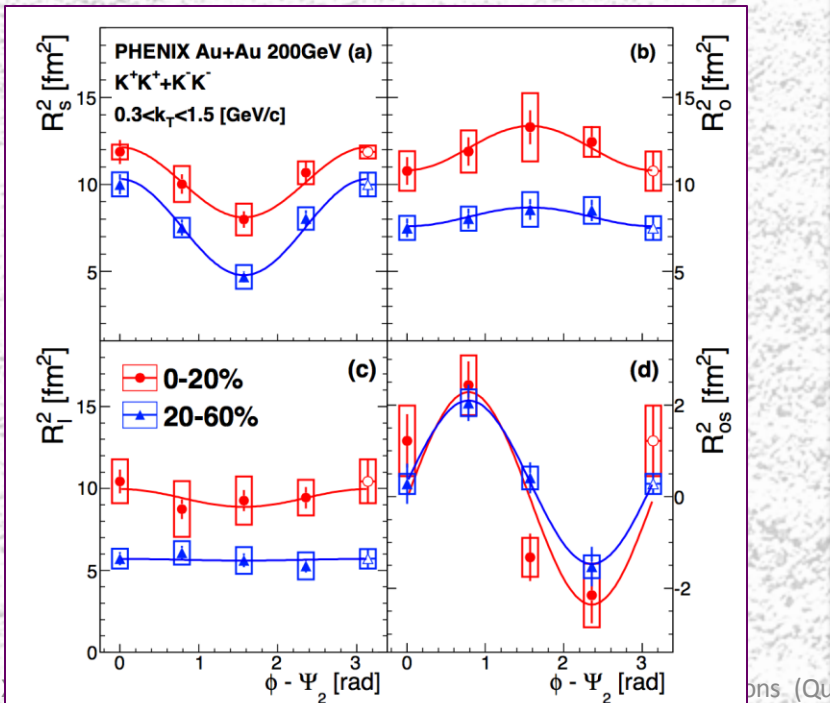
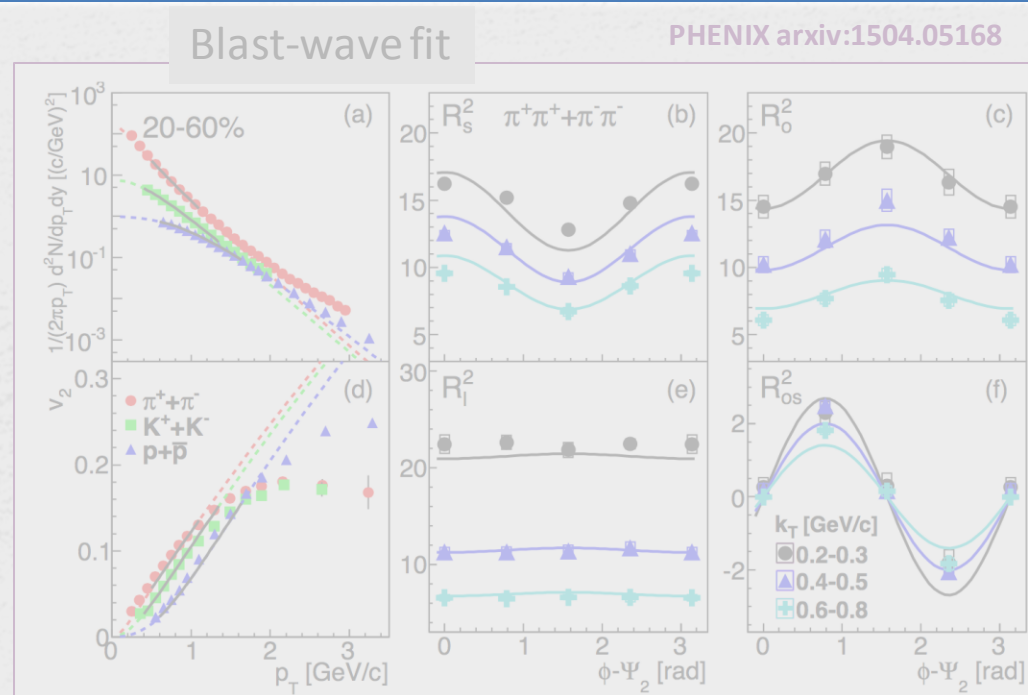
Dynamic *shape* evolution



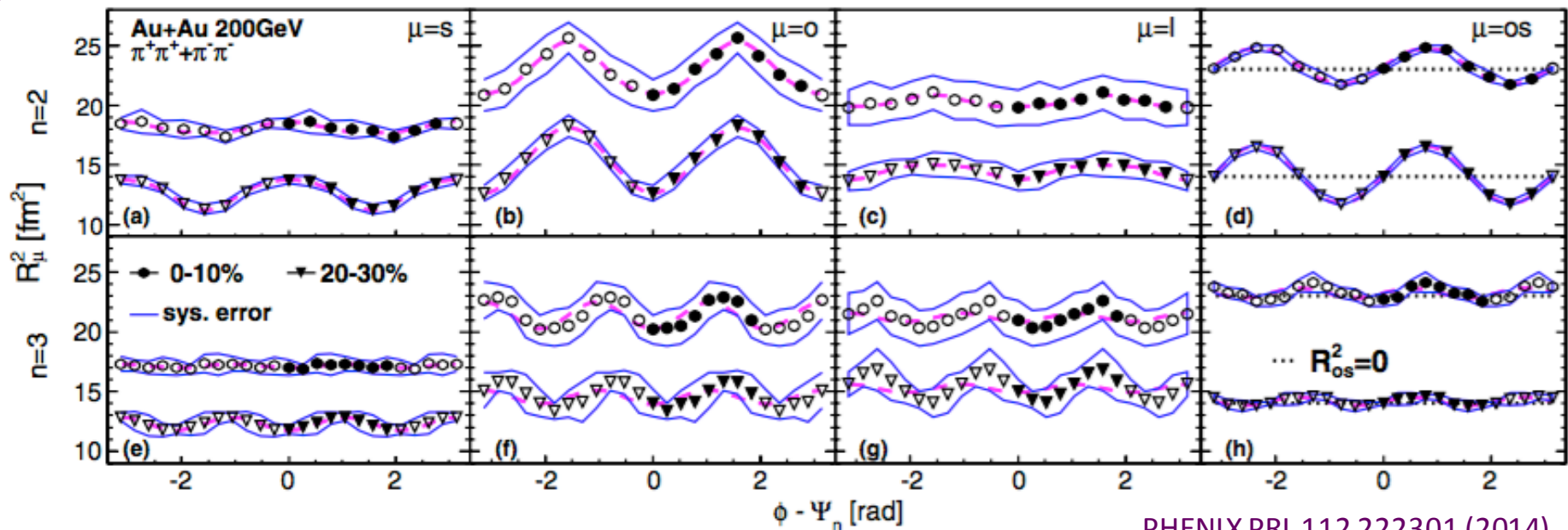
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- Full excitation function in decade 3
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...which is UrQMD (?!)
- final shape ~consistent with evolution time determined from R_{long} and flow velocity
- But must go beyond toy models

Extension to kaons

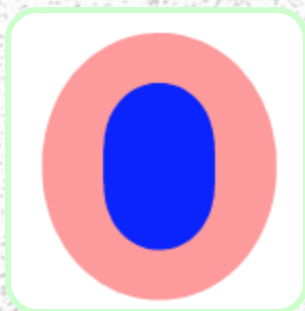
- kaon radius oscillations similar to pions
- m_T scaling for individual radii broken
- Blast-wave: decent fit to soft-sector data
- Hydro-kinetic model (HKM) and viscous hydro describe most femtosopic data



Higher-order geometrical substructure: n=3



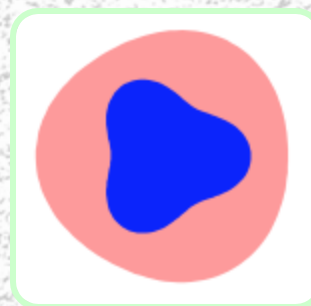
PHENIX PRL 112 222301 (2014)



transverse size increases $\sim x2$

n=2

- oscillations dominated by system geometry
- ellipticity reduced $\sim x3$
- \sim consistent w/ evolution lifetime (R_{long})



consistent w/ 3+1D E-by-E
viscous hydro calculations
(& predictions)
– Bozek 2014

n=3

- oscillations determined by flow gradients
- triangularity \sim eliminated / slightly reversed
- distinguishes differing flow scenarios
(Plumberg & Heinz, 2013)

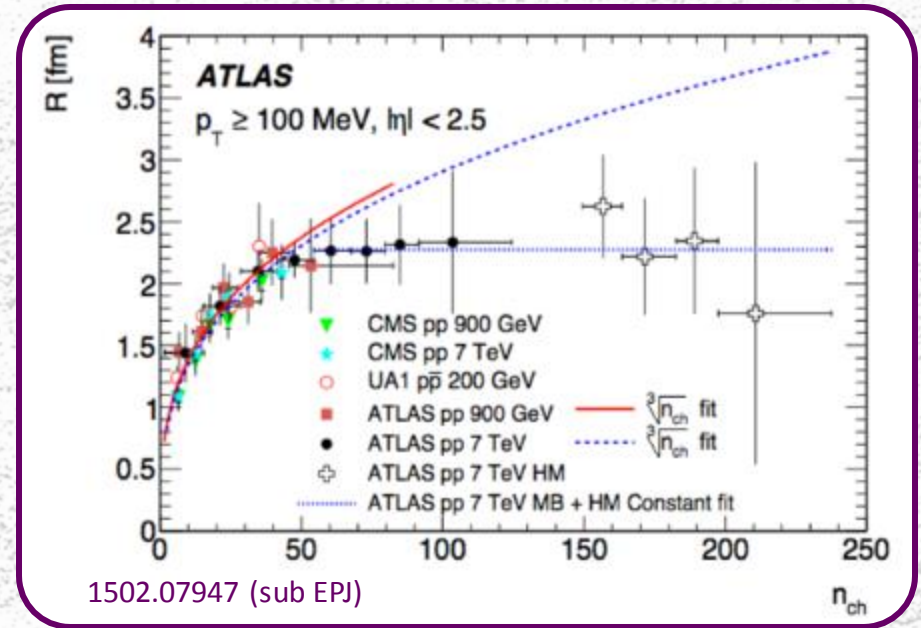
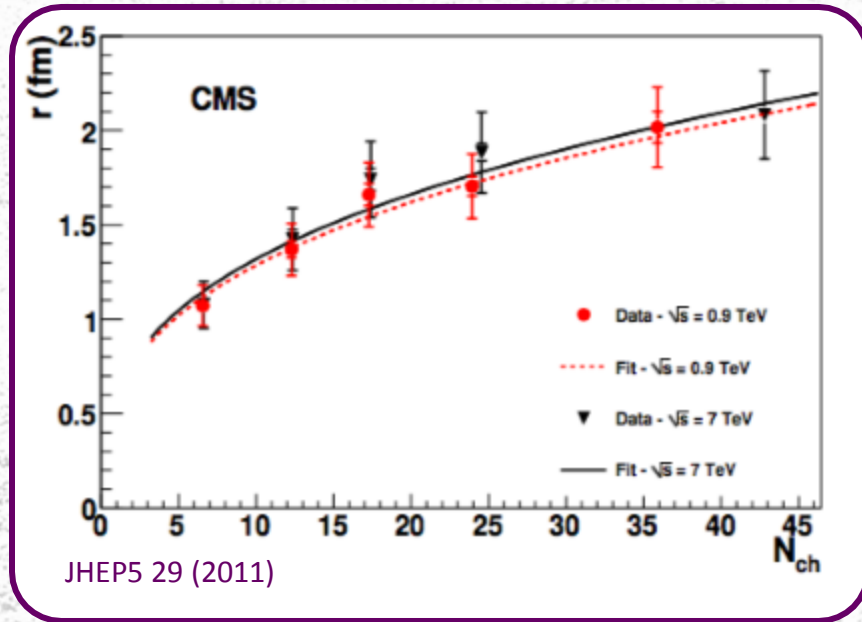
Femtoscopic heavy ion story in Decade 3

- New levels of detail with precision data and new techniques
 - dynamically-generated geometric substructure probed (asHBT)
 - SHD and images with spatially-extended tails
 - correlations between new species
 - residual correlations resolve “anomalies” in systematics
- Overall, very similar message over three orders of magnitude
 - freezeout radii $\sim \text{Mult}^{1/3}$, largely independent of collision energy
 - flow-dominated substructure revealed in m_T dependence of scales
 - at all root(s) & multiplicity ranges
 - for all species
 - shape evolution sensitive to geometric gradients and timescales
- Qualitatively most interesting physics: possible softening at $\sqrt{s} \sim 20$ GeV
 - may corroborate several other qualitatively intriguing signals in RHIC BES program
 - revealed by systematically varying colliding system with fixed detector, methods



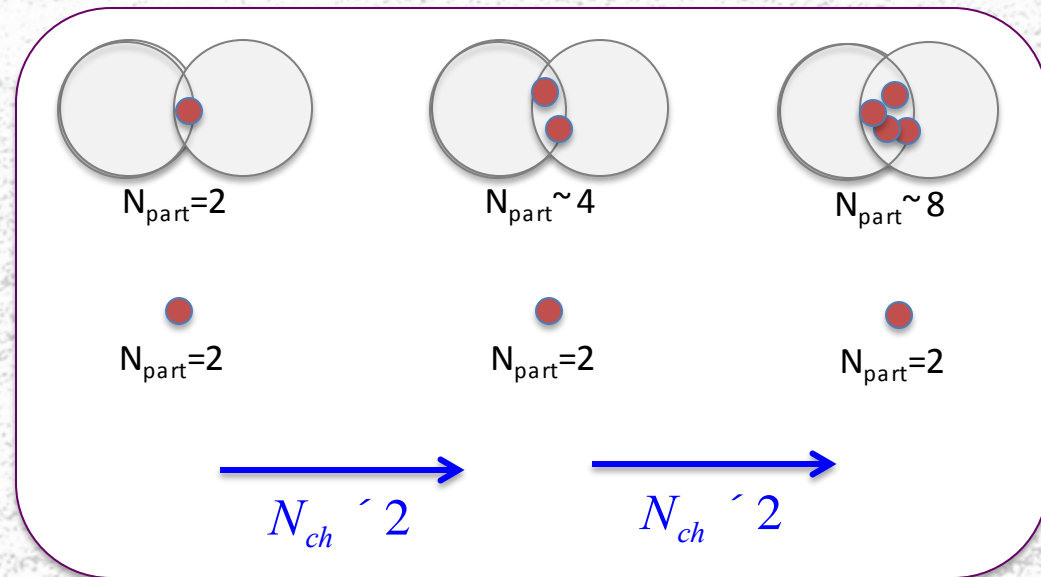
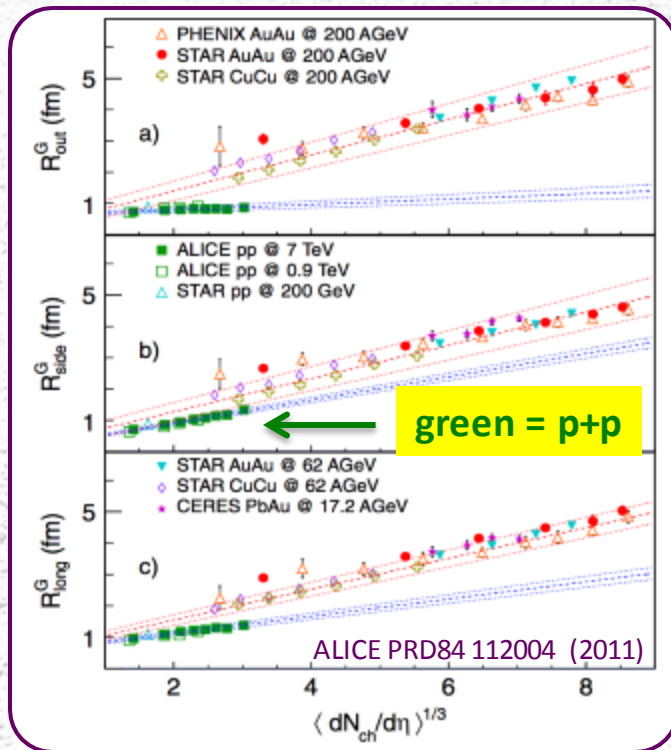
p+p versus A+A?

Femtoscscopy in p+p: Multiplicity dependence



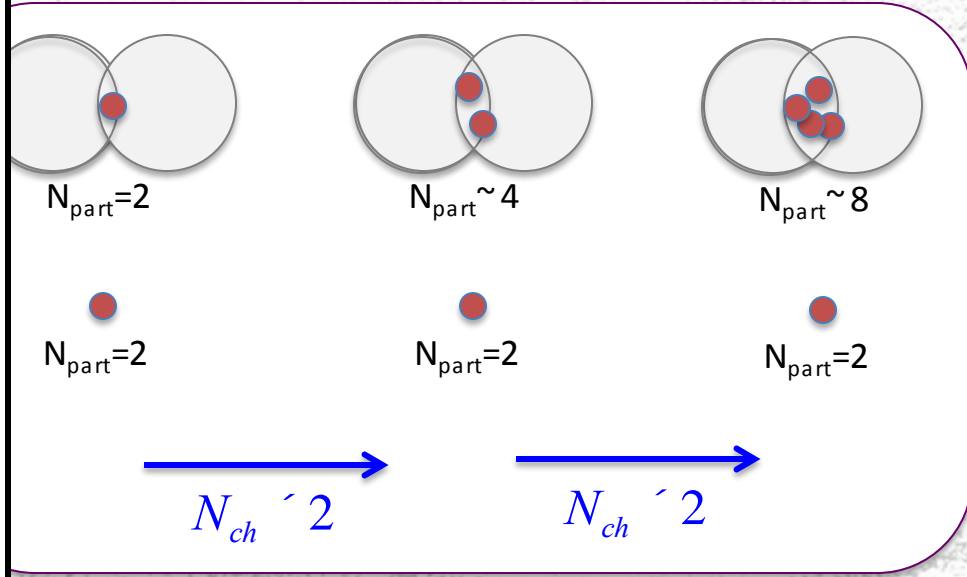
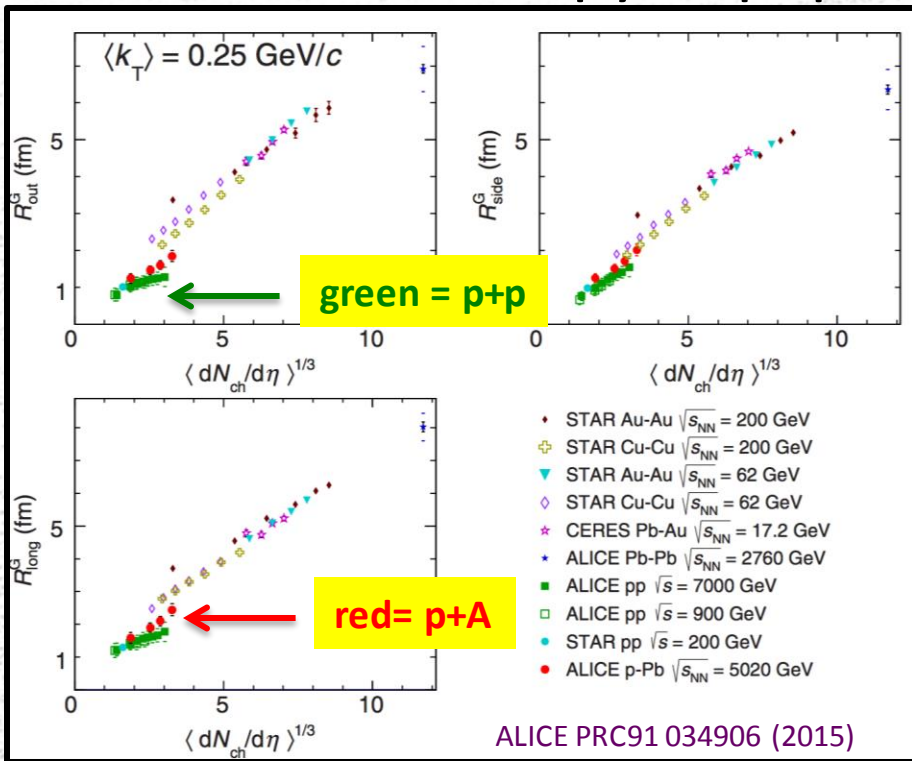
- Similar to A+A collisions, radii depend almost solely on multiplicity $\mu N_{ch}^{1/3}$, not \sqrt{s}
 - Fluctuations at extreme multiplicity violate the bulk trend
 - Pomeron picture? [Schegelsky et al, PLB (2011)]

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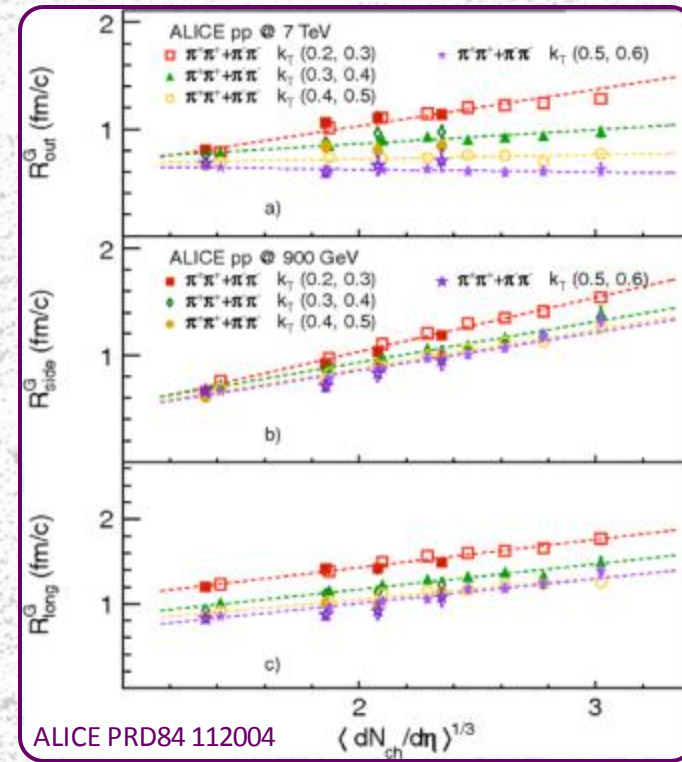
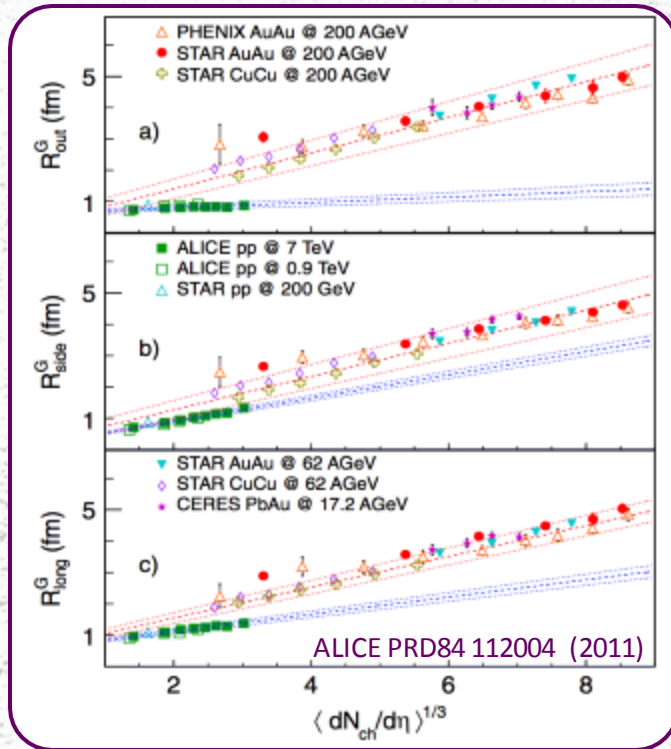
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- Proportionality constant differs from A+A case
 - relevance of initial geometry
 - p+p: multiplicity fluctuations in fixed (?) volume: expansion? jets? [Paic & Skowronski 2005]
 - A+A additional “trivial” initial geometry- N_{part} fluctuation

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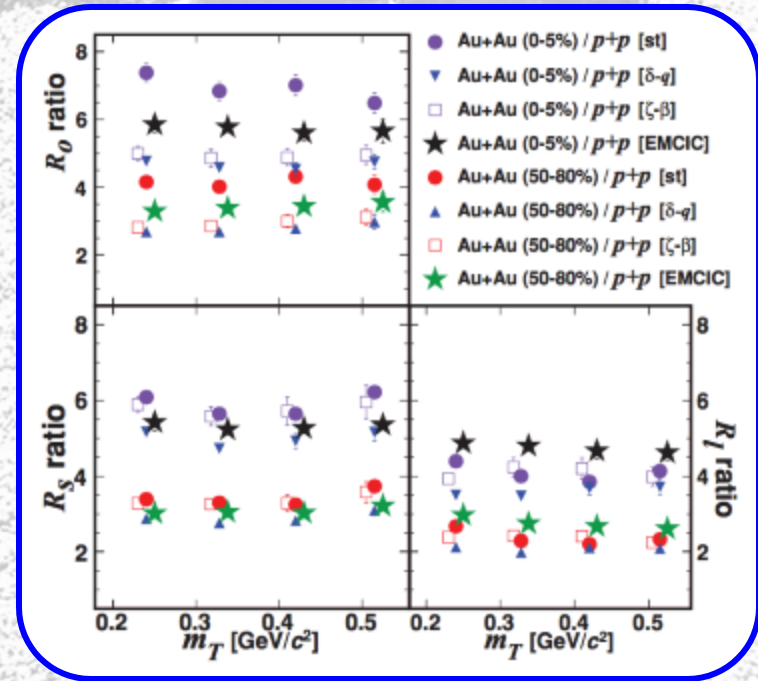
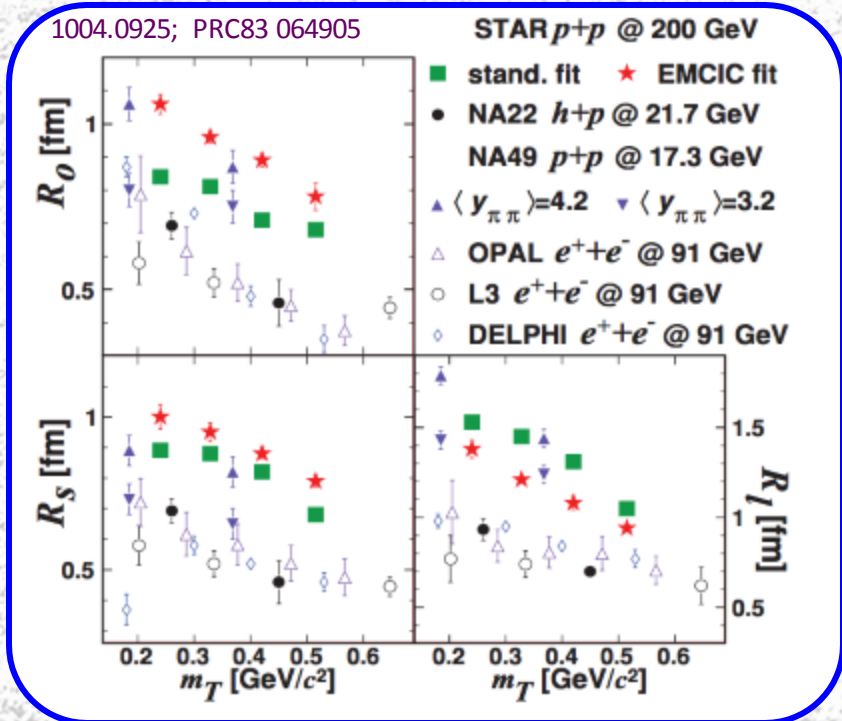
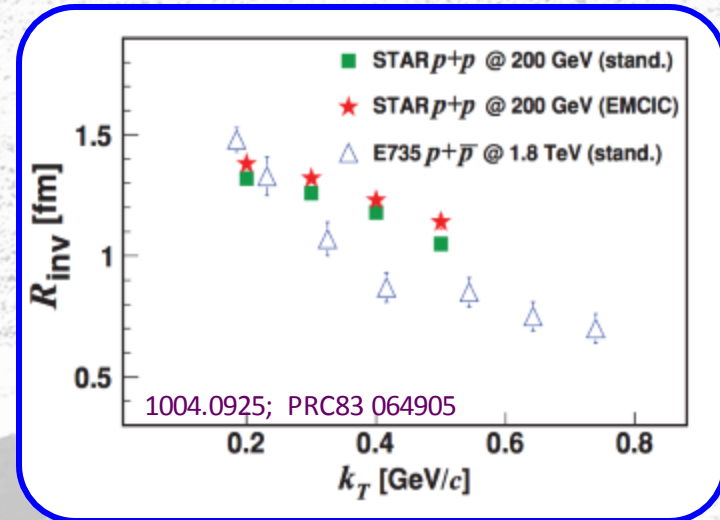


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- Similar to A+A: additional k_T -dependence \rightarrow **bulk dynamics?**

Strange suggestions from RHIC

STAR reports k_T dependence in *minbias* p+p @ 200 GeV

- requiring 2 midrapidity pions \rightarrow bias toward high mult
- quantitatively similar to A+A @ RHIC - **similar flow?**

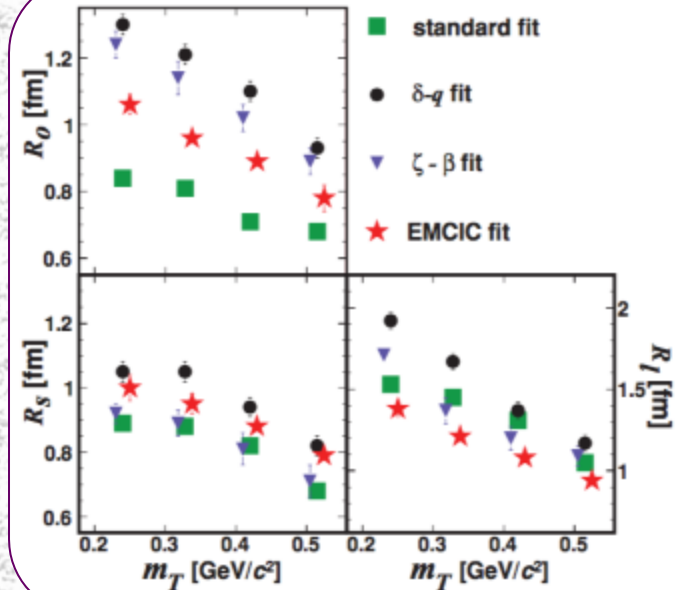
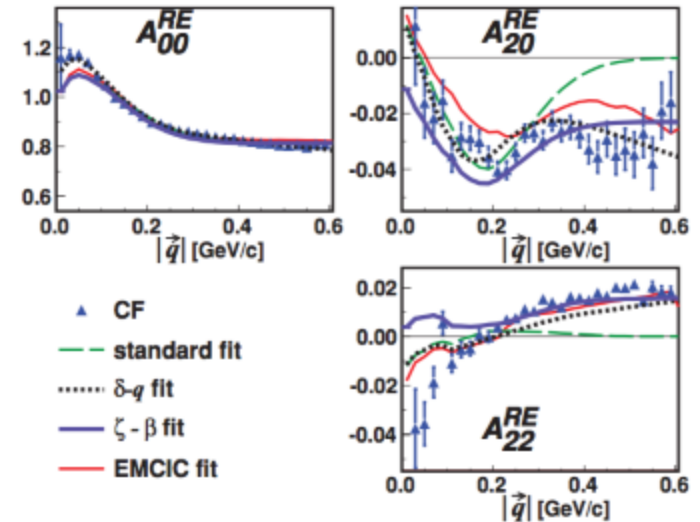
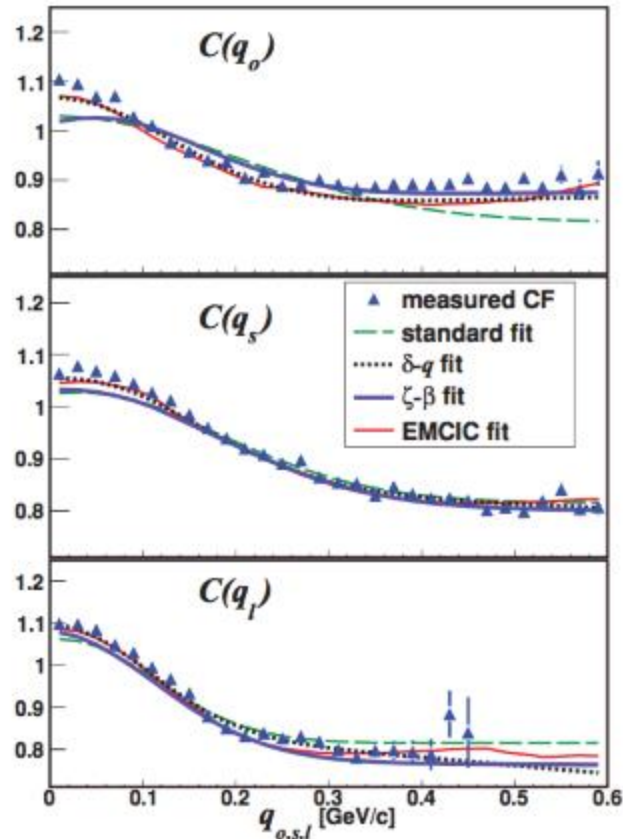


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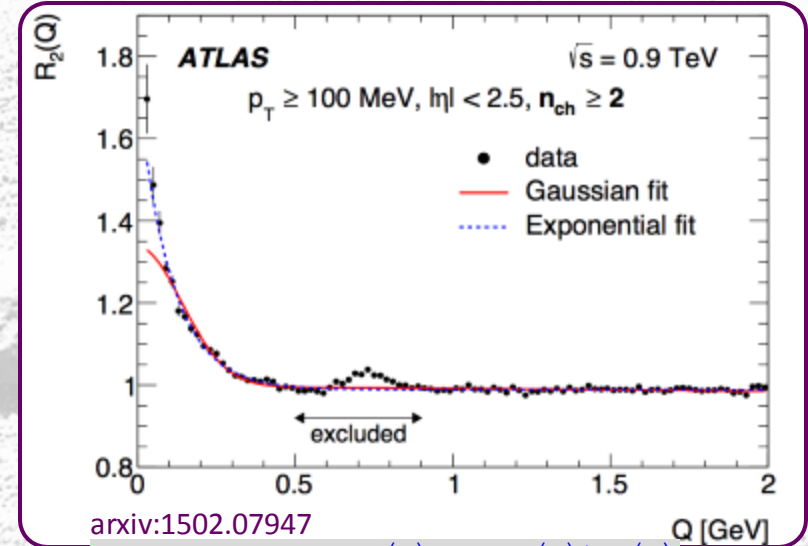
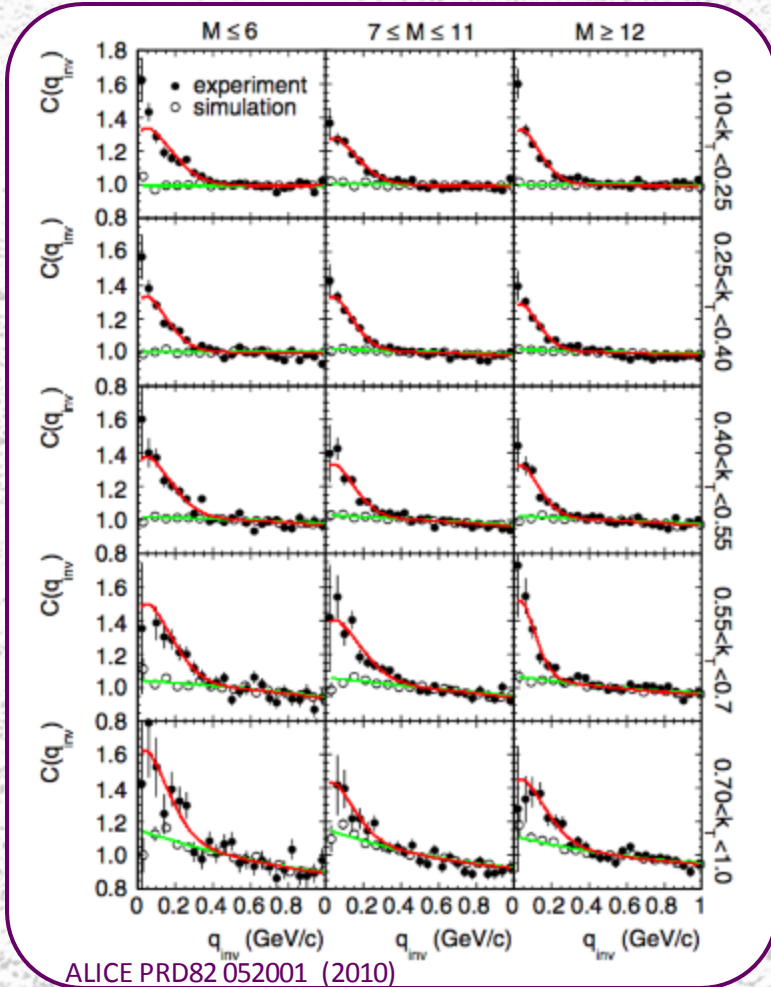
- requiring 2 midrapidity pions \rightarrow bias toward high mult
- quantitatively similar to A+A @ RHIC - **similar flow?**
- strong non-femto correlations \rightarrow poor fit
 - energy & momentum conservation also distorts spectra, masking flow effects? [Chajeccki & MAL PRC79 (2009)]

Harmonic decomposition \rightarrow

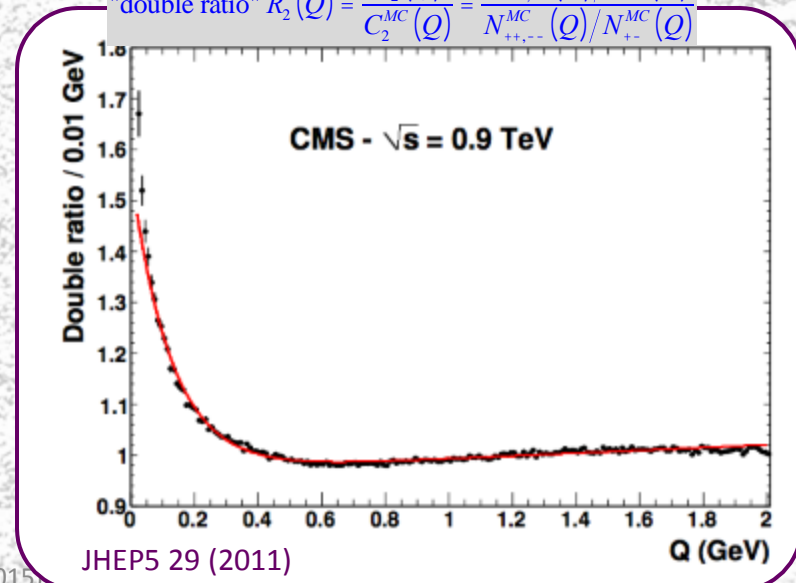


The message from LHC

- Much better (model-based) accounting of non-femto correlations
- 1D: exponential better than Gaussian fit



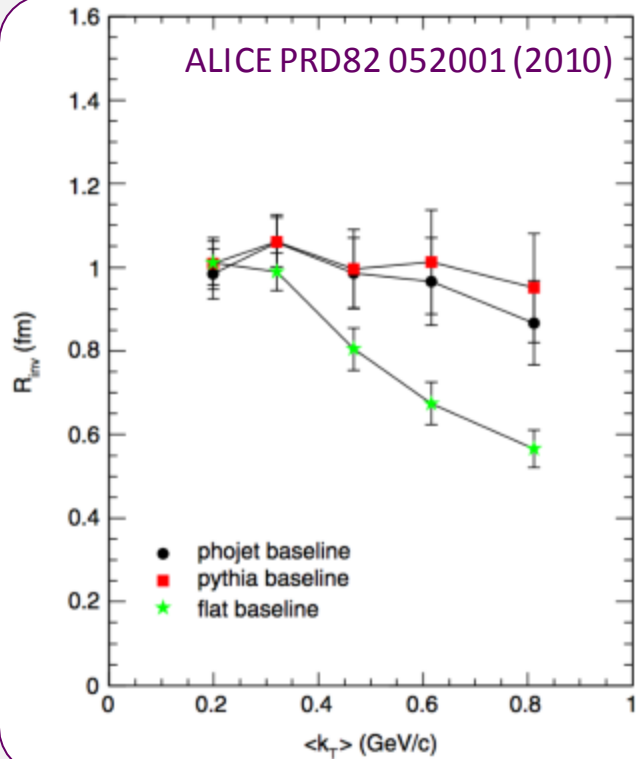
$$\text{"double ratio"} R_2(Q) = \frac{C_2(Q)}{C_2^{MC}(Q)} = \frac{N_{+,--}(Q)/N_{+,-}(Q)}{N_{+,--}^{MC}(Q)/N_{+,-}^{MC}(Q)}$$



Mixed message from LHC

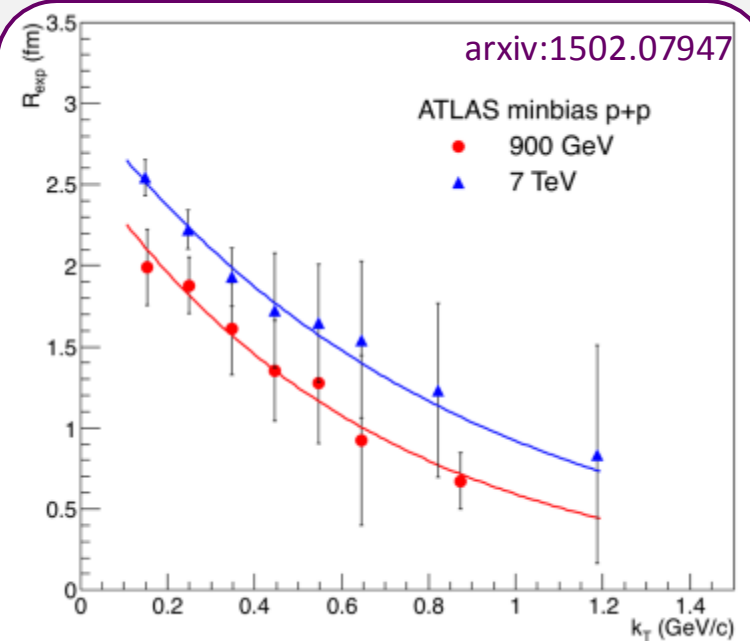
- Much better (model-based) accounting of non-femto correlations
- 1D: exponential better than Gaussian fit
- k_T -dependence in minbias?

ALICE - Gaussian radius



k_T dependence **if** non-femto background ignored
No k_T dependence if background is accounted for

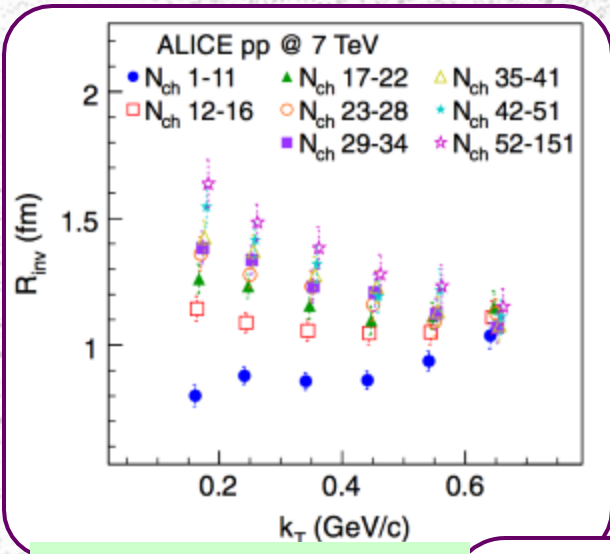
ATLAS - exponential radius



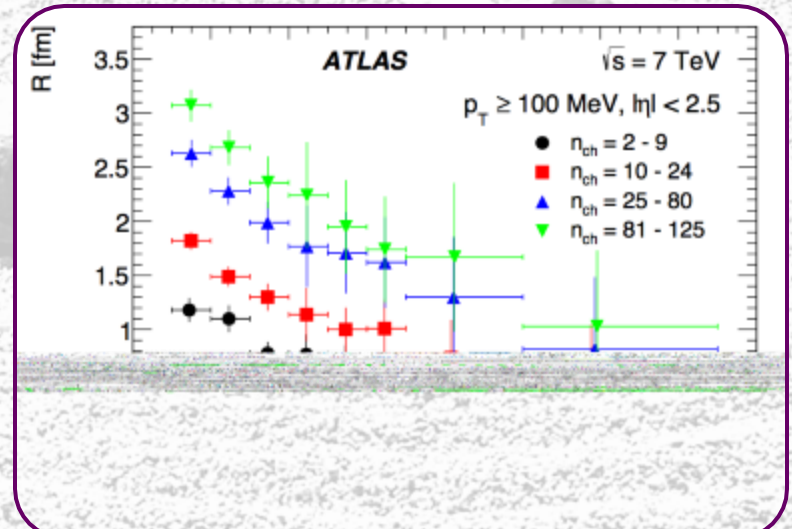
Strong k_T dependence (consistent w/ STAR, E735)
even accounting for non-femto bkgd

k_T dependence @ LHC – beyond minbias

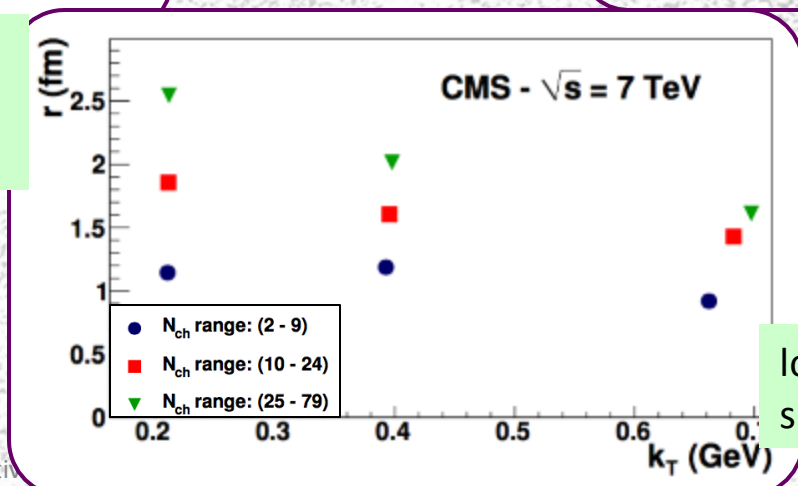
- Clearly a strong dependence for higher-multiplicity collisions
 - & need not go to “extreme” high multiplicity (good)
- what about the lowest multiplicity? (and implications for flow interpretations?)
 - mixed signals from ATLAS, ALICE, CMS (though techniques not identical)



low-multiplicity events
very different
rise with k_T



low-multiplicity events exhibit
strong k_T dependence

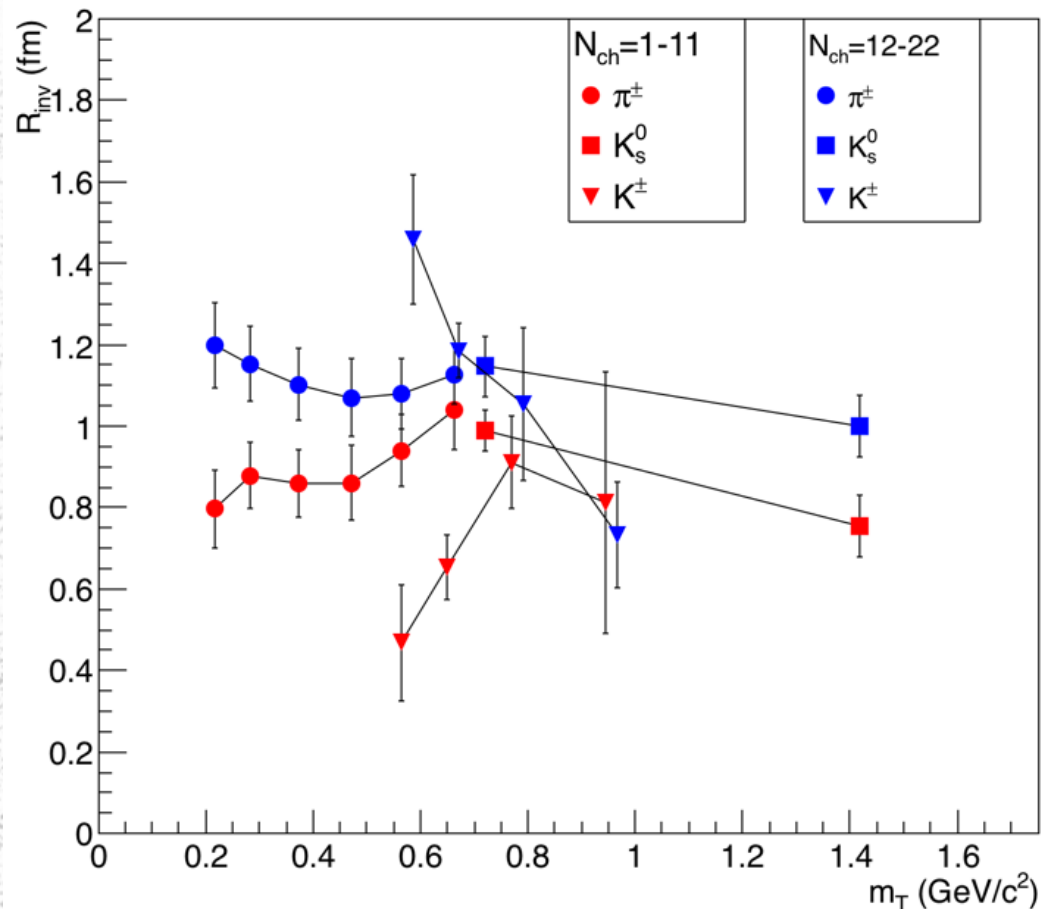


low-multiplicity events exhibit
small k_T dependence

k_T dependence @ LHC – beyond minbias

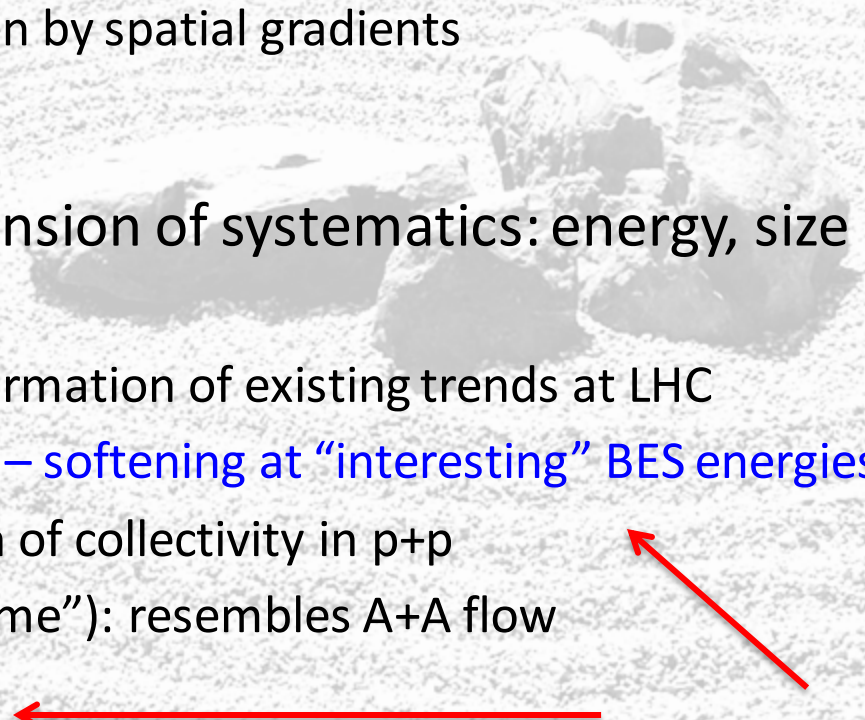
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Overall, a very confusing picture from low-multiplicity p+p collisions



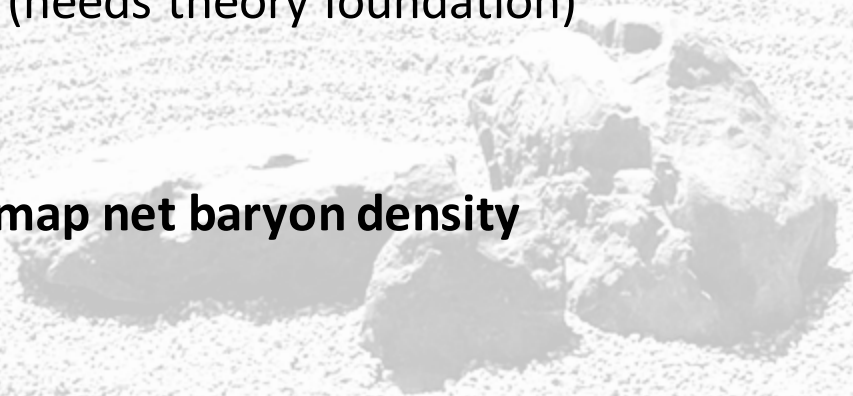
Summary

- Femtoscopy in RHI has taught us much of what we “know”
- Detailed geometric substructure paints a consistent picture
 - thermalized, flowing system, driven by spatial gradients
 - Evolution lifetime ~ 15 fm
- Third decade: tremendous extension of systematics: energy, size
 - improved statistics, techniques
 - heavy ion systematics: broad confirmation of existing trends at LHC
 - energy scan reveals crucial details – softening at “interesting” BES energies
 - an important light on the question of collectivity in p+p
 - high-multiplicity (not “extreme”): resembles A+A flow
 - low-multiplicity – flow?
 - experimentally extremely challenging
 - no clear picture as of now
- These two issues are the most compelling in the field overall



What's next? (Experimentally)

- RHIC BES-II
 - pion imaging – change of F.O. configuration?
 - asHBT[3] – probe of viscous effects (needs theory foundation)
 - asHBT[1] - tilt
 - non-id correlations (π -K)
 - **Baryon-AntiBaryon correlations – map net baryon density**
- p+p (RHIC & LHC)
 - **non-id (π -K)** – more strictly test flow hypothesis; huge non-femto problems
 - **3D radii from ATLAS** – do p-Pb fall faster than p+p (& Pb+Pb)?
- Pb+Pb (LHC)
 - N>2 correlations – stricter signal for coherent fraction
 - already a 2-sigma signal for 25% [ALICE PRC89 024911 (2014)]
 - physics impact?
 - new partner species – extract scattering parameters



Conference series discussing femtoscopic developments in detail

<https://indico.cern.ch/event/387606/>

-- registration still open!

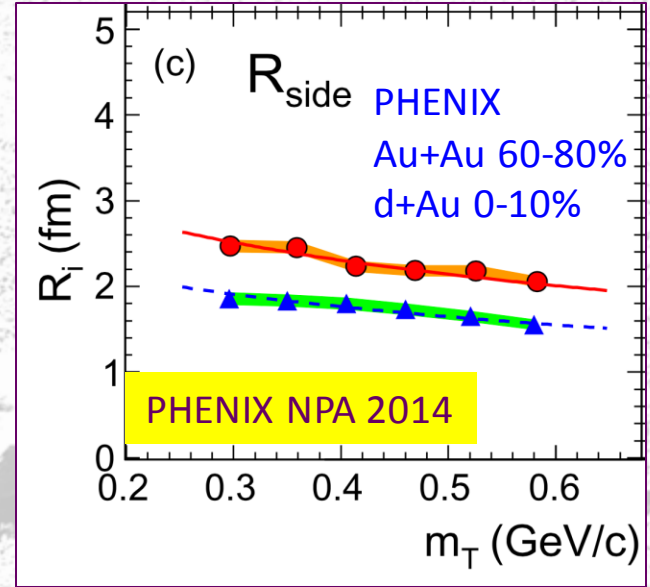
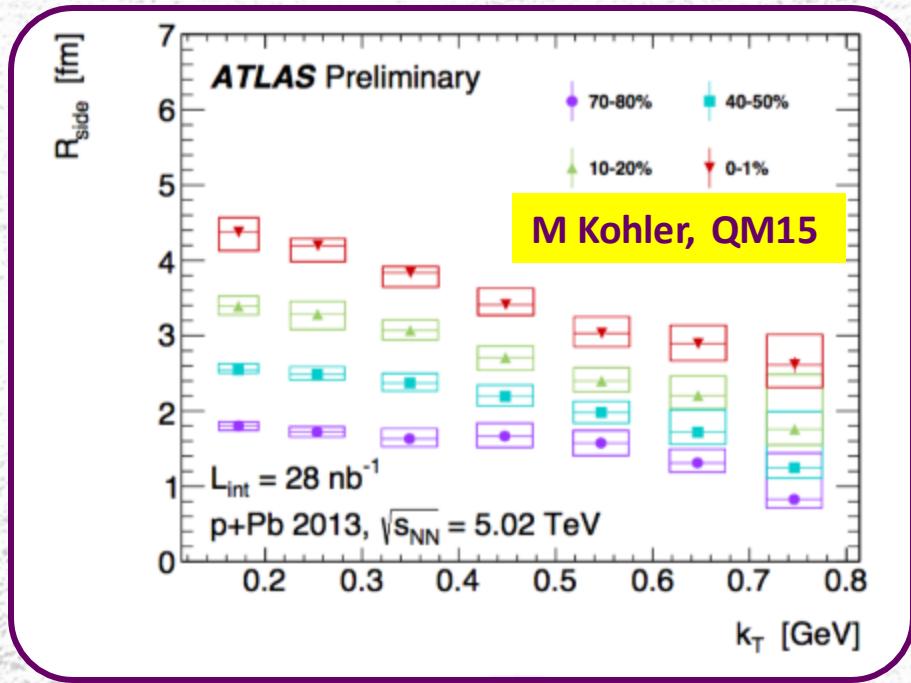
XI Workshop on Particle Correlations and Femtoscopy
Warsaw University of Technology, 3-7 November 2015



A photograph of a desert landscape with a large rock formation in the background and the word 'END' in red text overlaid.

END

More radial flow in small systems than AA?



STAR: ~identical dependence p+p, A+A

- identical flow

PHENIX: ~identical d+A, A+A

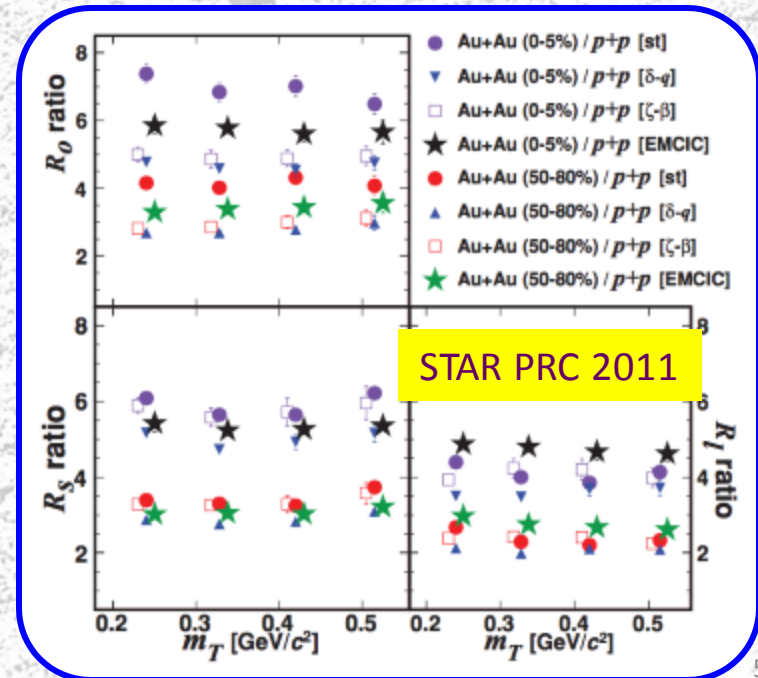
- identical flow

ALICE: high-mult pp ~ p+Pb, lower than central Pb+Pb

- slightly larger flow in Pb+Pb than p+Pb, p+p

ATLAS: only p+Pb available (exponential)

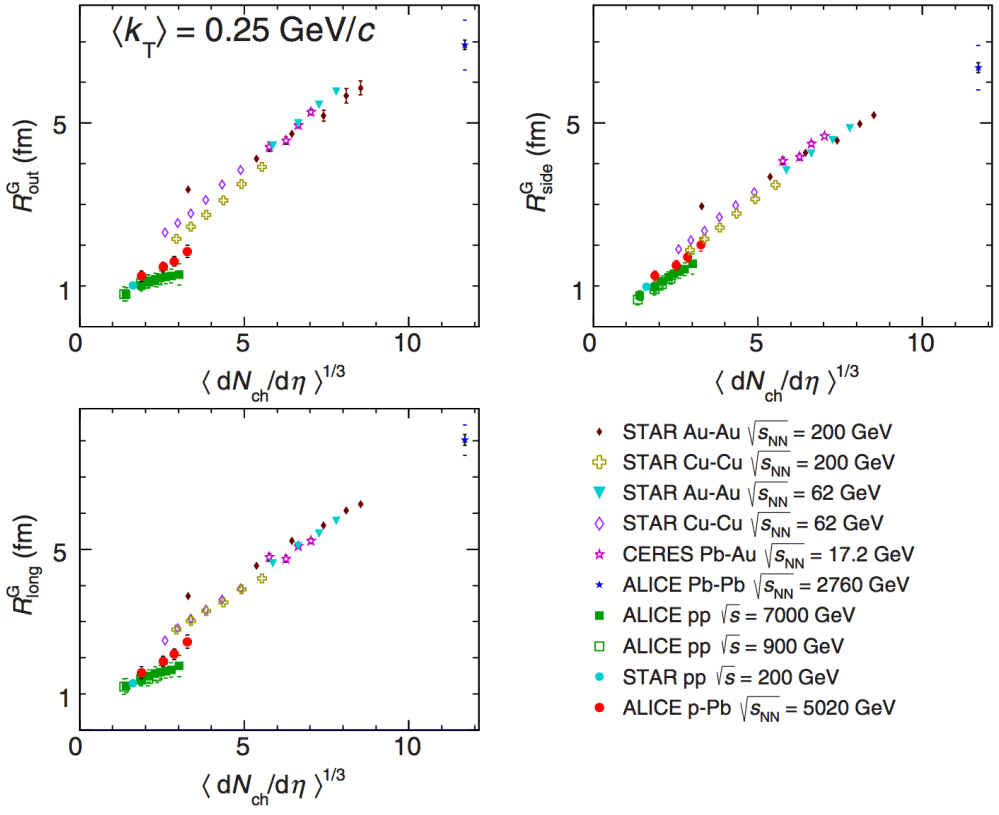
- larger than ALICE falloff for p+Pb
- 3D radii in p+p & A+A needed!



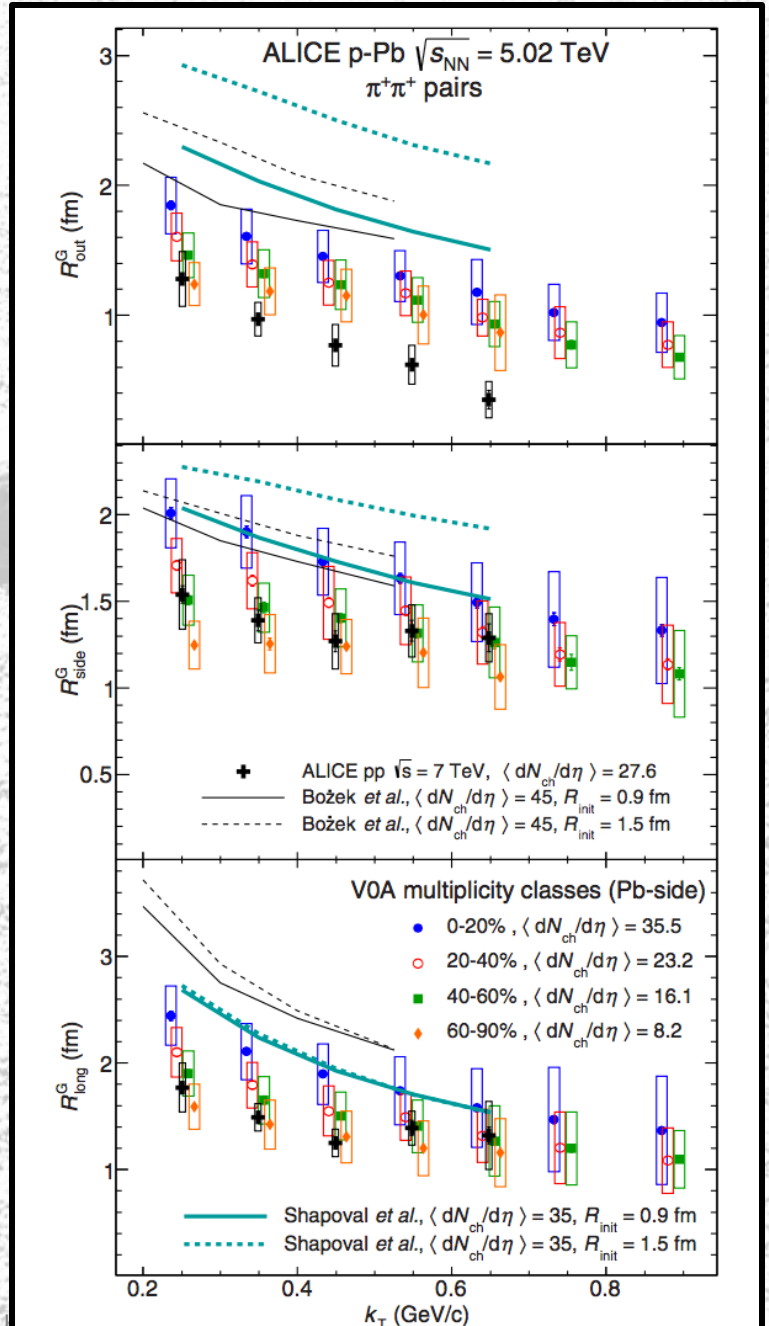


P+A

p+A



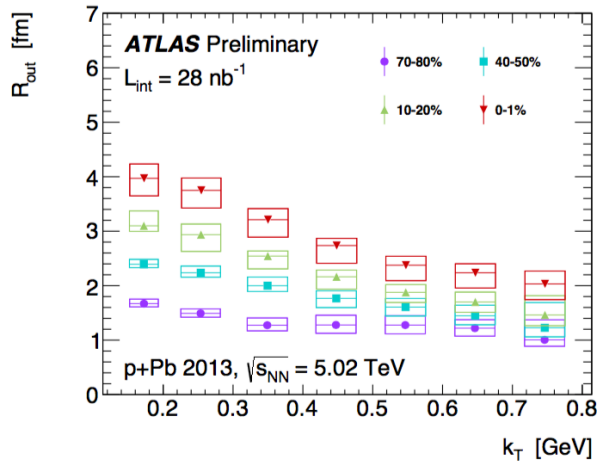
- Radii $\sim 20\%$ larger than p+p at equivalent mult.
- k_T -dependence similar
- scale with $N^{1/3}$, with slope between AA, pp
 - again, *initial* geometry matters



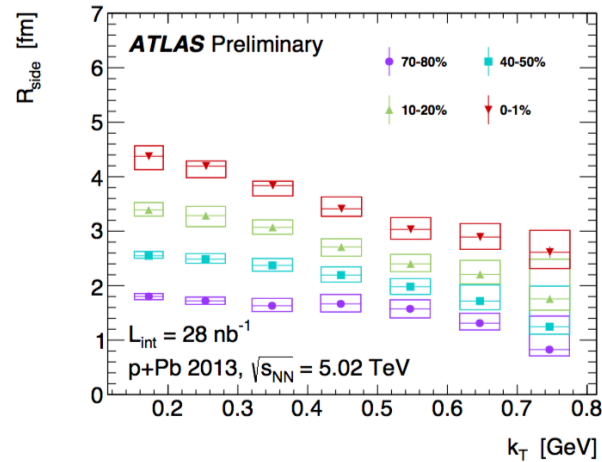
ATLAS p+Pb

M. Kohler, Tuesday

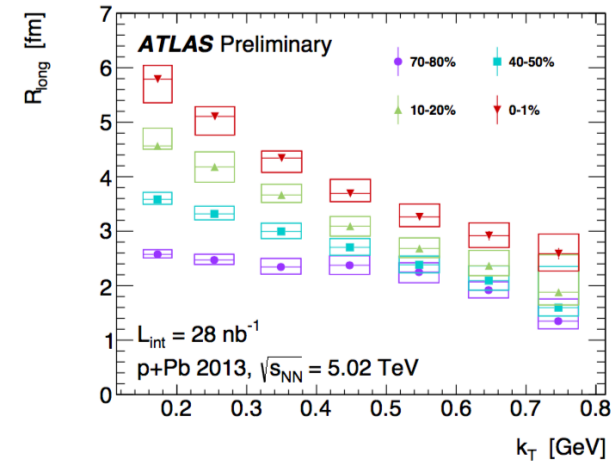
R_{out}

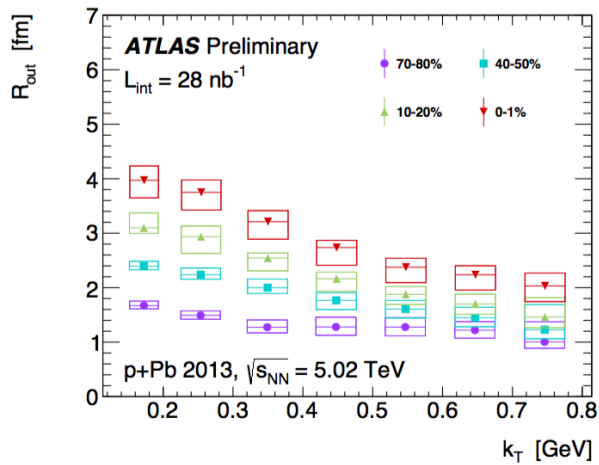
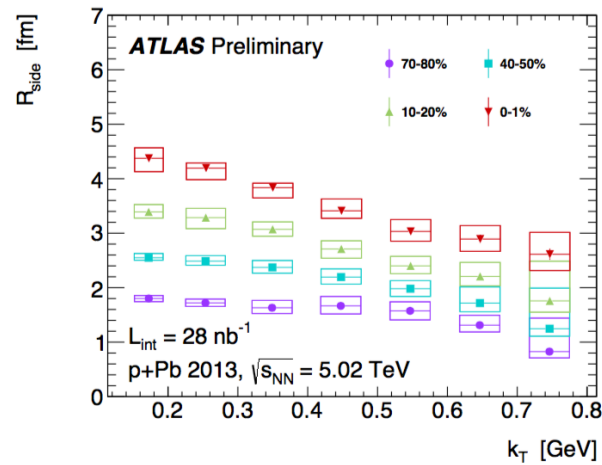
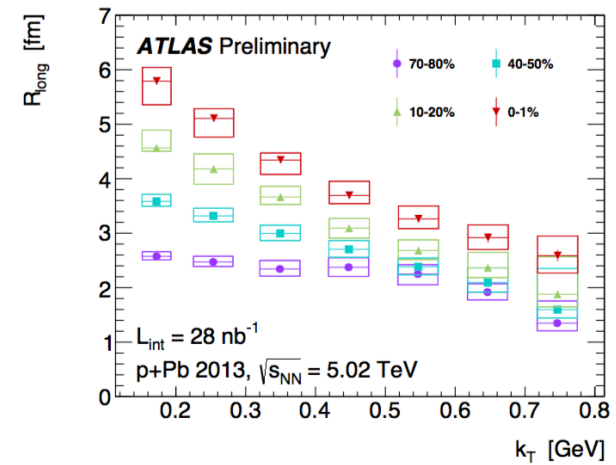


R_{side}



R_{long}



R_{out}  R_{side}  R_{long} 



IMAGING

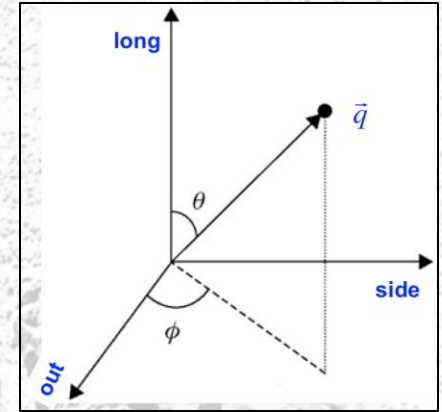
SHD & Imaging

Danielewicz & Pratt, PRC75 (2007)
 Chajeccki & Lisa PRC78, (2008)
 Kisiel & Brown PRC80 (2009)

Spherical (or Cartesian) Harmonic Decomposition

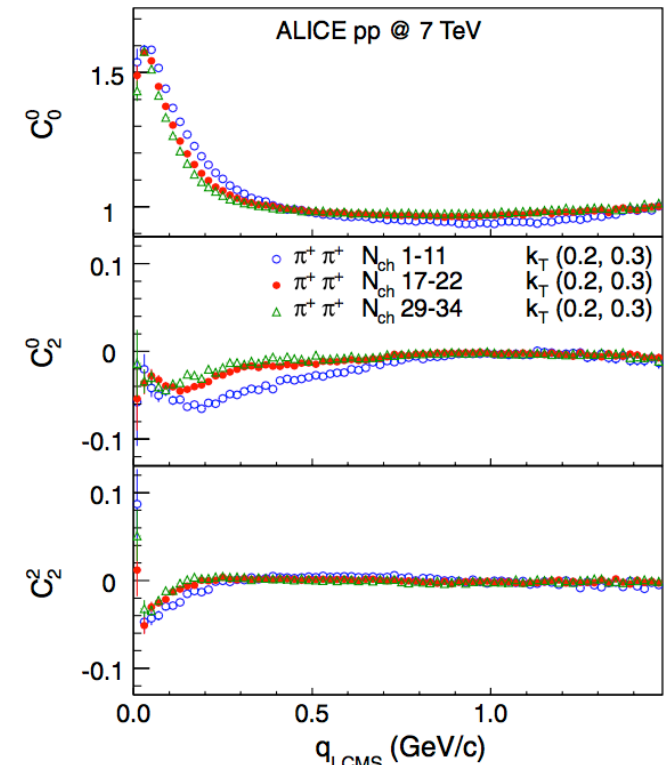
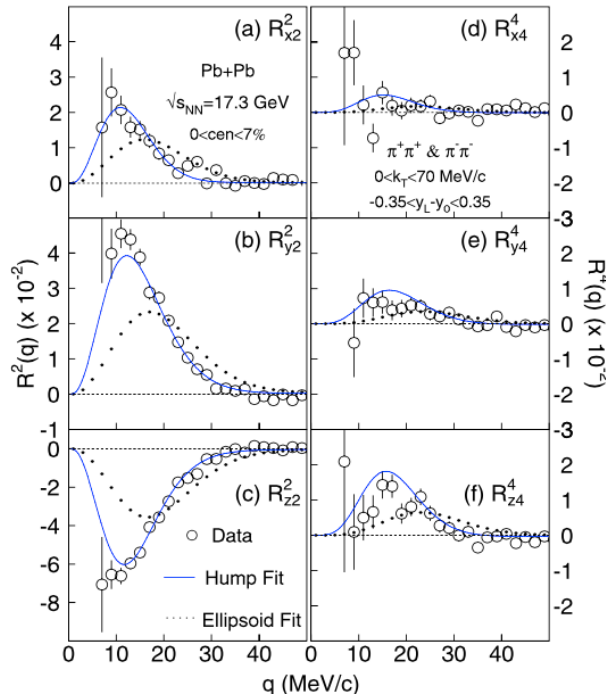
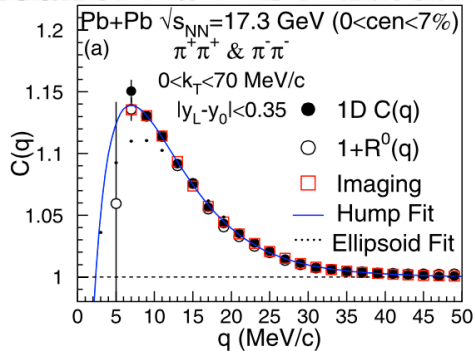
$$R_l^m(|\vec{q}|) = \frac{1}{\sqrt{4\pi}} \int d\phi d(\cos\theta) R(|\vec{q}|, \theta, \phi) Y_l^m(\theta, \phi)$$

- maximum use of symmetries
- efficient, simpler visualization
 - important for poorly-understood correlations hidden in 3D space



ALICE PRD (2011)

NA49 PLB (2010)



SHD & Imaging

Spherical (or Cartesian) Harmonic Decomposition

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$$S_l^m(|\vec{r}|) = \frac{1}{\sqrt{4\pi}} \int d\phi d(\cos\theta) S(|\vec{r}|, \theta, \phi) Y_l^m(\theta, \phi)$$

$$R_l^m(|\vec{q}|) = 4\pi \int dr r^2 K_l(|\vec{q}|, |\vec{r}|) S_l^m(|\vec{r}|)$$

3D Imaging

- Term-by-term ($l=0,1,2\dots$) correspondence between correlation function and 2-particle "image" moments
- reveals non-Gaussian tails (resonances, etc)
- restricts model freezeout parameters

NA49 PLB (2010)

