QCD in Heavy-Ion Collisions
(for non-experts)

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Student-day Lecture at QM15
Typical Scale of QCD Phenomena

\[ \alpha_s(Q^2) = \frac{1}{\beta_0 \ln(Q^2/\Lambda_{QCD}^2)} \]

\[ \Lambda_{QCD} \sim 200 \text{ MeV} \]

\[ M_B \sim N_c \Lambda_{QCD} \]

\[ T_c \sim \Lambda_{QCD} \]

\[ \rho_B \sim (\Lambda_{QCD})^3 \]
Typical Scale of QCD Phenomena

$x < 0.01$

$0.045 < Q^2 < 450 \text{ GeV}^2$

$\sigma_{\gamma^* p}(x, Q^2) \rightarrow \sigma_{\gamma^* p}(Q^2 / Q_s^2(x))$

$Q_s^2(x) = Q_0^2(x/x_0)^{-\lambda}$

Golec-Biernat-Wuesthoff
Stasto-Golec-Biernat-Kwiecinski Plot

Geometric Scaling

$Q_s(\text{RHIC}) = 1 \sim 2 \text{ GeV}$

$Q_s(\text{LHC}) \sim 2Q_s(\text{RHIC})$

$\Lambda_{\text{QCD}}$ and $Q_s$: scheme dependent
Four Regimes in HIC

Color Glass Condensate (CGC)
\[ \tau \lesssim 1/Q_s \sim 0.1\text{fm}/c \]

Color Glass + Plasma = Glasma
\[ \tau \lesssim \tau_0 \sim \Lambda_{\text{QCD}}^{-1} \]

(s) Quark-Gluon Plasma
\[ \tau \lesssim \tau_f \sim 10\text{fm}/c \]

Hadronization (quarks \rightarrow hadrons)
Glasma

Everything scales with $Q_s(x)$

Negative $P_L$

Flux tubes broken apart by instabilities → Positive $P_T$ → Hydrodynamization → Thermalization

Unsolved problems (enjoy talks on this during QM)
Four Regimes in HIC

Color Glass Condensate (CGC)
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QCD Phase Transitions

Hadronization (quarks → hadrons)
**QCD Phase Transitions**

**Confinement of Quarks and Gluons**

\[ \Lambda_{\text{QCD}}^{-1} \]

\[ T^{-1} \]

**Dynamical Generation of Mass (chiral sym.)**

"Constituent" Quark

\[ M_q \sim 350\text{MeV} \]

"Bare" Quark

\[ M_q \sim 3\sim5\text{MeV} \]
Confinement of Quarks (Wilson 1974)

\[ A_C = \text{(Area)} \]

\[ \langle W(C) \rangle = \langle \text{tr} P \exp \left[ i g \int_C d x^\mu A_\nu \right] \rangle \]

(Confinement)

\[ \Leftrightarrow \langle W(C) \rangle \sim \exp[-\#A_C] \]

Area Law
Order Parameter — Confinement

Confinement of Quarks (Wilson 1974)

Pair production of dynamical quarks

$$\langle W(C) \rangle \sim \exp[-\#L_C]$$

Perimeter Law
Extension to Finite-\(T\)

\[
Z = \text{tr} \ e^{-\hat{H}/T}
\]

\[
it \leftrightarrow \tau
\]

\[
A_\mu(\tau + \beta) = A_\mu(\tau)
\]

\[
\psi(\tau + \beta) = -\psi(\tau)
\]

\[
W(C) = \text{tr} L(0) \text{tr} L^\dagger(r)
\]

\[
\langle W(C) \rangle = \langle \text{tr} L(0) \text{tr} L^\dagger(r) \rangle
\]

\[
= \exp[-f_{q\bar{q}}(r)/T]
\]

\[
\rightarrow |\langle \text{tr} L \rangle|^2 \ (r \rightarrow \infty)
\]

\[
= \exp[-2f_q/T]
\]
**Extension to Finite-\(T\)**

\( W(C) = \text{tr} L(0) \text{tr} L^\dagger(r) \)

\( \langle W(C) \rangle = \langle \text{tr} L(0) \text{tr} L^\dagger(r) \rangle \)

\( = \exp[-f_{q\bar{q}}(r)/T] \)

\( \rightarrow |\langle \text{tr} L \rangle|^2 (r \rightarrow \infty) \)

\( = \exp[-2f_q/T] \)

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**Polyakov Loop**

\( Z = \text{tr} \, e^{-\hat{H}/T} \)

\( it \leftrightarrow \tau \)

\( A_\mu(\tau + \beta) = A_\mu(\tau) \)

\( \psi(\tau + \beta) = -\psi(\tau) \)
Extension to Finite-$T$

Quenched Case (no dynamical quarks)
\[ f_{q\bar{q}}(r) \sim \sigma r \rightarrow \infty \quad (r \rightarrow \infty) \]
\[ f_q \rightarrow \infty \quad (\text{in conf. phase}) \]

Dynamical Case (quarks with finite mass)
\[ f_{q\bar{q}}(r) \rightarrow 2(\text{hadron mass}) \quad (r \rightarrow \infty) \]
\[ f_q \rightarrow (\text{hadron mass}) \quad (\text{in “conf.” phase}) \]
Extension to Finite-$T$

Dynamical Case

\[ f_{q\bar{q}}(r) \rightarrow 2(\text{hadron mass}) \quad (r \rightarrow \infty) \]

\[ f_q \rightarrow (\text{hadron mass}) \quad (\text{in “conf.” phase}) \]

Linear potential is “screened” at large distances

Lattice from Tokyo group

No way to define confinement at finite $T$
Examples from Lattice-QCD

Polyakov loop increases very smoothly:

There is no clear-cut $T_c$ for deconfinement
**Order Parameter — Chiral Sym.**

\[
\text{SU}(N_f)_L \times \text{SU}(N_f)_R \times U(1)_A \rightarrow \text{SU}(N_f)_V
\]

- Instantons — \(\eta'\) mass

3NG modes for \(N_f = 2\)

8NG modes for \(N_f = 3\)

Chiral Condensate

\[
\langle \bar{q}q \rangle
\]
Order Parameter — Chiral Sym.

\[ SU(N_f)_L \times SU(N_f)_R \times U(1)_A \rightarrow SU(N_f)_V \]

For \( N_f = 2 \)

Without axial-U(1)

\[ O(4) \rightarrow O(3) \quad \text{2nd-order expected} \]

With axial-U(1) (effective restoration)

\[ O(4) \times O(2) \rightarrow O(3) \quad \text{1st-order expected?} \]

Lattice from D’Elia et al.
Examples from Lattice-QCD

Chiral condensate decreases (relatively) steeply:

Lattice from BNL-Bielefeld (2010)  Lattice from Wuppertal-Budapest (2010)

$\Delta_{l,s}$ : Special combination free from the renormalization
Phase Structure with Quark Masses

Far from the quenched 1st

Close to the O(4) 2nd?

Crossover confirmed by finite-\(V\) analysis

Aoki et al. (2006)
Examples from Lattice-QCD

Chiral restoration is more like a real phase transition
Magnetic Scaling: $O(4)$ vs $O(2)$

$Lattice from BNL-Bielefeld (2009)$

$(Scaled)$ Chiral Cond.

$(Scaled)$ Quark Mass

$T_c$ is better-defined for chiral phase transition
Hypothetical Phase Diagram

Deconfinement
Chiral Transitions
located at close $T$

Baym (1984)
“RHIC: From dreams to beams in two decades,” hep-ph/0104138
Chiral is rather sharp, while deconfinement is blurred
Changing the collision energy the phase boundary is investigated:
— Beam Energy Scan

Andronic et al. (2010)
Thermal Model Fit

How to determine $T$ and $\mu$ “experimentally”

Andronic Braun-Munzinger Stachel…
**Thermal Model Fit**

\[ \mu_Q : \text{determined from the charge conservation} \]

\[ \mu_S : \text{determined from zero net strangeness} \]

Mesons (blue)
Baryons (red)
contained in
THERMUS2.1
(from manual)

\[ T \text{ and } \mu_B : \text{fitting parameters} \]

http://www.phy.uct.ac.za/people/wheaton/THERMUS/thermus_index.html
V3.0 is available now…
Chemical Freezeout Curve

\[(\text{Mapping}) \sqrt{s_{NN}} \leftrightarrow T, \mu_B\]
Freezeout vs Phase Boundary

Chemical freezeout ~ “Chemical reaction” turned off
~ Inter-particle distance drops down

Thermodynamics from THERMUS

Smallness of $e^{-m/T}$ compensated by entropy
Do not get confused with the “thermal freezeout"

By definition “temperature” is an exponential slope of the particle distribution in momentum space.

Momentum exchange still goes on after the “chemical reaction” stops (elastic scatterings).

Blast-wave model parameters include $T_{\text{kin}}$

Cleymans-Redlich (1999)
Signature for Deconfinement

Melting of heavy quarkonium

Many (confusing) discussions have been triggered by this…

Static potential model

↓

Real-time physics important

Heavy-flavor diffusion (Langevin dynamics)

A. Mocsy

Beam-energy Scan
Beam-energy Scan

QCD CP

End-point of 1st order phase transition
Densest Matter

Estimate from HRG

![Graph showing baryon density, chemical potential, and temperature relationships for HRG and RMF models. The graph includes a blue dashed line indicating the chemical freeze-out line for the HRG model, with a label \( \sqrt{s_{NN}} = 8 \text{GeV} \).

- RHIC BES
- FAIR
- NICA
- J-PARC

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“Strangest” Matter

More baryons
↓
More $\Lambda$
↓
More $K^+$
($n_s = 0$)
QCD Critical Point

IF this is the case in QCD:
IF the curvature is large enough:

QCD CP is a hypothesis (needs exp. confirmation)

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Finite density causes larger pressure with lighter mass (in favor of small $M$)

QCD vacuum favors a dynamical mass $M_0$
Known CP (nuclear matter)

Liquid-gas phase transition
(Nuclear matter is a self-bound fermionic system)
Mixed Phase

Self-bound fermionic systems have a preferred density. Diluteness is realized as a “mixed phase” of nuclei.

No argument about whether quarks are self-bound? Quark EoS is constrained by neutron stars > 2M⊙
Fluctuations

Smooth bulk $p$ (dominant) + Fine structure (sub-dominant)

Q : How to extract the difference ?
Fluctuations

Smooth bulk $p$
(dominant)

+ Fine structure
(sub-dominant)

Q : How to extract the difference ?

A : Take the (higher) derivative !

$$\chi_{B,S}^{(n)} \equiv \frac{\partial^n}{\partial (\mu_{B,S}/T)^n} \frac{p}{T^4}$$

enhanced near QCD CP

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Fluctuations

$$\frac{\sigma^2}{M} \equiv \frac{\chi_B^{(2)}}{\chi_B^{(1)}}, \quad S\sigma \equiv \frac{\chi_B^{(3)}}{\chi_B^{(2)}}, \quad \kappa \sigma^2 \equiv \frac{\chi_B^{(4)}}{\chi_B^{(2)}}$$

**Skewness**  **Kurtosis**

HRG (non-interacting hadrons) + Boltzmann approx.

$$S\sigma = \tanh(\mu_B/T), \quad \kappa \sigma^2 = 1$$

Karsch-Redlich (2011)
Fluctuations

\( \kappa \sim \) how sharp

\( S \sim \) how distorted

STAR

→ Lecture by Nu Xu
Figure 2: Expansion coefficients of the pressure at non-zero baryon chemical potential. The left hand figure shows the leading order correction and the right hand figure shows the relative contribution of the leading and next to leading order corrections.

Figure 3: The ratio of sixth and second order cumulants of net-baryon number fluctuations versus temperature. This ratio gives the NNLO correction to the Taylor expansion of the pressure. The insertion shows the temperature range in which this contribution is less than 1% for

Lattice from BNL-Bielefeld (thanks to Maezawa)

Susceptibility

\[ \chi^B_2 = \left. \frac{\partial^2 (P/T^4)}{\partial \mu_B^2} \right|_{\mu_B=0} \]

 continuum extrap. \quad free

\( N_T = 12 \)

\( N_T = 8 \)

\( N_T = 6 \)

PDG-HRG

Deconfinement of heavy flavors (strange, charm, ...)

Deviation from HRG

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Another \((P\text{-odd})\) Fluctuations

\[
\langle \cos(\phi_{\alpha} + \phi_{\beta} - 2\psi_{RP}) \rangle
\]

STAR (2009)

\[
\times 10^3
\]

\[
\begin{array}{c}
\text{STAR, 200 GeV} \\
\text{same charge, AuAu} \\
\text{opp charge, AuAu} \\
\text{same charge, CuCu} \\
\text{opp charge, CuCu}
\end{array}
\]

% Most Central

\[
70 \quad 60 \quad 50 \quad 40 \quad 30 \quad 20 \quad 10 \quad 0
\]

2

2'

L or B

\[
\begin{array}{c}
\text{v}_1 \\
\Delta \phi \\
\text{v}_2
\end{array}
\]

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Magnetic Field? Vorticity?

Electric Current

Strong field of the QCD scale
Short life-time (shorter than the QCD scale)
Vorticity with longer life time
Chiral Magnetic Effect

Right-handed particles
Momentum parallel to Spin

Left-handed particles
Momentum anti-parallel to Spin

If topological fluctuations exist in a QGP, it should be reflected in charge-sep. fluct.

CME not confirmed but not excluded in HIC

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The QGP we obtain at LHC is a strongly interacting fluid. Do we expect that due to the asymptotic freedom QGP at some higher collision energies will be an almost ideal gas of quarks and gluons? If yes then how high should be such collision energy?

“strongly interacting” and “strongly correlated” should be distinguished. Resumed pQCD works already at $\sim 3T_c$ but the dynamics always passes over the sQGP regime near freezeout.

I think that the QCD system is well described by a free gas, the Quark Gluon gas model at high temperature. However, the results of the lattice calculation don't much the results of the Quark Gluon gas model even at high temperature, and I have heard that QGP behaves like a perfect fluid. So I'm interested in an approximation method of the QGP.

Resumed pQCD works at $\sim 3T_c$ for bulk thermodynamic quantities but does not account for smallness of the shear viscosity. Lattice is marginal and functional RG might be getting better…
Where does the Cronin effect comes from and which are its consequences in collisions? Can it be influenced by the shadowing effect?

Redistribution of momenta gives enhanced $R_{pA}$ at small $p_T$ (Cronin effect) that may or may not be influenced by non-linear evolution.

What are the details of pQCD and lattice QCD calculations? I would like to see basic examples leading to more complicated ones.

Example: Resumed pQCD at high $T$ (Hard Thermal Loop)

$$
\Gamma_{\text{eff}} = \int d^4x \ m_E^2 \ tr A_0^2 + \int \frac{d\Omega}{4\pi} \ W(gA \cdot \hat{K})
$$

$$
p \sim T \int d^3p \sim Tm_M^3 \sim g^6T^4 \ (m_M \sim g^2T) \quad \text{(IR Catastrophe)}
$$
Are there some theoretical and practical evidences to believe that not only scalar-pseudoscalar channel is important for low-energy-QCD models, i.e. should we use Fierz complete approach for calculations or you believe that standard scalar-pseudoscalar ansatz for effective interaction is sufficient?

Just go beyond the mean-field approximation.

What is chiral symmetry and why is it expected to be restored in heavy-ion collisions?

Just like the disappearance of magnetization at Curie temperature.

How many critical points one can expect in the QCD phase diagram?

One we know for sure, two we hope to find, three theory may have.

What is the critical point in the QCD phase diagram and how would it be visible in measurements?

You are supposed to know now after this lecture… aren’t you?
Is QGP formed in high-multiplicity pPb collisions? By considering the measured particle multiplicity and the small size of the colliding system, it can be deduced that the initial energy density in high-multiplicity pPb events exceeds the one measured in PbPb collisions, and therefore the critical value for the QGP phase transition. However, could such a small and short-lived system reach thermal equilibrium fast enough to form a QGP-like droplet?

This is precisely the hottest topic that will be discussed. Go to the parallel and plenary talks and enjoy them!
How is the CSC state of QCD described mathematically, and do we expect that it actually manifests itself physically anywhere?

Mathematically it is defined by the chiral symmetry breaking pattern and this is why we can never distinguish CSC from nuclear matter, in principle, in a gauge invariant way. CSC may or may not be realized in the cores of neutron star.

Do we need to go to higher energies in heavy ion collisions i.e. more than LHC energy? If yes then what we would explore in the direction of QGP or QCD?

Depends on physics we are interested in. For small-x physics it is nicer and nicer to go to higher and higher energies, but to explore the phase structure physics is dominated at the chemical freezeout which is not changed by the collision energies.