

Confronting fluctuations of conserved charges in central nuclear collisions at the LHC with predictions from Lattice QCD

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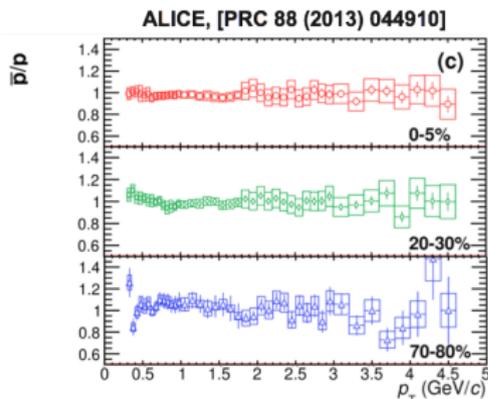
Quark Matter Kobe, Japan
28th of September 2015

Introduction: Ab-initio approaches in heavy-ion physics

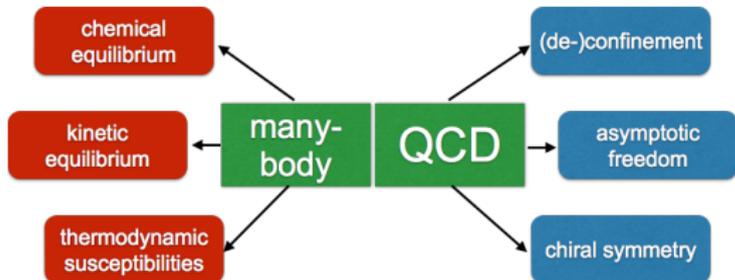
Ab-initio calculations in heavy-ion physics

- ▶ $\mu, T \ll \Lambda_{QCD}$:
 χ -perturbation theory
- ▶ $\mu, T \gg \Lambda_{QCD}$:
perturbative QCD
- ▶ $\mu_q < T$:
Lattice gauge theory

LHC (ALICE) data allows the most direct comparison to Lattice QCD calculations which correspond to $\mu_B = 0$. No extrapolation to finite μ_B is needed at LHC energies!



Introduction: Many-body QCD and light flavour hadron production



- ▶ Light flavour hadrons (u,d,s valence quarks) are produced in apparent chemical ($T_{chem} \approx 156$ MeV) and kinetic equilibrium ($T_{kin} \approx 100$ MeV).

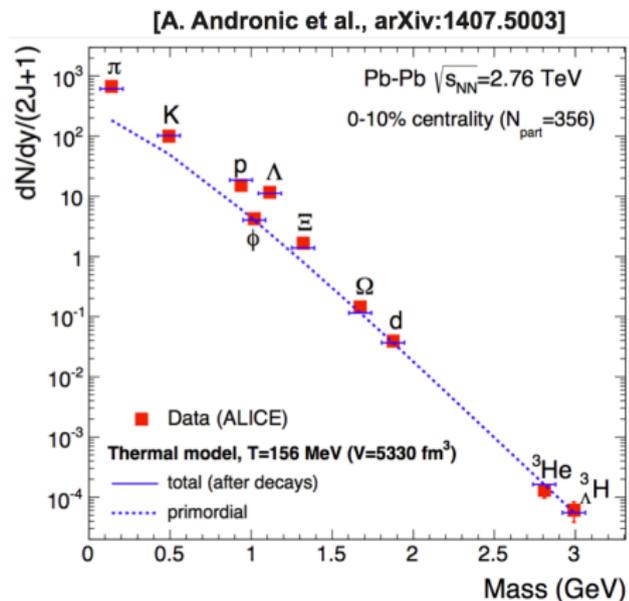
- ▶ 98% of all particles are produced with $p_T < 2$ GeV/c \Rightarrow thermal particle production in a non-perturbative regime. \rightarrow THERMODYNAMICS \rightarrow LATTICE QCD



central (0-5%) Pb-Pb collisions (LHC): $dN_{ch}/d\eta \approx 1600$

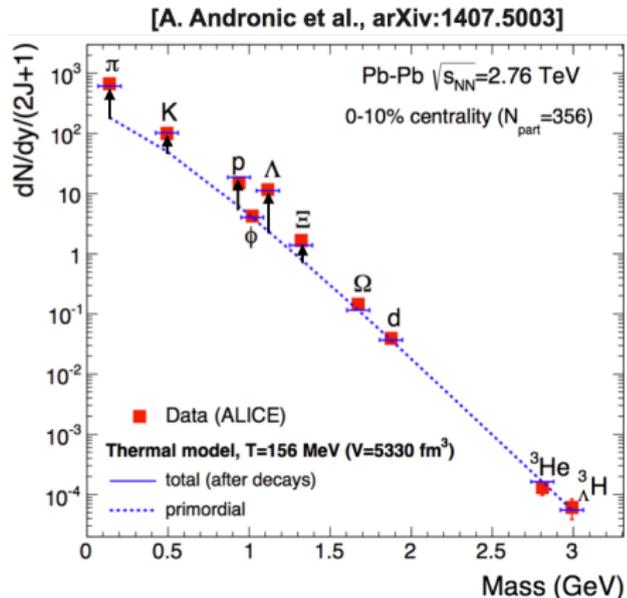
The experimental situation at LHC energies (1)

- ▶ After several years of data taking, the complete set of light flavour hadron **particle yields** has been measured.
- ▶ The thermal-statistical hadron resonance gas **model** describes the data well over several orders of magnitudes from the most abundantly to rarely produced particles. A beautiful picture from the experimental and theoretical side, but no sensitivity to criticality...
- ▶ Measurements of event-by-event fluctuations are very complicated (finite detection efficiencies, volume of phase space,..) and are only available for net-charge fluctuations at lower orders at LHC energies.



The experimental situation at LHC energies (2)

- ▶ The thermal model works nicely, but there are some remaining open questions.
- ▶ One of them: can the agreement between data and model be further improved by including feed-down from missing resonances?
- ▶ Light (anti-)nuclei are not affected by feed-down from resonances and are in agreement with the model.
- ▶ Unknown resonances are naturally *included* in LQCD calculations. **What can we learn from LQCD about particle yields?**



Thermodynamic susceptibilities

Thermodynamic susceptibilities

- ▶ Consider the conserved quantities in QCD: electric charge **Q**, baryon number **B**, strangeness **S**. In the grand-canonical ensemble, their conservation is guaranteed by the chemical potentials $\vec{\mu} = (\mu_B, \mu_S, \mu_Q)$.

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- ▶ **Fluctuations** and **correlations** of conserved quantities are quantified by the thermodynamic susceptibilities

$$\hat{\chi}_N = \frac{\chi_N}{T^2} = \frac{\partial^2 \hat{P}}{\partial \hat{\mu}_N^2} \quad \hat{\chi}_{NM} = \frac{\chi_{NM}}{T^2} = \frac{\partial^2 \hat{P}}{\partial \hat{\mu}_N \partial \hat{\mu}_M} \quad (1)$$

where $\hat{P} = P/T^4$ corresponds to the reduced pressure and $\hat{\mu} = \mu/T$ to the reduced chemical potential of the conserved quantity $N, M = (B, S, Q)$.

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- ▶ The susceptibility of a conserved charge can be also related to its variance

$$\hat{\chi}_N = \frac{1}{VT^3} \left(\langle N^2 \rangle - \langle N \rangle^2 \right) \quad (2)$$

where $N = N_q - N_{-q}$. With the probability distribution $P(N)$, the n -th moment $\langle N^n \rangle$ is calculated as $\langle N^n \rangle = \sum N^n P(N)$.

The Skellam distribution (1)

- ▶ **Baseline assumption of this study:** charge N_q and anti-charge N_{-q} are uncorrelated and follow a Poissonian distribution.
⇒ $P(N)$ follows a Skellam distribution.

$$P(N) = \left(\frac{\langle N_q \rangle}{\langle N_{-q} \rangle} \right)^{N/2} I_N \left(2\sqrt{\langle N_q \rangle \langle N_{-q} \rangle} \right) \exp[-(\langle N_q \rangle + \langle N_{-q} \rangle)] \quad (3)$$

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- ▶ Variance of the Skellam distribution:

$$\begin{aligned} \langle N^2 \rangle - \langle N \rangle^2 &= \langle (N_q - N_{-q})^2 \rangle - \langle (N_q - N_{-q}) \rangle^2 \\ &= \langle N_q^2 \rangle - 2\langle N_q \rangle \langle N_{-q} \rangle + \langle N_{-q}^2 \rangle - \langle N_q \rangle^2 + 2\langle N_q \rangle \langle N_{-q} \rangle - \langle N_{-q} \rangle^2 \\ &= \langle N_q^2 \rangle - \langle N_q \rangle^2 + \langle N_{-q}^2 \rangle - \langle N_{-q} \rangle^2 \\ &= \langle N_q \rangle + \langle N_{-q} \rangle . \end{aligned}$$

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- ▶ And thus the susceptibility is obtained as

$$\hat{\chi}_N = \frac{1}{VT^3} \left(\langle N_q \rangle + \langle N_{-q} \rangle \right) . \quad (4)$$

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- ▶ For strangeness and electric charge, there are hadrons with charge two and three. In this case, the Skellam probability distribution can be generalised:

$$\hat{\chi}_N = \frac{\chi_N}{T^2} = \frac{1}{VT^3} \sum_{n=1}^{|q|} n^2 (\langle N_n \rangle + \langle N_{-n} \rangle) \quad (5)$$

$$\hat{\chi}_{NM} = \frac{\chi_{NM}}{T^2} = \frac{1}{VT^3} \sum_{n=-q_N}^{q_N} \sum_{m=-q_M}^{q_M} nm \langle N_{n,m} \rangle \quad (6)$$

where $\langle N_{n,m} \rangle$, is the mean number of particles and resonances carrying charges $N = n$ and $M = m$.

Constructing net charge fluctuations from particle yields (1)

- ▶ Net baryon number susceptibility:

$$\frac{\chi_B}{T^2} = \frac{1}{VT^3} [\langle p \rangle + \langle N \rangle + \langle \Lambda + \Sigma^0 \rangle + \langle \Sigma^+ \rangle + \langle \Sigma^- \rangle + \langle \Xi^- \rangle + \langle \Xi^0 \rangle + \langle \Omega^- \rangle + \text{antiparticles}], \quad (7)$$

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- ▶ Net strangeness (we need to subtract S=0 contributions decaying into strange particles):

$$\begin{aligned} \frac{\chi_S}{T^2} \simeq & \frac{1}{VT^3} [(\langle K^+ \rangle + \langle K^0 \rangle + \langle \Lambda + \Sigma^0 \rangle + \langle \Sigma^+ \rangle + \langle \Sigma^- \rangle + 4\langle \Xi^- \rangle + 4\langle \Xi^0 \rangle + 9\langle \Omega^- \rangle + \text{antiparticles}) \\ & - (\Gamma_{\phi \rightarrow K^+} + \Gamma_{\phi \rightarrow K^-} + \Gamma_{\phi \rightarrow K^0} + \Gamma_{\phi \rightarrow \bar{K}^0}) \langle \phi \rangle]. \end{aligned} \quad (8)$$

Constructing net charge fluctuations from particle yields (2)

- ▶ Charge-strangeness correlation:

$$\begin{aligned} \frac{\chi_{QS}}{T^2} \simeq & \frac{1}{VT^3} [(\langle K^+ \rangle + 2\langle \Xi^- \rangle + 3\langle \Omega^- \rangle \\ & + \text{antiparticles}) - (\Gamma_{\phi \rightarrow K^+} + \Gamma_{\phi \rightarrow K^-})\langle \phi \rangle \\ & - (\Gamma_{K_0^* \rightarrow K^+} + \Gamma_{K_0^* \rightarrow K^-})\langle K_0^* \rangle], \end{aligned} \quad (9)$$

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- ▶ Upper limit for Baryon-number-strangeness correlation:

$$-\frac{\chi_{BS}}{T^2} > \frac{1}{VT^3} [2\langle \Lambda + \Sigma^0 \rangle + 4\langle \Sigma^+ \rangle + 8\langle \Xi \rangle + 6\langle \Omega^- \rangle]. \quad (10)$$

Contributions of strange baryonic resonances decaying into a non-strange baryon and a strange meson (like the decay of $\Sigma^* \rightarrow N\bar{K}$) are not (yet) determined experimentally.

Assumptions and approximations

- ▶ Protons and neutrons are produced in equal abundance: $\langle p \rangle = \langle N \rangle$. It is a very safe assumption and experimentally further substantiated by the fact that no surprises have been observed in the production of deuterons.

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- ▶ Measurements of the yield of the Σ -Baryon are still ongoing at the LHC. We have used a value of $\Sigma^0/\Lambda = 0.278 \pm 0.011 \pm 0.05$ from [PRD 36, 674 (1978)].
- ▶ Skellam distribution: Results from STAR at top RHIC energies show that fluctuations up to third order follow the Skellam distribution. We do not expect changes from RHIC to LHC in this respect and we have not seen dramatic differences in other soft particle production observables.

Results: Thermodynamic susceptibilities derived from particle yields

Further simplifications and inserting the values for the particle yields measured by ALICE gives the following results:

$$\begin{aligned}\frac{\chi_B}{T^2} &= \frac{1}{\sqrt{T^3}} [4\langle p \rangle + 2\langle (\Lambda + \Sigma^0) \rangle + 4\langle \Sigma^+ \rangle + 4\langle \Xi \rangle + 2\langle \Omega \rangle] \\ &= \frac{1}{\sqrt{T^3}} (203.7 \pm 11.44)\end{aligned}$$

$$\begin{aligned}\frac{\chi_S}{T^2} &\simeq \frac{1}{\sqrt{T^3}} [2\langle K^+ \rangle + 2\langle K^0 \rangle + 2\langle (\Lambda + \Sigma^0) \rangle + 4\langle \Sigma^+ \rangle \\ &\quad + 16\langle \Xi \rangle + 18\langle \Omega \rangle - 2(\Gamma_{\phi \rightarrow K^+} + \Gamma_{\phi \rightarrow K^0})\langle \phi \rangle] \\ &= \frac{1}{\sqrt{T^3}} (504.35 \pm 24.14)\end{aligned}$$

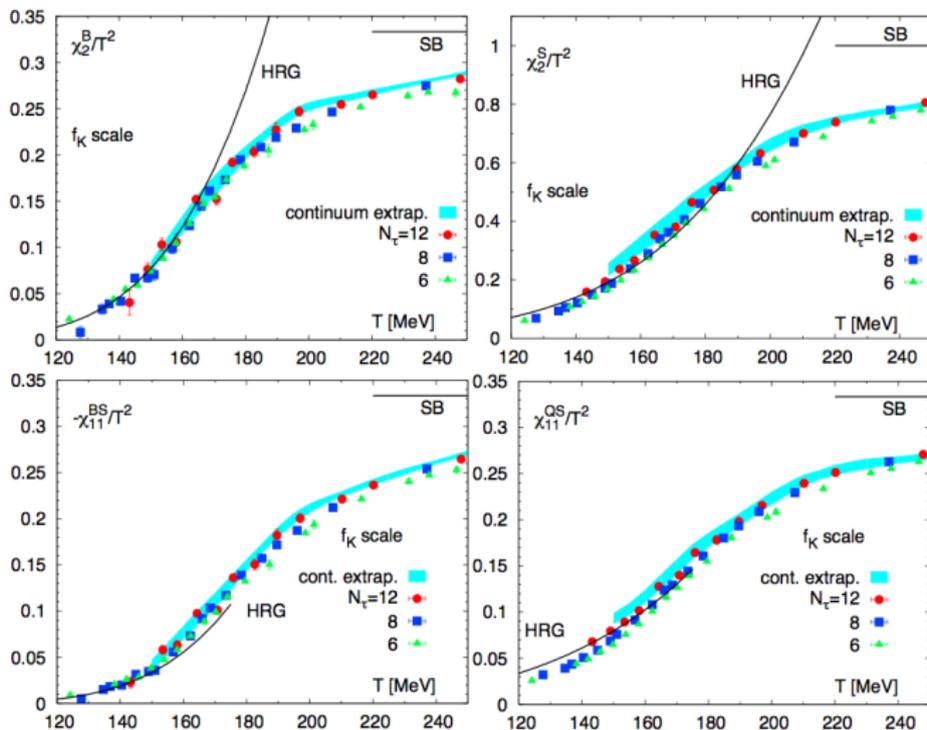
$$\begin{aligned}\frac{\chi_{QS}}{T^2} &\simeq \frac{1}{\sqrt{T^3}} [2\langle K^+ \rangle + 4\langle \Xi^- \rangle + 6\langle \Omega^- \rangle - 2\Gamma_{\phi \rightarrow K^+}\langle \phi \rangle - 2\Gamma_{K_0^* \rightarrow K^+}\langle K^* \rangle] \\ &= \frac{1}{\sqrt{T^3}} (178.5 \pm 17.14)\end{aligned}$$

Comparison to Lattice QCD results

The susceptibilities under study are directly calculated on the lattice:

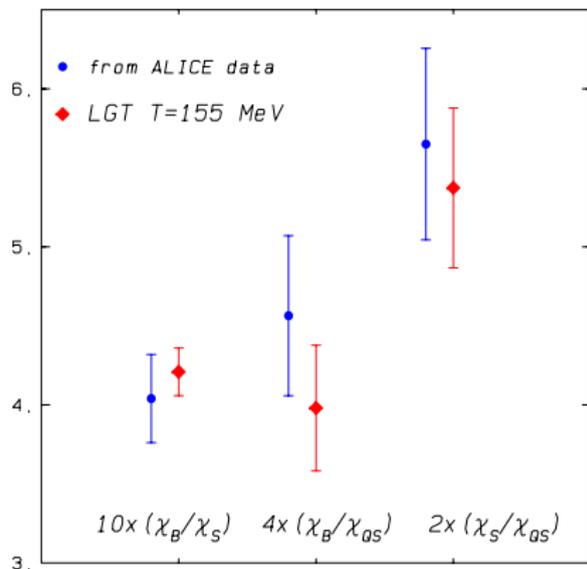
[A. Bazavov et al., Phys.Rev. D86 (2012) 034509]

[A. Bazavov et al., Phys.Rev.Lett. 113, 072001(2014)]



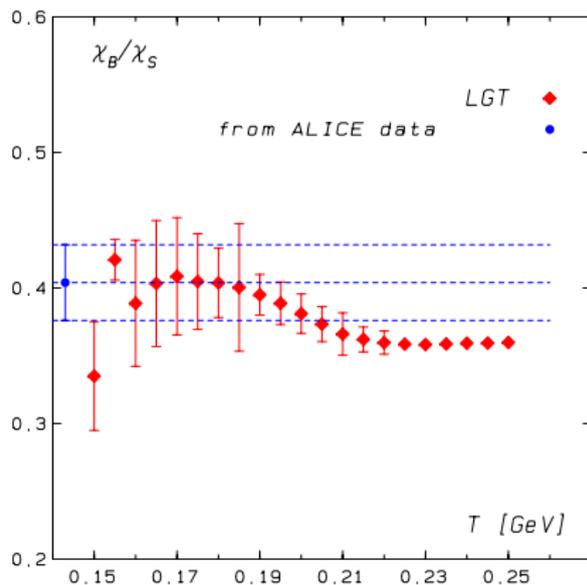
Comparison to Lattice QCD results at $T_c = 155$ MeV

- ▶ If there is a phase change from QGP to the hadronic phase, particle yields and fluctuations of conserved charges should be established at the chiral, pseudocritical temperature T_c .
- ▶ A value of $T_c = 155(1)(8)$ MeV was recently obtained in LQCD [T. Bhattacharya et al., PRL 113 082001 (2014)].
- ▶ A very good agreement is found between susceptibilities derived from ALICE particle yield data and Lattice QCD results at the chiral crossover.
- ▶ How unique is the determination of the temperature at which such an agreement holds?



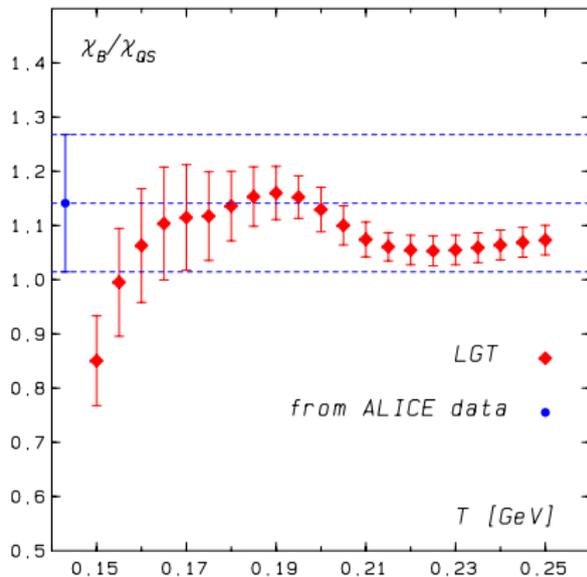
Temperature dependence of χ_B/χ_S

- ▶ The ratio χ_B/χ_S only shows a weak dependence on temperature in LQCD for $T > T_c$.
- ▶ We can reject the temperature range $T < 150$ MeV as a saturation regime for χ_B and χ_S .
- ▶ However, the upper limit can be as large as $T \approx 210$ MeV.



Temperature dependence of χ_B/χ_{QS}

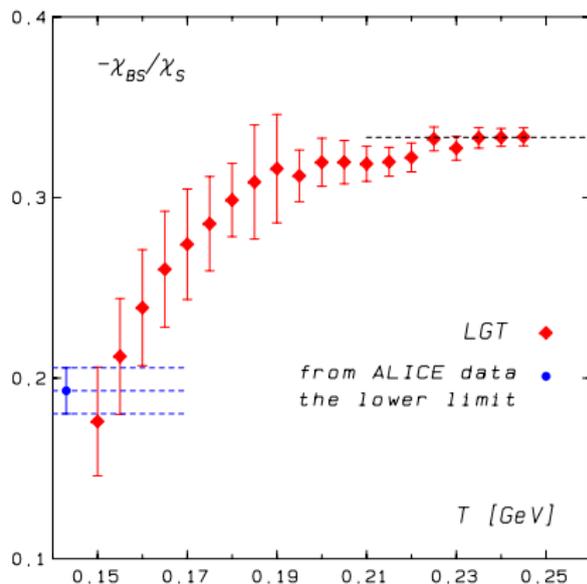
- ▶ Similarly, the ratio χ_B/χ_{QS} only shows only weak dependence on temperature in LQCD for $T > T_c$.
- ▶ Also in this case, we can reject the temperature range $T < 150$ MeV as a saturation regime for χ_B , χ_S , and χ_{QS} .
- ▶ At the same time, we know that Lattice QCD thermodynamics cannot be described anymore by hadronic degrees of freedom for $T > 163$ MeV.



Temperature dependence of χ_{BS}/χ_S

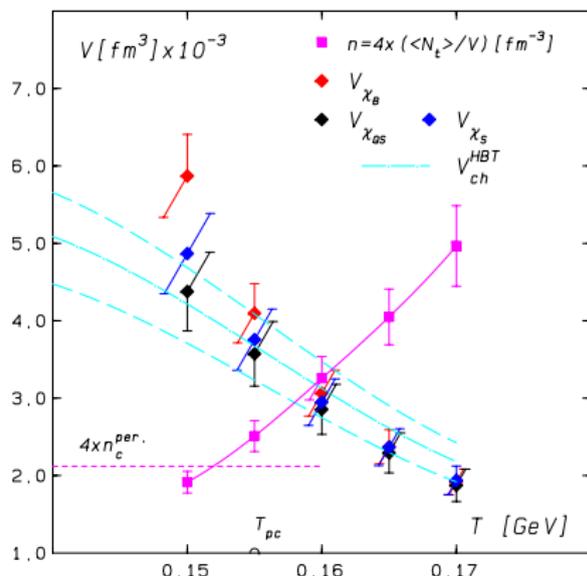
- ▶ χ_{BS}/χ_S shows a strong dependence on temperature.
- ▶ Unfortunately, only a lower limit can be set which corresponds to a lower limit of the temperature.
- ▶ We find again that $T < 150$ MeV can be rejected.

$$150 \text{ MeV} < T < 163 \text{ MeV}$$



Considerations on freeze-out volume: HBT

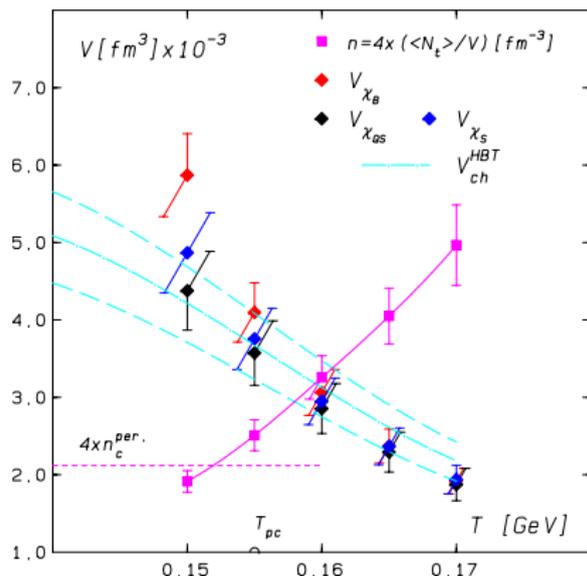
- ▶ If χ_B , χ_S and χ_{QS} are established in a common fireball, also the volumes V_{χ_B} , V_{χ_S} , and $V_{\chi_{QS}}$ must be equal.
- ▶ Can we make use of additional experimental information on the freeze-out volume and verify that a consistent picture is obtained?
- ▶ The volume of homogeneity at the last interaction can be obtained from pion HBT
 $V_{HBT} = (2\pi)^{3/2} R_l R_o R_s$, but must be corrected by 3D hydro back to the volume at chemical freeze-out.
- ▶ The volumina from HBT and from particle yields + LQCD nicely coincide, but do not allow a further restriction of the temperature range.



Considerations on freeze-out volume: percolation theory

- ▶ $V(T)$ can be used to calculate the density $n(T) = \langle N_t \rangle / V(T)$ at a given volume.
- ▶ The total number of particles can be again determined from particle yields as:

$$\begin{aligned} \langle N_t \rangle &= 3\langle \pi \rangle + 4\langle K \rangle + 4\langle p \rangle + 2\langle \Lambda \\ &\quad + \Sigma^0 \rangle + 4\langle \Sigma \rangle + 4\langle \Xi \rangle + 2\langle \bar{\Omega} \rangle \\ &= 2486 \pm 146 . \end{aligned}$$



- ▶ In percolation theory, one can calculate a critical density for objects of volume $V_0 = 4/3\pi R_0^3$ as $n_c = \frac{1.22}{V_0}$ [EPJC 59 (2009)].
- ▶ From the strong interaction radius of the proton, one obtains $R_0 \approx 0.82 \text{ fm} \Rightarrow n_c \approx 0.52 \text{ fm}^{-3} \Rightarrow T_c^p = 152 \text{ MeV}$.

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- ▶ The analysis study here provides the first direct link between LHC heavy ion data and predictions from LQCD.
- ▶ Looking very much forward to the first direct measurements of event-by-event fluctuations at the LHC:
 1. To see if the assumption that the variance is described by the Skellam distribution is valid.
 2. To investigate effects of critical chiral dynamics for higher order moments.